CHAPTER – 27
SPECIFIC HEAT CAPACITIES OF GASES

1. N = 1 mole, W = 20 g/mol, V = 50 m/s

K.E. of the vessel = Internal energy of the gas

$= (1/2)\, mv^2 = (1/2) \times 20 \times 10^{-3} \times 50 \times 50 = 25$ J

$25 = n\frac{3}{2}r(\Delta T) \Rightarrow 25 = 1 \times \frac{3}{2} \times 8.31 \times \Delta T \Rightarrow \Delta T = \frac{50}{3 \times 8.3} \approx 2$ k.

2. m = 5 g, $\Delta t = 25 - 15 = 10°C$

$C_V = 0.172$ cal/g-°CJ = 4.2 J/Cal.

dQ = du + dw

Now, V = 0 (for a rigid body)

So, dw = 0.

So dQ = du.

Q = msdt = 5 × 0.172 × 10 = 8.6 cal = 8.6 × 4.2 = 36.12 Joule.

3. $\gamma = 1.4$, w or piston = 50 kg., A of piston = 100 cm^2

Po = 100 kpa, g = 10 m/s^2, x = 20 cm.

$$dw = pdv = \left(\frac{mg}{A} + Po\right)Adx = \left(\frac{50 \times 10}{100 \times 10^{-4}} + 10^5\right)100 \times 10^{-4} \times 20 \times 10^{-2} = 1.5 \times 10^5 \times 20 \times 10^{-4} = 300 \text{ J.}$$

$$nRdt = 300 \Rightarrow dT = \frac{300}{nR}$$

$$dQ = nCpdT = nCp \times \frac{300}{nR} = \frac{n\gamma R300}{(\gamma - 1)nR} = \frac{300 \times 1.4}{0.4} = 1050 \text{ J.}$$

4. $C_VH_2 = 2.4$ Cal/g°C, $C_PH^2 = 3.4$ Cal/g°C

M = 2 g/ Mol, $R = 8.3 \times 10^7$ erg/mol-°C

We know, $C_P - C_V = 1$ Cal/g°C

So, difference of molar specific heats

$= C_P \times M - C_V \times M = 1 \times 2 = 2$ Cal/g°C

Now, $2 \times J = R \Rightarrow 2 \times J = 8.3 \times 10^7$ erg/mol-°C $\Rightarrow J = 4.15 \times 10^7$ erg/cal.

5. $\frac{C_P}{C_V} = 7.6$, n = 1 mole, $\Delta T = 50K$

(a) Keeping the pressure constant, dQ = du + dw,

$\Delta T = 50$ K, $\gamma = 7/6$, m = 1 mole,

$dQ = du + dw \Rightarrow nC_VdT = du + RdT \Rightarrow du = nCpdT - RdT$

$$= 1 \times \frac{R\gamma}{\gamma - 1} \times dT - RdT = \frac{R \times \frac{7}{6}}{\frac{7}{6} - 1}dT - RdT$$

$= DT - RdT = 7RdT - RdT = 6\,RdT = 6 \times 8.3 \times 50 = 2490$ J.

(b) Kipping Volume constant, $dv = nC_VdT$

$$= 1 \times \frac{R}{\gamma - 1} \times dt = \frac{1 \times 8.3}{\frac{7}{6} - 1} \times 50$$

$= 8.3 \times 50 \times 6 = 2490$ J

(c) Adiabetically dQ = 0, du = – dw

$$= \left[\frac{n \times R}{\gamma - 1}(T_1 - T_2)\right] = \frac{1 \times 8.3}{\frac{7}{6} - 1}(T_2 - T_1) = 8.3 \times 50 \times 6 = 2490 \text{ J}$$

6. m = 1.18 g, $V = 1 \times 10^3\ cm^3 = 1$ L T = 300 k, $P = 10^5$ Pa

$PV = nRT$ or $n = \frac{PV}{RT} = 10^5 = atm.$

PV = 1 1 1

$N = \frac{PV}{RT} = \frac{1}{8.2\times10^{-2}\times3\times10^{2}} = \frac{1}{8.2\times3} = \frac{1}{24.6}$

Now, $C_v = \frac{1}{n}\times\frac{Q}{dt} = 24.6 \times 2 = 49.2$

$C_p = R + C_v = 1.987 + 49.2 = 51.187$

$Q = nC_pdT = \frac{1}{24.6}\times51.187\times1 = 2.08$ Cal.

7. $V_1 = 100\ cm^3$, $V_2 = 200\ cm^3$ $P = 2 \times 10^5$ Pa, $\Delta Q = 50J$

(a) $\Delta Q = du + dw \Rightarrow 50 = du + 20\times 10^5(200 - 100 \times 10^{-6}) \Rightarrow 50 = du + 20 \Rightarrow du = 30$ J

(b) $30 = n \times \frac{3}{2}\times 8.3 \times 300$ [$U = \frac{3}{2}nRT$ for monoatomic]

$\Rightarrow n = \frac{2}{3\times83} = \frac{2}{249} = 0.008$

(c) $du = nC_vdT \Rightarrow C_v = \frac{dndTu}{} = \frac{30}{0.008\times300} = 12.5$

$C_p = C_v + R = 12.5 + 8.3 = 20.3$

(d) $C_v = 12.5$ (Proved above)

8. Q = Amt of heat given

Work done = $\frac{Q}{2}$, $\Delta Q = W + \Delta U$

for monoatomic gas $\Rightarrow \Delta U = Q - \frac{Q}{2} = \frac{Q}{2}$

$V = n\frac{3}{2}RT = \frac{Q}{2} = nT\times \frac{3}{2}R = 3R \times nT$

Again Q = n CpdT Where C_P > Molar heat capacity at const. pressure.

$3RnT = ndTC_P \Rightarrow C_P = 3R$

9. $P = KV \Rightarrow \frac{nRT}{V} = KV \Rightarrow RT = KV^2 \Rightarrow R\,\Delta T = 2KV\,\Delta U \Rightarrow \frac{R\Delta T}{2KV} = dv$

$dQ = du + dw \Rightarrow mcdT = C_VdT + pdv \Rightarrow msdT = C_V\,dT + \frac{PRdF}{2KV}$

$\Rightarrow ms = C_V + \frac{RKV}{2KV} \Rightarrow C_P + \frac{R}{2}$

10. $\frac{C_P}{C_V} = \gamma$, $C_P - C_V = R$, $C_V = \frac{r}{\gamma-1}$, $C_P = \frac{\gamma R}{\gamma-1}$

$Pdv = \frac{1}{b+1}(Rdt)$

$\Rightarrow 0 = C_VdT + \frac{1}{b+1}(Rdt) \Rightarrow \frac{1}{b+1} = \frac{-C_V}{R}$

$\Rightarrow b + 1 = \frac{-R}{C_V} = \frac{-(C_P - C_V)}{C_V} = -\gamma + 1 \Rightarrow b = -\gamma$

11. Considering two gases, in Gas(1) we have,

γ, Cp_1 (Sp. Heat at const. 'P'), Cv_1 (Sp. Heat at const. 'V'), n_1 (No. of moles)

$\frac{Cp_1}{Cv_1} = \gamma$ & $Cp_1 - Cv_1 = R$

$\Rightarrow \gamma Cv_1 - Cv_1 = R \Rightarrow Cv_1(\gamma - 1) = R$

$\Rightarrow Cv_1 = \frac{R}{\gamma-1}$ & $Cp_1 = \frac{\gamma R}{\gamma-1}$

In Gas(2) we have, γ, Cp_2 (Sp. Heat at const. 'P'), Cv_2 (Sp. Heat at const. 'V'), n_2 (No. of moles)

$\frac{Cp_2}{Cv_2} = \gamma$ & $Cp_2 - Cv_2 = R \Rightarrow \gamma Cv_2 - Cv_2 = R \Rightarrow Cv_2(\gamma - 1) = R \Rightarrow Cv_2 = \frac{R}{\gamma-1}$ & $Cp_2 = \frac{\gamma R}{\gamma-1}$

Given $n_1 : n_2 = 1 : 2$

$dU_1 = nCv_1\,dT$ & $dU_2 = 2nCv_2\,dT = 3nCvdT$

$$\Rightarrow C_V = \frac{Cv_1 + 2Cv_2}{3} = \frac{\frac{R}{\gamma-1} + \frac{2R}{\gamma-1}}{3} = \frac{3R}{3(\gamma-1)} = \frac{R}{\gamma-1} \quad \ldots(1)$$

$$\&Cp = \gamma Cv = \frac{\gamma r}{\gamma-1} \quad \ldots(2)$$

So, $\frac{Cp}{Cv} = \gamma$ [from (1) & (2)]

12. $Cp' = 2.5\ R$ $Cp'' = 3.5\ R$

$Cv' = 1.5\ R$ $Cv'' = 2.5\ R$

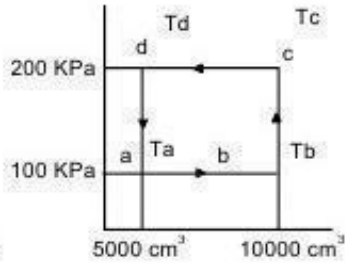
$n_1 = n_2 = 1$ mol $(n_1 + n_2)C_V dT = n_1\,C_v dT + n_2\,C_v dT$

$$\Rightarrow C_V = \frac{n_1 Cv' + n_2 Cv''}{n_1 + n_2} = \frac{1.5R + 2.5R}{2}\ 2R$$

$C_P = C_V + R = 2R + R = 3R$

$$\gamma = \frac{C_p}{C_V} = \frac{3R}{2R} = 1.5$$

13. $n = \frac{1}{2}$ mole, $R = \frac{25}{3}$ J/mol-k, $\gamma = \frac{5}{3}$



(a) Temp at A = T_a, $P_aV_a = nRT_a$

$$\Rightarrow T_a = \frac{P_aV_a}{nR} = \frac{5000\times10^{-6}\times100\times10^{3}}{\frac{1}{2}\times\frac{25}{3}} = 120\ k.$$

Similarly temperatures at point b = 240 k at C it is 480 k and at D it is 240 k.

(b) For ab process,

$dQ = nCpdT$ [since ab is isobaric]

$$= \frac{1}{2}\times\frac{R\gamma}{\gamma-1}(T_b - T_a) = \frac{1}{2}\times\frac{\frac{35}{3}\times\frac{5}{3}}{\frac{5}{3}-1}\times(240-120) = \frac{1}{2}\times\frac{125}{9}\times\frac{3}{2}\times120 = 1250\ J$$

For bc, $dQ = du + dw$ [dq = 0, Isochorie process]

$$\Rightarrow dQ = du = nC_v dT = \frac{nR}{\gamma-1}(T_c - T_a) = \frac{1}{2}\times\frac{\frac{25}{3}}{\left(\frac{5}{3}-1\right)}(240) = \frac{1}{2}\times\frac{25}{3}\times\frac{3}{2}\times240 = 1500\ J$$

(c) Heat liberated in cd = $-nC_pdT$

$$= \frac{-1}{2}\times\frac{nR}{\gamma-1}(T_d - T_c) = \frac{-1}{2}\times\frac{125}{3}\times\frac{3}{2}\times240 = 2500\ J$$

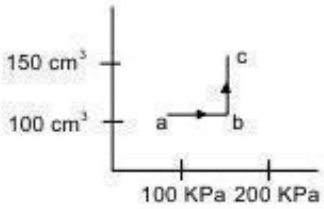
Heat liberated in da = $-nC_vdT$

$$= \frac{-1}{2}\times\frac{R}{\gamma-1}(T_a - T_d) = \frac{-1}{2}\times\frac{25}{2}\times(120-240) = 750\ J$$

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14. (a) For a, b 'V' is constant

So, $\frac{P_1}{T_1} = \frac{P_2}{T_2} \Rightarrow \frac{100}{300} = \frac{200}{T_2} \Rightarrow T_2 = \frac{200\times300}{100} = 600\ k$



For b,c 'P' is constant

So, $\frac{V_1}{T_1} = \frac{V_2}{T_2} \Rightarrow \frac{100}{600} = \frac{150}{T_2} \Rightarrow T_2 = \frac{600\times150}{100} = 900\ k$

(b) Work done = Area enclosed under the graph 50 cc × 200 kpa = $50 \times 10^{-6} \times 200 \times 10^{3}$ J = 10 J

(c) 'Q' Supplied = nC_vdT

Now, $n = \frac{PV}{RT}$ considering at pt. 'b'

$C_v = \frac{R}{\quad}dT$ = 300 a. b

$\frac{}{\gamma-1}$

$Q_{bc} = \frac{PV}{RT} \times \frac{R}{\gamma-1} dT = \frac{200\times10^3\times100\times10^{-6}}{600\times0.67} \times 300 = 14.925 \qquad (\because \gamma = 1.67)$

Q supplied to be $nC_p dT \qquad [\because C_p = \frac{\gamma R}{\gamma-1}]$

$= \frac{PV}{RT} \times \frac{\gamma R}{\gamma-1} dT = \frac{200\times10^3\times150\times10^{-6}}{8.3\times900} \times \frac{1.67\times8.3}{0.67} \times 300 = 24.925$

(d) $Q = \Delta U + w$

Now, $\Delta U = Q - w$ = Heat supplied – Work done = (24.925 + 14.925) – 1 = 29.850

15. In Joly's differential steam calorimeter

$C_v = \frac{m_2 L}{m_1(\theta_2 - \theta_1)}$

m_2 = Mass of steam condensed = 0.095 g, L = 540 Cal/g = 540 × 4.2 J/g

m_1 = Mass of gas present = 3 g, $\theta_1 = 20°C$, $\theta_2 = 100°C$

$\Rightarrow C_v = \frac{0.095\times540\times4.2}{3(100-20)} = 0.89 \approx 0.9$ J/g-K

16. $\gamma = 1.5$

Since it is an adiabatic process, So PV^γ = const.

(a) $P_1V_1^\gamma = P_2V_2^\gamma$ Given $V_1 = 4$ L, $V_2 = 3$ L, $\frac{P_2}{P_1} = ?$

$\Rightarrow \frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^\gamma = \left(\frac{4}{3}\right)^{1.5} = 1.5396 \approx 1.54$

(b) $TV^{\gamma-1}$ = Const.

$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1} \Rightarrow \frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = \left(\frac{4}{3}\right)^{0.5} = 1.154$

17. $P_1 = 2.5 \times 10^5$ Pa, $V_1 = 100$ cc, $T_1 = 300$ k

(a) $P_1V_1^\gamma = P_2V_2^\gamma$

$\Rightarrow 2.5 \times 10^5 \times V^{1.5} = \left(\frac{V}{2}\right)^{1.5} \times P_2$

$\Rightarrow P_2 = 2^{1.5} \times 2.5 \times 10^5 = 7.07 \times 10^5 \approx 7.1 \times 10^5$

(b) $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1} \Rightarrow 300 \times (100)^{1.5-1} = T_2 \times (50)^{1.5-1}$

$\Rightarrow T_2 = \frac{3000}{7.07} = 424.32$ k ≈ 424 k

(c) Work done by the gas in the process

$W = \frac{mR}{\gamma-1}[T_2 - T_1] = \frac{P_1V_1}{T(\gamma-1)}[T_2 - T_1]$

$= \frac{2.5\times10^5\times100\times10^{-6}}{300(1.5-1)}[424-300] = \frac{2.5\times10}{300\times0.5} \times 124 = 20.72 \approx 21$ J

18. $\gamma = 1.4$, $T_1 = 20°C = 293$ k, $P_1 = 2$ atm, $p_2 = 1$ atm

We know for adiabatic process,

$P_1^{1-\gamma} \times T_1^\gamma = P_2^{1-\gamma} \times T_2^\gamma$ or $(2)^{1-1.4} \times (293)^{1.4} = (1)^{1-1.4} \times T_2^{1.4}$

$\Rightarrow (2)^{0.4} \times (293)^{1.4} = T_2^{1.4} \Rightarrow 2153.78 = T_2^{1.4} \Rightarrow T_2 = (2153.78)^{1/1.4} = 240.3$ K

19. $P_1 = 100$ KPa $= 10^5$ Pa, $V_1 = 400$ cm^3 $= 400 \times 10^{-6}$ m^3, $T_1 = 300$ k,

$\gamma = \frac{C_P}{C_V} = 1.5$

(a) Suddenly compressed to $V_2 = 100$ cm^3

$P_1V_1^\gamma = P_2V_2^\gamma \Rightarrow 10^5 (400)^{1.5} = P_2 \times (100)^{1.5}$

$\Rightarrow P_2 = 10^5 \times (4)^{1.5} = 800$ KPa

$\frac{300\times20}{}$

$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1} \Rightarrow 300 \times (400)^{1.5-1} = T_2 \times (100)^{1.5-1} \Rightarrow T_2 = \frac{300 \times 20}{10} = 600\ K$

(b) Even if the container is slowly compressed the walls are adiabatic so heat transferred is 0.
Thus the values remain, $P_2 = 800$ KPa, $T_2 = 600$ K.

20. Given $\frac{C_P}{C_V} = \gamma$ P_0 (Initial Pressure), V_0 (Initial Volume)

(a) (i) Isothermal compression, $P_1V_1 = P_2V_2$ or, $P_0V_0 = \frac{P_2V_0}{2} \Rightarrow P_2 = 2P_0$

(ii) Adiabatic Compression $P_1V_1^{\gamma} = P_2V_2^{\gamma}$ or $2P_0\left(\frac{V_0}{2}\right)^{\gamma} = P1\left(\frac{V_0}{4}\right)^{\gamma}$

$\Rightarrow P' = \frac{V_o^{\gamma}}{2^{\gamma}} \times 2P_0 \times \frac{4^{\gamma}}{V_0^{\gamma}} = 2^{\gamma} \times 2\ P_0 \Rightarrow P_0 2^{\gamma+1}$

(b) (i) Adiabatic compression $P_1V_1^{\gamma} = P_2V_2^{\gamma}$ or $P_0V_0^{\gamma} = P'\left(\frac{V_0}{2}\right)^{\gamma} \Rightarrow P' = P_0 2^{\gamma}$

(ii) Isothermal compression $P_1V_1 = P_2V_2$ or $2^{\gamma}\ P_0 \times \frac{V_0}{2} = P_2 \times \frac{V_0}{4} \Rightarrow P_2 = P_0 2^{\gamma+1}$

21. Initial pressure = P_0
Initial Volume = V_0
$\gamma = \frac{C_P}{C_V}$

(a) Isothermally to pressure $\frac{P_0}{2}$

$P_0V_0 = \frac{P_0}{2}V_1 \Rightarrow V_1 = 2\ V_0$

Adiabetically to pressure = $\frac{P_0}{4}$

$\frac{P_0}{2}(V_1)^{\gamma} = \frac{P_0}{4}(V_2)^{\gamma} \Rightarrow \frac{P_0}{2}(2V_0)^{\gamma} = \frac{P_0}{4}(V_2)^{\gamma}$
$\Rightarrow 2^{\gamma+1}\ V_0^{\gamma} = V_2^{\gamma} \Rightarrow V_2 = 2^{(\gamma+1)/\gamma}\ V_0$
$\therefore$ Final Volume = $2^{(\gamma+1)/\gamma}\ V_0$

(b) Adiabetically to pressure $\frac{P_0}{2}$ to P_0

$P_0 \times (2^{\gamma+1}\ V_0^{\gamma}) = \frac{P_0}{2} \times (V')^{\gamma}$

Isothermal to pressure $\frac{P_0}{4}$

$\frac{P_0}{2} \times 2^{1/\gamma} V_0 = \frac{P_0}{4} \times V'' \Rightarrow V'' = 2^{(\gamma+1)/\gamma}\ V_0$

$\therefore$ Final Volume = $2^{(\gamma+1)/\gamma}\ V_0$

22. $PV = nRT$
Given $P = 150$ KPa $= 150 \times 10^3$ Pa, $V = 150\ cm^3 = 150 \times 10^{-6}\ m^3$, $T = 300$ k

(a) $n = \frac{PV}{RT} = \frac{150 \times 10^3 \times 150 \times 10^{-6}}{8.3 \times 300} = 9.036 \times 10^{-3} = 0.009$ moles.

(b) $\frac{C_P}{C_V} = \gamma \Rightarrow \frac{\gamma R}{(\gamma-1)C_V} = \gamma$ $\left[\therefore C_P = \frac{\gamma R}{\gamma-1}\right]$

$\Rightarrow C_V = \frac{R}{\gamma-1} = \frac{8.3}{1.5-1} = \frac{8.3}{0.5} = 2R = 16.6$ J/mole

(c) Given $P_1 = 150$ KPa $= 150 \times 10^3$ Pa, $P_2 = ?$
$V_1 = 150\ cm^3 = 150 \times 10^{-6}\ m^3$, $\gamma = 1.5$
$V_2 = 50\ cm^3 = 50 \times 10^{-6}\ m^3$, $T_1 = 300$ k, $T_2 = ?$

Since the process is adiabatic Hence – $P_1V_1^{\gamma} = P_2V_2^{\gamma}$

$\Rightarrow 150\times 10^3 (150 \times 10^{-6})^{\gamma} = P_2 \times (50 \times 10^{-6})^{\gamma}$

$\Rightarrow P_2 = 150 \times 10^3 \times \left(\frac{150\times10^{-6}}{50\times10^{-6}}\right)^{1.5} = 150000 \times 3^{1.5} = 779.422 \times 10^3 \text{ Pa} \approx 780 \text{ KPa}$

(d) $\Delta Q = W + \Delta U$ or $W = -\Delta U$ $[\because \Delta U = 0$, in adiabatic$]$

$= -nC_vdT = -0.009 \times 16.6 \times (520 - 300) = -0.009 \times 16.6 \times 220 = -32.8 \text{ J} \approx -33 \text{ J}$

(e) $\Delta U = nC_vdT = 0.009 \times 16.6 \times 220 \approx 33$ J

23. $V_A = V_B = V_C$

For A, the process is isothermal

$P_AV_A = P_A'V_A' \Rightarrow P_A' = P_A \frac{V_A}{V_A'} = P_A \times \frac{1}{2}$

For B, the process is adiabatic,

$P_A(V_B)^{\gamma} = P_A'(V_B)^{\gamma} = P_B' = P_B\left(\frac{V_B}{V_B'}\right)^{\gamma} = P_B \times \left(\frac{1}{2}\right)^{1.5} = \frac{P_B}{2^{1.5}}$

For, C, the process is isobaric

$\frac{V_C}{T_C} = \frac{V_C'}{T_C'} \Rightarrow \frac{V_C}{T_C} = \frac{2V_C'}{T_C'} \Rightarrow T_C' = \frac{2}{T_C}$

Final pressures are equal.

$= \frac{p_A}{2} = \frac{P_B}{2^{1.5}} = P_C \Rightarrow P_A : P_B : P_C = 2 : 2^{1.5} : 1 = 2 : 2\sqrt{2} : 1$

24. P_1 = Initial Pressure V_1 = Initial Volume P_2 = Final Pressure V_2 = Final Volume

Given, $V_2 = 2V_1$, Isothermal workdone = $nRT_1 \text{ Ln}\left(\frac{V_2}{V_1}\right)$

Adiabatic workdone = $\frac{P_1V_1 - P_2V_2}{\gamma - 1}$

Given that workdone in both cases is same.

Hence $nRT_1 \text{ Ln}\left(\frac{V_2}{V_1}\right) = \frac{P_1V_1 - P_2V_2}{\gamma - 1} \Rightarrow (\gamma - 1) \ln\left(\frac{V_2}{V_1}\right) = \frac{P_1V_1 - P_2V_2}{nRT_1}$

$\Rightarrow (\gamma - 1)\ln\left(\frac{V_2}{V_1}\right) = \frac{nRT_1 - nRT_2}{nRT_1} \Rightarrow (\gamma - 1)\ln 2 = \frac{T_1 - T_1}{T_1}$ ...(i) $[\because V_2 = 2V_1]$

We know $TV^{\gamma-1}$ = const. in adiabatic Process.

$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$, or $T_1(V_2)^{\gamma-1} = T_2 \times (2)^{\gamma-1} \times (V_1)^{\gamma-1}$

Or, $T_1 = 2^{\gamma-1} \times T_2$ or $T_2 = T_1^{1-\gamma}$ ...(ii)

From (i) & (ii)

$(\gamma - 1)\ln 2 = \frac{T_1 - T_1 \times 2^{1-\gamma}}{T_1} \Rightarrow (\gamma - 1)\ln 2 = 1 - 2^{1-\gamma}$

25. $\gamma = 1.5$, $T = 300$ k, $V = 1\text{Lv} = \frac{1}{2}$ l

(a) The process is adiabatic as it is sudden,

$P_1V_1^{\gamma} = P_2V_2^{\gamma} \Rightarrow P_1(V_0)^{\gamma} = P_2\left(\frac{V_0}{2}\right)^{\gamma} \Rightarrow P_2 = P_1\left(\frac{1}{1/2}\right)^{1.5} = P_1(2)^{1.5} \Rightarrow \frac{P_2}{P_1} = 2^{1.5} = 2\sqrt{2}$

(b) $P_1 = 100 \text{ KPa} = 10^5 \text{ Pa}$ $W = \frac{nR}{\gamma - 1}[T_1 - T_2]$

$T_1V_1^{\gamma-1} = P_2V_2^{\gamma-1} \Rightarrow 300 \times (1)^{1.5-1} = T_2(0.5)^{1.5-1} \Rightarrow 300 \times 1 = T_2 \sqrt{0.5}$

$T_2 = 300 \times \sqrt{\frac{1}{0.5}} = 300\sqrt{2}$ K

$P_1 V_1 = nRT_1 \Rightarrow n = \frac{P_1V_1}{RT_1} = \frac{10^5 \times 10^{-3}}{R \times 300} = \frac{1}{3R}$ (V in m^3)

$w = \frac{nR}{\gamma - 1}[T_1 - T_2] = \frac{1R}{3R(1.5-1)}\left[300 - 300\sqrt{2}\right] = \frac{300}{3 \times 0.5}\left(1 - \sqrt{2}\right) = -82.8\ J \approx -82\ J.$

(c) Internal Energy,

$dQ = 0, \quad \Rightarrow du = -dw = -(-82.8)J = 82.8\ J \approx 82\ J.$

(d) Final Temp = $300\sqrt{2} = 300 \times 1.414 \times 100 = 424.2\ k \approx 424\ k.$

(e) The pressure is kept constant. ∴ The process is isobaric.

Work done = $nRdT = \frac{1}{3R} \times R \times (300 - 300\sqrt{2})$ Final Temp = 300 K

$= -\frac{1}{3} \times 300\ (0.414) = -41.4\ J.$ Initial Temp = $300\sqrt{2}$

(f) Initial volume $\Rightarrow \frac{V_1}{T_1} = \frac{V_1'}{T_1'} = V_1' = \frac{V_1}{T_1} \times T_1' = \frac{1}{2 \times 300 \times \sqrt{2}} \times 300 = \frac{1}{2\sqrt{2}} L.$

Final volume = 1L

Work done in isothermal = $nRT \ln \frac{V_2}{V_1}$

$= \frac{1}{3R} \times R \times 300 \ln\left(\frac{1}{1/2\sqrt{2}}\right) = 100 \times \ln\left(2\sqrt{2}\right) = 100 \times 1.039 \approx 103$

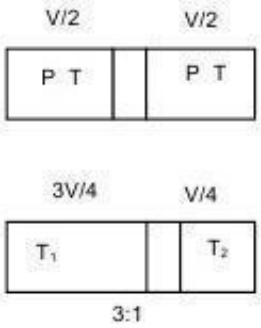
(g) Net work done = $W_A + W_B + W_C = -82 - 41.4 + 103 = -20.4\ J.$

26. Given $\gamma = 1.5$

We know fro adiabatic process $TV^{\gamma-1} = Const.$

So, $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$ ...(eq)

As, it is an adiabatic process and all the other conditions are same. Hence the above equation can be applied.



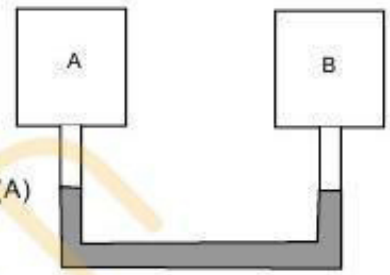
So, $T_1 \times \left(\frac{3V}{4}\right)^{1.5-1} = T_2 \times \left(\frac{V}{4}\right)^{1.5-1} \Rightarrow T_1 \times \left(\frac{3V}{4}\right)^{0.5} = T_2 \times \left(\frac{V}{4}\right)^{0.5}$

$\Rightarrow \frac{T_1}{T_2} = \left(\frac{V}{4}\right)^{0.5} \times \left(\frac{4}{3V}\right)^{0.5} = \frac{1}{\sqrt{3}}$ So, $T_1 : T_2 = 1 : \sqrt{3}$

27. $V = 200\ cm^3, \quad C = 12.5\ J/mol\text{-}k, \quad T = 300\ k, \quad P = 75\ cm$

(a) No. of moles of gas in each vessel,

$\frac{PV}{RT} = \frac{75 \times 13.6 \times 980 \times 200}{8.3 \times 10^7 \times 300} = 0.008$



(b) Heat is supplied to the gas but dv = 0

$dQ = du \Rightarrow 5 = nC_V dT \Rightarrow 5 = 0.008 \times 12.5 \times dT \Rightarrow dT = \frac{5}{0.008 \times 12.5}$ for (A)

For (B) $dT = \frac{10}{0.008 \times 12.5}$ $\because \frac{P}{T} = \frac{P_A}{T_A}$ [For container A]

$\Rightarrow \frac{75}{300} = \frac{P_A \times 0.008 \times 12.5}{5} \Rightarrow P_A = \frac{75 \times 5}{300 \times 0.008 \times 12.5} = 12.5$ cm of Hg.

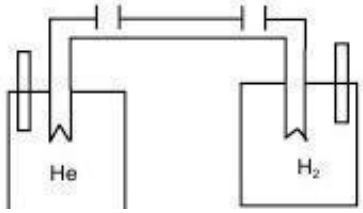
$\because \frac{P}{T} = \frac{P_B}{T_B}$ [For Container B] $\Rightarrow \frac{75}{300} = \frac{P_B \times 0.008 \times 12.5}{10} \Rightarrow P_B = 2\ P_A = 25$ cm of Hg.

Mercury moves by a distance $P_B - P_A = 25 - 12.5 = 12.5$ Cm.

28. $mHe = 0.1\ g, \quad \gamma = 1.67, \quad \mu = 4\ g/mol, \quad mH_2 = ?$

$\mu = 28/mol \ \gamma_2 = 1.4$

Since it is an adiabatic surrounding



He $dQ = nC_V dT = \frac{0.1}{4} \times \frac{R}{\gamma - 1} \times dT = \frac{0.1}{4} \times \frac{R}{(1.67-1)} \times dT$ ...(i)

$\frac{m}{} \quad \frac{R}{} \quad \frac{m}{} \quad \frac{R}{}$

$H_2 = nC_V dT = \frac{m}{2} \times \frac{R}{\gamma - 1} \times dT = \frac{m}{2} \times \frac{R}{1.4 - 1} \times dT$ [Where m is the rqd. Mass of H_2]

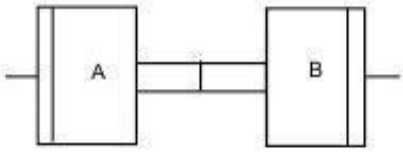
Since equal amount of heat is given to both and ΔT is same in both.

Equating (i) & (ii) we get

$$\frac{0.1}{4} \times \frac{R}{0.67} \times dT = \frac{m}{2} \times \frac{R}{0.4} \times dT \Rightarrow m = \frac{0.1}{2} \times \frac{0.4}{0.67} = 0.0298 \approx 0.03 \text{ g}$$

29. Initial pressure = P_0, Initial Temperature = T_0

Initial Volume = V_0



$$\frac{C_P}{C_V} = \gamma$$

(a) For the diathermic vessel the temperature inside remains constant

$P_1 V_1 - P_2 V_2 \Rightarrow P_0 V_0 = P_2 \times 2V_0 \Rightarrow P_2 = \frac{P_0}{2}$, Temperature = T_o

For adiabatic vessel the temperature does not remains constant. The process is adiabatic

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1} \Rightarrow T_0 V_0^{\gamma-1} = T_2 \times (2V_0)^{\gamma-1} \Rightarrow T_2 = T_0 \left(\frac{V_0}{2V_0}\right)^{\gamma-1} = T_0 \times \left(\frac{1}{2}\right)^{\gamma-1} = \frac{T_0}{2^{\gamma-1}}$$

$$P_1 V_1^{\gamma} = P_2 V_2^{\gamma} \Rightarrow P_0 V_0^{\gamma} = p_1 (2V_0)^{\gamma} \Rightarrow P_1 = P_0 \left(\frac{V_0}{2V_0}\right)^{\gamma} = \frac{P_0}{2^{\gamma}}$$

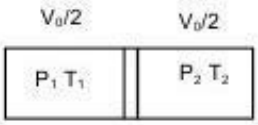
(b) When the values are opened, the temperature remains T_0 through out

$P_1 = \frac{n_1 R T_0}{4V_0}$, $P_2 = \frac{n_2 R T_0}{4V_0}$ [Total value after the expt = $2V_0 + 2V_0 = 4V_0$]

$$P = P_1 + P_2 = \frac{(n_1 + n_2) R T_0}{4V_0} = \frac{2nRT_0}{4V_0} = \frac{nRT_0}{2V} = \frac{P_0}{2}$$

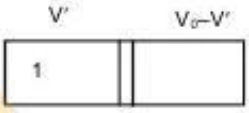
30. For an adiabatic process, Pv^{γ} = Const.

There will be a common pressure 'P' when the equilibrium is reached



Hence $P_1 \left(\frac{V_0}{2}\right)^{\gamma} = P(V')^{\gamma}$

For left $P = P_1 \left(\frac{V_0}{2}\right)^{\gamma} (V')^{\gamma}$ ...(1)



For Right $P = P_2 \left(\frac{V_0}{2}\right)^{\gamma} (V_0 - V')^{\gamma}$ ...(2)

Equating 'P' for both left & right

$$= \frac{P_1}{(V')^{\gamma}} = \frac{P_2}{(V_0 - V')^{\gamma}} \text{ or } \frac{V_0 - V'}{V'} = \left(\frac{P_2}{P_1}\right)^{1/\gamma}$$

$$\Rightarrow \frac{V_0}{V'} - 1 = \frac{P_2^{1/\gamma}}{P_1^{1/\gamma}} \Rightarrow \frac{V_0}{V'} = \frac{P_2^{1/\gamma} + P_1^{1/\gamma}}{P_1^{1/\gamma}} \Rightarrow V' = \frac{V_0 P_1^{1/\gamma}}{P_1^{1/\gamma} + P_2^{1/\gamma}}$$ For left .....(3)

Similarly $V_0 - V' = \frac{V_0 P_2^{1/\gamma}}{P_1^{1/\gamma} + P_2^{1/\gamma}}$ For right .....(4)

(b) Since the whole process takes place in adiabatic surroundings. The separator is adiabatic. Hence heat given to the gas in the left part = Zero.

(c) From (1) Final pressure $P = \frac{P_1 \left(\frac{V_0}{2}\right)^{\gamma}}{(V')^{\gamma}}$

Again from (3) $V' = \frac{V_0 P_1^{1/\gamma}}{P_1^{1/\gamma} + P_2^{1/\gamma}}$ or $P = \frac{P_1 (V_0)^{\gamma}}{2^{\gamma}} \times \frac{\left(P_1^{1/\gamma} + P_2^{1/\gamma}\right)^{\gamma}}{(V_0)^{\gamma} P_1} = \left(\frac{P_1^{1/\gamma} + P_2^{1/\gamma}}{2}\right)^{\gamma}$

$$\left(\overline{P_1^{1/\gamma} + P_2^{1/\gamma}}\right)$$

31. $A = 1\ cm^2 = 1 \times 10^{-4}\ m^2$, $M = 0.03\ g = 0.03 \times 10^{-3}\ kg$,
$P = 1\ atm = 10^5$ pascal, $L = 40\ cm = 0.4\ m$.
$L_1 = 80\ cm = 0.8\ m$, $P = 0.355\ atm$
The process is adiabatic

$$P(V)^{\gamma} = P(V')^{\gamma} = \Rightarrow 1 \times (AL)^{\gamma} = 0.355 \times (A2L)^{\gamma} \Rightarrow 1\ \ 1 = 0.355\ \ 2^{\gamma} \Rightarrow \frac{1}{0.355} = 2^{\gamma}$$

$$= \gamma \log 2 = \log\left(\frac{1}{0.355}\right) = 1.4941$$

$$V = \sqrt{\frac{\gamma P}{f}} = \sqrt{\frac{1.4941 \times 10^5}{m/v}} = \sqrt{\frac{1.4941 \times 10^5}{\left(\frac{0.03 \times 10^{-3}}{10^{-4} \times 1 \times 0.4}\right)}} = \sqrt{\frac{1.441 \times 10^5 \times 4 \times 10^{-5}}{3 \times 10^{-5}}} = 446.33 \approx 447\ m/s$$

32. $V = 1280\ m/s$, $T = 0°C$, $foH_2 = 0.089\ kg/m^3$, $rR = 8.3\ J/mol\text{-}k$,
At STP, $P = 10^5\ Pa$, We know

$$V_{sound} = \sqrt{\frac{\gamma P}{fo}} \Rightarrow 1280 = \sqrt{\frac{\gamma \times 10^5}{0.089}} \Rightarrow (1280)^2 = \frac{\gamma \times 10^5}{0.089} \Rightarrow \gamma = \frac{0.089 \times (1280)^2}{10^5} \approx 1.458$$

Again,

$$C_V = \frac{R}{\gamma - 1} = \frac{8.3}{1.458 - 1} = 18.1\ J/mol\text{-}k$$

Again, $\frac{C_P}{C_V} = \gamma$ or $C_P = \gamma C_V = 1.458 \times 18.1 = 26.3\ J/mol\text{-}k$

33. $\mu = 4g = 4 \times 10^{-3}\ kg$, $V = 22400\ cm^3 = 22400 \times 10^{-6}\ m^3$
$C_P = 5\ cal/mol\text{-}ki = 5 \times 4.2\ J/mol\text{-}k = 21\ J/mol\text{-}k$

$$C_P = \frac{\gamma R}{\gamma - 1} = \frac{\gamma \times 8.3}{\gamma - 1}$$

$$\Rightarrow 21(\gamma - 1) = \gamma\ (8.3) \Rightarrow 21\ \gamma - 21 = 8.3\ \gamma \Rightarrow \gamma = \frac{21}{12.7}$$

Since the condition is STP, $P = 1\ atm = 10^5\ pa$

$$V = \sqrt{\frac{\gamma f}{f}} = \sqrt{\frac{\frac{21}{12.7} \times 10^5}{\frac{4 \times 10^{-3}}{22400 \times 10^{-6}}}} = \sqrt{\frac{21 \times 10^5 \times 22400 \times 10^{-6}}{12.7 \times 4 \times 10^{-3}}} = 962.28\ m/s$$

34. Given $fo = 1.7 \times 10^{-3}\ g/cm^3 = 1.7\ kg/m^3$, $P = 1.5 \times 10^5\ Pa$, $R = 8.3\ J/mol\text{-}k$,
$f = 3.0\ KHz$.

Node separation in a Kundt'' tube $= \frac{\lambda}{2} = 6\ cm, \Rightarrow \lambda = 12\ cm = 12 \times 10^{-3}\ m$

So, $V = f\lambda = 3 \times 10^3 \times 12 \times 10^{-2} = 360\ m/s$

We know, Speed of sound $= \sqrt{\frac{\gamma P}{fo}} \Rightarrow (360)^2 = \frac{\gamma \times 1.5 \times 10^5}{1.7} \Rightarrow \gamma = \frac{(360)^2 \times 1.7}{1.5 \times 10^5} = 1.4688$

But $C_V = \frac{R}{\gamma - 1} = \frac{8.3}{1.488 - 1} = 17.72\ J/mol\text{-}k$

Again $\frac{C_P}{C_V} = \gamma$ So, $C_P = \gamma C_V = 17.72 \times 1.468 = 26.01 \approx 26\ J/mol\text{-}k$

35. $f = 5 \times 10^3\ Hz$, $T = 300\ Hz$, $\frac{\lambda}{2} = 3.3\ cm \Rightarrow \lambda = 6.6 \times 10^{-2}\ m$

$V = f\lambda = 5 \times 10^3 \times 6.6 \times 10^{-2} = (66 \times 5)\ m/s$

$V = \frac{\lambda P}{f}$ [$Pv = nRT \Rightarrow P = \frac{m}{mV} \times Rt \Rightarrow PM = foRT \Rightarrow \frac{P}{fo} = \frac{RT}{m}$]

$$= \sqrt{\frac{\gamma RT}{m}}(66\times 5) = \sqrt{\frac{\gamma\times 8.3\times 300}{32\times 10^{-3}}} \Rightarrow (66\times 5)^2 = \frac{\gamma\times 8.3\times 300}{32\times 10^{-3}} \Rightarrow \gamma = \frac{(66\times 5)^2\times 32\times 10^{-3}}{8.3\times 300} = 1.3995$$

$$C_v = \frac{R}{\gamma - 1} = \frac{8.3}{0.3995} = 20.7 \text{ J/mol-k},$$

$C_P = C_V + R = 20.77 + 8.3 = 29.07$ J/mol-k.