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Ch-11
Trigonometric Identities
Exercise 11.1

Q.1 $(1-\cos^2 A)\operatorname{cosec}^2 A = 1$

LHS $= (1-\cos^2 A)\operatorname{cosec}^2 A$ $\quad (1-\cos^2 A = \sin^2 A)$

$= \sin^2 A \operatorname{cosec}^2 A$

$= \boxed{1} = \text{RHS}$

Hence, proved

Q.2 $(1+\cot^2 A)\sin^2 A = 1$

LHS $= (1+\cot^2 A)\sin^2 A$ $\quad (1+\cot^2 A = \operatorname{cosec}^2 A)$

$= \operatorname{cosec}^2 A \sin^2 A$

$= \boxed{1} = \text{RHS}$

Hence, proved

Q.3 $\tan^2\theta \cos^2\theta = 1-\cos^2\theta$

LHS $= \tan^2\theta \cos^2\theta$

$= \sin^2\theta \cos^2\theta$

$\cos^2\theta$

$= \boxed{1-\cos^2\theta}$ $\quad (1-\cos^2\theta = \sin^2\theta)$

$= \text{RHS}$

Hence, proved

Q.4 $\operatorname{cosec}\theta \sqrt{1-\cos^2\theta} = 1$

LHS $= \operatorname{cosec}\theta \sqrt{1-\cos^2\theta}$

$= \operatorname{cosec}\theta \sqrt{\sin^2\theta}$ $\quad (1-\cos^2\theta = \sin^2\theta)$

$= \operatorname{cosec}\theta \sin\theta$

$= \boxed{1} = \text{RHS}$

Hence, proved

Q.5 $(\sec^2\theta - 1)(\operatorname{cosec}^2\theta - 1) = 1$

LHS $= (\sec^2\theta - 1)(\operatorname{cosec}^2\theta - 1)$ $\left| \begin{array}{l} \sec^2\theta - 1 = \tan^2\theta \\ \operatorname{cosec}^2\theta - 1 = \cot^2\theta \end{array} \right.$

$= \tan^2\theta \cot^2\theta$

$= \boxed{1} = \text{RHS}$

Hence, proved

Q.6 $\tan\theta + \dfrac{1}{\tan\theta} = \sec\theta \operatorname{cosec}\theta$

LHS $= \tan\theta + \dfrac{1}{\tan\theta}$

$= \dfrac{\sin\theta}{\cos\theta} + \dfrac{1}{\frac{\sin\theta}{\cos\theta}}$

$= \dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}$

$= \dfrac{\sin^2\theta + \cos^2\theta}{\cos\theta \sin\theta}$ $\left(\sin^2\theta + \cos^2\theta = 1\right)$

$= \dfrac{1}{\cos\theta \sin\theta} = \dfrac{1}{\cos\theta} \times \dfrac{1}{\sin\theta}$

$= \boxed{\sec\theta \operatorname{cosec}\theta} = \text{RHS}$

Hence, proved

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Q.13 $\frac{\sin\theta}{1-\cos\theta} = \operatorname{cosec}\theta + \cot\theta$

LHS $= \frac{\sin\theta}{1-\cos\theta}$

$= \frac{\sin\theta(1+\cos\theta)}{1-\cos\theta(1+\cos\theta)}$

$= \frac{\sin\theta + \sin\theta\cos\theta}{1-\cos^2\theta}$

$= \frac{\sin\theta + \sin\theta\cos\theta}{\sin^2\theta}$ $\left(1-\cos^2\theta = \sin^2\theta\right)$

$= \frac{1+\cos\theta}{\sin\theta}$

$= \frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta} = \boxed{\operatorname{cosec}\theta + \cot\theta} = \text{RHS}$

Hence, proved

Q.14 $\frac{1-\sin\theta}{1+\sin\theta} = (\sec\theta - \tan\theta)^2$

LHS $= \frac{1-\sin\theta}{1+\sin\theta}$

$= \frac{(1-\sin\theta)(1-\sin\theta)}{(1+\sin\theta)(1-\sin\theta)} = \frac{(1-\sin\theta)^2}{1-\sin^2\theta}$

$= \frac{(1-\sin\theta)^2}{\cos^2\theta} = \frac{1}{\cos^2\theta}$ $\left(1-\sin^2\theta = \cos^2\theta\right)$

$= \left(\frac{1}{\cos^2\theta} - \frac{\sin\theta}{\cos^2\theta}\right)^2 = \left(\frac{1-\sin\theta}{\cos\theta}\right)^2$

$= \left(\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}\right)^2 = \boxed{(\sec\theta - \tan\theta)^2} = \text{RHS}$

Hence, proved

Q.15 $\frac{(1+\cot^2\theta)\tan\theta}{\sec^2\theta} = \cot\theta$

LHS $= \frac{(1+\cot^2\theta)\tan\theta}{\sec^2\theta}$

$= \frac{\operatorname{cosec}^2\theta \tan\theta}{\sec^2\theta}$ $\quad (1+\cot^2\theta = \operatorname{cosec}^2\theta)$

$= \frac{1}{\sin^2\theta} \cdot \frac{\sin\theta}{\cos\theta} = \frac{\cos\theta \sin\theta}{\sin^2\theta \cos\theta}$

$\frac{1}{\cos^2\theta}$

$= \frac{\cos\theta}{\sin\theta} = \boxed{\cot\theta} = \text{RHS}$

Hence, proved

Q.16 $\tan^2\theta - \sin^2\theta = \tan^2\theta \sin^2\theta$

LHS $= \tan^2\theta - \sin^2\theta$

$= \frac{\sin^2\theta}{\cos^2\theta} - \frac{\sin^2\theta}{1}$

$= \frac{\sin^2\theta - \sin^2\theta\cos^2\theta}{\cos^2\theta}$

$= \frac{\sin^2\theta(1-\cos^2\theta)}{\cos^2\theta}$

$= \sin^2\theta \times \frac{\sin^2\theta}{\cos^2\theta}$ $\quad (1-\cos^2\theta = \sin^2\theta)$

$= \boxed{\sin^2\theta \tan^2\theta} = \text{RHS}$

Hence proved

Q.17 $(\sec\theta + \cos\theta)(\sec\theta - \cos\theta) = \tan^2\theta + \sin^2\theta$

LHS $= (\sec\theta + \cos\theta)(\sec\theta - \cos\theta)$

$= \sec^2\theta - \cos^2\theta$

$= (1 + \tan^2\theta) - (1 - \sin^2\theta)$ | $\sec^2\theta = 1 + \tan^2\theta$, $\cos^2\theta = 1 - \sin^2\theta$

$= 1 + \tan^2\theta - 1 + \sin^2\theta$

$= \boxed{\tan^2\theta + \sin^2\theta}$ = RHS

Hence, proved

Q.18 $(\operatorname{cosec}\theta + \sin\theta)(\operatorname{cosec}\theta - \sin\theta) = \cot^2\theta + \cos^2\theta$

LHS $= (\operatorname{cosec}\theta + \sin\theta)(\operatorname{cosec}\theta - \sin\theta)$

$= \operatorname{cosec}^2\theta - \sin^2\theta$

$= (1 + \cot^2\theta) - (1 - \cos^2\theta)$ | $\operatorname{cosec}^2\theta = 1 + \cot^2\theta$, $\sin^2\theta = 1 - \cos^2\theta$

$= 1 + \cot^2\theta - 1 + \cos^2\theta$

$= \boxed{\cot^2\theta + \cos^2\theta}$ = RHS

Hence, proved

Q.19 $\sec A\,(1 - \sin A)(\sec A + \tan A) = 1$

LHS $= \sec A\,(1 - \sin A)(\sec A + \tan A)$

$= \frac{1}{\cos A}(1 - \sin A)\left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)$

$= \frac{(1 - \sin A)(1 + \sin A)}{\cos A \times \cos A}$

$= \frac{(1 - \sin^2 A)}{\cos^2 A}$ $\quad (1 - \sin^2 A = \cos^2 A)$

$= \frac{\cos^2 A}{\cos^2 A} = \boxed{1}$ = RHS

Hence, proved

Q.20 $(\operatorname{cosec} A - \sin A)(\sec A - \cos A)(\tan A + \cot A) = 1$

LHS $= (\operatorname{cosec} A - \sin A)(\sec A - \cos A)(\tan A + \cot A)$

$= \left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right)\left(\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}\right)$

$$= \left(\frac{1-\sin^2 A}{\sin A}\right)\left(\frac{1-\cos^2 A}{\cos A}\right)\left(\frac{\sin^2 A+\cos^2 A}{\sin A \cos A}\right)$$

$$= \left(\frac{\cos^2 A}{\sin A}\right)\left(\frac{\sin^2 A}{\cos A}\right)\left(\frac{\sin^2 A+\cos^2 A}{\sin A \cos A}\right) \quad \begin{bmatrix} 1-\sin^2 A = \cos^2 A \\ 1-\cos^2 A = \sin^2 A \end{bmatrix}$$

$$= \frac{\cos^2 A \times \sin^2 A}{\sin A \cos A} \times \frac{1}{\sin A \cos A}$$

$$= \frac{\cancel{\cos^2 A} \times \cancel{\sin^2 A}}{\cancel{\sin^2 A} \times \cancel{\cos^2 A}} = \boxed{1} = \text{RHS}$$

Hence, proved

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Q.21 $(1+\tan^2\theta)(1-\sin\theta)(1+\sin\theta) = 1$

LHS $= (1+\tan^2\theta)(1-\sin\theta)(1+\sin\theta)$

$= (1+\tan^2\theta)(1-\sin^2\theta)$

$= \sec^2\theta \cos^2\theta$ } $1+\tan^2\theta = \sec^2\theta$, $1-\sin^2\theta = \cos^2\theta$

$= \cancel{\sec^2\theta} \dfrac{1}{\cancel{\sec^2\theta}}$

$= \boxed{1} = $ RHS

Hence, proved

Q.22 $\sin^2 A \cot^2 A + \cos^2 A \tan^2 A = 1$

LHS $= \sin^2 A \cot^2 A + \cos^2 A \tan^2 A$

$= \cancel{\sin^2 A} \dfrac{\cos^2 A}{\cancel{\sin^2 A}} + \cancel{\cos^2 A} \dfrac{\sin^2 A}{\cancel{\cos^2 A}}$

$= \cos^2 A + \sin^2 A$ $\quad (\cos^2 A + \sin^2 A = 1)$

$= \boxed{1} = $ RHS

Hence, proved

Q.23

(i) $\cot\theta - \tan\theta = \dfrac{2\cos^2\theta - 1}{\sin\theta\cos\theta}$

LHS $= \cot\theta - \tan\theta$

$= \dfrac{\cos\theta}{\sin\theta} - \dfrac{\sin\theta}{\cos\theta}$

$= \dfrac{\cos^2\theta - \sin^2\theta}{\sin\theta\cos\theta} = \dfrac{\cos^2\theta - (1-\cos^2\theta)}{\sin\theta\cos\theta}$ $\left[\sin^2\theta = 1-\cos^2\theta\right]$

$= \dfrac{\cos^2\theta - 1 + \cos^2\theta}{\sin\theta\cos\theta} = \boxed{\dfrac{2\cos^2\theta - 1}{\sin\theta\cos\theta}} = $ RHS

Hence, proved

(ii) $\tan\theta - \cot\theta = \dfrac{2\sin^2\theta - 1}{\sin\theta\cos\theta}$

LHS $= \tan\theta - \cot\theta$

$= \dfrac{\sin\theta}{\cos\theta} - \dfrac{\cos\theta}{\sin\theta}$

$= \dfrac{\sin^2\theta - \cos^2\theta}{\sin\theta\cos\theta}$

$= \dfrac{\sin^2\theta - (1 - \sin^2\theta)}{\sin\theta\cos\theta}$ $\quad [\cos^2\theta = 1 - \sin^2\theta]$

$= \dfrac{\sin^2\theta - 1 + \sin^2\theta}{\sin\theta\cos\theta}$

$= \boxed{\dfrac{2\sin^2\theta - 1}{\sin\theta\cos\theta}} = \text{RHS}$

Hence, proved

(iii) $\dfrac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A} = \tan A$

LHS $= \dfrac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A}$

$= \dfrac{\sin A - \sin^3 A + \sin^3 A}{\cos^3 A + \cos^3 A - \cos A}$

$= \dfrac{\sin A}{\cos A}\left(\dfrac{1 - 2\sin^2 A}{2\cos^2 A - 1}\right)$

$= \dfrac{\sin A}{\cos A}\left(\dfrac{\sin^2 A + \cos^2 A - 2\sin^2 A}{2\cos^2 A - \sin^2 A - \cos^2 A}\right)$ $\quad [\sin^2 A + \cos^2 A = 1$

$= \dfrac{\sin A}{\cos A}\left(\dfrac{\cos^2 A - \sin^2 A}{\cos^2 A - \sin^2 A}\right)$

$= \dfrac{\sin A}{\cos A} = \boxed{\tan A} = \text{RHS}$

Hence, proved

Q.24 $\frac{\cos^2\theta}{\sin\theta} - \operatorname{cosec}\theta + \sin\theta = 0$

LHS $= \frac{\cos^2\theta}{\sin\theta} - \operatorname{cosec}\theta + \sin\theta$

$= \frac{\cos^2\theta}{\sin\theta} - \frac{1}{\sin\theta} + \sin\theta$

$= \frac{(\cos^2\theta - 1)}{\sin\theta} + \frac{\sin\theta}{1}$ $\left[\cos^2\theta - 1 = -\sin^2\theta\right]$

$= \frac{-\sin^2\theta}{\sin\theta} + \sin\theta$

$= -\sin\theta + \sin\theta = \boxed{0} = \text{RHS}$

Hence, proved

Q.25 $\frac{1}{1+\sin A} + \frac{1}{1-\sin A} = 2\sec^2 A$

LHS $= \frac{1}{1+\sin A} + \frac{1}{1-\sin A}$

$= \frac{(1-\sin A) + (1+\sin A)}{(1+\sin A)(1-\sin A)} = \frac{1 - \sin A + 1 + \sin A}{1 - \sin^2 A}$

$= \frac{2}{\cos^2 A}$ $\left[1 - \sin^2 A = \cos^2 A\right]$

$= \boxed{2\sec^2 A} = \text{RHS}$

Hence, proved

Q.27 $\frac{(1+\sin\theta)^2 + (1-\sin\theta)^2}{2\cos^2\theta} = \frac{1+\sin^2\theta}{1-\sin^2\theta}$

LHS $= \frac{(1+\sin\theta)^2 + (1-\sin\theta)^2}{2\cos^2\theta}$

$= \frac{1 + \sin^2\theta + 2\sin\theta + 1 + \sin^2\theta - 2\sin\theta}{2\cos^2\theta}$

$= \frac{2 + 2\sin^2\theta}{2\cos^2\theta} = \frac{1+\sin^2\theta}{\cos^2\theta}$

$$= \frac{1+\sin^2\theta}{1-\sin^2\theta} = \text{RHS} \quad [\cos^2\theta = 1-\sin^2\theta]$$

Hence, proved

Q.30 $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \tan\theta + \cot\theta$

LHS $= \frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta}$

$= \frac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\sin\theta}{\cos\theta}}$

$= \frac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta-\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{\frac{\cos\theta-\sin\theta}{\cos\theta}}$

$= \frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)} + \frac{\cos^2\theta}{\sin\theta(\cos\theta-\sin\theta)}$

$= \frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)} - \frac{\cos^2\theta}{\sin\theta(\sin\theta-\cos\theta)}$

$= \frac{\sin^3\theta - \cos^3\theta}{\sin\theta\cos\theta(\sin\theta-\cos\theta)}$

$= \frac{(\sin\theta-\cos\theta)(\sin^2\theta+\cos^2\theta+\sin\theta\cos\theta)}{\sin\theta\cos\theta(\sin\theta-\cos\theta)}$

$= \frac{1+\sin\theta\cos\theta}{\sin\theta\cos\theta} \quad [\sin^2\theta+\cos^2\theta = 1]$

Q.20 $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1+\tan\theta+\cot\theta$

LHS $= \frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta}$

$= \frac{\tan\theta}{1-\frac{1}{\tan\theta}} + \frac{\frac{1}{\tan\theta}}{1-\tan\theta}$

$= \frac{\tan\theta}{\frac{\tan\theta-1}{\tan\theta}} + \frac{1}{\tan\theta(1-\tan\theta)}$

$= \frac{\tan^2\theta}{\tan\theta-1} + \frac{1}{\tan\theta(1-\tan\theta)}$

$= \frac{\tan^2\theta}{\tan\theta-1} - \frac{1}{\tan\theta(\tan\theta-1)}$

$= \frac{\tan^3\theta-1}{\tan\theta(\tan\theta-1)} = \frac{(\tan\theta-1)(\tan^2\theta+\tan\theta+1)}{\tan\theta(\tan\theta-1)}$

$= \frac{\tan^2\theta+\tan\theta+1}{\tan\theta}$

[Using identity, $a^3-b^3=(a-b)(a^2+b^2+ab)$]

$= \frac{\tan^2\theta}{\tan\theta} + \frac{\tan\theta}{\tan\theta} + \frac{1}{\tan\theta}$

$= \tan\theta + 1 + \cot\theta$

$= \boxed{1+\tan\theta+\cot\theta} = \text{RHS}$

Hence, proved

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Q.31 $\sec^6\theta = \tan^6\theta + 3\tan^2\theta \sec^2\theta + 1$

RHS $= \tan^6\theta + 3\tan^2\theta \sec^2\theta + 1$

$= \left(\frac{\sin\theta}{\cos\theta}\right)^6 + 3\left(\frac{\sin\theta}{\cos\theta}\right)^2\left(\frac{1}{\cos^2\theta}\right) + 1$

$= \frac{\sin^6\theta}{\cos^6\theta} + 3\frac{\sin^2\theta}{\cos^2\theta} \times \frac{1}{\cos^2\theta} + 1$

$= \frac{\sin^6\theta}{\cos^6\theta} + \frac{3\sin^2\theta}{\cos^4\theta} + 1$

$= \frac{\sin^6\theta + 3\sin^2\theta\cos^2\theta + \cos^6\theta}{\cos^6\theta}$

$= \frac{\sin^6\theta + \cos^6\theta + 3\sin^2\theta\cos^2\theta}{\cos^6\theta}$

$= \frac{1}{\cos^6\theta}\left[\sin^6\theta + \cos^6\theta + 3\sin^2\theta\cos^2\theta\right]$

$= \frac{1}{\cos^6\theta}\left[(\sin^2\theta)^3 + (\cos^2\theta)^3 + 3\sin^2\theta\cos^2\theta\right]$ (*)

(*) $= \frac{1}{\cos^6\theta} \times [(\sin^2\theta)^3 + (\cos^2\theta)^3 + 3\sin^2\theta\cos^2\theta(\sin^2\theta + \cos^2\theta)]$

$= \frac{1}{\cos^6\theta}\left[(\sin^2\theta + \cos^2\theta)(\sin^4\theta + \cos^4\theta - \sin^2\theta\cos^2\theta) + 3\sin^2\theta\cos^2\theta\right]$

$= \frac{1}{\cos^6\theta}\left[1\left(\sin^4\theta + \cos^4\theta - \sin^2\theta\cos^2\theta + 3\sin^2\theta\cos^2\theta\right)\right]$

$= \sec^6\theta(\sin^4\theta + \cos^4\theta + 2\sin^2\theta\cos^2\theta)^3$

$= \frac{1}{\cos^6\theta}\left[(\sin^2\theta)^2 + (\cos^2\theta)^2 + 2\sin^2\theta\cos^2\theta\right]$

$= \frac{1}{\cos^6\theta}\left[(\sin^2\theta + \cos^2\theta)^2\right]$

$= \frac{1}{\cos^6\theta}\left[(1)^2\right]$ $\left(\sin^2\theta + \cos^2\theta = 1\right)$

$= \sec^6\theta$

$=$ LHS

$= \frac{1}{\cos^6\theta} = \boxed{\sec^6\theta} = LHS$

Hence, proved

Q.32 $\operatorname{cosec}^6\theta = \cot^6\theta + 3\cot^2\theta \operatorname{cosec}^2\theta + 1$

RHS $= \cot^6\theta + 3\cot^2\theta \operatorname{cosec}^2\theta + 1$

$= \left(\frac{\cos^6\theta}{\sin^6\theta}\right) + 3\left(\frac{\cos^2\theta}{\sin^2\theta}\right)\left(\frac{1}{\sin^2\theta}\right) + 1$

$= \frac{\cos^6\theta}{\sin^6\theta} + \frac{3\cos^2\theta}{\sin^4\theta} + 1$

$= \frac{\cos^6\theta + 3\cos^2\theta\sin^2\theta + \sin^6\theta}{\sin^6\theta}$

$= \frac{\cos^6\theta + \sin^6\theta + 3\cos^2\theta\sin^2\theta}{\sin^6\theta}$

$= \frac{1}{\sin^6\theta}\left[(\sin^2\theta)^3 + (\cos^2\theta)^3 + 3\sin^2\theta\cos^2\theta\right]$

<same as previous question>

$= \frac{1}{\sin^6\theta}\left[(\sin^2\theta + \cos^2\theta)(\sin^4\theta + \cos^4\theta - \sin^2\theta\cos^2\theta) + 3\sin^2\theta\cos^2\theta\right]$

$= \frac{1}{\sin^6\theta}\left[1(\sin^4\theta + \cos^4\theta - \sin^2\theta\cos^2\theta + 3\sin^2\theta\cos^2\theta)\right]$

$= \frac{1}{\sin^6\theta}\left[(\sin^2\theta)^2 + (\cos^2\theta)^2 + 2\sin^2\theta\cos^2\theta\right]$

$= \frac{1}{\sin^6\theta}\left[(\sin^2\theta + \cos^2\theta)^2\right]$

$= \frac{1}{\sin^6\theta}\left[(1)^2\right] \quad (\sin^2\theta + \cos^2\theta = 1)$

$= \frac{1}{\sin^6\theta} = \boxed{\operatorname{cosec}^6\theta} = LHS$

Hence, proved

Q.33 $\frac{(1+\tan^2\theta)\cot\theta}{\operatorname{cosec}^2\theta} = \tan\theta$

LHS $= \frac{(1+\tan^2\theta)\cot\theta}{\operatorname{cosec}^2\theta}$

$= \frac{\sec^2\theta \cot\theta}{\operatorname{cosec}^2\theta}$ $\quad [1+\tan^2\theta = \sec^2\theta]$

$= \frac{\sin^2\theta \times \cancel{\cos\theta}}{\cos^2\theta \times \cancel{\sin\theta}}$

$= \frac{\sin\theta}{\cos\theta} = \boxed{\tan\theta} = $ RHS

Hence, proved

Q.34 $\frac{1+\cos A}{\sin^2 A} = \frac{1}{1-\cos A}$

LHS $= \frac{1+\cos A}{\sin^2 A}$

$= \frac{1+\cos A}{1-\cos^2 A}$ $\quad [\sin^2 A = 1-\cos^2 A]$

$= \frac{\cancel{(1+\cos A)}}{\cancel{(1+\cos A)}(1-\cos A)}$

$= \boxed{\frac{1}{1-\cos A}} = $ RHS

Hence, proved

Q.35 $\frac{\sec A - \tan A}{\sec A + \tan A} = \frac{\cos^2 A}{(1+\sin A)^2}$

LHS $= \frac{\sec A - \tan A}{\sec A + \tan A}$

$= \frac{\frac{1}{\cos A} - \frac{\sin A}{\cos A}}{\frac{1}{\cos A} + \frac{\sin A}{\cos A}}$

$$= \frac{\frac{1-\sin A}{\cos A}}{\frac{1+\sin A}{\cos A}} = \frac{1-\sin A}{1+\sin A}$$

$$= \frac{(1-\sin A)(1-\sin A)}{(1+\sin A)(1-\sin A)}$$

$$= \frac{(1-\sin A)^2}{1-\sin^2 A}$$

$$= \frac{(1-\sin A)(1+\sin A)}{(1+\sin A)(1+\sin A)}$$

$$= \frac{(1-\sin^2 A)}{(1+\sin A)^2} = \boxed{\frac{\cos^2 A}{(1+\sin A)^2}} \quad [1-\sin^2 A = \cos^2 A]$$

$= RHS$

Hence, proved

Q.36 $\frac{1+\cos A}{\sin A} = \frac{\sin A}{1-\cos A}$

$$LHS = \frac{1+\cos A}{\sin A}$$

$$= \frac{(1+\cos A)(1-\cos A)}{(\sin A)(1-\cos A)}$$

$$= \frac{(1-\cos^2 A)}{\sin A(1-\cos A)}$$

$$= \frac{\sin^2 A}{\sin A(1-\cos A)} \quad [1-\cos^2 A = \sin^2 A]$$

$$= \boxed{\frac{\sin A}{1-\cos A}} = RHS$$

Hence, proved

Q.37 (ii) $\sqrt{\frac{1-\cos A}{1+\cos A}} + \sqrt{\frac{1+\cos A}{1-\cos A}} = 2\,\text{cosec}\,A$

LHS $= \sqrt{\frac{1-\cos A}{1+\cos A}} + \sqrt{\frac{1+\cos A}{1-\cos A}}$

$= \sqrt{\frac{(1-\cos A)(1-\cos A)}{(1+\cos A)(1-\cos A)}} + \sqrt{\frac{(1+\cos A)(1+\cos A)}{(1-\cos A)(1+\cos A)}}$

$= \sqrt{\frac{(1-\cos A)^2}{(1-\cos^2 A)}} + \sqrt{\frac{(1+\cos A)^2}{(1-\cos^2 A)}}$

$= \sqrt{\frac{(1-\cos A)^2}{\sin^2 A}} + \sqrt{\frac{(1+\cos A)^2}{\sin^2 A}}$ $\left[1-\cos^2 A = \sin^2 A\right]$

$= \frac{(1-\cos A)}{\sin A} + \frac{(1+\cos A)}{\sin A}$

$= \frac{1-\cos A+1+\cos A}{\sin A}$

$= \frac{2}{\sin A} = \boxed{2\,\text{cosec}\,A} = \text{RHS}$

Hence, proved

Q.38 (i) $\sqrt{\frac{\sec\theta-1}{\sec\theta+1}} + \sqrt{\frac{\sec\theta+1}{\sec\theta-1}} = 2\,\text{cosec}\,\theta$

LHS $= \sqrt{\frac{\sec\theta-1}{\sec\theta+1}} + \sqrt{\frac{\sec\theta+1}{\sec\theta-1}}$

$= \sqrt{\frac{(\sec\theta-1)(\sec\theta-1)}{(\sec\theta+1)(\sec\theta-1)}} + \sqrt{\frac{(\sec\theta+1)(\sec\theta+1)}{(\sec\theta-1)(\sec\theta+1)}}$

$= \sqrt{\frac{(\sec\theta-1)^2}{\sec^2\theta-1}} + \sqrt{\frac{(\sec\theta+1)^2}{\sec^2\theta-1}}$

$= \sqrt{\frac{(\sec\theta-1)^2}{\tan^2\theta}} + \sqrt{\frac{(\sec\theta+1)^2}{\tan^2\theta}}$ $\left[\tan^2\theta+1 = \sec^2\theta\right]$

$= \frac{(\sec\theta-1)}{\tan\theta} + \frac{(\sec\theta+1)}{\tan\theta}$

$$= \frac{\sec\theta - 1 + \sec\theta + 1}{\tan\theta}$$

$$= \frac{2\sec\theta}{\tan\theta}$$

$$= \frac{2 \cdot \frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}} = \frac{2}{\sin\theta} = \boxed{2\operatorname{cosec}\theta} = \text{RHS}$$

Hence, proved

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Exercise 11.1

Q.38

(ii) $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} + \sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = 2\sec\theta$

$$\text{LHS} = \sqrt{\frac{1+\sin\theta}{1-\sin\theta}} + \sqrt{\frac{1-\sin\theta}{1+\sin\theta}}$$

$$= \sqrt{\frac{(1+\sin\theta)(1+\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}} + \sqrt{\frac{(1-\sin\theta)(1-\sin\theta)}{(1+\sin\theta)(1-\sin\theta)}}$$

$$= \sqrt{\frac{(1+\sin\theta)^2}{1-\sin^2\theta}} + \sqrt{\frac{(1-\sin\theta)^2}{1-\sin^2\theta}}$$

$$= \sqrt{\frac{(1+\sin\theta)^2}{\cos^2\theta}} + \sqrt{\frac{(1-\sin\theta)^2}{\cos^2\theta}} \quad \left[\sin^2\theta + \cos^2\theta = 1\right]$$

$$= \frac{(1+\sin\theta)}{\cos\theta} + \frac{(1-\sin\theta)}{\cos\theta}$$

$$= \frac{1+\sin\theta+1-\sin\theta}{\cos\theta} = \frac{2}{\cos\theta}$$

$$= \boxed{2\sec\theta} = \text{RHS}$$

Hence, proved

(iii) $\sqrt{\frac{1+\cos\theta}{1-\cos\theta}} + \sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = 2\operatorname{cosec}\theta$

$$\text{LHS} = \sqrt{\frac{1+\cos\theta}{1-\cos\theta}} + \sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$$

$$= \sqrt{\frac{(1+\cos\theta)(1+\cos\theta)}{(1-\cos\theta)(1+\cos\theta)}} + \sqrt{\frac{(1-\cos\theta)(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}}$$

$$= \sqrt{\frac{(1+\cos\theta)^2}{1-\cos^2\theta}} + \sqrt{\frac{(1-\cos\theta)^2}{1-\cos^2\theta}}$$

$$= \sqrt{\frac{(1+\cos\theta)^2}{\sin^2\theta}} + \sqrt{\frac{(1-\cos\theta)^2}{\sin^2\theta}} \qquad \left[\sin^2\theta + \cos^2\theta = 1\right]$$

$$= \frac{(1+\cos\theta)}{\sin\theta} + \frac{(1-\cos\theta)}{\sin\theta}$$

$$= \frac{1+\cos\theta+1-\cos\theta}{\sin\theta} = \frac{2}{\sin\theta}$$

$$= \boxed{2\operatorname{cosec}\theta} = \text{RHS}$$

Hence, proved

(iv) $\dfrac{\sec\theta - 1}{\sec\theta + 1} = \left(\dfrac{\sin\theta}{1+\cos\theta}\right)^2$

LHS $= \dfrac{\sec\theta - 1}{\sec\theta + 1}$

$$= \frac{\frac{1}{\cos\theta} - 1}{\frac{1}{\cos\theta} + 1} = \frac{\frac{1-\cos\theta}{\cos\theta}}{\frac{1+\cos\theta}{\cos\theta}}$$

$$= \frac{1-\cos\theta}{1+\cos\theta} = \frac{(1-\cos\theta)(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}$$

$$= \frac{(1-\cos\theta)^2}{(1+\cos\theta)(1+\cos\theta)} = \frac{1-\cos^2\theta}{(1+\cos\theta)^2}$$

$$= \frac{\sin^2\theta}{(1+\cos\theta)^2} \qquad \left[\sin^2\theta + \cos^2\theta = 1\right]$$

$$= \boxed{\left(\frac{\sin\theta}{1+\cos\theta}\right)^2} = \text{RHS}$$

Hence, proved

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Q.39 $(\sec A - \tan A)^2 = \dfrac{1-\sin A}{1+\sin A}$

$$\text{LHS} = (\sec A - \tan A)^2$$
$$= \left(\frac{1}{\cos A} - \frac{\sin A}{\cos A}\right)^2$$
$$= \frac{(1-\sin A)^2}{\cos^2 A}$$
$$= \frac{(1-\sin A)^2}{(1-\sin^2 A)} \quad [\sin^2\theta + \cos^2\theta = 1]$$
$$= \frac{(1-\sin A)(1-\sin A)}{(1-\sin A)(1+\sin A)}$$
$$= \left[\frac{1-\sin A}{1+\sin A}\right] = \text{RHS}$$

Hence proved

Q.40 $\dfrac{1-\cos A}{1+\cos A} = (\cot A - \operatorname{cosec} A)^2$

$$\text{RHS} = (\cot A - \operatorname{cosec} A)^2$$
$$= \left(\frac{\cos A}{\sin A} - \frac{1}{\sin A}\right)^2$$
$$= \left(\frac{\cos A + 1}{\sin A}\right)^2$$
$$= \frac{(\cos A - 1)^2}{\sin^2 A} = \frac{[-(1-\cos A)]^2}{\sin^2 A}$$
$$= \frac{(1-\cos A)^2}{1-\cos^2 A} \quad [\sin^2\theta + \cos^2\theta = 1]$$

$$= \frac{(1-\cos A)(1-\cos A)}{(1-\cos A)(1+\cos A)}$$

$$= \left|\frac{1-\cos A}{1+\cos A}\right| = \text{LHS}$$

Hence, proved

Q.41 $\frac{1}{\sec A - 1} + \frac{1}{\sec A + 1} = 2\,\text{cosec}\,A \cot A$

$$\text{LHS} = \frac{1}{\sec A - 1} + \frac{1}{\sec A + 1}$$

$$= \frac{\sec A + 1 + \sec A - 1}{(\sec A - 1)(\sec A + 1)}$$

$$= \frac{2\sec A}{\sec^2 A - 1} = \frac{2\sec A}{\tan^2} \quad \left[1 + \tan^2 A = \sec^2 A\right]$$

$$= \frac{2}{\frac{\cos A}{\sin^2 A}} = \frac{2\cos A}{\sin^2 A}$$

$$= \frac{2\cos A \times 1}{\sin A \quad \sin A}$$

$$= |2\,\text{cosec}\,A \cot A| = \text{RHS}$$

Hence, proved

Q.42 $\frac{\cos A}{1-\tan A} + \frac{\sin A}{1-\cot A} = \sin A + \cos A$

$$\text{LHS} = \frac{\cos A}{1-\tan A} + \frac{\sin A}{1-\cot A}$$

$$= \frac{\cos A}{1-\frac{\sin A}{\cos A}} + \frac{\sin A}{1-\frac{\cos A}{\sin A}}$$

$$= \frac{\cos A}{\frac{\cos A - \sin A}{\cos A}} + \frac{\sin A}{\frac{\sin A - \cos A}{\sin A}}$$

$$= \frac{\cos^2 A}{\cos A - \sin A} + \frac{\sin^2 A}{\sin A - \cos A}$$

$$= \frac{-\cos^2 A}{\sin A - \cos A} + \frac{\sin^2 A}{\sin A - \cos A}$$

$$= \frac{\sin^2 A - \cos^2 A}{\sin A - \cos A}$$

$$= \frac{(\sin A + \cos A)(\sin A - \cos A)}{(\sin A - \cos A)}$$

$$= \boxed{\sin A + \cos A} = \text{RHS}$$

Hence proved

Q.43 $\frac{\operatorname{cosec} A}{\operatorname{cosec} A - 1} + \frac{\operatorname{cosec} A}{\operatorname{cosec} A + 1} = 2\sec^2 A$

LHS $= \frac{\operatorname{cosec} A}{\operatorname{cosec} A - 1} + \frac{\operatorname{cosec} A}{\operatorname{cosec} A + 1}$

$$= \frac{\operatorname{cosec}^2 A + \operatorname{cosec} A + \operatorname{cosec}^2 A - \operatorname{cosec} A}{(\operatorname{cosec} A - 1)(\operatorname{cosec} A + 1)}$$

$$= \frac{2\operatorname{cosec}^2 A}{\operatorname{cosec}^2 A - 1}$$

$$= \frac{2\operatorname{cosec}^2 A}{\cot^2 A} \quad \left[\cot^2 A = \operatorname{cosec}^2 A - 1\right]$$

$$= \frac{2 \quad \frac{1}{\sin^2 A}}{\frac{\cos^2 A}{\sin^2 A}} = \frac{2}{\cos^2 A}$$

$$= \boxed{2\sec^2 A} = \text{RHS}$$

Hence proved

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Q.44 $\frac{\tan^2 A}{1+\tan^2 A} + \frac{\cot^2 A}{1+\cot^2 A} = 1$

LHS $= \frac{\tan^2 A}{1+\tan^2 A} + \frac{\cot^2 A}{1+\cot^2 A}$

$= \frac{\tan^2 A}{\sec^2 A} + \frac{\cot^2 A}{\operatorname{cosec}^2 A}$ $\left[1+\tan^2 A = \sec^2 A,\ 1+\cot^2 A = \operatorname{cosec}^2 A\right]$

$= \frac{\frac{\sin^2 A}{\cos^2 A}}{\frac{1}{\cos^2 A}} + \frac{\frac{\cos^2 A}{\sin^2 A}}{\frac{1}{\sin^2 A}}$

$= \sin^2 A + \cos^2 A$ $\left[\sin^2 A + \cos^2 A = 1\right]$

$= \boxed{1} = \text{RHS}$

Hence, proved

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Q.46 $\dfrac{1+\cos\theta-\sin^2\theta}{\sin\theta(1+\cos\theta)} = \cot\theta$

LHS $= \dfrac{1+\cos\theta-\sin^2\theta}{\sin\theta(1+\cos\theta)}$

$= \dfrac{1-\sin^2\theta+\cos\theta}{\sin\theta(1+\cos\theta)}$

$= \dfrac{\cos^2\theta+\cos\theta}{\sin\theta(1+\cos\theta)}$ $\quad[\sin^2\theta+\cos^2\theta=1]$

$= \dfrac{\cos\theta(\cos\theta+1)}{\sin\theta(1+\cos\theta)}$

$= \dfrac{\cos\theta}{\sin\theta} = \boxed{\cot\theta} = \text{RHS}$

Hence proved

Q.47 (i) $\dfrac{1+\cos\theta+\sin\theta}{1+\cos\theta-\sin\theta} = \dfrac{1+\sin\theta}{\cos\theta}$

LHS $= \dfrac{1+\cos\theta+\sin\theta}{1+\cos\theta-\sin\theta}$

$= \dfrac{(1+\cos\theta)+\sin\theta}{(1+\cos\theta)-\sin\theta} \times \dfrac{(1+\cos\theta)+\sin\theta}{(1+\cos\theta)+\sin\theta}$

$= \dfrac{[(1+\cos\theta)+\sin\theta]^2}{(1+\cos\theta)^2-\sin^2\theta} = \dfrac{(1+\cos\theta)^2+\sin^2\theta+2\sin\theta(1+\cos\theta)}{1+\cos^2\theta+2\cos\theta-1+\cos^2\theta}$

$[\sin^2\theta+\cos^2\theta=1]$

$$= \frac{1+\cos^2\theta + 2\cos\theta + 1 - \cos^2\theta + 2\sin\theta + 2\cos\theta\sin\theta}{2\cos\theta + 2\cos^2\theta}$$

$$= \frac{2 + 2\cos\theta + 2\sin\theta + 2\cos\theta\sin\theta}{2\cos\theta(1+\cos\theta)}$$

$$= \frac{2(1+\cos\theta) + 2\sin\theta(1+\cos\theta)}{2\cos\theta(1+\cos\theta)}$$

$$= \frac{(1+\cos\theta)(2+2\sin\theta)}{2\cos\theta(1+\cos\theta)}$$

$$= \frac{2(1+\sin\theta)}{2\cos\theta} = \boxed{\frac{1+\sin\theta}{\cos\theta}} = \text{RHS}$$

Hence proved

(ii) $\dfrac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \dfrac{1}{\sec\theta - \tan\theta}$

$$\text{LHS} = \frac{(\sin\theta - \cos\theta) + 1}{(\sin\theta + \cos\theta) - 1}$$

$$= \frac{(\sin\theta - \cos\theta) + 1}{(\sin\theta + \cos\theta) - 1} \times \frac{(\sin\theta + \cos\theta) + 1}{(\sin\theta + \cos\theta) + 1}$$

$$= \frac{(\sin\theta + 1)^2 - \cos^2\theta}{(\sin\theta + \cos\theta)^2 - 1}$$

$$= \frac{\sin^2\theta + 1 + 2\sin\theta - \cos^2\theta}{\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta - 1}$$

$$= \frac{\sin^2\theta + 2\sin\theta + (1 - \cos^2\theta)}{(\sin^2\theta + \cos^2\theta) + 2\sin\theta\cos\theta - 1}$$

$$= \frac{\sin^2\theta + 2\sin\theta + \sin^2\theta}{1 + 2\sin\theta\cos\theta - 1} \qquad [\sin^2\theta + \cos^2\theta = 1]$$

$$= \frac{2\sin\theta + 2\sin^2\theta}{2\sin\theta\cos\theta}$$

$$= \frac{2\sin\theta(1+\sin\theta)}{2\sin\theta\cos\theta} = \frac{1+\sin\theta}{\cos\theta}$$

$$= \left(\frac{1+\sin\theta}{\cos\theta}\right)\left(\frac{1-\sin\theta}{1-\sin\theta}\right)$$

$$= \frac{1-\sin^2\theta}{\cos\theta - \cos\theta\sin\theta} \qquad \left[\sin^2\theta + \cos^2\theta = 1\right]$$

$$= \frac{\cos^2\theta}{\cos\theta(1-\sin\theta)}$$

$$= \frac{\cos\theta}{1-\sin\theta}$$

$$= \frac{\frac{\cos\theta}{\cos\theta}}{\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}} \qquad \left[\text{Divide numerator \& denominator by } \cos\theta\right]$$

$$= \boxed{\frac{1}{\sec\theta - \tan\theta}} = \text{RHS}$$

Hence, proved

Q.48 $\dfrac{1}{\sec A - \tan A} - \dfrac{1}{\cos A} = \dfrac{1}{\cos A} - \dfrac{1}{\sec A + \tan A}$

$$\text{LHS} = \frac{1}{\sec A - \tan A} - \frac{1}{\cos A}$$

$$\Rightarrow \frac{1}{\sec A - \tan A} + \frac{1}{\sec A + \tan A} = \frac{1}{\cos A} + \frac{1}{\cos A}$$

$$\text{LHS} = \frac{1}{\sec A - \tan A} + \frac{1}{\sec A + \tan A}$$

$$= \frac{\sec A + \tan A + \sec A - \tan A}{(\sec A - \tan A)(\sec A - \tan A)}$$

$$= \frac{2\sec A}{\sec^2 A - \tan^2 A} \qquad \left[1 + \tan^2\theta = \sec^2\theta\right]$$

$$= \frac{2\sec A}{1} = \frac{2}{\cos A} = \boxed{\frac{1}{\cos A} + \frac{1}{\cos A}} = \text{RHS}$$

Hence, proved

Q. 44 $\tan^2 A + \cot^2 A = \sec^2 A \operatorname{cosec}^2 A - 2$

$\Rightarrow \tan^2 A + \cos^2 A + 2 = \sec^2 A \operatorname{cosec}^2 A$

~~LHS~~ $= \tan^2 A + \cot^2 A$

$= \frac{\sin^2 A}{\cos^2 A} + \frac{\cos^2 A}{\sin^2 A}$

LHS $= \tan^2 A + \cot^2 A + 2$

$= (1 + \tan^2 A) + (1 + \cot^2 A)$ $\quad [1 + \tan^2 A = \sec^2 A, \; 1 + \cot^2 A = \operatorname{cosec}^2 A]$

$= \sec^2 A + \operatorname{cosec}^2 A$

$= \frac{1}{\cos^2 A} + \frac{1}{\sin^2 A}$

$= \frac{\sin^2 A + \cos^2 A}{\cos^2 A \sin^2 A}$ $\quad [\sin^2 \theta + \cos^2 \theta = 1]$

$= \frac{1}{\cos^2 A \sin^2 A}$

$= \frac{1}{\cos^2 A} \times \frac{1}{\sin^2 A}$

$= \boxed{\sec^2 A \operatorname{cosec}^2 A} = \text{RHS}$

Hence, proved

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Q.50 $\frac{\tan A}{1+\sec A} - \frac{\tan A}{1-\sec A} = 2\operatorname{cosec} A$

LHS $= \frac{\tan A}{1+\sec A} - \frac{\tan A}{1-\sec A}$

$= \frac{\tan A(1-\sec A) - \tan A(1+\sec A)}{(1+\sec A)(1-\sec A)}$

$= \frac{\tan A(1-\sec A - 1-\sec A)}{1-\sec^2 A}$

$= \frac{\tan A(-2\sec A)}{1-\sec^2 A}$ [$\sec^2 A - 1 = \tan^2 A$]

$= \frac{\tan A(2\sec A)}{\sec^2 A - 1}$

$= \frac{2\tan A \sec A}{\tan^2 A} = \frac{2\sec A}{\tan A}$

$= \frac{2\frac{1}{\cos A}}{\frac{\sin A}{\cos A}} = \frac{2}{\sin A} = 2\operatorname{cosec} A = RHS$

Hence, proved.

Q.51 $\frac{1+\cot^2\theta}{1+\operatorname{cosec}\theta} = \operatorname{cosec}\theta$

LHS $= \frac{1+\cot^2\theta}{1+\operatorname{cosec}\theta}$

$= \frac{1+\frac{\cos^2\theta}{\sin^2\theta}}{1+\frac{1}{\sin\theta}} = \frac{1+\frac{\cos^2\theta}{\sin^2\theta}}{\frac{\sin\theta+1}{\sin\theta}}$

$$= \frac{1+\cos^2\theta}{\sin\theta(1+\sin\theta)}$$

$$= \frac{\sin\theta(1+\sin\theta)+\cos^2\theta}{\sin\theta(1+\sin\theta)}$$

$$= \frac{\sin^2\theta+\sin\theta+\cos^2\theta}{\sin\theta(1+\sin\theta)}$$

$$= \frac{1+\sin\theta}{\sin\theta(1+\sin\theta)} \quad \left[\sin^2\theta+\cos^2\theta=1\right]$$

$$= \frac{1}{\sin\theta} = \boxed{\operatorname{cosec}\theta} = \text{RHS}$$

Hence, proved

Q.52 $\dfrac{\cos\theta}{\operatorname{cosec}\theta+1} + \dfrac{\cos\theta}{\operatorname{cosec}\theta-1} = 2\tan\theta$

$$\text{LHS} = \frac{\cos\theta}{\operatorname{cosec}\theta+1} + \frac{\cos\theta}{\operatorname{cosec}\theta-1}$$

$$= \frac{\cos\theta(\operatorname{cosec}\theta-1)+\cos\theta(\operatorname{cosec}\theta+1)}{\operatorname{cosec}^2\theta-1}$$

$$= \frac{\cos\theta(\operatorname{cosec}\theta-1+\operatorname{cosec}\theta+1)}{\cot^2\theta} \quad \left[\operatorname{cosec}^2\theta-1=\tan^2\theta\right]$$

$$= \frac{2\cos\theta\operatorname{cosec}\theta}{\cot^2\theta}$$

$$= \frac{2\cos\theta\cdot\frac{1}{\sin\theta}}{\frac{\cos^2\theta}{\sin^2\theta}} = \frac{2\cos\theta}{\sin\theta}\times\frac{\sin^2\theta}{\cos^2\theta}$$

$$= \frac{2\sin\theta}{\cos\theta} = \boxed{2\tan\theta} = \text{RHS}$$

Hence, proved

Q.53 $(1+\tan^2 A)\left(1+\frac{1}{\tan^2 A}\right) = \frac{1}{\sin^2 A - \sin^4 A}$

LHS $= (1+\tan^2 A)\left(1+\frac{1}{\tan^2 A}\right)$

$= (1+\tan^2 A)(1+\cot^2 A)$ $\quad [1+\tan^2 A = \sec^2 A,\ 1+\cot^2 A = \operatorname{cosec}^2 A]$

$= \sec^2 A \operatorname{cosec}^2 A$

$= \frac{1}{\cos^2 A \sin^2 A}$

$= \frac{1}{\sin^2 (1-\sin^2 A)}$ $\quad [\cos^2 A = 1 - \sin^2 A]$

$= \frac{1}{\sin^2 A - \sin^4 A}$ $= RHS$

Hence, proved

Q.54 $\sin^2 A \cos^2 B - \cos^2 A \sin^2 B = \sin^2 A - \sin^2 B$

LHS $= \sin^2 A \cos^2 B - \cos^2 A \sin^2 B$

$= \sin^2 A (1-\sin^2 B) - (1-\sin^2 A)\sin^2 B$

$= \sin^2 A - \sin^2 A \sin^2 B - \sin^2 B + \sin^2 A \sin^2 B$

$= \sin^2 A - \sin^2 B = RHS$

Hence, proved

Q.55

(i) $\frac{\cot A + \tan B}{\cot B + \tan A} = \cot A \tan B$

LHS $= \frac{\cot A + \tan B}{\cot B + \tan A}$

$= \frac{\frac{\cos A}{\sin A} + \frac{\sin B}{\cos B}}{\frac{\cos B}{\sin B} + \frac{\sin A}{\cos A}} = \frac{\cos A \cos B + \sin A \sin B}{\sin A \cos B} \Big/ \frac{\cos A \cos B + \sin A \sin B}{\sin B \cos A}$

$= \frac{\sin B \cos A}{\sin A \cos B}$

$= \frac{\sin B}{\cos B} \frac{\cos A}{\sin A}$

$= \boxed{\tan A \cot A} = RHS$

Hence, proved

(ii) $\frac{\tan A + \tan B}{\cot A + \cot B} = \tan A \tan B$

$LHS = \frac{\tan A + \tan B}{\cot A + \cot B}$

$= \frac{\frac{\sin A}{\cos A} + \frac{\sin B}{\cos B}}{\frac{\cos A}{\sin A} + \frac{\cos B}{\sin B}} = \frac{\sin A \cos B + \sin B \cos A}{\cos A \cos B} \Big/ \frac{\sin B \cos A + \sin A \cos B}{\sin A \sin B}$

$= \frac{\sin A \sin B}{\cos A \cos B} = \boxed{\tan A \tan B} = RHS$

Hence, proved

Q. 56 $\cot^2 A \operatorname{cosec}^2 B - \cot^2 B \operatorname{cosec}^2 A = \cot^2 A - \cot^2 B$

$LHS = \cot^2 A (1 + \cot^2 B) - \cot^2 B (1 + \cot^2 A)$

$LHS = \cot^2 A \operatorname{cosec}^2 B - \cot^2 B \operatorname{cosec}^2 A$

$= \frac{\cos^2 A}{\sin^2 A} \times \frac{1}{\sin^2 B} - \frac{\cos^2 B}{\sin^2 B} \times \frac{1}{\sin^2 A}$

$= \frac{\cos^2 A}{\sin^2 A \sin^2 B} - \frac{\cos^2 B}{\sin^2 B \sin^2 A}$

$= \frac{\cos^2 A - \cos^2 B}{\sin^2 A \sin^2 B} \longrightarrow (1)$

$= \cot^2 A + \cot^2 A \cot^2 B - \cot^2 B - \cot^2 A \cot^2 B$

$= \cot^2 A - \cot^2 B = RHS$

$RHS = \cot^2 A - \cot^2 B$

$= \frac{\cos^2 A}{\sin^2 A} - \frac{\cos^2 B}{\sin^2 B}$

$$= \frac{\cos^2 A \sin^2 B - \cos^2 B \sin^2 A}{\sin^2 A \sin^2 B}$$

$$= \frac{\cos^2 A (1 - \cos^2 B) - \cos^2 B (1 - \cos^2 A)}{\sin^2 A \sin^2 B}$$

$[\sin^2\theta + \cos^2\theta = 1]$

$$= \frac{\cos^2 A - \cancel{\cos^2 A \cos^2 B} - \cos^2 B + \cancel{\cos^2 A \cos^2 B}}{\sin^2 A \sin^2 B}$$

$$= \frac{\cos^2 A - \cos^2 B}{\sin^2 A \sin^2 B} \quad \text{——— (2)}$$

From (1) & (2),

LHS = RHS

Hence proved

Q. 57 $\tan^2 A \sec^2 B - \sec^2 A \tan^2 B = \tan^2 A - \tan^2 B$

Do by same method as above

LHS $= \tan^2 A \sec^2 B - \sec^2 A \tan^2 B$

$$= \frac{\sin^2 A}{\cos^2 A} \times \frac{1}{\cos^2 B} - \frac{1}{\cos^2 A} \times \frac{\sin^2 B}{\cos^2 B}$$

$$= \frac{\sin^2 A}{\cos^2 A \cos^2 B} - \frac{\sin^2 B}{\cos^2 A \cos^2 B}$$

$$= \frac{\sin^2 A - \sin^2 B}{\cos^2 A \cos^2 B} \quad \text{——— (1)}$$

RHS $= \tan^2 A - \tan^2 B$

$$= \frac{\sin^2 A}{\cos^2 A} - \frac{\sin^2 B}{\cos^2 B}$$

$$= \frac{\sin^2 A \cos^2 B - \sin^2 B \cos^2 A}{\cos^2 A \cos^2 B}$$

$$= \frac{\sin^2 A (1 - \sin^2 B) - \sin^2 B (1 - \sin^2 A)}{\cos^2 A \cos^2 B}$$

$[\sin^2\theta + \cos^2\theta = 1]$

$$= \sin^2 A - \sin^2 A \sin^2 B - \sin^2 B + \sin^2 A \sin^2 B$$
$$\cos^2 A \cos^2 B$$

$$= \frac{\sin^2 A - \sin^2 B}{\cos^2 A \cos^2 B} \quad \text{——— (2)}$$

From (1) & (2),

LHS = RHS

Hence, proved

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Q.58 $x = a\sec\theta + b\tan\theta$ } given
$y = a\tan\theta + b\sec\theta$ }

To prove – $x^2 - y^2 = a^2 - b^2$

LHS $= x^2 - y^2$
$= (a\sec\theta + b\tan\theta)^2 - (a\tan\theta + b\sec\theta)^2$
$= a^2\sec^2\theta + b^2\tan^2\theta + 2ab\sec\theta\tan\theta - a^2\tan^2\theta$
$- b^2\sec^2\theta - 2ab\sec\theta\tan\theta$
$= a^2(\sec^2\theta - \tan^2\theta) + b^2(\tan^2\theta - \sec^2\theta)$
$= a^2(\sec^2\theta - \tan^2\theta) - b^2(\sec^2\theta - \tan^2\theta)$
$= a^2(1) - b^2(1)$ [$\sec^2\theta - \tan^2\theta = 1$]
$= a^2 - b^2$ = RHS

Hence, proved

Q.59 $3\sin\theta + 5\cos\theta = 5$ (given)

To prove –
$5\sin\theta - 3\cos\theta = \pm 3$

Proof – $3\sin\theta + 5\cos\theta = 5$
Squaring both sides
$(3\sin\theta + 5\cos\theta)^2 = (5)^2$
$\Rightarrow 9\sin^2\theta + 25\cos^2\theta + 30\sin\theta\cos\theta = 25$
$\Rightarrow 9(1 - \cos^2\theta) + 25(1 - \sin^2\theta) + 30\sin\theta\cos\theta = 25$
$\Rightarrow 9 - 9\cos^2\theta + 25 - 25\sin^2\theta + 30\sin\theta\cos\theta = 25$
$\Rightarrow -25\sin^2\theta - 9\cos^2\theta + 30\sin\theta\cos\theta = 25 - 34$
$\Rightarrow -25\sin^2\theta - 9\cos^2\theta + 30\sin\theta\cos\theta = -9$
$\Rightarrow 25\sin^2\theta + 9\cos^2\theta - 30\sin\theta\cos\theta = 9$

$\Rightarrow (5\sin\theta)^2 + (3\cos\theta)^2 - (2\times 5\sin\theta \times 3\cos\theta) = 9$

$\Rightarrow (5\sin\theta - 3\cos\theta)^2 = (\pm 3)^2$

$\Rightarrow \boxed{5\sin\theta - 3\cos\theta = \pm 3}$

Hence, proved.

Q.60 Given - $\operatorname{cosec}\theta + \cot\theta = m$

$\operatorname{cosec}\theta - \cot\theta = n$

To prove - $mn = 1$

Proof - mn

$= (\operatorname{cosec}\theta + \cot\theta)(\operatorname{cosec}\theta - \cot\theta)$

$= \operatorname{cosec}^2\theta - \cot^2\theta$

$= \boxed{1}$ $\quad [\operatorname{cosec}^2\theta - \cot^2\theta = 1]$

Hence, proved

Q.61 $\dfrac{\tan^3\theta}{1+\tan^2\theta} + \dfrac{\cot^3\theta}{1+\cot^2\theta} = \sec\theta\operatorname{cosec}\theta - 2\sin\theta\cos\theta$

LHS $= \dfrac{\tan^3\theta}{1+\tan^2\theta} + \dfrac{\cot^3\theta}{1+\cot^2\theta}$

$= \dfrac{\frac{\sin^3\theta}{\cos^3\theta}}{\sec^2\theta} + \dfrac{\frac{\cos^3\theta}{\sin^3\theta}}{\operatorname{cosec}^2\theta}$ $\quad \left[\begin{array}{l} 1+\tan^2\theta = \sec^2\theta \\ 1+\cot^2\theta = \operatorname{cosec}^2\theta \end{array}\right]$

$= \dfrac{\sin^3\theta \times \cancel{\cos^2\theta}}{\cancel{\sec^2}\cos^5\theta} + \dfrac{\cos^3\theta \cancel{\sin^2\theta}}{\sin^5\theta}$

$= \dfrac{\sin^3\theta}{\cos\theta} + \dfrac{\cos^3\theta}{\sin\theta}$

$= \dfrac{\sin^4\theta + \cos^4\theta}{\cos\theta\sin\theta}$ $\quad \left[a^4+b^4 = (a^2+b^2)^2 - 2a^2b^2\right.$

$= \dfrac{(\sin^2\theta+\cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta}{\sin\theta\cos\theta}$ $\quad \left[\sin^2\theta+\cos^2\theta = 1\right.$

$= \dfrac{1 - 2\sin^2\theta\cos^2\theta}{\sin\theta\cos\theta}$

$$= \frac{1}{\sin\theta\cos\theta} - \frac{2\sin^2\theta\cos^2\theta}{\sin\theta\cos\theta}$$

$$= \boxed{\operatorname{cosec}\theta\sec\theta - 2\sin\theta\cos\theta}$$

Hence proved

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Exercise 11.1

Q.62 $T_n = \sin^n\theta + \cos^n\theta$ (Given)

To prove - $\frac{T_3 - T_5}{T_1} = \frac{T_5 - T_7}{T_3}$

Proof - LHS = $\frac{T_3 - T_5}{T_1}$

$= \frac{(\sin^3\theta + \cos^3\theta) - (\sin^5\theta + \cos^5\theta)}{\sin\theta + \cos\theta}$

$= \frac{\sin^3\theta + \cos^3\theta - \sin^5\theta - \cos^5\theta}{\sin\theta + \cos\theta}$

$= \frac{\sin^3\theta - \sin^5\theta + \cos^3\theta - \cos^5\theta}{\sin\theta + \cos\theta}$

$= \frac{\sin^3\theta(1 - \sin^2\theta) + \cos^3\theta(1 - \cos^2\theta)}{\sin\theta + \cos\theta}$

$= \frac{\sin^3\theta\cos^2\theta + \cos^3\theta\sin^2\theta}{\sin\theta + \cos\theta}$ $\left[\sin^2\theta + \cos^2\theta = 1\right]$

$= \sin^2\theta + \cos^2\theta$

$= \frac{\sin^2\theta\cos^2\theta(\sin\theta + \cos\theta)}{\sin\theta + \cos\theta}$

$= \sin^2\theta\cos^2\theta$ ——— (1)

RHS = $\frac{T_5 - T_7}{T_3}$

$= \frac{(\sin^5\theta + \cos^5\theta) - (\sin^7\theta + \cos^7\theta)}{\sin^3\theta + \cos^3\theta}$

$= \frac{\sin^5\theta + \cos^5\theta - \sin^7\theta - \cos^7\theta}{\sin^3\theta + \cos^3\theta}$

$= \frac{\sin^5\theta(1 - \sin^2\theta) + \cos^5\theta(1 - \cos^2\theta)}{\sin^3\theta + \cos^3\theta}$

$$= \frac{\sin^5\theta \cos^2\theta + \cos^5\theta \sin^2\theta}{\sin^3\theta + \cos^3\theta}$$

$$= \sin^2\theta \cos^2\theta \frac{(\sin^3\theta + \cos^3\theta)}{\sin^3\theta + \cos^3\theta}$$

$$= \sin^2\theta \cos^2\theta \quad \text{——— (2)}$$

From (1) & (2)

LHS = RHS

Hence proved

Q.63 $\left(\tan\theta + \frac{1}{\cos\theta}\right)^2 + \left(\tan\theta - \frac{1}{\cos\theta}\right)^2 = 2\left(\frac{1+\sin^2\theta}{1-\sin^2\theta}\right)$

$$\text{LHS} = \left(\tan\theta + \frac{1}{\cos\theta}\right)^2 + \left(\tan\theta - \frac{1}{\cos\theta}\right)^2$$

$$= \left(\frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}\right)^2 + \left(\frac{\sin\theta}{\cos\theta} - \frac{1}{\cos\theta}\right)^2$$

$$= \frac{(\sin\theta+1)^2}{\cos^2\theta} + \frac{(\sin\theta-1)^2}{\cos^2\theta}$$

$$= \frac{\sin^2\theta + 1 + 2\sin\theta + \sin^2\theta + 1 - 2\sin\theta}{\cos^2\theta}$$

$$= \frac{2\sin^2\theta + 2}{\cos^2\theta}$$

$$= 2\left(\frac{1+\sin^2\theta}{\cos^2\theta}\right) \quad \left[1 - \sin^2\theta = \cos^2\theta\right]$$

$$= \left[2\left(\frac{1+\sin^2\theta}{1-\sin^2\theta}\right)\right] = \text{RHS}$$

Hence proved

Q.64 $\left(\frac{1}{\sec^2\theta - \cos^2\theta} + \frac{1}{\operatorname{cosec}^2\theta - \sin^2\theta}\right) \sin^2\theta\cos^2\theta = \frac{1 - \sin^2\theta\cos^2\theta}{2 + \sin^2\theta\cos^2\theta}$

LHS $= \left(\frac{1}{\sec^2\theta - \cos^2\theta} + \frac{1}{\operatorname{cosec}^2\theta - \sin^2\theta}\right) \sin^2\theta\cos^2\theta$

$= \left(\frac{1}{\frac{1}{\cos^2\theta} - \cos^2\theta} + \frac{1}{\frac{1}{\sin^2\theta} - \sin^2\theta}\right) \sin^2\theta\cos^2\theta$

$= \left(\frac{\cos^2\theta}{1 - \cos^4\theta} + \frac{\sin^2\theta}{1 - \sin^4\theta}\right) \sin^2\theta\cos^2\theta$

$= \left[\frac{\cos^2\theta}{(1+\cos^2\theta)(1-\cos^2\theta)} + \frac{\sin^2\theta}{(1+\sin^2\theta)(1-\sin^2\theta)}\right] \sin^2\theta\cos^2\theta$

$= \left[\frac{\cos^2\theta}{(1+\cos^2\theta)\sin^2\theta} + \frac{\sin^2\theta}{(1+\sin^2\theta)\cos^2\theta}\right] \sin^2\theta\cos^2\theta$

(Using $\sin^2\theta + \cos^2\theta = 1$)

$= \left[\frac{\cos^2\theta\cos^2\theta(1+\sin^2\theta) + \sin^2\theta\sin^2\theta(1+\cos^2\theta)}{\sin^2\theta\cos^2\theta(1+\cos^2\theta)(1+\sin^2\theta)}\right] \sin^2\theta\cos^2\theta$

$= \frac{\cos^4\theta + \cos^4\theta\sin^2\theta + \sin^4\theta + \sin^4\theta\cos^2\theta}{(1+\cos^2\theta)(1+\sin^2\theta)}$

$= \frac{\cos^4\theta + \sin^4\theta + \cos^2\theta\sin^2\theta(\cos^2\theta + \sin^2\theta)}{(1+\cos^2\theta)(1+\sin^2\theta)}$

$= \frac{(\cos^2\theta + \sin^2\theta)^2 - 2\cos^2\theta\sin^2 + \cos^2\theta\sin^2\theta}{(\cos^2\theta + \sin^2\theta)}$

$\frac{(1+\cos^2\theta)(1+\sin^2\theta)}{}$

$= \frac{(1)^2 - 2\cos^2\theta\sin^2\theta + \cos^2\theta\sin^2\theta(1)}{(1+\cos^2\theta)(1+\sin^2\theta)}$

[Using, $\sin^2\theta + \cos^2\theta = 1$]

$$= \frac{1 - 2\sin\theta\cos^2\theta + \cos^2\theta\sin^2\theta}{(1+\cos^2\theta)(1+\sin^2\theta)}$$

$$= \frac{1 - \cos^2\theta\sin^2\theta}{1 + \sin^2\theta + \cos^2\theta + \cos^2\theta\sin^2\theta}$$

$$= \frac{1 - \cos^2\theta\sin^2\theta}{1 + 1 + \cos^2\theta\sin^2\theta} \quad \left[\sin^2\theta + \cos^2\theta = 1\right]$$

$$= \boxed{\frac{1 - \cos^2\theta\sin^2\theta}{2 + \cos^2\sin^2\theta}} = \text{RHS}$$

Hence, proved

Q.65 $\left[\frac{1+\sin\theta-\cos\theta}{1+\sin\theta+\cos\theta}\right]^2 = \frac{1-\cos\theta}{1+\cos\theta}$

LHS $= \left[\frac{(1+\sin\theta)-\cos\theta}{(1+\sin\theta)+\cos\theta} \times \frac{(1+\sin\theta)+\cos\theta}{(1+\sin\theta)-\cos\theta}\right]^2$

$$= \left\{\frac{[(1+\sin\theta)-\cos\theta]^2}{(1+\sin\theta)^2 - \cos^2\theta}\right\}^2$$

$$= \left[\frac{(1+\sin\theta)^2 + \cos^2\theta - 2\cos\theta(1+\sin\theta)}{1+\sin^2+2\sin\theta-\cos^2\theta}\right]^2$$

$$= \left[\frac{1+\sin^2\theta+2\sin\theta+\cos^2\theta-2\cos\theta-2\cos\theta\sin\theta}{1-\cos^2\theta+\sin^2\theta+2\sin\theta}\right]^2$$

$$= \left[\frac{1+1+2\sin\theta-2\cos\theta-2\cos\theta\sin\theta}{\sin^2\theta+\sin^2\theta+2\sin\theta}\right]^2$$

$$= \left[\frac{2+2\sin\theta-2\cos\theta-2\cos\theta\sin\theta}{2\sin^2\theta+2\sin\theta}\right]^2$$

$$= \left[\frac{1+\sin\theta-\cos\theta-\cos\theta\sin\theta}{\sin^2\theta+\sin\theta}\right]^2$$

$$= \left[\frac{1(1+\sin\theta)-\cos\theta(1+\sin\theta)}{\sin\theta(1+\sin\theta)}\right]^2$$

$$= \left[\frac{(1+\sin\theta)(1-\cos\theta)}{\sin\theta(1+\sin\theta)}\right]^2$$

$$= \frac{(1-\cos\theta)^2}{\sin^2\theta} = \frac{(1-\cos\theta)^2}{1-\cos^2\theta} \quad \left[\sin^2\theta + \cos^2\theta = 1\right]$$

$$= \frac{(1-\cos\theta)^2}{(1+\cos\theta)(1-\cos\theta)}$$

$$= \frac{(1-\cos\theta)(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}$$

$$= \boxed{\frac{1-\cos\theta}{1+\cos\theta}} = \text{RHS}$$

Hence, proved

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Exercise 11.1

Q.69 $(\sec A - \operatorname{cosec} A)(1 + \tan A + \cot A) = \tan A \sec A - \cot A \operatorname{cosec} A$

LHS $= (\sec A - \operatorname{cosec} A)(1 + \tan A + \cot A)$

$= \left(\frac{1}{\cos A} - \frac{1}{\sin A}\right)\left(1 + \frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}\right)$

$= \left(\frac{\sin A - \cos A}{\cos A \sin A}\right)\left(\frac{\cos A \sin A + \sin^2 A + \cos^2 A}{\cos A \sin A}\right)$

$= \frac{(\sin A - \cos A)(\sin^2 A + \cos^2 A + \cos A \sin A)}{\cos^2 A \sin^2 A}$

$= \frac{\sin^3 A - \cos^3 A}{\cos^2 A \sin^2 A}$ [$a^3 - b^3 = (a-b)(a^2 + b^2 + ab)$] — ①

RHS $= \tan A \sec A - \cot A \operatorname{cosec} A$

$= \frac{\sin A}{\cos A}\cdot\frac{1}{\cos A} - \frac{\cos A}{\sin A}\cdot\frac{1}{\sin A}$

$= \frac{\sin A}{\cos^2 A} - \frac{\cos A}{\sin^2 A} = \frac{\sin^3 A - \cos^3 A}{\cos^2 A \sin^2 A}$ — ②

From ① & ②

LHS = RHS

Hence, proved

Q.70 $\frac{\cos A \operatorname{cosec} A - \sin A \sec A}{\cos A + \sin A} = \operatorname{cosec} A - \sec A$

LHS $= \frac{\cos A \operatorname{cosec} A - \sin A \sec A}{\cos A + \sin A}$

$$= \frac{\frac{\cos A}{\sin A} \cdot \frac{1}{\cos A} - \frac{\sin A}{\cos A} \cdot \frac{1}{\cos A}}{\cos A + \sin A}$$

$$= \frac{\frac{\cos A}{\sin A} - \frac{\sin A}{\cos A}}{\cos A + \sin A}$$

$$= \frac{\cos^2 A - \sin^2 A}{\sin A \cos A (\cos A + \sin A)}$$

$$= \frac{(\cos A + \sin A)(\cos A - \sin A)(\cos A - \sin A)}{\sin A \cos A (\cos A + \sin A)}$$

$$= \frac{\cos A - \sin A}{\sin A \cos A}$$

$$= \frac{\cos A}{\sin A \cos A} - \frac{\sin A}{\sin A \cos A}$$

$$= \frac{1}{\sin A} - \frac{1}{\cos A}$$

$$= \boxed{\operatorname{cosec} A - \sec A} = \text{RHS}$$

Hence proved

Q.71 $\dfrac{\sin A}{\sec A + \tan A - 1} + \dfrac{\cos A}{\operatorname{cosec} A + \cot A - 1} = 1$

LHS $= \dfrac{\sin A}{\sec A + \tan A - 1} + \dfrac{\cos A}{\operatorname{cosec} A + \cot A - 1}$

$$= \frac{\sin A}{\frac{1}{\cos A} + \frac{\sin A}{\cos A} - 1} + \frac{\cos A}{\frac{1}{\sin A} + \frac{\cos A}{\sin A} - 1}$$

$$= \frac{\sin A}{\frac{1 + \sin A - \cos A}{\cos A}} + \frac{\cos A}{\frac{1 + \cos A - \sin A}{\sin A}}$$

$$= \frac{\sin A \cos A}{1+\sin A-\cos A} + \frac{\sin A \cos A}{1+\cos A-\sin A}$$

$$= \sin A \cos A \left(\frac{1}{1+\sin A-\cos A} + \frac{1}{1+\cos A-\sin A} \right)$$

$$= \sin A \cos A \left[\frac{1+\cos A-\sin A+1+\sin A-\cos A}{(1+\sin A-\cos A)(1+\cos A-\sin A)} \right]$$

$$= \sin A \cos A \left(\frac{2}{1+\cos A-\sin A+\sin A+\sin A \cos A-\sin^2 A-\cos A-\cos^2 A+\cos A \sin A} \right)$$

$$= \sin A \cos A \left(\frac{2}{1-\sin^2 A-\cos^2 A+2\sin A \cos A} \right)$$

$$= \sin A \cos A \left[\frac{2}{1-(\sin^2 A+\cos^2 A)+2\sin A \cos A} \right]$$

$$= \sin A \cos A \left[\frac{2}{1-1+2\sin A \cos A} \right] \quad [\sin^2 A+\cos^2 A=1]$$

$$= \sin A \cos A \left[\frac{2}{2\sin A \cos A} \right]$$

$$= \boxed{1} = \text{RHS}$$

Hence, proved

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Exercise 11.1

Q.73 $\sec^4 A (1-\sin^4 A) - 2\tan^2 A = 1$

LHS $= \sec^4 A (1-\sec^4 A) - 2\tan^2 A$
$= \sec^4 A (-\tan^4 A) - 2\tan^2 A$ $\quad (1-\sec^2\theta = -\tan^2\theta)$
$= (\sec^2 A)^2 - \tan^4 A - 2\tan^2 A$
$= (1+$

LHS $= \sec^4 A (1-\sin^4 A) - 2\tan^2 A$
$= \sec^4 A (1-\sin^2 A)(1+\sin^2 A) - 2\tan^2 A$
$= \sec^4 A (\cos^2 A)(1+\sin^2 A) - 2\tan^2 A$ $\quad [\sin^2\theta + \cos^2\theta = 1]$
$= \frac{1}{\cos^4 A} \cos^2 A (1+\sin^2 A) - 2\tan^2 A$
$= \sec^2 A + \sin^2 A \sec^2 A - 2\tan^2 A$
$= \sec^2 A + \sin^2 A \frac{1}{\cos^2 A} - 2\tan^2 A$
$= \sec^2 A + \tan^2 A - 2\tan^2 A$
$= \sec^2 A - \tan^2 A$ $\quad [\sec^2 A = 1+\tan^2 A]$
$= \boxed{1} = RHS$

Hence, proved

Q.74 $\frac{\cot^2 A (\sec A - 1)}{1+\sin A} = \sec^2 A \left[\frac{1-\sin A}{1+\sec A}\right]$

LHS $= \frac{\cot^2 A (\sec A - 1)}{1+\sin A}$
$= \frac{\frac{\cos^2 A}{\sin^2 A}\left(\frac{1}{\cos A} - 1\right)}{1+\sin A} = \frac{\frac{\cos^2 A}{\sin^2 A}\left(\frac{1-\cos A}{\cos A}\right)}{1+\sin A}$

$$= \frac{\cos^2 A}{1-\cos^2 A}\left(\frac{1-\cos A}{\cos A}\right)$$

$$1+\sin A$$

$$= \frac{\cos A}{(1+\cos A)(1-\cos A)}(1-\cos A)$$

$$= \frac{\cos A}{(1+\cos A)(1+\sin A)}(1+\sin A) \quad \text{---- (1)}$$

RHS $= \sec^2 A\left[\frac{1-\sin A}{1+\sec A}\right]$

$$= \frac{1}{\cos^2 A}\left[\frac{1-\sin A}{1+\frac{1}{\cos A}}\right]$$

$$= \frac{1}{\cos^2 A}\left[\frac{1-\sin A}{\frac{\cos A+1}{\cos A}}\right]$$

$$= \frac{1}{\cos^2 A}\left[\frac{(1-\sin A)\cos A}{\cos A+1}\right]$$

$$= \frac{1-\sin A}{\cos A(1+\cos A)}$$

Multiply by $(1+\sin A)$

$$= \frac{(1-\sin A)(1+\sin A)}{\cos A(1+\cos A)(1+\sin A)}$$

$$= \frac{1-\sin^2 A}{\cos A(1+\cos A)(1+\sin A)}$$

$$= \frac{\cos^2 A}{\cos A(1+\cos A)(1+\sin A)}$$

$$= \frac{\cos A}{(1+\cos A)(1+\sin A)} \quad \text{---- (2)}$$

From ① & ②

LHS = RHS

Hence, proved

Q.75 $(1+\cot A+\tan A)(\sin A-\cos A) = \frac{\sec A}{\operatorname{cosec}^2 A} - \frac{\operatorname{cosec} A}{\sec^2 A} =$

$\sin A \tan A - \cot A \cos A$

① $(1+\cot A+\tan A)(\sin A-\cos A)$

$= \sin A + \cot A \sin A + \sin A \tan A - \cos A - \cot A \cos A - \tan A \cos A$

$= \sin A - \cos A + \frac{\cos A}{\sin A} \times \sin A \cot A \cos A + \sin A \tan A - \frac{\sin A}{\cos A} \times \cos A$

$= \sin A - \cos A + \cos A - \cot A \cos A + \sin A \tan A - \sin A$

$= \boxed{\sin A \tan A - \cot A \cos A}$ ——— ①

② $\frac{\sec A}{\operatorname{cosec}^2 A} - \frac{\operatorname{cosec} A}{\sec^2 A}$

$= \frac{\frac{1}{\cos A}}{\frac{1}{\sin^2 A}} - \frac{\frac{1}{\sin A}}{\frac{1}{\cos^2 A}} = \frac{\sin^2 A}{\cos A} - \frac{\cos^2 A}{\sin A}$

$= \sin A \left(\frac{\sin A}{\cos A}\right) - \cos A \left(\frac{\cos A}{\sin A}\right)$

$= \boxed{\sin A \tan A - \cos A \cot A}$ ——— ②

③ $\sin A \tan A - \cot A \cos A$ ——— ③

From ①, ②, ③

All are same.

Hence, proved

Q. 76 $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$ ——— (1)

$\frac{x}{a}\sin\theta - \frac{y}{b}\cos\theta = 1$ ——— (2) } Given

To prove – $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2$

Proof – Squaring (1) & (2) and then add.

$$\left(\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta\right)^2 + \left(\frac{x}{a}\sin\theta - \frac{y}{b}\cos\theta\right)^2 = (1)^2 + (1)^2$$

$$\Rightarrow \frac{x^2}{a^2}\cos^2\theta + \frac{y^2}{b^2}\sin^2\theta + 2\frac{x}{a}\cos\theta\frac{y}{b}\sin\theta + \frac{x^2}{a^2}\sin^2\theta + \frac{y^2}{b^2}\cos^2\theta - 2\frac{x}{a}\sin\theta\frac{y}{b}\cos\theta = 1+1$$

$$\Rightarrow \frac{x^2}{a^2}\cos^2\theta + \frac{x^2}{a^2}\sin^2\theta + \frac{y^2}{b^2}\sin^2\theta + \frac{y^2}{b^2}\cos^2\theta = 2$$

$$\Rightarrow \frac{x^2}{a^2}\left(\sin^2\theta + \cos^2\theta\right) + \frac{y^2}{b^2}\left(\sin^2\theta + \cos^2\theta\right) = 2$$

$$\Rightarrow \frac{x^2}{a^2}(1) + \frac{y^2}{b^2}(1) = 2 \qquad [\sin^2\theta + \cos^2\theta = 1]$$

$$\Rightarrow \boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2}$$

Hence proved

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Exercise 11.1

Q.11 $\operatorname{cosec}\theta - \sin\theta = a^3$
$\sec\theta - \cos\theta = b^3$ } Given

To prove :- $a^2b^2(a^2+b^2) = 1$

Proof :- $\operatorname{cosec}\theta - \sin\theta = a^3$

$\Rightarrow \frac{1}{\sin\theta} - \sin\theta = a^3$

$\Rightarrow \frac{1-\sin^2\theta}{\sin\theta} = a^3$ [$\sin\theta + \cos\theta = 1$]

$\Rightarrow \frac{\cos^2\theta}{\sin\theta} = a^3$

$\Rightarrow a = \left(\frac{\cos^2\theta}{\sin\theta}\right)^{1/3}$

$\sec\theta - \cos\theta = b^3$

$\Rightarrow \frac{1}{\cos\theta} - \cos\theta = b^3$

$\Rightarrow \frac{1-\cos^2\theta}{\cos\theta} = b^3$

$\Rightarrow \frac{\sin^2\theta}{\cos\theta} = b^3$

$\Rightarrow b = \left(\frac{\sin^2\theta}{\cos\theta}\right)^{1/3}$

LHS $= a^2b^2(a^2+b^2)$

$= a^4b^2 + a^2b^4$

$= \left(\frac{\cos^2\theta}{\sin\theta}\right)^{4/3}\left(\frac{\sin^2\theta}{\cos\theta}\right)^{2/3} + \left(\frac{\cos^2\theta}{\sin\theta}\right)^{2/3}\left(\frac{\sin^2\theta}{\cos\theta}\right)^{4/3}$

$= (\cos\theta)^{8/3-2/3} \times (\sin\theta)^{4/3-4/3} + (\sin\theta)^{8/3-2/3} \times (\cos\theta)^{4/3-4/3}$

$= \cos^2\theta + \sin^2\theta$ $\quad [\sin^2\theta + \cos^2\theta = 1]$

$= \boxed{1} = \text{RHS}$

Hence, proved

Q.78 $a\cos^3\theta + 3a\cos\theta\sin^2\theta = m$ —①
$a\sin^3\theta + 3a\cos^2\theta\sin\theta = n$ —②] Given

To prove – $(m+n)^{2/3} + (m-n)^{2/3} = 2a^{2/3}$

Proof – $m+n = a\cos^3\theta + 3a\cos\theta\sin^2\theta + a\sin^3\theta + 3a\cos^2\theta\sin\theta$

$= a(\cos^3\theta + \sin^3\theta) + 3a\cos\theta\sin\theta(\cos\theta + \sin\theta)$

$= a(\cos\theta + \sin\theta)^3$ $\quad [(a+b)^3 = a^3 + b^3 + 3ab(a+b)]$

Similarly,

$m-n = a(\cos\theta - \sin\theta)^3$

LHS =

$(m+n)^{2/3} + (m-n)^{2/3}$

$= [a(\cos\theta+\sin\theta)^3]^{2/3} + [a(\cos\theta-\sin\theta)^3]^{2/3}$

$= a^{2/3}[(\cos\theta+\sin\theta)^2 + (\cos\theta-\sin\theta)^2]$

$= a^{2/3}[\cos^2\theta + \sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta + \sin^2\theta - 2\sin\theta\cos\theta]$

$= a^{2/3}[2(\sin^2\theta + \cos^2\theta)]$

$= a^{2/3}[2(1)]$ $\quad [\sin^2\theta + \cos^2\theta = 1]$

$= \boxed{2a^{2/3}} = \text{RHS}$

Hence, proved

Q.79 $x = a\cos^3\theta$
$y = b\sin^3\theta$] given

To prove – $\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3} = 1$

Proof :- LHS $= \left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3}$

$= \left(\frac{a\cos^3\theta}{a}\right)^{2/3} + \left(\frac{b\sin^3\theta}{b}\right)^{2/3}$ [given]

$= \cos^2\theta + \sin^2\theta$ [$\cos^2\theta + \sin^2\theta = 1$]

$= \boxed{1}$ = RHS

Hence, proved

Q.80 $a\cos\theta + b\sin\theta = m$ — (1)
$a\sin\theta - b\cos\theta = n$ — (2)] given

To prove – $a^2 + b^2 = m^2 + n^2$

Squaring both given eq.

$\therefore (a\cos\theta + b\sin\theta)^2 = m^2$

$\Rightarrow a^2\cos^2\theta + b^2\sin^2\theta + 2ab\cos\theta\sin\theta = m^2$ —— (3)

$\therefore (a\sin\theta - b\cos\theta)^2 = n^2$

$\Rightarrow a^2\sin^2\theta + b^2\cos^2\theta - 2ab\sin\theta\cos\theta = n^2$ —— (4)

Add (3) & (4)

$a^2\cos^2\theta + b^2\sin^2\theta + 2ab\cos\theta\sin\theta + a\sin^2\theta + b^2\cos^2\theta - 2ab\sin\theta\cos\theta = m^2 + n^2$

$a^2(\cos^2\theta + \sin^2\theta) + b^2(\sin^2\theta + \cos^2\theta) = m^2 + n^2$

$a^2(1) + b^2(1) = m^2 + n^2$ [$\sin^2\theta + \cos^2\theta = 1$]

$\boxed{a^2 + b^2 = m^2 + n^2}$

Hence, proved