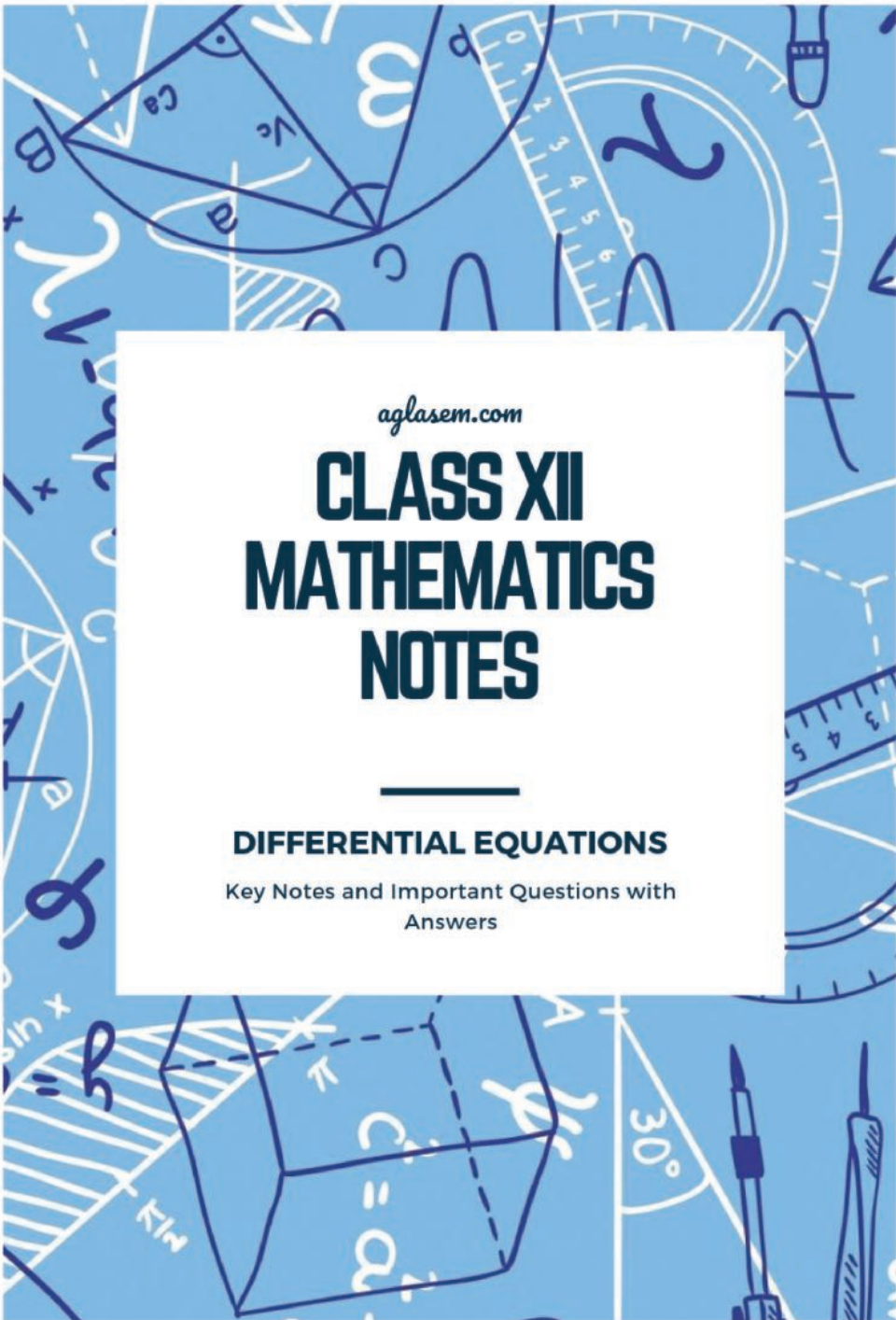




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CLASS XII MATHEMATICS NOTES

DIFFERENTIAL EQUATIONS

Key Notes and Important Questions with
Answers

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Key Notes and Important Questions with Answers

CHAPTER-9

DIFFERENTIAL EQUATIONS

POINTS TO REMEMBER

- **Differential Equation:** Equation containing derivatives of a dependent variable with respect to an independent variable is called differential equation.
- **Order of a Differential Equation:** The order of a differential equation is defined to be the order of the highest order derivative occurring in the differential equation.
- **Degree of a Differential Equation:** Highest power of highest order derivative involved in the equation is called degree of differential equation where equation is a polynomial equation in differential coefficients.
- **Formation of a Differential Equation:** We differentiate the family of curves as many times as the number of arbitrary constant in the given family of curves. Now eliminate the arbitrary constants from these equations.

After elimination, the equation obtained is differential equation.

- **Solution of Differential Equation**

- (i) **Variable Separable Method**

$$\frac{dy}{dx} = f(x, y).$$

We Separate the variables and get

$$f(x)dx = g(y)dy$$

Then $\int f(x)dx = \int g(y)dy + c$ is the required solutions.

- (ii) **Homogeneous Differential Equation:** A differential equation of the form $\frac{dy}{dx} = \frac{f(x,y)}{g(x,y)}$ where $f(x, y)$ and $g(x, y)$ are both homogeneous functions of the same degree in x and y i.e., of the form $\frac{dy}{dx} = F\left(\frac{y}{x}\right)$ is called a homogeneous differential equation.

For solving this type of equations we substitute $y = vx$ and then $\frac{dy}{dx} = v + x \frac{dv}{dx}$. The equation can be solved by variables separable method.

A homogeneous differential equation can be of the form $\frac{dx}{dy} = F\left(\frac{x}{y}\right)$

To solve this equation, we substitute $x = vy$ and then $\frac{dx}{dy} = v + y \frac{dv}{dy}$ then the equation can be solved by variable separate method.

- (iii) **Linear Differential Equation:** An equation of the form $\frac{dy}{dx} + Py = Q$

where P and Q are constant or functions of x only is called a linear differential equation. For finding solution of this type of equations, we find integrating factor (I.F.) = $e^{\int P dx}$

Solution is $y (I.F.) = \int Q \cdot (I.F.) dx + c$

Similarly, differential equations of the type $\frac{dx}{dy} + Px = Q$ where P and Q are constants or functions of y only can be solved.

Here, I.F. = $e^{\int P dy}$ and the solution is $x (I.F.) = \int Q \times (I.F.) dy + C$

Very Short Answer Type Questions (1 Mark)

- Write the order and degree of the following differential equations.

$$(i) \quad \frac{dy}{dx} + \cos y = 0$$

$$(ii) \quad \left(\frac{dy}{dx}\right)^2 + 3 \frac{d^2y}{dx^2} = 4$$

$$(iii) \quad \frac{d^4y}{dx^4} + \sin x = \left(\frac{d^2y}{dx^2}\right)^5$$

$$(iv) \quad \frac{d^5y}{dx^5} + \log\left(\frac{dy}{dx}\right) = 0$$

$$(v) \quad \sqrt{1 + \frac{dy}{dx}} = \left(\frac{d^2y}{dx^2}\right)^{1/3}$$

$$(vi) \quad \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2} = k \frac{d^2y}{dx^2}$$

$$(vii) \quad \left(\frac{d^3y}{dx^3}\right)^2 + \left(\frac{d^2y}{dx^2}\right)^3 = \sin x$$

$$(viii) \quad \frac{dy}{dx} + \tan\left(\frac{dy}{dx}\right) = 0$$

2. Write integrating factor differential equations :

$$(I) \quad \frac{dy}{dx} + y \cos x = \sin x$$

$$(II) \quad \frac{dy}{dx} + y \sec^2 x = \sec x + \tan x$$

$$(III) \quad x^2 \frac{dy}{dx} + y = x^4$$

$$(IV) \quad x \frac{dy}{dx} + y \log x = x + y$$

$$(V) \quad x \frac{dy}{dx} - 3y = x^3$$

$$(VI) \quad \frac{dy}{dx} + y \tan x = \sec x$$

$$(VII) \quad \frac{dy}{dx} + \frac{1}{1+x^2} y = \sin x$$

3. Write order of the differential equation of the family of following curves.

$$(I) \quad y = Ae^x + Be^{x+c}$$

$$(II) \quad Ay = Bx^2$$

$$(III) \quad (x-a)^2 + (y-b)^2 = 9$$

$$(IV) \quad Ax + By^2 = Bx^2 - Ay$$

$$(V) \quad \frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$$

$$(VI) \quad y = a \cos (ax + b)$$

$$(VII) \quad y = a + be^{x+c}$$

Very Short Answer Type Questions [2 Marks]

4. Write the general solution of the following differential equations.

$$(i) \quad \frac{dy}{dx} = x^5 + x^2 - \frac{2}{x}$$

$$(ii) \quad (e^x + e^{-x}) dy = (e^x - e^{-x}) dx$$

$$(iii) \quad \frac{dy}{dx} = x^3 + e^x + x^e$$

$$(iv) \quad \frac{dy}{dx} = 5^{x+y}$$

$$(v) \quad \frac{dy}{dx} = \frac{1 - \cos 2x}{1 + \cos 2y}$$

$$(vi) \quad \frac{dy}{dx} = \frac{1 - 2y}{3x + 1}$$

Short Answer Type Questions (4 Marks)

5. (I) Show that $y = e^{m \sin^{-1} x}$ is a solution of

$$(1 - x^2) \frac{d^2 y}{dx^2} - x \frac{dy}{dx} - m^2 y = 0$$

- (II) Show that $y = \sin(\sin x)$ is a solution of differential equation

$$\frac{d^2 y}{dx^2} + (\tan x) \frac{dy}{dx} + y \cos^2 x = 0$$

- (III) Show that $y = Ax + \frac{B}{x}$ is a solution of $x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} - y = 0$

- (IV) Show that $y = a \cos(\log x) + b \sin(\log x)$ is a solution of

$$x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = 0$$

- (V) Verify that $y = \log(x + \sqrt{x^2 + a^2})$ satisfies the differential

$$\text{equation: } (a^2 + x^2) \frac{d^2 y}{dx^2} + x \frac{dy}{dx} = 0$$

(VI) Find the differential equation of the family of curves

$$y = e^x(A \cos x + B \sin x), \text{ where } A \text{ and } B \text{ are arbitrary constants.}$$

(VII) Find the differential equation of an ellipse with major and minor axes $2a$ and $2b$ respectively.

(VIII) Form the differential equation representing the family of curves $(y - b)^2 = 4(x - a)$.

6. Solve the following differential equations.

(I) $(1 - x^2) \frac{dy}{dx} - xy = x^2$, given that $x = 0, y = 2$

(II) $x \frac{dy}{dx} + 2y = x^2 \log x$

(III) $\frac{dy}{dx} + \frac{1}{x} y = \cos x + \frac{\sin x}{x}, \quad x > 0$

(IV) $dy = \cos x (2 - y \operatorname{cosec} x) dx$; given that $x = \frac{\pi}{2}, y = 2$

(V) $y dx + (x - y^3) dy = 0$

(VI) $ye^y dx = (y^3 + 2xe^y) dy$

7. Solve each of the following differential equations:

(I) $y - x \frac{dy}{dx} = 2 \left(y^2 + \frac{dy}{dx} \right)$

(II) $\cos y \, dx + (1 + 2e^{-x}) \sin y \, dy = 0$

$$(III) \quad x\sqrt{1-y^2}dx + y\sqrt{1-x^2}dy = 0$$

$$(IV) \quad \sqrt{(1-x^2)(1-y^2)}dy + xy \, dx = 0$$

$$(V) \quad (xy^2 + x)dx + (yx^2 + y)dy = 0; y(0) = 1$$

$$(VI) \quad \frac{dy}{dx} - y \sin^3 x \cos^3 x + xy e^x = 0$$

$$(VII) \quad \tan x \tan y \, dx + \sec^2 x \sec^2 y \, dy = 0$$

$$(VIII) \quad \frac{dy}{dx} = x - 1 + xy - y$$

8. Solve the following differential equations:

$$(I) \quad x^2 y \, dx - (x^3 + y^3) \, dy = 0$$

$$(II) \quad x^2 \frac{dy}{dx} = x^2 + xy + y^2$$

$$(III) \quad (x^2 - y^2)dx + 2xy \, dy = 0, \quad y(1) = 1$$

$$(IV) \quad \left(y \sin \frac{x}{y}\right) dx = \left(x \sin \frac{x}{y} - y\right) dy$$

$$(V) \quad \frac{dy}{dx} = \frac{y}{x} + \tan\left(\frac{y}{x}\right)$$

$$(VI) \quad x \frac{dy}{dx} = y (\log y - \log x + 1)$$

$$(VII) \quad \frac{dy}{dx} = e^{x+y} + x^2 e^y$$

$$(VIII) \frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$$

$$(IX) (3xy + y^2)dx + (x^2 + xy)dy = 0$$

9. (I) Form the differential equation of the family of circles touching y-axis at (0, 0).
- (II) Form the differential equation of family of parabolas having vertex at (0,0) and axis along the (i) positive y-axis (ii) positive x-axis.
- (III) Form differential equation of family of circles passing through origin and whose centres lie on x-axis.
- (IV) Form the differential equation of the family of circles in the first quadrant and touching the coordinate axes.
10. Show that the differential equation $\frac{dy}{dx} = \frac{x+2y}{x-y}$ is homogeneous and solve it.
11. Show that the differential equation:
 $(x^2 + 2xy - y^2)dx + (y^2 + 2xy - x^2)dy = 0$ is homogeneous and solve it.
12. Solve the following differential equations:
- (I) $\frac{dy}{dx} - 2y = \cos 3x$
- (II) $\sin x \frac{dy}{dx} + y \cos x = 2 \sin^2 x \cos x$ if $y\left(\frac{\pi}{2}\right) = 1$
- (III) $\log\left(\frac{dy}{dx}\right) = px + qy$
13. Solve the following differential equations:
- (I) $(x^3 + y^3) dx = (x^2y + xy^2)dy$

$$(II) \quad x \, dy - y \, dx = \sqrt{x^2 + y^2} \, dx$$

$$(III) \quad y \left\{ x \cos \left(\frac{y}{x} \right) + y \sin \left(\frac{y}{x} \right) \right\} dx$$

$$-x \left\{ y \sin \left(\frac{y}{x} \right) - x \cos \left(\frac{y}{x} \right) \right\} dy = 0$$

$$(IV) \quad x^2 dy + y(x + y) \, dx = 0 \text{ given that } y=1 \text{ when } x=1.$$

$$(V) \quad x e^{\frac{y}{x}} - y + x \frac{dy}{dx} = 0 \text{ if } y(e) = 0$$

$$(VI) \quad (x^3 - 3xy^2) \, dx = (y^3 - 3x^2y) \, dy$$

$$(VII) \quad \frac{dy}{dx} - \frac{y}{x} + \operatorname{cosec} \left(\frac{y}{x} \right) = 0 \text{ given that } y = 0 \text{ when } x = 1$$

14. Solve the following differential equations:

$$(I) \quad \cos^2 x \frac{dy}{dx} = \tan x - y$$

$$(II) \quad x \cos x \frac{dy}{dx} + y (x \sin x + \cos x) = 1$$

$$(III) \quad \left(1 + e^{\frac{x}{y}} \right) dx + e^{\frac{x}{y}} \left(1 - \frac{x}{y} \right) dy = 0$$

$$(IV) \quad (y - \sin x) \, dx + \tan x \, dy = 0, \, y(0) = 0$$

Long Answer Type Questions (6 Marks)

15. Solve the following differential equations:

$$(I) \quad (x \, dy - y \, dx)y \sin\left(\frac{y}{x}\right) = (y \, dx + x \, dy)x \cos\left(\frac{y}{x}\right)$$

$$(II) \quad 3e^x \tan y \, dx + (1 - e^x)\sec^2 y \, dy = 0 \text{ given that } y = \frac{\pi}{4}, \text{ when } x = 1$$

$$(III) \quad \frac{dy}{dx} + y \cot x = 2x + x^2 \cot x \text{ given that } y(0) = 0.$$

16. Show that the differential equation

$2y e^{\frac{x}{y}} dx + \left(y - 2x e^{\frac{x}{y}}\right) dy = 0$ is homogenous. Find the particular solution of this differential equation given that $x = 0$ when $y = 1$.

ANSWERS

1.
 - (i) order = 1, degree is not defined
 - (ii) order = 2, degree = 1
 - (iii) order = 4, degree = 1
 - (iv) order = 5, degree is not defined.
 - (v) order = 2, degree = 2
 - (vi) order = 2, degree = 2
 - (vii) order = 3, degree = 2
 - (viii) order = 1, degree is not defined

2.

(I) $e^{\sin x}$	(II) $e^{\tan x}$
(III) $e^{-1/x}$	(IV) $e^{\frac{(\log x)^2}{2}}$
(V) $\frac{1}{x^3}$	(VI) $\sec x$
(VII) $e^{\tan^{-1} x}$	

3. (I) 1 (II) 1
(III) 2 (IV) 1
(V) 1 (VI) 2
(VII) 2
4. (I) $y = \frac{x^6}{6} + \frac{x^3}{3} - 2 \log |x| + c$ (II) $y = \log_e |e^x + e^{-x}| + c$
(III) $y = \frac{x^4}{4} + e^x + \frac{x^{e+1}}{e+1} + c$ (IV) $5^x + 5^{-y} = c$
(V) $2(y - x) + \sin 2y + \sin 2x = c$
(VI) $2 \log |3x + 1| + 3 \log |1 - 2y| = c$
5. (VI) $\frac{d^2y}{dx^2} - 2 \frac{dy}{dx} + 2y = 0$
(VII) $x \left(\frac{dy}{dx} \right)^2 + xy \frac{d^2y}{dx^2} = y \frac{dy}{dx}$
(VIII) $2 \frac{d^2y}{dx^2} + \left(\frac{dy}{dx} \right)^3 = 0$
6. (I) $y\sqrt{1-x^2} + \frac{x\sqrt{1-x^2}}{2} = \frac{\sin^{-1} x}{2} + 2$
(II) $y = \frac{x^2(4 \log_e x - 1)}{16} + \frac{c}{x^2}$
(III) $y = \sin x + \frac{c}{x}, x > 0$
(IV) $2y \sin x = 3 - \cos 2x$
(V) $xy = \frac{y^4}{4} + c$
(VI) $x = -y^2 e^{-y} + cy^2$

7. (I) $cy = (x + 2)(1 - 2y)$
- (II) $(e^x + 2) \sec y = c$
- (III) $\sqrt{1 - x^2} + \sqrt{1 - y^2} = c$
- (IV) $\frac{1}{2} \log \left| \frac{\sqrt{1-y^2}-1}{\sqrt{1-y^2}+1} \right| = \sqrt{1-x^2} - \sqrt{1-y^2} + c$
- (V) $(x^2 + 1)(y^2 + 1) = 2$
- (VI) $\log y = -\frac{1}{4} \cos^4 x + \frac{1}{6} \cos^6 x - xe^x + e^x + c$
- $$= \frac{1}{16} \left[\frac{\cos^3 2x}{3} - \cos 2x \right] - (x - 1)e^x + c$$
- (VII) $\log |\tan y| - \frac{\cos 2x}{4} = c$
- (VIII) $\log |y + 1| = \frac{x^2}{2} - x + c$
8. (I) $\frac{-x^3}{y^3} + 3 \log |y| = c$
- (II) $\tan^{-1} \left(\frac{y}{x} \right) = \log |x| + c$
- (III) $x^2 + y^2 = 2x$
- (IV) $y = ce^{\cos(x/y)}$
- (V) $\sin \left(\frac{y}{x} \right) = cx$
- (VI) $\log |y/x| = cx$
- (VII) $-e^{-y} = e^x + \frac{x^3}{3} + c$

$$(VIII) \quad \sin^{-1} y = \sin^{-1} x + c$$

$$(IX) \quad (2xy + y^2) = \frac{c}{x^2}$$

$$9. \quad (I) \quad x^2 - y^2 + 2xy \frac{dy}{dx} = 0$$

$$(II) \quad 2y = x \frac{dy}{dx}, \quad y = 2x \frac{dy}{dx}$$

$$(III) \quad x^2 - y^2 + 2xy \frac{dy}{dx} = 0$$

$$(IV) \quad (x - y)^2 (1 + y'^2) = (x + yy')^2$$

$$10. \quad \log|x^2 + xy + y^2| = 2\sqrt{3} \tan^{-1} \left(\frac{x+2y}{\sqrt{3x}} \right) + c$$

$$11. \quad x^2 + y^2 = c(x + y)$$

$$12. \quad (I) \quad y = \frac{3 \sin 3x}{13} - \frac{2 \cos 3}{13} + Ce^{2x}$$

$$(II) \quad y = \frac{2}{3} \sin^2 x + \frac{1}{3} \operatorname{cosec} x$$

$$(III) \quad \frac{e^{px}}{p} + \frac{e^{-qy}}{q} = c$$

$$13. \quad (I) \quad -y = x \log\{c(x - y)\}$$

$$(II) \quad cx^2 = y + \sqrt{x^2 + y^2}$$

$$(III) \quad xy \cos \left(\frac{y}{x} \right) = c$$

- (IV) $3x^2y = y + 2x$
- (V) $y = -x \log(\log|x|), x \neq 0$
- (VI) $c(x^2 + y^2) = \sqrt{x^2 - y^2}$
- (VII) $\cos \frac{y}{x} = \log|x| + 1$
14. (I) $y = \tan x - 1 + ce^{-\tan x}$
- (II) $y = \frac{\sin x}{x} + c \frac{\cos x}{x}$
- (III) $x + ye^{\frac{x}{y}} = c$
- (IV) $2y = \sin x$
15. (I) $cxy = \sec\left(\frac{y}{x}\right)$
- (II) $(1 - e)^3 \tan y = (1 - e^x)^3$
- (III) $y = x^2$
16. $e^{x/y} = -\frac{1}{2} \log|y| + 1$