UNIT 3
INTEGERS

(A) Main Concepts and Results

- The collection of numbers 0, +1, –1, +2, –2, +3, –3, ... is called **integers**.
- The numbers +1, +2, +3, +4, ... are referred to as **positive integers**.
- The numbers –1, –2, –3, –4, ... are referred to as **negative integers**.
- The numbers 0, +1, +2, +3, ... are called **non-negative integers**.
- The integers are represented on the number line as follows :



Fig. 3.1

- All the positive integers lie to the right of 0 and the negative integers to the left of 0 on the number line.
- All non negative integers are the same as whole numbers and hence all the opertations on them are done as in the case of whole numbers.
- To add two negative integers, we add the corresponding positive integers and retain the negative sign with the sum.
- To add a positive integer and a negative integer, we ignore the signs and subtract integer with smaller numerical value from the integer with larger numerical value and take the sign of the larger one.
- Two integers whose sum is zero are called **additive inverses** of each other. They are also called the **negatives** of each other.

- Additive inverse of an integer is obtained by changing the sign of the integer. For example, the additive inverse of +5 is –5 and the additive inverse of –3 is +3.
- To subtract an integer from a given integer, we add the additive inverse of the integer to the given integer.
- To compare two integers on the number line, we locate their positions on the number line and the integer lying to the right of the other is always greater.

(B) Solved Examples

Example 1: Write the correct answer from the given four options:

Sania and Trapi visited Leh and Tawang respectively during winter. Sania reported that she had experienced –4°C on Sunday, while Trapi reported that she had experienced –2°C on that day. On that Sunday

(A) Leh was cooler than Tawang.

(B) Leh was hotter than Tawang.

(C) Leh was as cool as Tawang.

(D) Tawang was cooler than Leh.

Solution: The correct answer is (A).

Example 2: State whether each of the following statements is true or false:

(a) Every positive integer is greater than 0.

(b) Every integer is either positive or negative.

Solution: (a) True (b) False

Example 3: Fill in the blank using <, > or = to make the statement correct

3 + (–2) ____ 3 + (–3)

Solution : 3 + (–2) > 3 + (–3)

Example 4: Represent the following using integers with proper sign:

(a) 3 km above sea level (b) A loss of Rs 500

Solution: (a) +3

(b) –500

Example 5: Find the sum of the pairs of integers:

(a) – 6, – 4 (b) +3, – 4 (c) +4, –2

Solution: (a) – 6 and – 4 both have negative signs.

So, – 6 + (– 4) = – (6 + 4) = –10

(b) + 3 and – 4 have opposite signs.

As 4 – 3 = 1, therefore + 3 + (– 4) = –1

(c) + 4 and –2 have opposite signs.

So, 4 + (–2) = 4 – 2 = 2

Example 6: Find the sum of –2 and –3, using the number line.

Solution: To add –2 and –3, on the number line, we first move 2 steps to the left of 0, reaching –2. Then we move 3 steps to the left of –2 and reach –5. (Fig. 3.2)

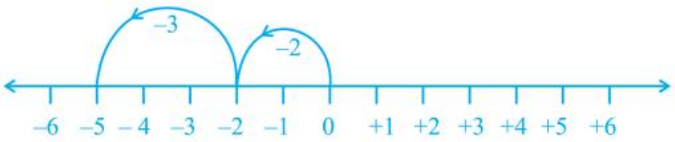


Fig. 3.2

Thus, –2 + (–3) = –5.

Example 7: Subtract : (i) 3 from –4 (ii) –3 from –4

Solution: (a) The additive inverse of 3 is –3.

So, – 4 – 3 = – 4 + (–3) = – (4 + 3) = –7

(b) The additive inverse of –3 is + 3.

So, – 4 – (–3) = – 4 + (+3) = –1

Example 8: Using the number line, subtract : (a) 2 from –3 (b) –2 from –3.

Solution: (a) To subtract 2 from –3, we move 2 steps to the left of – 3 on the number line and reach –5. (Fig. 3.3)

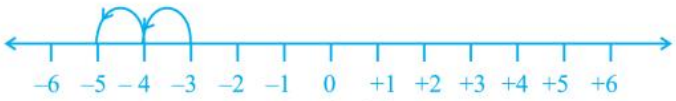


Fig. 3.3

So, –3 – 2 = –5.

(b) To subtract –2 from –3, we observe that 2 is the addtive inverse of –2.

So, we add 2 to –3 using the number line and reach at –1.

So, –3 – (–2) = –3 + (+ 2) = –1

Example 9: How many integers are there between –9 and –2 ?

Solution: The integers –8, –7, –6, –5, –4 and –3 lie between –9 and –2. So, there are six integers between – 9 and –2.

Example10: Calculate:

1 – 2 + 3 – 4 + 5 – 6 + 7 – 8 + 9 – 10

Solution: 1 – 2 + 3 – 4 + 5 – 6 + 7 – 8 + 9 – 10

= (1 + 3 + 5 + 7 + 9) – (2 + 4 + 6 + 8 + 10)

= 25 – 30

= –5.

Alternatively, 1 – 2 + 3 – 4 + 5 – 6 + 7 –8 + 9 – 10

= (1 – 2) + (3 – 4) + (5 – 6) + (7 – 8) + (9 – 10)

= (–1) + (–1) + (–1) + (–1) + (–1)

= –5.

Example 11: The sum of two integers is 47. If one of the integers is – 24, find the other.

Solution: As the sum is 47, the other integer is obtained by subtracting –24 from 47. So, the required integer

= 47 – (–24)

= 47 + 24

= 71.

Example 12: Write the digits 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9 in this order and insert '+ 'or '–' between them to get the result
(a) 5 (b) –3

Solution: (a) $0 + 1 - 2 + 3 - 4 + 5 - 6 + 7 - 8 + 9 = 5$

(b) $0 - 1 - 2 + 3 + 4 - 5 + 6 - 7 + 8 - 9 = -3$

Example 13: Write five distinct integers whose sum is 5.

Solution: As the required sum is 5, keep 5 as one of the integers and write two pairs of integers which are additive inverses of each other.

For example, $5 + [2+(-2)]+[3+(-3)]= 5$.

Thus, the required five integers are 5, 2, –2, 3, –3

There can be many combinatioins of five integers, such as 5, 3, –3, 6, –6 or 4, 2, 3, –3, –1 etc., whose sum is 5.

(C) Exercise

In questions 1 to 17, only one of the four options is correct. Write the correct one.

1. Every integer less than 0 has the sign

(A) + (B) – (C) × (D) ÷

2. The integer '5 units to the right of 0 on the number line' is

(A) +5 (B) –5 (C) +4 (D) – 4

3. The predecessor of the integer –1 is

(A) 0 (B) 2 (C) –2 (D) 1

4. Number of integers lying between –1 and 1 is

(A) 1 (B) 2 (C) 3 (D) 0

5. Number of whole numbers lying between –5 and 5 is

(A) 10 (B) 3 (C) 4 (D) 5

6. The greatest integer lying between –10 and –15 is

(A) –10 (B) –11 (C) –15 (D) –14

7. The least integer lying between –10 and –15 is

(A) –10 (B) –11 (C) –15 (D) –14

8. On the number line, the integer 5 is located

(A) to the left of 0 (B) to the right of 0

(C) to the left of 1 (D) to the left of –2

9. In which of the following pairs of integers, the first integer is not on the left of the other integer on the number line?

(A) (–1, 10) (B) (–3, –5) (C) (–5, –3) (D) (–6, 0)

10. The integer with negative sign (–) is always less than

(A) 0 (B) –3 (C) –1 (D) –2

11. An integer with positive sign (+) is always greater than

(A) 0 (B) 1 (C) 2 (D) 3

12. The successor of the predecessor of –50 is

(A) –48 (B) –49 (C) –50 (D) –51

13. The additive inverse of a negative integer

(A) is always negative (B) is always positive

(C) is the same integer (D) zero

14. Amulya and Amar visited two places A and B respectively in Kashmir and recorded the minimum temperatures on a particular day as –4°C at A and –1°C at B. Which of the following statement is true?

(A) A is cooler than B

(B) B is cooler than A

(C) There is a difference of 2°C in the temperature

(D) The temperature at A is 4°C higher than that at B.

15. When a negative integer is subtracted from another negative integer, the sign of the result

(A) is always negative (B) is always positive

(C) is never negative (D) depends on the numerical value of the integers

16. The statement "When an integer is added to itself, the sum is greater than the integer" is

(A) always true

(B) never true

(C) true only when the integer is positive

(D) true for non-negative integers

17. Which of the following shows the maximum rise in temperature?

(A) 0°C to 10°C (B) –4°C to 8°C

(C) –15°C to –8°C (D) –7°C to 0°C

In questions 18 to 39, state whether the given statements are true (T) or false (F) :

18. The smallest natural number is zero.

19. Zero is not an integer as it is neither positive nor negative.

20. The sum of all the integers between –5 and –1 is –6.

21. The successor of the integer 1 is 0.

22. Every positive integer is larger than every negative integer.

23. The sum of any two negative integers is always greater than both the integers.

24. The sum of any two negative integers is always smaller than both the integers.

25. The sum of any two positive integers is greater than both the integers.

26. All whole numbers are integers.

27. All integers are whole numbers.

28. Since $5 > 3$, therefore $-5 > -3$

29. Zero is less than every positive integer.

30. Zero is larger than every negative integer.

31. Zero is neither positive nor negative.

32. On the number line, an integer on the right of a given integer is always larger than the integer.

33. –2 is to the left of –5 on the number line.

34. The smallest integer is 0.

35. 6 and –6 are at the same distance from 0 on the number line.

36. The difference between an integer and its additive inverse is always even.

37. The sum of an integer and its additive inverse is always zero.

38. The sum of two negative integers is a positive integer.

39. The sum of three different integers can never be zero.

In questions 40 to 49, fill in the blanks to make the statements true:

40. On the number line, –15 is to the _____ of zero.

41. On the number line, 10 is to the _____ of zero.

42. The additive inverse of 14 is _____.

43. The additive inverse of –1 is _____.

44. The additive inverse of 0 is _____.

45. The number of integers lying between –5 and 5 is _____.

46. (–11) + (–2) + (–1) = _____.

47. _____ + (–11) + 111 = 130

48. (–80) + 0 + (–90) = _____

49. _____ –3456 = –8910

In questions 50 to 58, fill in the blanks using <, = or > :

50. (–11) + (–15) _____ 11 + 15

51. (–71) + (+9) _____ (–81) + (–9)

52. 0 _____ 1

53. –60 _____ 50

54. –10 _____ –11

55. –101 _____ –102

56. (–2) + (–5) + (–6) _____ (–3) + (–4) + (–6)

57. 0 _____ –2

58. 1 + 2 + 3 _____ (–1) + (–2) + (–3)

59. Match the items of Column I with that of Column II:

	Column I	Column II
(i)	The additive inverse of +2	(A) 0
(ii)	The greatest negative integer	(B) –2
(iii)	The greatest negative even integer	(C) 2
(iv)	The smallest integer greater than every negative integer	(D) 1
(v)	Sum of predecessor and successor of –1	(E) –1

60. Compute each of the following:

(a) 30 + (–25) + (–10)
(b) (–20) + (–5)
(c) 70 + (–20) + (–30)
(d) –50 + (–60) + 50
(e) 1 + (–2) + (– 3) + (– 4)
(f) 0 + (– 5) + (– 2)
(g) 0 – (–6) – (+6)
(h) 0 – 2 – (–2)

61. If we denote the height of a place above sea level by a positive integer and depth below the sea level by a negative integer, write the following using integers with the appropriate signs:

(a) 200 m above sea level
(b) 100 m below sea level
(c) 10 m above sea level
(d) sea level

62. Write the opposite of each of the following:

(a) Decrease in size
(b) Failure
(c) Profit of Rs.10
(d) 1000 A.D.
(e) Rise in water level
(f) 60 km south
(g) 10 m above the danger mark of river Ganga
(h) 20 m below the danger mark of the river Brahmaputra
(i) Winning by a margin of 2000 votes
(j) Depositing Rs.100 in the Bank account
(k) 20°C rise in temperature.

63. Temperature of a place at 12:00 noon was +5°C. Temperature increased by 3°C in first hour and decreased by 1°C in the second hour. What was the temperature at 2:00 pm?

64. Write the digits 0, 1, 2, 3, ..., 9 in this order and insert '+' or '–' between them to get the result 3.

65. Write the integer which is its own additive inverse.

66. Write six distinct integers whose sum is 7.

67. Write the integer which is 4 more than its additive inverse.

68. Write the integer which is 2 less than its additive inverse.

69. Write two integers whose sum is less than both the integers.

70. Write two distinct integers whose sum is equal to one of the integers.

71. Using number line, how do you compare

(a) two negative integers? (b) two positive integers?

(c) one positive and one negative integer?

72. Observe the following :

$1 + 2 - 3 + 4 + 5 - 6 - 7 + 8 - 9 = -5$

Change one '–' sign as '+' sign to get the sum 9.

73. Arrange the following integers in the ascending order :

–2, 1, 0, –3, +4, –5

74. Arrange the following integers in the descending order :

–3, 0, –1, –4, –3, –6

75. Write two integers whose sum is 6 and difference is also 6.

76. Write five integers which are less than –100 but greater than –150.

77. Write four pairs of integers which are at the same distance from 2 on the number line.

78. The sum of two integers is 30. If one of the integers is –42, then find the other.

79. Sum of two integers is –80. If one of the integers is –90, then find the other.

80. If we are at 8 on the number line, in which direction should we move to reach the integer

(a) –5 (b) 11 (c) 0?

81. Using the number line, write the integer which is

(a) 4 more than –5

(b) 3 less than 2

(c) 2 less than –2

82. Find the value of

49 – (–40) – (–3) + 69

83. Subtract –5308 from the sum [(–2100) + (–2001)]

(D) Activities

Activity I : The faces of two dice are marked +1, +2, +3, +4, +5, +6 and –1, –2, –3, –4, –5, –6, respectively.

Two players throw the pair of dice alternately and record the sum of the numbers that turn up each time and keep adding their scores separately. The player whose score reaches 20 or more first, wins the game.

(i) What can be the possible scores in a single throw of the pair of dice?

(ii) What is the maximum score?

(iii) What is the minimum score?

(iv) A player gets his score 20 as follows:

(5) + (–4) + (6) + (2) + (+5) + (4) + (2)

Is he a winner?

(v) What is the minimum number of throws needed to win the game?

Activity II : Repeat Activity I by taking two dice marked with numbers +1, –2, +3, –4, +5, –6 and –1, +2, –3, +4, –5, +6, respectively.

DO YOU KNOW?

I. Indians were the first to use negative numbers. Brahmagupta used negative numbers in 628 A.D. He stated rules for operations on negative numbers. European Mathematicians of 16th and 17th century did not accept the idea of negative numbers and referred them as absurd and fiction. John Wallis believed that negative numbers were greater than infinity.

II. The scientists believe that the lowest temperature attainable is about –273°C. At this temperature, the molecules and the atoms of a substance have the least possible energy.

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