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FOR CBSE CLASS 12 PREPARATION

CBSE Class 12 Sample Paper 2027 Solution (Marking Scheme) for Maths

EXAM	YEAR	TYPE	SUBJECT
CBSE Class 12	2027	Solutions	Maths

MATHEMATICS (041)
MARKING SCHEME
CLASS XII (2026-27)

SECTION: A (Solution of MCQs of 1 Mark each, no part marks)

Q. No.	ANS	HINTS/SOLUTION	Marks
1.	(D)	$\pi + \cos^{-1} x$	1
	(D)	**For Visually Impaired:** $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right] - \{\pi\}$	1
2.	(D)	$\left(-\infty, -\frac{1}{2}\right] \cup \left[\frac{1}{2}, \infty\right)$	1
3.	(A)	$\cot^{-1}\left(2\cos\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)\right) = \cot^{-1}\left(2\cos\left(\frac{1}{2}\times\frac{\pi}{3}\right)\right)$ $= \cot^{-1}\left(2\cos\left(\frac{\pi}{6}\right)\right) = \cot^{-1}\left(2\times\frac{\sqrt{3}}{2}\right) = \cot^{-1}(\sqrt{3}) = \frac{\pi}{6}$	1
4.	(B)	$3a = 1, b = -2, 2c = 4 \Rightarrow \sqrt{6a + 2b + 3c} = \sqrt{2 - 4 + 6} = \sqrt{4} = 2$	1
5.	(C)	$\lvert adj(PQ)\rvert = \lvert PQ\rvert = \lvert P\rvert\lvert Q\rvert = 2 \times 3 = 6$ ($\because$ order of P and $Q = 2 \times 2$)	1
6.	(D)	$\begin{bmatrix}1 & x\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & 2\end{bmatrix}\begin{bmatrix}4\\ -1\end{bmatrix} = O \Rightarrow \begin{bmatrix}1 & 2x\end{bmatrix}\begin{bmatrix}4\\ -1\end{bmatrix} = O \Rightarrow 4 - 2x = 0 \Rightarrow x = 2$	1
7.	(A)	Let $y = \cos^{-1}(2x^2 - 1)$ and $u = \cos^{-1}(x)$ $\Rightarrow y = \cos^{-1}(2\cos^2 u - 1)$ as $\cos u = x$ $\Rightarrow y = \cos^{-1}(\cos(2u)) = 2u$ $\Rightarrow \frac{dy}{du} = 2$	1
8.	(D)	$f(x)$ is differentiable $\Rightarrow f(x)$ is continuous $\Rightarrow c = 0$ and $a = 2b$ $\Rightarrow (a, b, c)$ becomes $(2b, b, 0)$ which is satisfied by option (D) $(3, \frac{3}{2}, 0)$ only	1
9.	(D)	y is decreasing $\Rightarrow \frac{dy}{dx} < 0 \Rightarrow a\{1 + 2\cos\left(\frac{x}{a}\right)\} < 0 \Rightarrow \cos\left(\frac{x}{a}\right) < -\frac{1}{2}$ $\Rightarrow \frac{x}{a} \in \left(\frac{2\pi}{3}, \pi\right] \Rightarrow x \in \left(\frac{2\pi a}{3}, \pi a\right]$	1
10.	(A)	$\frac{dy}{dx} > y \Rightarrow 4x^3 > x^4 \Rightarrow x^3(4 - x) > 0 \Rightarrow x(x - 4) < 0 \Rightarrow 0 < x < 4$	1
11.	(B)	The order is 2 and the term containing $\frac{d^2y}{dx^2}$ has the maximum power 2 in the equation, so the degree is 2	1
12.	(D)	$\int \frac{1}{1+e^{2x}}dx = \int\left(\frac{1+e^{2x}}{1+e^{2x}} - \frac{e^{2x}}{1+e^{2x}}\right)dx = x - \frac{1}{2}log\lvert 1 + e^{2x}\rvert + C$ So $P = \frac{1}{2}$	1
13.	(C)	Magnitude of $2\hat{\imath} - u\hat{\jmath} + 3\hat{k} = 4 \Rightarrow \sqrt{4 + u^2 + 9} = 4$ $\Rightarrow u^2 = 16 - 13 = 3 \Rightarrow u = \pm\sqrt{3}$	1
14.	(B)	Direction cosines of the vector $\hat{\imath} + \hat{\jmath} + \sqrt{2}\,\hat{k}$ are $\frac{1}{2}, \frac{1}{2}, \frac{1}{\sqrt{2}}$ Let angle = γ. Then $\cos\gamma = \frac{1}{\sqrt{2}} \Rightarrow \gamma = \frac{\pi}{4}$	1
15.	(B)	Both the vectors $\hat{\imath} + \hat{\jmath}$ and $\hat{\imath} - \hat{\jmath}$ are in XY-plane So a vector perpendicular to both is $3\hat{k}$	1

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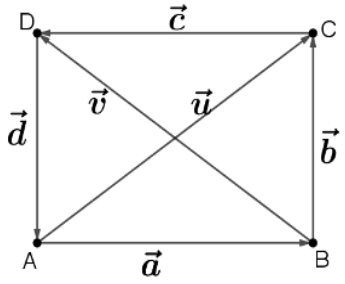
16.	**(D)**	The probability that it rains on a particular day is $\frac{1}{5}$. Probability that it rains on three consecutive days = $\frac{1}{5}\times\frac{1}{5}\times\frac{1}{5}=\frac{1}{125}$	**1**
17.	**(D)**	Let the events be A: sum of the numbers appeared is a multiple of 5 B: at least one number is 4 $A=\{(1,4),(2,3),(3,2),(4,1),(4,6),(5,5),(6,4)\}$ $A\cap B=\{(1,4),(4,1),(4,6),(6,4)\}$ So required probability = $P(B\|A)=\frac{4}{7}$	**1**
18.	**(A)**	Probability of occurrence of any one of them is $\frac{1}{3}$ $\Rightarrow P(A'\cap B)+P(A\cap B')=\frac{1}{3}\Rightarrow P(A')P(B)+P(A)P(B')=\frac{1}{3}$ $\Rightarrow\left(1-\frac{1}{6}\right)k+\frac{1}{6}(1-k)=\frac{1}{3}\Rightarrow k\left(\frac{5}{6}-\frac{1}{6}\right)=\frac{1}{3}-\frac{1}{6}\Rightarrow k=\frac{1}{6}\times\frac{6}{4}=\frac{1}{4}$	**1**
19.	**(B)**	Both assertion and reason are true, but reason is not the correct explanation of the assertion $\int_{-1}^{1}\|x-2\|\,dx=\int_{-1}^{1}(-x+2)\,dx=4$	**1**
20.	**(A)**	Both assertion and reason are true, and reason is the correct explanation of the assertion.	**1**

Section –B

[This section comprises of solution of very short answer type questions (VSA) of 2 marks each]

21(A).	$I=\int\frac{1}{x\sqrt{x^6-1}}dx=\frac{1}{6}\int\frac{6x^5}{x^6\sqrt{x^6-1}}dx$ Let $x^6-1=t^2 \quad\Rightarrow 6x^5dx=2tdt$ $I=\frac{1}{3}\int\frac{1}{(t^2+1)}dt=\frac{1}{3}\tan^{-1}\left(\sqrt{x^6-1}\right)+C$	½ 1+½
	OR	
21(B).	Let $I=\int\frac{\cos x-\sin x}{\sqrt{\sin(2x)}}\,dx=\int\frac{\cos x-\sin x}{\sqrt{1+\sin(2x)-1}}\,dx=\int\frac{\cos x-\sin x}{\sqrt{(sinx+cosx)^2-1}}\,dx$ Let $sinx+cosx=t \quad\Rightarrow(\cos x-\sin x)dx=dt$ $I=\int\frac{1}{\sqrt{t^2-1}}\,dt=log\left\|t+\sqrt{t^2-1}\right\|+C$ $=log\left\|sinx+cosx+\sqrt{(sinx+cosx)^2-1}\right\|+C$ $=log\left\|sinx+cosx+\sqrt{\sin(2x)}\right\|+C$	½ ½ ½ ½
22.	$y=\sin(x+y)\Rightarrow\sin^{-1}y=x+y$ $\Rightarrow\frac{1}{\sqrt{1-y^2}}=\frac{dx}{dy}+1\ \Rightarrow\ \frac{dx}{dy}=\frac{1-\sqrt{1-y^2}}{\sqrt{1-y^2}}$ $\Rightarrow\frac{dy}{dx}=\frac{\sqrt{1-y^2}}{1-\sqrt{1-y^2}}$ So, $\frac{dy}{dx}=0\ \Rightarrow y=\pm1$	1 1

	Alternative way: $y = \sin(x+y) \Rightarrow \frac{dy}{dx} = \cos(x+y)\left(1+\frac{dy}{dx}\right)$ $\Rightarrow \frac{dy}{dx}(1-\cos(x+y)) = \cos(x+y)$ $\Rightarrow \cos(x+y) = 0$ as $\frac{dy}{dx} = 0$ $\Rightarrow \sin(x+y) = \pm 1 \Rightarrow y = \pm 1$	**1** **1**
23(A).	Given points are $A(2,6,5), B(3,p,7), C(-4,3,4)$ and $D(-1,0,q)$ Direction ratios of the line $\overleftrightarrow{AB}$ are $\langle 1,\ p-6,\ 2\rangle$ and Direction ratios of the line $\overleftrightarrow{CD}$ are $\langle 3,-3,q-4\rangle$ As the lines are parallel, so we have $\frac{1}{3} = \frac{p-6}{-3} = \frac{2}{q-4}$ $\Rightarrow -1 = p-6 \ and\ q-4 = 3\times 2$ $\Rightarrow p = -1+6 = 5,\ q = 6+4 = 10$ Thus, $\frac{q}{p} = \frac{10}{5} = 2$	**1** **½** **½**
	OR	
23 (B).	 Let vectors representing the four sides of the rectangle are as follows $\overrightarrow{AB} = \vec{a}, \overrightarrow{BC} = \vec{b}, \overrightarrow{CD} = \vec{c}, and\ \overrightarrow{DA} = \vec{d}$ Then $\overrightarrow{AC} = \vec{u} = \vec{a}+\vec{b}$ …(i)and $\overrightarrow{BD} = \vec{v} = \vec{b}+\vec{c} = \vec{b}-\vec{a}$ …(ii) Solving (i) and (ii), $\vec{a} = \frac{1}{2}(\vec{u}-\vec{v})\ and\ \vec{b} = \frac{1}{2}(\vec{u}+\vec{v})$ Also, $\vec{c} = -\vec{a} = \frac{1}{2}(\vec{v}-\vec{u})$ and $\vec{d} = -\vec{b} = -\frac{1}{2}(\vec{u}+\vec{v})$	**½** **1** **½**
24.	Given $\lvert\vec{a}\rvert = \sqrt{6}$, $\vec{a}\cdot\vec{b} = 1$ and $\vec{b}\times\vec{a} = 3\hat{\imath}+4\hat{\jmath}-2\hat{k}$ So $\lvert\vec{a}\times\vec{b}\rvert = \lvert\vec{b}\times\vec{a}\rvert = \sqrt{29}$ We know, $\lvert\vec{a}\times\vec{b}\rvert^2 + \lvert\vec{a}\cdot\vec{b}\rvert^2 = \lvert\vec{a}\rvert^2\lvert\vec{b}\rvert^2$ $\Rightarrow 29+1 = 6\lvert\vec{b}\rvert^2 \Rightarrow \lvert\vec{b}\rvert^2 = \frac{30}{6} = 5 \Rightarrow \lvert\vec{b}\rvert = \sqrt{5}$	**½** **½** **1**
25.	Constraints are $3x+2y \geq 10, x+2y \leq 6,$ $y \geq 0$	**1½** **½**

	For Visually Impaired: As maximum value exists at (p, q) and $\left(2p, \frac{q}{2}\right)$ So $Z\ at\ (p,q) = Z\ at\left(2p, \frac{q}{2}\right)$ $\Rightarrow 3p + 4q = 3 \times 2p + 4 \times \frac{q}{2}$ $\Rightarrow 3p + 4q = 6p + 2q$ $\Rightarrow 6p - 3p = 4q - 2q$ $\Rightarrow 3p = 2q$	**1** **1**

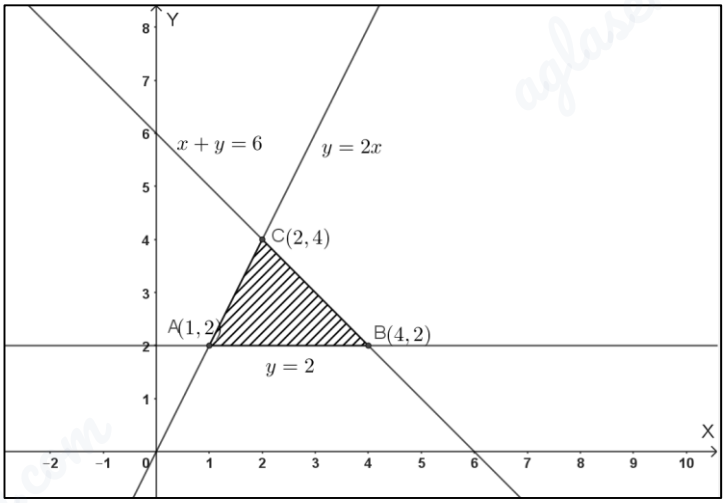
Section –C

[This section comprises of solution of short answer type questions (SA) of 3 marks each]

26(A)	Let $A = PQ$ and $B = QP$. Then $A = PQ = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 2 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 2 & 2 \end{bmatrix}$ $B = QP = \begin{bmatrix} 1 & 0 \\ 0 & 2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 4 \\ 2 & 0 & 0 \end{bmatrix}$ Here $\lvert A\rvert = 4 \neq 0$ So, A is a non-singular matrix, hence its inverse exists. Now $A^{-1} = \frac{1}{\lvert A\rvert} adj\ A = \frac{1}{4}\begin{bmatrix} 2 & 0 \\ -2 & 2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$ Here $\lvert B\rvert = 0$ So, B is a singular matrix, hence its inverse does not exist.	**1** **1** **½** **½**
27.	Let A be the area of the circular region covered by the drone and let r be its radius. Let h be the height of the drone at any time t. Given $\frac{dA}{dh} = 44\ m^2\ per\ metre$ and $\frac{dr}{dt} = 6\ m/s$ To find $\frac{dh}{dt}$ when r = 14 m. $A = \pi r^2 \quad \Rightarrow \frac{dA}{dt} = 2\pi r \frac{dr}{dt} \quad \Rightarrow \frac{dA}{dh}.\frac{dh}{dt} = 2\pi r \frac{dr}{dt}$ $44\ \frac{dh}{dt} = 2(\frac{22}{7})6r \quad \Rightarrow \left.\frac{dh}{dt}\right]_{r=14m} = 12$ m/s The height of the drone increases at the rate of 12 m/s at that moment.	**½** **1 ½** **1**
28(A).	$\frac{dx}{dt} = 3sin^2t\ cos\ t$ and $\frac{dy}{dt} = -3\ cos^2t\ sin\ t$ $\frac{dy}{dx} = \left.\frac{dy}{dt}\right/\frac{dx}{dt} = -cot\ t$ $\frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{dy}{dx}\right).\frac{dt}{dx} = cosec^2t\left(\frac{1}{3sin^2t\ \cos t}\right) = \frac{1}{3}(sec\ t.cosec^4t)$ At the point $\left(\frac{1}{2\sqrt{2}}, \frac{1}{2\sqrt{2}}\right)$, $sin^3t = \frac{1}{2\sqrt{2}} \Rightarrow t = \frac{\pi}{4}$ $\left.\frac{d^2y}{dx^2}\right]_{t=\frac{\pi}{4}} = \frac{4\sqrt{2}}{3}$	**1** **1** **1**

	OR	
28(B).	Let $y = x^{logx} + (logx)^x = u + v$ where $u = x^{logx}$ and $v = (logx)^x$ $log\ u = (log\ x)^2$ and $log\ v = x.log(log\ x)$ $\frac{1}{u}\frac{du}{dx} = 2(\ log\ x).\frac{1}{x}$ and $\frac{1}{v}\frac{dv}{dx} = log(log\ x) + \frac{1}{log\ x}$ $\frac{du}{dx} = x^{logx}\left(2(\ log\ x).\frac{1}{x}\right)$ and $\frac{dv}{dx} = (logx)^x\left(log(log\ x) + \frac{1}{log\ x}\right)$ $\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ $\Rightarrow \frac{dy}{dx} = \frac{2}{x}\ x^{logx}\ log\ x + \ (logx)^x\left(log(log\ x) + \frac{1}{log\ x}\right)$ **Alternative Solution:** Since $x = e^{logx}$ $y = x^{logx} + (logx)^x = e^{\log(x^{logx})} + e^{\log((logx)^x)}$ $= e^{(\log x)^2} + e^{x\log(\log x)}$ $\frac{dy}{dx} = e^{(\log x)^2}\left[\frac{2\log x}{x}\right] + e^{x\log(\log x)}[\log(\log x) + x\left(\frac{1}{\log x}\right)\left(\frac{1}{x}\right)]$ $OR\ \ x^{logx}\left[\frac{2\log x}{x}\right] + (logx)^x[\log(\log x) + \left(\frac{1}{\log x}\right)]$	**1** **1** **1** **1** **1+1**
29(A).	$xdy + (y - x^2e^x)dx = 0$ is a linear differential equation given by $\frac{dy}{dx} + \frac{1}{x}\ y = xe^x$ I.F = $e^{\int\frac{1}{x}dx} = \ e^{log\ x} = x$ Solution is $y.x = \int x^2e^x\ dx$ using by parts $yx = x^2e^x - \int 2xe^x\ dx$ using by parts again $yx = x^2e^x - 2\{xe^x - e^x\} + C$ y (1) = 0 $\Rightarrow$ $C = -e$ Particular solution is $yx = x^2e^x - 2\{xe^x - e^x\} - e = e^x\{x^2 - 2x + 2\} - e$	**½** **½** **½** **½** **½** **½**
	OR	
29(B).	$\frac{dy}{dx} = \frac{xy}{x^2+y^2}$ is a homogeneous differential equation. Let $y = v\ x$ $\Rightarrow \frac{dy}{dx} = v + x\ \frac{dv}{dx}$ So, equation reduces to	

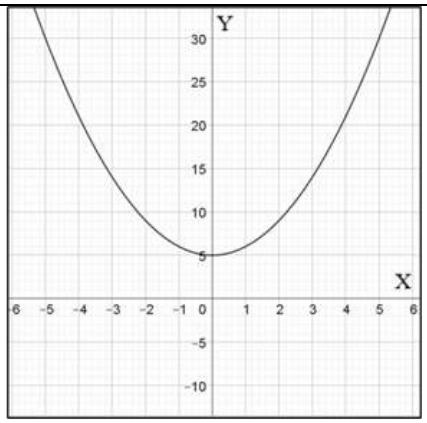
	$v + x\frac{dv}{dx} = \frac{v}{1+v^2} \quad \Rightarrow x\frac{dv}{dx} = \frac{v-v-v^3}{1+v^2}$ $\int \frac{1+v^2}{v^3} dv = -\int \frac{dx}{x}$ $\int (\frac{1}{v^3} + \frac{1}{v})\ dv = -log\ \lvert x\rvert + \log C \Rightarrow \frac{-1}{2v^2} + \log\lvert v\rvert = -\log\lvert x\rvert + \log C$ $\log\left\lvert\frac{y}{C}\right\rvert = \frac{x^2}{2y^2}$ or $y = C\, e^{\frac{x^2}{2y^2}}$ $C = 1$ when $x = 0$ and $y = 1$. The particular solution is $y = e^{\frac{x^2}{2y^2}}$	**1** **1** **1**
30(A).	A point on the line $x - 2 = \frac{2y+3}{4} = 2 - z = k$ *is* $P\left(k+2, \frac{4k-3}{2}, 2-k\right)$ And direction ratios of the given line are $\langle 1, 2, -1\rangle$. Direction Ratios of the line joining the points $A(2,1,4)$ and this point P are $\langle k+2-2, \frac{4k-3}{2} - 1, 2-k-4\rangle = \langle k, \frac{4k-5}{2}, -k-2\rangle$ As $\overrightarrow{AP}$ is perpendicular to the given line, So $1 \times k + 2 \times \frac{4k-5}{2} + (-1) \times (-k-2) = 0$ $\Rightarrow k + 4k - 5 + k + 2 = 0 \Rightarrow 6k - 3 = 0 \Rightarrow k = \frac{1}{2}$ So, foot of the perpendicular is $P\left(k+2, \frac{4k-3}{2}, 2-k\right) = \left(\frac{1}{2}+2, \frac{2-3}{2}, 2-\frac{1}{2}\right) = \left(\frac{5}{2}, \frac{-1}{2}, \frac{3}{2}\right)$ Direction ratios of the line $\overleftrightarrow{AP}$ are $\langle k, \frac{4k-5}{2}, -k-2\rangle = \langle \frac{1}{2}, -\frac{3}{2}, -\frac{5}{2}\rangle$ or $\langle 1, -3, -5\rangle$ And equation of $\overleftrightarrow{AP}$ is $\frac{x-2}{1} = \frac{y-1}{-3} = \frac{z-4}{-5}$ OR $x - 2 = \frac{1-y}{3} = \frac{4-z}{5}$	½ ½ ½ ½ ½ ½
	OR	
30(B).	Given lines are $\frac{x-1}{2} = \frac{y-1}{2} = \frac{z-2}{1}$ and $\frac{x-5}{2} = y - \frac{1}{2} = \frac{3-z}{2} \Rightarrow \frac{x-5}{2} = \frac{y-\frac{1}{2}}{1} = \frac{z-3}{-2}$ Let the angle between the lines be θ, then $\cos\theta = \frac{2\times2+2\times1+1\times(-2)}{\sqrt{9}\times\sqrt{9}} = \frac{4}{9} \Rightarrow \theta = \cos^{-1}\left(\frac{4}{9}\right)$ Now required line will be in the direction of the vector $\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 2 & 1 \\ 2 & 1 & -2 \end{vmatrix} = -5\hat{\imath} + 6\hat{\jmath} - 2\hat{k}$ So required line is $\frac{x-1}{-5} = \frac{y-1}{6} = \frac{z-1}{-2}$ OR $\frac{x-1}{5} = \frac{y-1}{-6} = \frac{z-1}{2}$	½ **1** **1** ½

31.	 Feasible region is bounded with three vertices $A(1,2), B(4,2), C(2,4)$ Now $Z_A = 100, Z_B = 220, Z_C = 200$ So, Maximum Value of Z is 220 at the point $B(4,2)$.	**1 ½** **1** **½**
	For Visually Impaired: The vertices of the feasible region are $A(20,0), B(40,0), C(30,10),\ D(0,20)$ and $E(0,10)$. Now $Z_A = 50 \times 20 - 20 \times 0 = 1000$ $Z_B = 50 \times 40 - 20 \times 0 = 2000$ $Z_C = 50 \times 30 - 20 \times 10 = 1300$ $Z_D = 50 \times 0 - 20 \times 20 = -400$ $Z_E = 50 \times 0 - 20 \times 10 = -200$ Minimum value of Z is -400 at $D(0,20)$ Maximum Value of Z is 2000 at $B(40,0)$	**2** **1**

Section –D

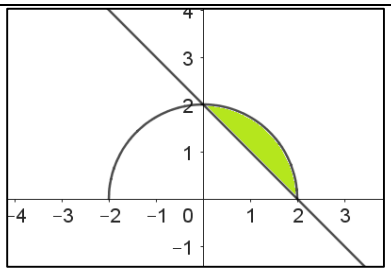
[This section comprises of solution of long answer type questions (LA) of 5 marks each]

32(A).	Given relation R is defined on $A = N \times N$ such that $(a,b)R(c,d)\ if\ and\ only\ if\ a^2 + d^2 = b^2 + c^2$ Reflexive: Let $(p,q) \in A = N \times N$ Then as $p, q \in N$, so $p^2 + q^2 = q^2 + p^2$ $\Rightarrow (p,q)R(p,q)\ \forall (p,q) \in N \times N$ $\Rightarrow R$ is a reflexive relation. Symmetric: Let $(p,q)\ and\ (r,s) \in A = N \times N$ such that $(p,q)\ R\ (r,s)$ $\Rightarrow p^2 + s^2 = q^2 + r^2$ $\Rightarrow q^2 + r^2 = p^2 + s^2$ $\Rightarrow r^2 + q^2 = s^2 + p^2$ $\Rightarrow (r,s)\ R\ (p,q)$ Thus, R is a symmetric relation. Transitive:	**1 ½** **1 ½**

	Let $(p,q),(r,s),(u,v) \in A$ such that $(p,q)\ R\ (r,s)\ and\ (r,s)R(u,v)$ $\Rightarrow p^2+s^2=q^2+r^2$ ...(i) and $r^2+v^2=s^2+u^2$ ...(ii) Adding (i) and (ii), $p^2+s^2+r^2+v^2=q^2+r^2+s^2+u^2$ $\Rightarrow p^2+v^2=q^2+u^2$ $\Rightarrow (p,q)\ R\ (u,v)$ Thus, R is a transitive relation So, R is a reflexive, symmetric and transitive relation. Hence R is an equivalence relation	**1 ½** **½**
	OR	
32(B).	 Yes, R is a function as the graph shows every element of the set $(-\infty,\infty)$ has a unique image in the set $[0,\infty)$. Now, let us denote this function by $f:(-\infty,\infty)\to[0,\infty)$ as $f(x)=x^2+5$ Injective Function: Here $f(-1)=(-1)^2+5=6$ and $f(1)=1^2+5=6$ So $f(-1)=f(1)=6$ Thus f is a many-one function, so not an injective function. Surjective Function: Range of $f(x)=\ [5,\infty) \neq$ co-domain$=[0,\infty)$ So, it is not surjective. **Alternate Method:** Correct graph as above Yes, R is a function as the graph shows every element of the set $(-\infty,\infty)$ has a unique image in the set $[0,\infty)$. Now, let us denote this function by $f:(-\infty,\infty)\to[0,\infty)$ as $f(x)=x^2+5$ Injective Function: Let $a,b \in (-\infty,\infty)$, such that $f(a)=f(b)$ $\Rightarrow a^2+5=b^2+5 \Rightarrow a^2=b^2 \Rightarrow a=\pm b$ So f is not injective. Surjective Function: For $y=1$ there is no pre-image as if $1=x^2+5 \Rightarrow x^2=-4$ not possible. So, it is not surjective. **For Visually Impaired:** For every element a of the set $(-\infty,\infty)$, $a^2 \in [0,\infty)$ and therefore $a^2+7 \in [7,\infty)$ Hence every element of the set $(-\infty,\infty)$ has a unique image in the set $[0,\infty)$.	**1 for correct graph** **1** **1 ½** **1 ½** **1** **1** **1 ½** **1 ½**

	So, $f:(-\infty,\infty)\to[0,\infty)$, $f(x)=x^2+7$ is a function. Now, let us denote this function by $f:(-\infty,\infty)\to[0,\infty)$ as $f(x)=x^2+7$ Injective Function: Here $f(-1)=(-1)^2+7=8$ and $f(1)=1^2+7=8$ So $f(-1)=f(1)=8$ Thus f is a many-one function, so not an injective function. Surjective Function: Range of $f(x)=\ [7,\infty)\neq$ co-domain$=[0,\infty)$ So, it is not surjective. Thus $f(x)$ is neither injective nor surjective and hence not bijective	1 1 ½ 1 ½ 1
33.	Let $I=\int_{-a}^{a}\sqrt{\frac{a-x}{a+x}}\,dx$ $I=\int_{-a}^{a}\sqrt{\frac{a-x}{a+x}\times\frac{a-x}{a-x}}\,dx$ $=\int_{-a}^{a}\frac{a-x}{\sqrt{a^2-x^2}}dx=\int_{-a}^{a}\frac{a}{\sqrt{a^2-x^2}}dx-\int_{-a}^{a}\frac{x}{\sqrt{a^2-x^2}}dx=I_1-I_2$ $I_1=a\int_{-a}^{a}\frac{1}{\sqrt{a^2-x^2}}dx$ $=2a\int_{0}^{a}\frac{1}{\sqrt{a^2-x^2}}dx$ using $\int_{-a}^{a}f(x)\,dx=2\int_{0}^{a}f(x)\,dx$ if $f(-x)=f(x)$ $I_1=2a\ \left[\sin^{-1}\left(\frac{x}{a}\right)\right]_0^a=2a\left(\frac{\pi}{2}\right)=\pi a$ and $I_2=\int_{-a}^{a}\frac{x}{\sqrt{a^2-x^2}}dx=0$ using $\int_{-a}^{a}f(x)\,dx=0$ if $f(-x)=-f(x)$ $I=\ I_1-I_2\ =\pi a$ **Alternate Method:** Let $x=a\cos 2\theta\ \Rightarrow dx=-2\,a\sin(2\,\theta)\,d\theta$ When $x=-a,\theta=\frac{\pi}{2}$ and $x=a,\theta=0$ $I=\int_{-a}^{a}\sqrt{\frac{a-x}{a+x}}\,dx\ =-2\,a\int_{\frac{\pi}{2}}^{0}\tan\theta\sin(2\theta)\,d\theta$ $I=\ 2\,a\int_{0}^{\frac{\pi}{2}}\tan\theta\sin(2\theta)\,d\theta$ .....(i) using $\int_a^b f(x)dx=-\int_b^a f(x)dx$ $I=\ 2a\int_{0}^{\frac{\pi}{2}}\tan\left(\frac{\pi}{2}-\theta\right)\sin(\pi-2\,\theta)d\theta$ using $\int_0^a f(x)dx=\int_0^a f(a-x)dx$ $I=\ 2a\int_{0}^{\frac{\pi}{2}}\cot(\theta)\sin(2\theta)\,d\theta$ .....(ii) Adding equations (i) and (ii) $2I=2a\int_{0}^{\frac{\pi}{2}}(\tan\theta+\cot\theta)\sin(2\theta)\,d\theta$ $\Rightarrow I=\ 2a\int_{0}^{\frac{\pi}{2}}(sin^2\theta+cos^2\theta)\,d\theta$ $\Rightarrow I=2a\int_{0}^{\frac{\pi}{2}}1\,d\theta=2a\left(\frac{\pi}{2}\right)=\pi a$	½ 1 1 1 1 ½ ½ ½ 1 1 1 1

34(A).



1 mark for correct figure

1

$y = \sqrt{4 - x^2}$ is a semi-circle with radius 2 units and centre (0,0)

$x = 2 - y$ which is x + y = 2 is a line with intercepts 2 on each axis.

Curves meet at (0,2) and (2,0) **1**

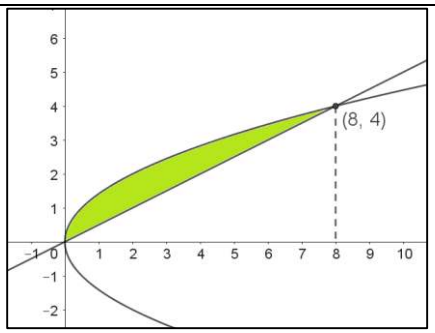
Area = $\int_0^2 \left(\sqrt{4 - x^2} - (2 - x)\right) dx$ **1**

$= \left[\frac{x}{2}\sqrt{4 - x^2} + 2 \sin^{-1}\left(\frac{x}{2}\right) - 2x + \frac{x^2}{2}\right]_0^2$ **1**

$= \pi - 4 + 2 = (\pi - 2)$ sq. units **1**

OR

34(B).



1 mark for correct figure. **1**

$y^2 = 2x$ represents a parabola, whose points of intersection with the given line $x = 2y$ are $(0, 0)$, $(8, 4)$ **1**

Area = $\int_0^8 \left(\sqrt{2x} - \frac{x}{2}\right) dx$ **1**

$= \left[\sqrt{2}\frac{2}{3}x^{\frac{3}{2}} - \frac{x^2}{4}\right]_0^8$ **1**

$= \frac{64}{3} - 16 = \frac{16}{3}$ sq. units **1**

34(A).

For Visually Impaired:

$25x^2 + 16y^2 = 400$ is an Ellipse with the standard equation as $\frac{x^2}{16} + \frac{y^2}{25} = 1$ **½**

$\Rightarrow y = \frac{5}{4}\sqrt{4^2 - x^2}$ **1**

Area = $4\int_0^4 \left(\frac{5}{4}\sqrt{4^2 - x^2}\right) dx$

$= 5\int_0^4 \left(\sqrt{4^2 - x^2}\right) dx$ **1**

$= 5\left[\frac{x}{2}\sqrt{16 - x^2} + 8 \sin^{-1}\left(\frac{x}{4}\right)\right]_0^4$ **1½**

$= 5[4(\pi)] = 20\pi$ sq. units **1**

OR

34(B).	$y^2 = 8x$ represents a parabola, and the area of the region above the x-axis, below the parabola, between the lines $x = 2$ and $x = 5$ is given by Area = $\int_2^5 (\sqrt{8x})\ dx$ $= \left[2\sqrt{2}\frac{2}{3}x^{\frac{3}{2}}\right]_2^5$ $= \sqrt{2}\left(\frac{4}{3}\right)[5\sqrt{5} - 2\sqrt{2}] = \frac{4}{3}[5\sqrt{10} - 4]$ sq. units	½ **2** **1 ½** **1**
35.	Let the events be E_1 : User uses Alpha smartwatch E_2 : User uses Beta smartwatch E_3 : User uses Gamma smartwatch E_4 : User uses Delta smartwatch A : User gets a false heart-rate alert Clearly $\{E_1, E_2, E_3, E_4\}$ is a partition of the sample space. Then $P(E_1)$=0.45, $P(E_2) = 0.3, P(E_3) = 0.15, P(E_4) = 0.1$ Now $P(A\|E_1) = 0.01,\ P(A\|E_2) = 0.02, P(A\|E_3) = 0.05, P(A\|E_4) = 0.01$ (i) By Theorem of Total Probability, The total probability that a randomly selected user gets a false alert $= P(A)$ $= P(E_1)P(A\|E_1) + P(E_2)P(A\|E_2) + P(E_3)P(A\|E_3) + P(E_4)P(A\|E_4)$ $= 0.45 \times 0.01 + 0.3 \times 0.02 + 0.15 \times 0.05 + 0.1 \times 0.01$ $= 0.0045 + 0.006 + 0.0075 + 0.001 = 0.019$ (ii) By Bayes Theorem, The probability that false alert came from a Beta smartwatch $= P(E_2\|A)$ $= \frac{P(E_2)P(A\|E_2)}{P(E_1)P(A\|E_1)+P(E_2)P(A\|E_2)+P(E_3)P(A\|E_3)+P(E_4)P(A\|E_4)}$ $= \frac{0.3\times0.02}{0.019} = \frac{0.006}{0.019} = \frac{6}{19}$	**1** **1** **1 ½** **1 ½**

Section –E

[This section comprises solution of 3 case- study/passage-based questions of 4 marks]

36.	(i) Given lines are $l_1: \vec{r} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k} + \lambda(\hat{\imath} + 3\hat{\jmath} - \hat{k}) \equiv \vec{a} + \lambda\vec{b}$ and $l_2: \vec{r} = 2\hat{\imath} - \hat{\jmath} + \hat{k} + \mu(2\hat{\imath} + 3\hat{\jmath} + \hat{k}) \equiv \vec{c} + \mu\vec{d}$ Here $\vec{a} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k},\ \vec{b} = \hat{\imath} + 3\hat{\jmath} - \hat{k},\ \vec{c} = 2\hat{\imath} - \hat{\jmath} + \hat{k},\ \vec{d} = 2\hat{\imath} + 3\hat{\jmath} + \hat{k}$ For direction vectors $\vec{b}$ and $\vec{d}$, $\vec{b} \neq \lambda\vec{d}$, for any $\lambda \in R$ $\Rightarrow$ $\vec{b}$ is not parallel to $\vec{d}$ So, lines are not parallel. (ii) Given lines are $l_1: \vec{r} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k} + \lambda(\hat{\imath} + 3\hat{\jmath} - \hat{k}) \equiv \vec{a} + \lambda\vec{b}$ and $l_3: \vec{r} = 3\hat{\imath} - \hat{\jmath} + \hat{k} + \gamma(\hat{\imath} + 3\hat{\jmath} - \hat{k}) \equiv \vec{e} + \gamma\vec{f}$	**1** **1**

	Since $\vec{b} = \vec{f}$, the lines l_1 and l_3 are parallel.	
	(iii) (a) $\vec{c} - \vec{a} = 2\hat{\imath} - \hat{\jmath} + \hat{k} - (\hat{\imath} + 2\hat{\jmath} + 3\hat{k}) = \hat{\imath} - 3\hat{\jmath} - 2\hat{k}$ $\vec{b} \times \vec{d} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 3 & -1 \\ 2 & 3 & 1 \end{vmatrix} = 6\hat{\imath} - 3\hat{\jmath} - 3\hat{k} = 3(2\hat{\imath} - \hat{\jmath} - \hat{k})$ $(\vec{c} - \vec{a}).(\vec{b} \times \vec{d}) = (\hat{\imath} - 3\hat{\jmath} - 2\hat{k}).3(2\hat{\imath} - \hat{\jmath} - \hat{k}) = 3(2 + 3 + 2) = 21 \neq 0$ So, the lines are skew lines. $\lvert\vec{b} \times \vec{d}\rvert = 3\sqrt{2^2 + (-1)^2 + (-1)^2} = 3\sqrt{6}$ Thus, the shortest distance between the given lines is $D = \left\lvert \frac{(\vec{c} - \vec{a}).(\vec{b} \times \vec{d})}{\lvert\vec{b} \times \vec{d}\rvert} \right\rvert = \left\lvert \frac{21}{3\sqrt{6}} \right\rvert = \frac{7}{\sqrt{6}} = \frac{7\sqrt{6}}{6}\ units$ **OR** (b) $l_1 : \vec{r} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k} + \lambda(\hat{\imath} + 3\hat{\jmath} - \hat{k}) \equiv \vec{a} + \lambda\,\vec{b}$ and $l_3 : \vec{r} = 3\hat{\imath} - \hat{\jmath} + \hat{k} + \gamma(\hat{\imath} + 3\hat{\jmath} - \hat{k}) \equiv \vec{e} + \gamma\,\vec{b}$ Since the lines are parallel, the shortest distance = $\frac{\lvert\vec{b} \times (\vec{e} - \vec{a})\rvert}{\lvert\vec{b}\rvert}$ $= \frac{1}{\sqrt{11}} \left\lVert \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 3 & -1 \\ 2 & -3 & -2 \end{vmatrix} \right\rVert = \frac{\lvert -9\hat{\imath} - 9\hat{k}\rvert}{\sqrt{11}} = \frac{9\sqrt{22}}{11} units$	1 1 1 1 1
37.	(i) $W(x) = 35 + 1.6x - 0.02x^2$ $W'(x) = 1.6 - 0.04x = 0 \Rightarrow x = 40$ Therefore, the value of x is 40 crores at the stationary point of $W(x)$. (ii) $W''(x) = -0.04 < 0 \Rightarrow W(x)$ is maximum at $x = 40$ by the second derivative test. Thus, the WQI is maximum when ₹ 40 crores are spent on the revival. Max. WQI = $W(40) = 35 + 64 - 32 = 67$ (iii) (a) If the investment is ₹35 crores, then the WQI = $W(35)$ $W(35) = 35 + (1.6)(35) - (0.02)(35)^2$ $= (2.6)(35) - (0.02)(1225)$ $= 91 - 24.5 = 66.5$ Also, 67-66.5 = 0.5 So, the WQI is short by 0.5 units of its maximum level. **OR** (b) $W(60) = 35 + 96 - 72 = 59$ 59 is less than the maximum value 67 of WQI, even when the investment has been increased from 40 crores to 60 crores. This shows that the returns are diminishing.	1 ½ ½ 1 1 1 1

38.

(i) Taking price of each kg of apples, oranges and pomegranate as ₹x, ₹y, ₹z respectively.

Matrix form is $AX = B$,

where $A = \begin{pmatrix} 5 & 10 & 5 \\ 10 & 10 & 5 \\ 8 & 15 & 7 \end{pmatrix}, X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, B = \begin{pmatrix} 2750 \\ 3500 \\ 4100 \end{pmatrix}$ **1**

Here $|A| = 25 \neq 0$

So, the system is consistent and has unique solution

Adjoint of the matrix A = adjA = $\begin{pmatrix} -5 & 5 & 0 \\ -30 & -5 & 25 \\ 70 & 5 & -50 \end{pmatrix}$ **1**

(ii) Unique solution of the system is given by

$X = A^{-1}B = \frac{1}{25}\begin{pmatrix} -5 & 5 & 0 \\ -30 & -5 & 25 \\ 70 & 5 & -50 \end{pmatrix}\begin{pmatrix} 2750 \\ 3500 \\ 4100 \end{pmatrix}$ **1**

$\Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 150 \\ 100 \\ 200 \end{pmatrix} \Rightarrow x = 150, y = 100, z = 200$

So the price of Apples, Oranges and Pomegranate are respectively ₹150 per kg, ₹100 per kg, ₹200 per kg. **1**