

CHAPTER – 23

HEAT AND TEMPERATURE

EXERCISES

1. Ice point = 20° (L_0) $L_1 = 32^\circ$

Steam point = 80° (L_{100})

$$T = \frac{L_1 - L_0}{L_{100} - L_0} \times 100 = \frac{32 - 20}{80 - 20} \times 100 = 20^\circ\text{C}$$

2. $P_{tr} = 1.500 \times 10^4 \text{ Pa}$

$$P = 2.050 \times 10^4 \text{ Pa}$$

We know, For constant volume gas Thermometer

$$T = \frac{P}{P_{tr}} \times 273.16 \text{ K} = \frac{2.050 \times 10^4}{1.500 \times 10^4} \times 273.16 = 373.31$$

3. Pressure Measured at M.P = $2.2 \times$ Pressure at Triple Point

$$T = \frac{P}{P_{tr}} \times 273.16 = \frac{2.2 \times P_{tr}}{P_{tr}} \times 273.16 = 600.952 \text{ K} \approx 601 \text{ K}$$

4. $P_{tr} = 40 \times 10^3 \text{ Pa}$, $P = ?$

$$T = 100^\circ\text{C} = 373 \text{ K}, \quad T = \frac{P}{P_{tr}} \times 273.16 \text{ K}$$

$$\Rightarrow P = \frac{T \times P_{tr}}{273.16} = \frac{373 \times 40 \times 10^3}{273.16} = 54620 \text{ Pa} = 5.462 \times 10^4 \text{ Pa} \approx 55 \text{ K Pa}$$

5. $P_1 = 70 \text{ K Pa}$,

$$P_2 = ?$$

$$T_1 = 273 \text{ K},$$

$$T_2 = 373 \text{ K}$$

$$T = \frac{P_1}{P_{tr}} \times 273.16$$

$$\Rightarrow 273 = \frac{70 \times 10^3}{P_{tr}} \times 273.16$$

$$\Rightarrow P_{tr} = \frac{70 \times 273.16 \times 10^3}{273}$$

$$T_2 = \frac{P_2}{P_{tr}} \times 273.16$$

$$\Rightarrow 373 = \frac{P_2 \times 273}{70 \times 273.16 \times 10^3}$$

$$\Rightarrow P_2 = \frac{373 \times 70 \times 10^3}{273} = 95.6 \text{ K Pa}$$

6. $P_{\text{ice point}} = P_{0^\circ} = 80 \text{ cm of Hg}$

$$P_{\text{steam point}} = P_{100^\circ} = 90 \text{ cm of Hg}$$

$$P_0 = 100 \text{ cm}$$

$$t = \frac{P - P_0}{P_{100} - P_0} \times 100^\circ = \frac{80 - 100}{90 - 100} \times 100 = 200^\circ\text{C}$$

7. $T' = \frac{V}{V - V'} T_0$ $T_0 = 273,$

$$V = 1800 \text{ CC}, \quad V' = 200 \text{ CC}$$

$$T' = \frac{1800}{1600} \times 273 = 307.125 \approx 307$$

8. $R_t = 86\Omega$; $R_{0^\circ} = 80\Omega$; $R_{100^\circ} = 90\Omega$

$$t = \frac{R_t - R_0}{R_{100} - R_0} \times 100 = \frac{86 - 80}{90 - 80} \times 100 = 60^\circ\text{C}$$

9. R at ice point (R_0) = 20Ω

$$R \text{ at steam point } (R_{100}) = 27.5\Omega$$

$$R \text{ at Zinc point } (R_{420}) = 50\Omega$$

$$R_\theta = R_0 (1 + \alpha\theta + \beta\theta^2)$$

$$\Rightarrow R_{100} = R_0 + R_0 \alpha\theta + R_0 \beta\theta^2$$

$$\Rightarrow \frac{R_{100} - R_0}{R_0} = \alpha\theta + \beta\theta^2$$

$$\Rightarrow \frac{27.5 - 20}{20} = \alpha \times 100 + \beta \times 10000$$

$$\Rightarrow \frac{7.5}{20} = 100\alpha + 10000\beta$$

$$R_{420} = R_0 (1 + \alpha\theta + \beta\theta^2) \Rightarrow \frac{50 - R_0}{R_0} = \alpha\theta + \beta\theta^2$$

$$\Rightarrow \frac{50 - 20}{20} = 420 \times \alpha + 176400 \times \beta \quad \Rightarrow \frac{3}{2} = 420 \alpha + 176400 \beta$$

$$\Rightarrow \frac{7.5}{20} = 100 \alpha + 10000 \beta \quad \Rightarrow \frac{3}{2} = 420 \alpha + 176400 \beta$$

10. $L_1 = ?$, $L_0 = 10 \text{ m}$, $\alpha = 1 \times 10^{-5} / ^\circ\text{C}$, $t = 35$

$$L_1 = L_0 (1 + \alpha t) = 10(1 + 10^{-5} \times 35) = 10 + 35 \times 10^{-4} = 10.0035 \text{ m}$$

11. $t_1 = 20^\circ\text{C}$, $t_2 = 10^\circ\text{C}$, $L_1 = 1 \text{ cm} = 0.01 \text{ m}$, $L_2 = ?$

$$\alpha_{\text{steel}} = 1.1 \times 10^{-5} / ^\circ\text{C}$$

$$L_2 = L_1 (1 + \alpha_{\text{steel}} \Delta T) = 0.01(1 + 101 \times 10^{-5} \times 10) = 0.01 + 0.01 \times 1.1 \times 10^{-4} \\ = 10^4 \times 10^{-6} + 1.1 \times 10^{-6} = 10^{-6} (10000 + 1.1) = 10001.1 \\ = 1.00011 \times 10^{-2} \text{ m} = 1.00011 \text{ cm}$$

12. $L_0 = 12 \text{ cm}$, $\alpha = 11 \times 10^{-5} / ^\circ\text{C}$

$$t_w = 18^\circ\text{C} \quad t_s = 48^\circ\text{C}$$

$$L_w = L_0(1 + \alpha t_w) = 12(1 + 11 \times 10^{-5} \times 18) = 12.002376 \text{ m}$$

$$L_s = L_0(1 + \alpha t_s) = 12(1 + 11 \times 10^{-5} \times 48) = 12.006336 \text{ m}$$

$$\Delta L = 12.006336 - 12.002376 = 0.00396 \text{ m} \approx 0.4 \text{ cm}$$

13. $d_1 = 2 \text{ cm} = 2 \times 10^{-2}$

$$t_1 = 0^\circ\text{C}, \quad t_2 = 100^\circ\text{C}$$

$$\alpha_{\text{al}} = 2.3 \times 10^{-5} / ^\circ\text{C}$$

$$d_2 = d_1 (1 + \alpha \Delta t) = 2 \times 10^{-2} (1 + 2.3 \times 10^{-5} \times 10^2) \\ = 0.02 + 0.000046 = 0.020046 \text{ m} = 2.0046 \text{ cm}$$

14. $L_{\text{st}} = L_{\text{Al}}$ at 20°C $\alpha_{\text{Al}} = 2.3 \times 10^{-5} / ^\circ\text{C}$

$$\text{So, } L_{\text{st}}(1 - \alpha_{\text{st}} \times 20) = L_{\text{Al}}(1 - \alpha_{\text{Al}} \times 20) \quad \alpha_{\text{st}} = 1.1 \times 10^{-5} / ^\circ\text{C}$$

$$(a) \Rightarrow \frac{L_{\text{st}}}{L_{\text{Al}}} = \frac{(1 - \alpha_{\text{Al}} \times 20)}{(1 - \alpha_{\text{st}} \times 20)} = \frac{1 - 2.3 \times 10^{-5} \times 20}{1 - 1.1 \times 10^{-5} \times 20} = \frac{0.99954}{0.99978} = 0.999$$

$$(b) \Rightarrow \frac{L_{40\text{st}}}{L_{40\text{Al}}} = \frac{(1 - \alpha_{\text{Al}} \times 40)}{(1 - \alpha_{\text{st}} \times 40)} = \frac{1 - 2.3 \times 10^{-5} \times 20}{1 - 1.1 \times 10^{-5} \times 20} = \frac{0.99954}{0.99978} = 0.999$$

$$= \frac{L_{\text{Al}}}{L_{\text{st}}} \times \frac{1 + 2.3 \times 10^{-5} \times 10}{273} = \frac{0.99977 \times 1.00092}{1.00044} = 1.0002496 \approx 1.00025$$

$$\frac{L_{100\text{Al}}}{L_{100\text{St}}} = \frac{(1 + \alpha_{\text{Al}} \times 100)}{(1 + \alpha_{\text{st}} \times 100)} = \frac{0.99977 \times 1.00092}{1.00011} = 1.00096$$

15. (a) Length at $16^\circ\text{C} = L$

$$L = ? \quad T_1 = 16^\circ\text{C}, \quad T_2 = 46^\circ\text{C}$$

$$\alpha = 1.1 \times 10^{-5} / ^\circ\text{C}$$

$$\Delta L = L \alpha \Delta \theta = L \times 1.1 \times 10^{-5} \times 30$$

$$\% \text{ of error} = \left(\frac{\Delta L}{L} \times 100 \right) \% = \left(\frac{L \alpha \Delta \theta}{L} \times 100 \right) \% = 1.1 \times 10^{-5} \times 30 \times 100 \% = 0.033 \%$$

(b) $T_2 = 6^\circ\text{C}$

$$\% \text{ of error} = \left(\frac{\Delta L}{L} \times 100 \right) \% = \left(\frac{L \alpha \Delta \theta}{L} \times 100 \right) \% = -1.1 \times 10^{-5} \times 10 \times 100 = -0.011 \%$$

16. $T_1 = 20^\circ\text{C}$, $\Delta L = 0.055 \text{ mm} = 0.55 \times 10^{-3} \text{ m}$

$$t_2 = ? \quad \alpha_{\text{st}} = 11 \times 10^{-6} / ^\circ\text{C}$$

We know,

$$\Delta L = L_0 \alpha \Delta T$$

In our case,

$$0.055 \times 10^{-3} = 1 \times 11 \times 10^{-6} \times (T_1 + T_2)$$

$$0.055 = 11 \times 10^{-3} \times 20 \pm 11 \times 10^{-3} \times T_2$$

$$T_2 = 20 + 5 = 25^\circ\text{C} \quad \text{or } 20 - 5 = 15^\circ\text{C}$$

$$r_2 = 20 + 0 = 20 \text{ cm} \quad \text{or } 20 - 0 = 20 \text{ cm}$$

The expt. Can be performed from 15 to 25°C

17. $f_{0^\circ\text{C}} = 0.098 \text{ g/m}^3$, $f_{4^\circ\text{C}} = 1 \text{ g/m}^3$

$$f_{0^\circ\text{C}} = \frac{f_{4^\circ\text{C}}}{1 + \gamma \Delta T} \Rightarrow 0.998 = \frac{1}{1 + \gamma \times 4} \Rightarrow 1 + 4\gamma = \frac{1}{0.998}$$

$$\Rightarrow 4 + \gamma = \frac{1}{0.998} - 1 \Rightarrow \gamma = 0.0005 \approx 5 \times 10^{-4}$$

As density decreases $\gamma = -5 \times 10^{-4}$

18. Iron rod Aluminium rod

$$L_{\text{Fe}}$$

$$L_{\text{Al}}$$

$$\alpha_{\text{Fe}} = 12 \times 10^{-6} / ^\circ\text{C}$$

$$\alpha_{\text{Al}} = 23 \times 10^{-6} / ^\circ\text{C}$$

Since the difference in length is independent of temp. Hence the difference always remains constant.

$$L'_{\text{Fe}} = L_{\text{Fe}}(1 + \alpha_{\text{Fe}} \times \Delta T) \quad \dots (1)$$

$$L'_{\text{Al}} = L_{\text{Al}}(1 + \alpha_{\text{Al}} \times \Delta T) \quad \dots (2)$$

$$L'_{\text{Fe}} - L'_{\text{Al}} = L_{\text{Fe}} - L_{\text{Al}} + L_{\text{Fe}} \times \alpha_{\text{Fe}} \times \Delta T - L_{\text{Al}} \times \alpha_{\text{Al}} \times \Delta T$$

$$\frac{L_{\text{Fe}}}{L_{\text{Al}}} = \frac{\alpha_{\text{Al}}}{\alpha_{\text{Fe}}} = \frac{23}{12} = 23 : 12$$

19. $g_1 = 9.8 \text{ m/s}^2$,

$$g_2 = 9.788 \text{ m/s}^2$$

$$T_1 = 2\pi \frac{\sqrt{l_1}}{g_1}$$

$$T_2 = 2\pi \frac{\sqrt{l_2}}{g_2} = 2\pi \frac{\sqrt{l_1(1 + \Delta T)}}{g}$$

$$\alpha_{\text{Steel}} = 12 \times 10^{-6} / ^\circ\text{C}$$

$$T_1 = 20^\circ\text{C}$$

$$T_2 = ?$$

$$T_1 = T_2$$

$$\Rightarrow 2\pi \frac{\sqrt{l_1}}{g_1} = 2\pi \frac{\sqrt{l_1(1 + \Delta T)}}{g_2}$$

$$\Rightarrow \frac{l_1}{g_1} = \frac{l_1(1 + \Delta T)}{g_2}$$

$$\Rightarrow \frac{1}{9.8} = \frac{1 + 12 \times 10^{-6} \times \Delta T}{9.788}$$

$$\Rightarrow \frac{9.788}{9.8} = 1 + 12 \times 10^{-6} \times \Delta T$$

$$\Rightarrow \frac{9.788}{9.8} - 1 = 12 \times 10^{-6} \Delta T$$

$$\Rightarrow \Delta T = \frac{-0.00122}{12 \times 10^{-6}}$$

$$\Rightarrow T_2 - 20 = -101.6$$

$$\Rightarrow T_2 = -101.6 + 20 = -81.6 \approx -82^\circ\text{C}$$

20. Given

$$d_{\text{Si}} = 2.005 \text{ cm},$$

$$d_{\text{Al}} = 2.000 \text{ cm}$$

$$\alpha_{\text{S}} = 11 \times 10^{-6} / ^\circ\text{C}$$

$$\alpha_{\text{Al}} = 23 \times 10^{-6} / ^\circ\text{C}$$

$$d's = 2.005 (1 + \alpha_{\text{S}} \Delta T) \text{ (where } \Delta T \text{ is change in temp.)}$$

$$\Rightarrow d's = 2.005 + 2.005 \times 11 \times 10^{-6} \Delta T$$

$$d'_{\text{Al}} = 2(1 + \alpha_{\text{Al}} \Delta T) = 2 + 2 \times 23 \times 10^{-6} \Delta T$$

The two will slip i.e the steel ball will fall when both the diameters become equal.

So,

$$\Rightarrow 2.005 + 2.005 \times 11 \times 10^{-6} \Delta T = 2 + 2 \times 23 \times 10^{-6} \Delta T$$

$$\Rightarrow (46 - 22.055)10^{-6} \times \Delta T = 0.005$$

$$\Rightarrow \Delta T = \frac{0.005 \times 10^6}{23.945} = 208.81$$

Now $\Delta T = T_2 - T_1 = T_2 - 10^\circ\text{C}$ [$\therefore T_1 = 10^\circ\text{C}$ given]

$$\Rightarrow T_2 = \Delta T + T_1 = 208.81 + 10 = 218.81$$

21. The final length of aluminium should be equal to final length of glass.

Let the initial length of aluminium = l

$$l(1 - \alpha_{\text{Al}}\Delta T) = 20(1 - \alpha_{\text{G}}\Delta T)$$

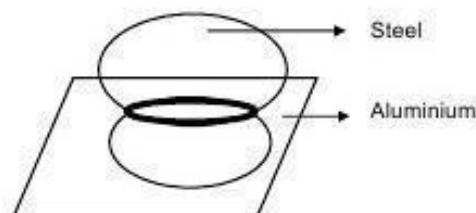
$$\Rightarrow l(1 - 24 \times 10^{-6} \times 40) = 20(1 - 9 \times 10^{-6} \times 40)$$

$$\Rightarrow l(1 - 0.00096) = 20(1 - 0.00036)$$

$$\Rightarrow l = \frac{20 \times 0.99964}{0.99904} = 20.012 \text{ cm}$$

Let initial breadth of aluminium = b

$$b(1 - \alpha_{\text{Al}}\Delta T) = 30(1 - \alpha_{\text{G}}\Delta T)$$



$$\Rightarrow b = \frac{30 \times (1 - 9 \times 10^{-6} \times 40)}{(1 - 24 \times 10^{-6} \times 40)} = \frac{30 \times 0.99964}{0.99904} = 30.018 \text{ cm}$$

22. $V_g = 1000 \text{ CC}, \quad T_1 = 20^\circ\text{C}$
 $V_{Hg} = ? \quad \gamma_{Hg} = 1.8 \times 10^{-4} / ^\circ\text{C}$
 $\gamma_g = 9 \times 10^{-6} / ^\circ\text{C}$

ΔT remains constant

Volume of remaining space = $V'_g - V'_{Hg}$

Now

$$V'_g = V_g(1 + \gamma_g \Delta T) \quad \dots(1)$$

$$V'_{Hg} = V_{Hg}(1 + \gamma_{Hg} \Delta T) \quad \dots(2)$$

Subtracting (2) from (1)

$$V'_g - V'_{Hg} = V_g - V_{Hg} + V_g \gamma_g \Delta T - V_{Hg} \gamma_{Hg} \Delta T$$

$$\Rightarrow \frac{V_g}{V_{Hg}} = \frac{\gamma_{Hg}}{\gamma_g} \Rightarrow \frac{1000}{V_{Hg}} = \frac{1.8 \times 10^{-4}}{9 \times 10^{-6}}$$

$$\Rightarrow V_{Hg} = \frac{9 \times 10^{-3}}{1.8 \times 10^{-4}} = 500 \text{ CC.}$$

23. Volume of water = 500 cm^3

Area of cross section of can = 125 m^2

Final Volume of water

$$= 500(1 + \gamma \Delta \theta) = 500[1 + 3.2 \times 10^{-4} \times (80 - 10)] = 511.2 \text{ cm}^3$$

The aluminium vessel expands in its length only so area expansion of base can be neglected.

Increase in volume of water = 11.2 cm^3

Considering a cylinder of volume = 11.2 cm^3

$$\text{Height of water increased} = \frac{11.2}{125} = 0.089 \text{ cm}$$

24. $V_0 = 10 \times 10 \times 10 = 1000 \text{ CC}$

$\Delta T = 10^\circ\text{C}, \quad V'_{Hg} - V'_g = 1.6 \text{ cm}^3$

$\alpha_g = 6.5 \times 10^{-6} / ^\circ\text{C}, \quad \gamma_{Hg} = ?, \quad \gamma_g = 3 \times 6.5 \times 10^{-6} / ^\circ\text{C}$

$$V'_{Hg} = V_{Hg}(1 + \gamma_{Hg} \Delta T) \quad \dots(1)$$

$$V'_g = V_g(1 + \gamma_g \Delta T) \quad \dots(2)$$

$$V'_{Hg} - V'_g = V_{Hg} - V_g + V_{Hg} \gamma_{Hg} \Delta T - V_g \gamma_g \Delta T$$

$$\Rightarrow 1.6 = 1000 \times \gamma_{Hg} \times 10 - 1000 \times 6.5 \times 3 \times 10^{-6} \times 10$$

$$\Rightarrow \gamma_{Hg} = \frac{1.6 + 6.3 \times 3 \times 10^{-2}}{10000} = 1.789 \times 10^{-4} \approx 1.8 \times 10^{-4} / ^\circ\text{C}$$

25. $f_w = 880 \text{ Kg/m}^3, \quad f_b = 900 \text{ Kg/m}^3$
 $T_1 = 0^\circ\text{C}, \quad \gamma_w = 1.2 \times 10^{-3} / ^\circ\text{C},$
 $\gamma_b = 1.5 \times 10^{-3} / ^\circ\text{C}$

The sphere begins to sink when,

$(mg)_{\text{sphere}} = \text{displaced water}$

$$\Rightarrow V f'_w g = V f'_b g$$

$$\Rightarrow \frac{f_w}{1 + \gamma_w \Delta \theta} = \frac{f_b}{1 + \gamma_b \Delta \theta}$$

$$\Rightarrow \frac{880}{1 + 1.2 \times 10^{-3} \Delta \theta} = \frac{900}{1 + 1.5 \times 10^{-3} \Delta \theta}$$

$$\Rightarrow 880 + 880 \times 1.5 \times 10^{-3} (\Delta \theta) = 900 + 900 \times 1.2 \times 10^{-3} (\Delta \theta)$$

$$\Rightarrow (880 \times 1.5 \times 10^{-3} - 900 \times 1.2 \times 10^{-3}) (\Delta \theta) = 20$$

$$\Rightarrow (1320 - 1080) \times 10^{-3} (\Delta \theta) = 20$$

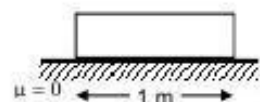
$$\Rightarrow \Delta \theta = 83.3^\circ\text{C} \approx 83^\circ\text{C}$$

26. $\Delta L = 100^\circ\text{C}$

A longitudinal strain develops if and only if, there is an opposition to the expansion.

Since there is no opposition in this case, hence the longitudinal strain here = Zero.

27. $\theta_1 = 20^\circ\text{C}, \quad \theta_2 = 50^\circ\text{C}$



$$\alpha_{\text{steel}} = 1.2 \times 10^{-5} / ^\circ\text{C}$$

Longitudinal strain = ?

$$\text{Strain} = \frac{\Delta L}{L} = \frac{L \alpha \Delta \theta}{L} = \alpha \Delta \theta$$

$$= 1.2 \times 10^{-5} \times (50 - 20) = 3.6 \times 10^{-4}$$

$$28. A = 0.5 \text{ mm}^2 = 0.5 \times 10^{-6} \text{ m}^2$$

$$T_1 = 20^\circ\text{C}, T_2 = 0^\circ\text{C}$$

$$\alpha_s = 1.2 \times 10^{-5} / ^\circ\text{C}, Y = 2 \times 10^{11} \text{ N/m}^2$$

$$\text{Decrease in length due to compression} = L \alpha \Delta \theta \quad \dots (1)$$

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{F}{A} \times \frac{L}{\Delta L} \Rightarrow \Delta L = \frac{FL}{AY} \quad \dots (2)$$

Tension is developed due to (1) & (2)

Equating them,

$$L \alpha \Delta \theta = \frac{FL}{AY} \Rightarrow F = \alpha \Delta \theta AY$$

$$= 1.2 \times 10^{-5} \times (20 - 0) \times 0.5 \times 10^{-6} \times 2 \times 10^{11} = 24 \text{ N}$$

$$29. \theta_1 = 20^\circ\text{C}, \theta_2 = 100^\circ\text{C}$$

$$A = 2 \text{ mm}^2 = 2 \times 10^{-6} \text{ m}^2$$

$$\alpha_{\text{steel}} = 12 \times 10^{-6} / ^\circ\text{C}, Y_{\text{steel}} = 2 \times 10^{11} \text{ N/m}^2$$

Force exerted on the clamps = ?

$$\left(\frac{F}{A} \right)_{\text{Strain}} = Y \Rightarrow F = \frac{Y \times \Delta L}{L} \times L = \frac{Y L \alpha \Delta \theta A}{L} = Y A \alpha \Delta \theta$$

$$= 2 \times 10^{11} \times 2 \times 10^{-6} \times 12 \times 10^{-6} \times 80 = 384 \text{ N}$$

$$30. \text{ Let the final length of the system at system of temp. } 0^\circ\text{C} = \ell_0$$

$$\text{Initial length of the system} = \ell_0$$

When temp. changes by θ .

$$\text{Strain of the system} = \ell_1 - \frac{\ell_0}{\ell_0}$$

$$\text{But the total strain of the system} = \frac{\text{total stress of system}}{\text{total young's modulus of system}}$$

Now, total stress = Stress due to two steel rod + Stress due to Aluminium

$$= \gamma_s \alpha_s \theta + \gamma_s ds \theta + \gamma_{al} \text{ at } \theta = 2\gamma_s \alpha_s \theta + \gamma_{al} \text{ at } \theta$$

$$\text{Now young's modulus of system} = \gamma_s + \gamma_s + \gamma_{al} = 2\gamma_s + \gamma_{al}$$

$$\therefore \text{Strain of system} = \frac{2\gamma_s \alpha_s \theta + \gamma_{al} \alpha_{al} \theta}{2\gamma_s + \gamma_{al}}$$

$$\Rightarrow \frac{\ell_0 - \ell_0}{\ell_0} = \frac{2\gamma_s \alpha_s \theta + \gamma_{al} \alpha_{al} \theta}{2\gamma_s + \gamma_{al}}$$

$$\Rightarrow \ell_0 = \ell_0 \left[\frac{1 + \alpha_{al} \gamma_{al} + 2\alpha_s \gamma_s \theta}{\gamma_{al} + 2\gamma_s} \right]$$

$$31. \text{ The ball tries to expand its volume. But it is kept in the same volume. So it is kept at a constant volume.}$$

So the stress arises

$$\frac{P}{\left(\frac{\Delta V}{V} \right)} = B \Rightarrow P = B \frac{\Delta V}{V} = B \times \gamma \Delta \theta$$

$$= B \times 3\alpha \Delta \theta = 1.6 \times 10^{11} \times 10^{-6} \times 3 \times 12 \times 10^{-6} \times (120 - 20) = 57.6 \times 19^7 \approx 5.8 \times 10^8 \text{ pa.}$$

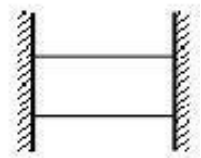
$$32. \text{ Given}$$

$$I_0 = \text{Moment of Inertia at } 0^\circ\text{C}$$

$$\alpha = \text{Coefficient of linear expansion}$$

$$\text{To prove, } I = I_0 (1 + 2\alpha \theta)$$

$$\text{Let the temp. change to } \theta \text{ from } 0^\circ\text{C}$$



Steel
Aluminium
Steel

$$\Delta T = \theta$$

Let 'R' be the radius of Gyration,

$$\text{Now, } R' = R(1 + \alpha\theta), \quad I_0 = MR^2 \quad \text{where M is the mass.}$$

$$\text{Now, } I' = MR'^2 = MR^2(1 + \alpha\theta)^2 \approx MR^2(1 + 2\alpha\theta)$$

[By binomial expansion or neglecting $\alpha^2 \theta^2$ which given a very small value.]

$$\text{So, } I = I_0(1 + 2\alpha\theta) \quad (\text{proved})$$

33. Let the initial m.I. at 0°C be I_0

$$T = 2\pi \sqrt{\frac{I}{K}}$$

$$I = I_0(1 + 2\alpha\Delta\theta) \quad (\text{from above question})$$

$$\text{At } 5^\circ\text{C}, \quad T_1 = 2\pi \sqrt{\frac{I_0(1 + 2\alpha\Delta\theta)}{K}} = 2\pi \sqrt{\frac{I_0(1 + 2\alpha 5)}{K}} = 2\pi \sqrt{\frac{I_0(1 + 10\alpha)}{K}}$$

$$\text{At } 45^\circ\text{C}, \quad T_2 = 2\pi \sqrt{\frac{I_0(1 + 2\alpha 45)}{K}} = 2\pi \sqrt{\frac{I_0(1 + 90\alpha)}{K}}$$

$$\frac{T_2}{T_1} = \sqrt{\frac{1 + 90\alpha}{1 + 10\alpha}} = \sqrt{\frac{1 + 90 \times 2.4 \times 10^{-5}}{1 + 10 \times 2.4 \times 10^{-5}}} = \sqrt{\frac{1.00216}{1.00024}}$$

$$\% \text{ change} = \left(\frac{T_2}{T_1} - 1 \right) \times 100 = 0.0959\% = 9.6 \times 10^{-2}\%$$

34. $T_1 = 20^\circ\text{C}, \quad T_2 = 50^\circ\text{C}, \quad \Delta T = 30^\circ\text{C}$

$$\alpha = 1.2 \times 10^{-5} / ^\circ\text{C}$$

ω remains constant

$$(I) \omega = \frac{V}{R} \quad (II) \omega = \frac{V'}{R'}$$

$$\text{Now, } R' = R(1 + \alpha\Delta\theta) = R + R \times 1.2 \times 10^{-5} \times 30 = 1.00036R$$

From (I) and (II)

$$\frac{V}{R} = \frac{V'}{R'} = \frac{V'}{1.00036R}$$

$$\Rightarrow V' = 1.00036 V$$

$$\% \text{ change} = \frac{(1.00036V - V)}{V} \times 100 = 0.00036 \times 100 = 3.6 \times 10^{-2}$$