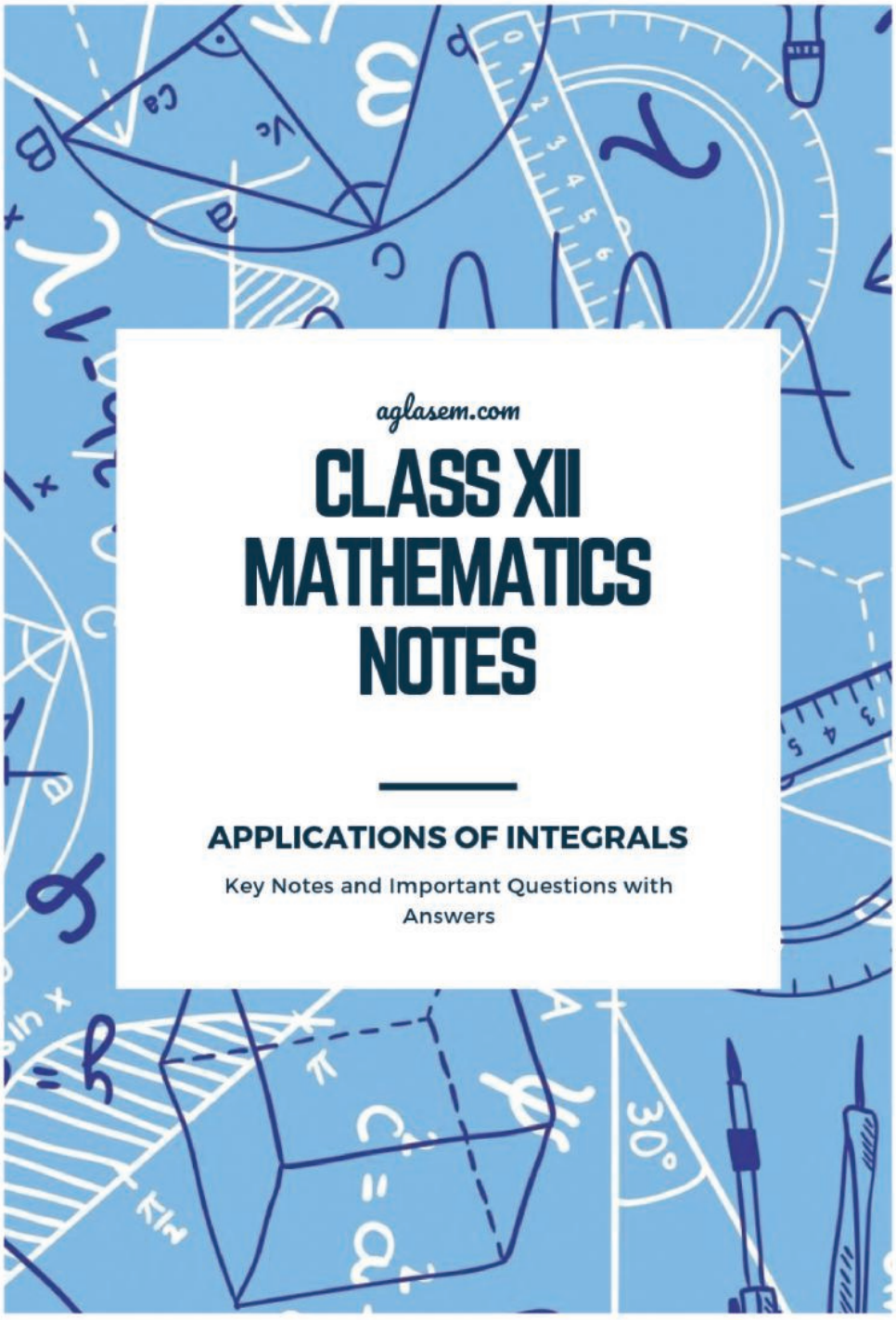


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CLASS XII MATHEMATICS NOTES

APPLICATIONS OF INTEGRALS

Key Notes and Important Questions with Answers

CHAPTER 8

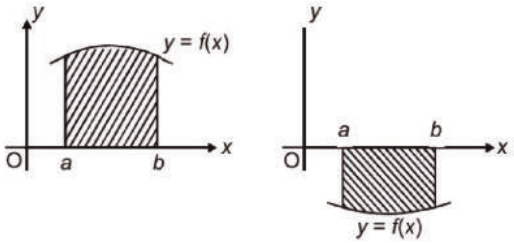
APPLICATIONS OF INTEGRALS

POINT TO REMEMBER

AREA OF BOUNDED REGION

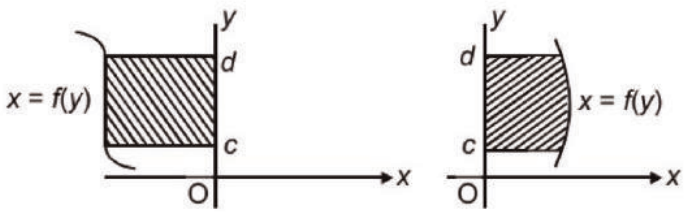
- Area bounded by the curve $y = f(x)$, the x axis and between the ordinates, $x = a$ and $x = b$ is given by

$$Area = \left| \int_a^b f(x)dx \right|$$

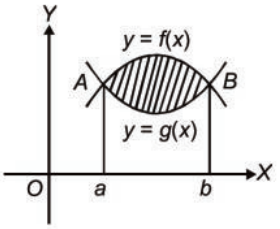


- Area bounded by the curve $x = f(y)$, the y-axis and between the abscissas, y = c and y = d is given by

$$Area = \left| \int_c^d f(y)dy \right|$$

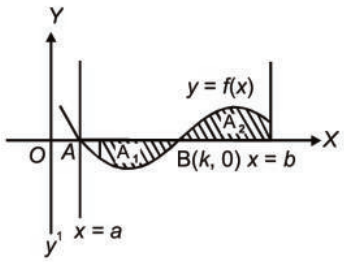


- Area bounded by two curves $y = f(x)$ and $y = g(x)$ such that $0 \le g(x) \le f(x)$ for all $x \in [a, b]$ and between the ordinates $x = a$ and $x = b$ is given by



$$\text{Area} = \int_a^b [f(x) - g(x)]dx$$

- Area of the following shaded region = $\left|\int_a^k f(x)dx\right| + \int_k^b f(x)dx$



LONG ANSWER TYPE QUESTIONS (6 MARKS)

1. Find the area of the parabola $y^2 = 4ax$ bounded by its Latus rectum.

2. Find the area of the region $\{(x, y): x^2 \le y \le |x|\}$.

3. Find the area of region in the first quadrant enclosed by x–axis, the line $y = x$ and the circle $x^2 + y^2 = 32$.

4. Find the area of region $\{(x, y) : y^2 \leq 4x, 4x^2 + 4y^2 \leq 9\}$

5. Prove that the curve $y = x^2$ and, $x = y^2$ divide the square bounded by $x = 0$, $y = 0$, $x = 1$, $y = 1$ into three equal parts.

6. Find the area of the smaller region enclosed between ellipse $b^2x^2 + a^2y^2 = a^2b^2$ and the line $bx + ay = ab$.

7. Find the common area bounded by the circles $x^2 + y^2 = 4$ and $(x - 2)^2 + y^2 = 4$.

8. Using integration, find the area of the triangle whose sides are given by $2x + y = 4,\ 3x - 2y = 6\ and\ x - 3y + 5 = 0$.

9. Using integration, find the area of the triangle whose vertices are (–1, 0), (1, 3) and (3, 2).

10. Find the area of the region $\{(x, y) : x^2 + y^2 \leq 1 \leq x + y\}$.

11. Find the area of the region bounded by the curve $x^2 = 4y$ and the line $x = 4y - 2$.

12. Find the area lying above x-axis and included between the circle $x^2 + y^2 = 8x$ and inside the parabola $y^2 = 4x$.

13. Using integration, find the area enclosed by the curve $y = \cos x$, $y = \sin x$ and x-axis in the interval $[0, \pi/2]$.

14. Using integration, find the area of the following region: $\{(x, y) : |x - 1| \leq y \leq \sqrt{5 - x^2}\}$

15. Using integration, find the area of the triangle formed by positive x-axis and tangent and normal to the circle $x^2 + y^2 = 4$ at $(1, \sqrt{3})$.

16. Using integration, find the area of the region bounded by the line $x - y + 2 = 0$, the curve $x = \sqrt{y}$ and y-axis.

17. Find the area of the region bounded by the curves $ay^2 = x^3$, the y-axis and the lines y = a and y = 2a.

18. Find the area bounded by x-axis, the curve $y = 2x^2$ and tangent to the curve at the point whose abscissa is 2.

19. Using integration, find the area of the region bounded by the curve$y = 1 + |x + 1|$ and lines $x = -3, x = 3, y = 0$.

20. Find the area of the region $\{(x, y): y^2 \geq 6x,\ x^2 + y^2 \leq 16\}$

21. Find the area of the region enclosed between curves $y = |x - 1|$ and$y = 3 - |x|$.

22. If the area bounded by the parabola $y^2 = 16$ ax and the line y = 4 mx is $\frac{a^2}{12}$sq unit then using integration find the value of m.

ANSWERS

1. $\frac{8}{3}a^2$ sq. units
2. $\frac{1}{3}$ sq. units
3. 4π sq. units
4. $\left[\frac{\sqrt{2}}{6} + \frac{9\pi}{8} - \frac{9}{4}\sin^{-1}\left(\frac{1}{3}\right)\right]$ sq. units
5. ....
6. $\left(\frac{\pi-2}{4}\right)$ab sq. units
7. $\left(\frac{8\pi}{3} - 2\sqrt{3}\right)$ sq. units
8. 3.5 sq. units
9. 4 sq. units
10. $\left(\pi - \frac{1}{2}\right)$ sq. units
11. $\frac{9}{8}$ sq. units
12. $\frac{4}{3}(8 + 3\pi)$ sq. units
13. $\left(2 - \sqrt{2}\right)$ sq. units
14. $\left(\frac{5\pi}{4} - \frac{1}{2}\right)$ sq. units
15. $2\sqrt{3}$ sq. units
16. $\frac{10}{3}$ sq. units
17. $\frac{3}{5}a^2\left[(32)^{\frac{1}{3}} - 1\right]$ sq. units

18. $\frac{4}{3}$ sq. units

19. 16 sq. units

20. $\frac{32\pi-4\sqrt{3}}{3}$ sq. units

21. 4 sq. units

22. m = 2