UNIT 5

UNDERSTANDING QUADRILATERALS AND PRACTICAL GEOMETRY

(A) Main Concepts and Results

- A simple closed curve made up of only line segments is called a **polygon**.
- A diagonal of a polygon is a line segment connecting two non-consecutive vertices.
- A convex polygon is a polygon in which no portion of its any diagonal is in its exterior.
- A **quadrilateral** is a polygon having only four sides.
- A **regular polygon** is a polygon whose all sides are equal and also all angles are equal.
- The sum of interior angles of a polygon of n sides is $(n\text{-}2)$ straight angles.
- The sum of interior angles of a quadrilateral is 360°.
- The sum of exterior angles, taken in an order, of a polygon is 360°.
- **Trapezium** is a quadrilateral in which a pair of opposite sides is parallel.
- **Kite** is a quadrilateral which has two pairs of equal consecutive sides.
- A **parallelogram** is a quadrilateral in which each pair of opposite sides is parallel.

- A **rhombus** is a parallelogram in which adjacent sides are equal.
- A **rectangle** is a parallelogram in which one angle is of 90^0.
- A **square** is a parallelogram in which adjacent sides are equal and one angle is of 90^0.
- In a parallelogram, opposite sides are equal, opposite angles are equal and diagonals bisect each other.
- In a rhombus diagonals intersect at right angles.
- In a rectangle diagonals are equal.
- Five measurements can determine a quadrilateral uniquely.
- A quadrilateral can be constructed uniquely if the lengths of its four sides and a diagonal are given.
- A quadrilateral can be constructed uniquely if the lengths of its three sides and two diagonals are given.
- A quadrilateral can be constructed uniquely if its two adjacent sides and three angles are given.
- A quadrilateral can be constructed uniquely if its three sides and two included angles are given.

(B) Solved Examples

In examples 1 to 8, there are four options out of which one is correct. Write the correct answer.

Example 1 : The number of diagonals in a polygon of n sides is

(a) $\frac{n(n-1)}{2}$ (b) $\frac{n(n-2)}{2}$ (c) $\frac{n(n-3)}{2}$ (d) $n\,(n-3)$.

Solution : The correct answer is (c).

Example 2 : The angles of a quadrilateral ABCD taken in an order are in the ratio 3 : 7 : 6 : 4. Then ABCD is a

(a) kite (b) parallelogram

(c) rhombus (d) trapezium

Solution : The correct answer is (d).

Example 3 : If the diagonals of a quadrilateral bisect each other at right angles, it will be a

(a) rhombus (b) trapezium

(c) rectangle (d) kite

Solution : The correct answer is (a).

Example 4 : The sum of the angles of a quadrilateral is

(a) 180° (b) 270° (c) 360° (d) 300°

Solution : The correct answer is (c).

Example 5 : In a square ABCD, the diagonals meet at point O. The ΔAOB is

(a) isosceles right triangle

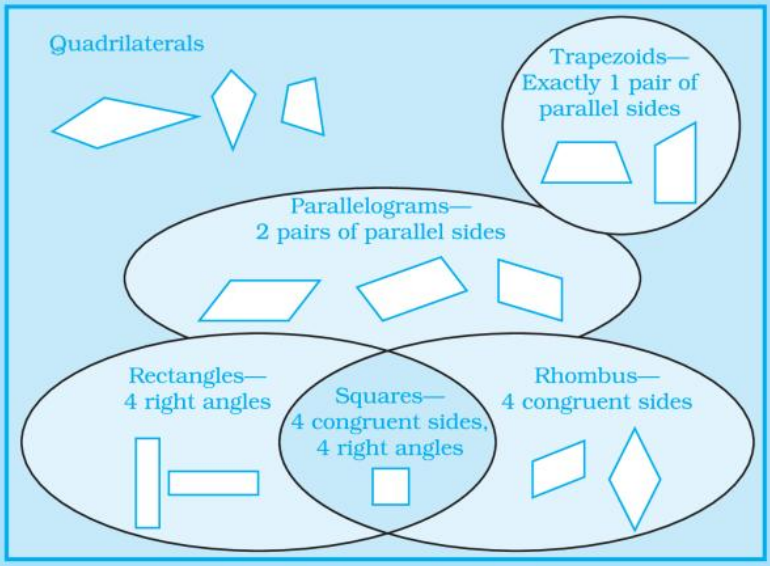
(b) equilateral triangle

(c) isosceles triangle but not right triangle

(d) scalene right triangle.

Solution : The correct answer is (a).

Quadrilaterals with certain properties are given additional names. A trapezium has exactly 1 pair of parallel sides. A parallelogram has 2 pairs of parallel sides. A rectangle has 4 right angles. A rhombus has 4 congruent sides. A square has 4 congruent sides and 4 right angles.



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Example 6 : ABCD is a quadrilateral in which AB = 5 cm, CD = 8 cm and the sum of angle A and angle D is 180°. What is the name of this quadrilateral?

(a) Parallelogram (b) Trapezium

(c) Rhombus (d) Can not be determined

Solution : The correct answer is (b).

Example 7 : Rukmini has a farm land which is triangular in shape. What is the sum of all the exterior angles taken in an order of the farm land?

(a) 90° (b) 180° (c) 360°

(d) Can not be determined.

Solution : The correct answer is (c).

Example 8 : How many sides does an octagon have?

(A) 7 (b) 8 (c) 9 (d) 10

Solution : The correct answer is (b)

In examples 9 and 13, fill in the blanks to make the statements true.

Example 9 : The diagonals of a rhombus bisect each other at _____ angles.

Solution : Right.

Example 10 : For getting diagonals through vertex A of a pentagon ABCDE, A is joined to _____.

Solution : C and D.

Example 11 : For constructing a unique quadrilateral at least _____ measurements are required.

Solution : Five.

Example 12 : If diagonals of a quadrilateral bisect at right angles it is a _____.

Solution : Rhombus (or square).

Example 13 : The diagonals of a _____ intersect at right angles.

Solution : Kite.

In examples 14 to 23, state whether the statements are true (T) or false (F).

Example 14 : Every rectangle is a parallelogram.

Solution : True.

Example 15 : Every rhombus is a kite.

Solution : True.

Example 16 : Every parallelogram is a trapezuim.

Solution : True.

Example 17 : Every kite is a trapezium.

Solution : False.

Example 18 : Every kite is a parallelogram.

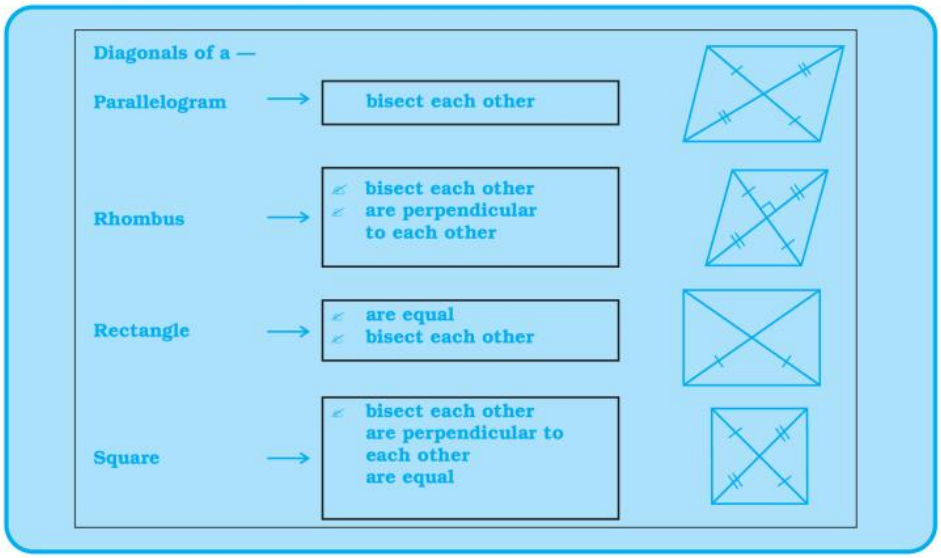
Solution : False.

Example 19 : Diagonals of a rectangle are perpendicular to each other.

Solution : False.

Example 20 : For constructing a unique parallelogram lengths of only two sides should be given.

Solution : False.



Example 21 :  is a simple closed curve.

Solution : False.

Example 22 :  is a concave polygon.

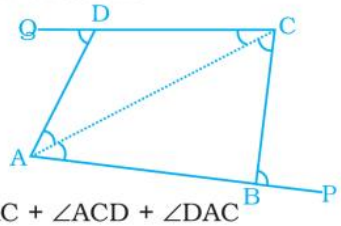
Solution : True.

Example 23 : A triangle is not a polygon.

Solution : False.

Example 24 : The sides AB and CD of a quadrilateral ABCD are extended to points P and Q respectively. Is $\angle ADQ + \angle CBP = \angle A + \angle C$? Give reason.

Solution : Join AC, then



$\angle CBP = \angle BCA + \angle BAC$ and

$\angle ADQ = \angle ACD + \angle DAC$

(Exterior angles of triangles)

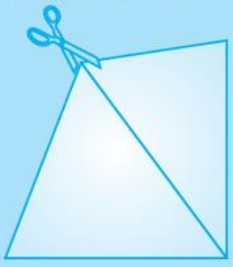
Therefore, $\angle CBP + \angle ADQ = \angle BCA + \angle BAC + \angle ACD + \angle DAC$

$= (\angle BCA + \angle ACD) + (\angle BAC + \angle DAC)$

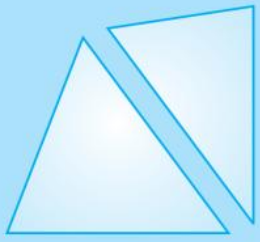
$= \angle C + \angle A$

Angles in a Quadrilateral

A diagonal of a quadrilateral is a segment that joins two vertices of the quadrilateral but is not a side. You can use a diagonal of a quadrilateral to show that the sum of the angle measures in a quadrilateral is 360°.



Cut a quadrilateral along a diagonal to form two triangles.



The sum of the angle measures in each triangle is 180°.

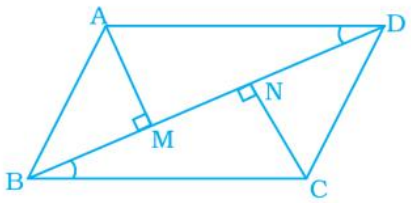


Quadrilateral with 2 pairs of parallel sides.

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Example 25 : If AM and CN are perpendiculars on the diagonal BD of a parallelogram ABCD, Is $\Delta AMD \cong \Delta CNB$? Give reason.

Solution :



In triangles AMD and CNB,

AD = BC (opposite sides of parallelogram)

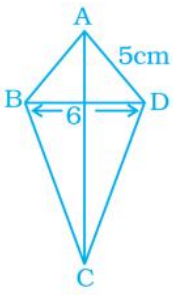
$\angle AMB = \angle CNB = 90^0$

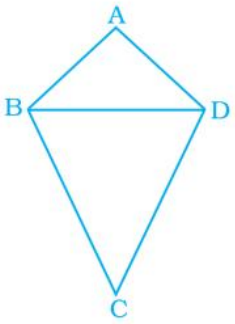
$\angle ADM = \angle NBC$ (AD || BC and BD is transversal.)

So, $\Delta AMD \cong \Delta CNB$ (AAS)

Example 26 : Construct a quadrilateral ABCD in which AB = AD = 5cm, BC = CD = 7cm and BD = 6cm. What type of quadrilateral is this?

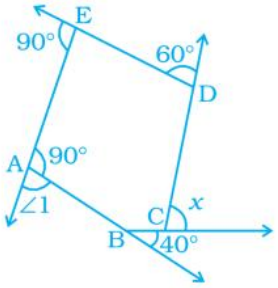
Solution : Looking at the rough figure, draw a line segment BD = 6cm. Taking B and D as centres and 5 cm radius, draw arcs to intersect at the point A, then taking B and D as centres and 7 cm radius, draw arcs in the opposite side of A to intersect at the point C. Join AB, AD and BC, DC. Then ABCD is the required quadrilateral. It is a kite.





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Example 27 : Find x in the following figure.



Solution : In the given figure $\angle 1 + 90° = 180°$ (linear pair)

$\angle 1 = 90°$

Now, sum of exterior angles of a polygon is 360°,

therefore, $x + 60° + 90° + 90° + 40° = 360°$

$x + 280° = 360°$

$x = 80°$

Classifying Plane Figures

Triangle	Trapezoid	Parallelogram
 Closed figure with 3 straight sides that connect 3 points	 Quadrilateral with 1 pair of parallel sides	 Quadrilateral with 2 pairs of parallel sides

Rhombus	Rectangle	Square	Circle
 Parallelogram with 4 sides of equal length	 Parallelogram with 4 right angles	 Parallelogram with 4 sides of equal length and 4 right angles	 Set of all points in a plane that are at the same distance from a fixed point

Example 28 : Two adjacent angles of a parallelogram are in the ratio 4:5. Find their measures.

Solution : Let the angles be $4x$ and $5x$.

Then, $4x + 5x = 180°$

$9x = 180°$

$x = 20°$

So, angles are $4 \times 20° = 80°$ and $5 \times 20° = 100°$.

Example 29 : The four angles of a quadrilateral are in the ratio 3 : 4 : 5 : 6. Find the angles.

Solution : Let angles be $3x$, $4x$, $5x$, $6x$.

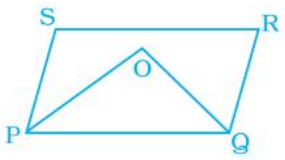
Thus, $3x + 4x + 5x + 6x = 360°$ since sum of the angles of a quadrilateral is 360°.

So, $18x = 360°$

or, $x = 20°$

Thus, angles are 60°, 80°, 100°, 120°.

Example 30 : In a parallelogram PQRS, the bisectors of $\angle P$ and $\angle Q$ meet at O. Find $\angle POQ$.



Solution : Since OP and OQ are the bisectors of $\angle P$ and $\angle Q$ respectively (see figure on the right),

so, $\angle OPQ = \frac{1}{2} \angle P$ and $\angle OQP = \frac{1}{2} \angle Q$

In ΔPOQ,

$\angle OPQ + \angle PQO + \angle POQ = 180°$ (Angle sum property)

i.e. $\frac{1}{2}\angle P + \angle POQ + \frac{1}{2}\angle Q = 180°$

i.e. $\angle POQ = 180° - \frac{1}{2}(\angle P + \angle Q)$

$= 180° - \frac{1}{2} \times 180°$

$= 90°$

Example 31 : Three angles of a quadrilateral are 50°, 40° and 123°. Find its fourth angle.

Solution : Let fourth angle be x. Then $50^0 + 40^0 + 123^0 + x = 360^0$.

or $x = 360^0 - 50^0 - 40^0 - 123^0$

$= 360^0 - 213^0 = 147^0$.

A quadrilateral is a closed plane figure with four sides that are line segments. The figures below are special types of quadrilaterals.

Special Quadrilaterals	Diagram
Trapezium A trapezium is a quadrilateral with exactly 1 pair of parallel sides.	
Parallelogram A Parallelogram is a quadrilateral with 2 pairs of parallel sides.	
Rhombus A rhombus is a parallelogram with 4 sides of equal length.	
Rectangle A rectangle is a parallelogram with 4 right angles.	
Square A square is a parallelogram with 4 sides of equal length and 4 right angles.	

Example 32 : The ratio of exterior angle to interior angle of a regular polygon is 1:4. Find the number of sides of the polygon.

Solution : Let the exterior angle of the polygon be x

Then, the interior angle of polygon = $180° - x$

According to question,

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$$\frac{x}{180° - x} = \frac{1}{4}$$

or, $4x = 180° - x$

or, $5x = 180°$

or, $x = \frac{180°}{5}$

So, $x = 36°$

Number of sides of polygon = $\frac{360°}{\text{exterior angle}}$

$= \frac{360°}{36°} = 10$

Example 33 : Each interior angle of a polygon is 108°. Find the number of sides of the polygon.

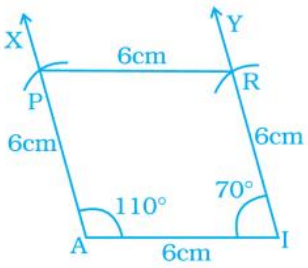
Solution : Since interior angle = 108°

so, exterior angle = $180^0 - 108^0 = 72°$

Number of sides = $\frac{360°}{\text{exterior angle}} = \frac{360^0}{72^0} = 5$

Example 34 : Construct a rhombus PAIR, given that PA = 6 cm and angle $\angle A = 110°$.

Solution :



Since in a rhombus, all sides are equal so, PA = AI = IR = RP = 6cm

Also, rhombus is a parallelogram

so, adjacent angle, $\angle I = 180° - 110° = 70°$

Steps of construction

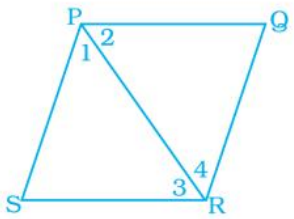
1. Draw AI = 6 cm
2. Draw ray $\overrightarrow{AX}$ such that ∠IAX = 110° and draw $\overrightarrow{IY}$ such that ∠AIY = 70°.
3. With A and I as centres and radius 6cm draw arcs intersecting AX and IY at P and R respectively.
4. Join PR.

   Thus, PAIR is the required rhombus.

Example 35 : One of the diagonals of a rhombus and its sides are equal. Find the angles of the rhombus.

Solution : Let PQRS be a rhombus such that its diagonal PR is equal to its side, that is, PQ = QR = RS = PS = PR

So, ΔPRS and ΔPQR are equilateral.



∠S = ∠Q = 60° [Each angle of an equilateral triangle is 60°.]

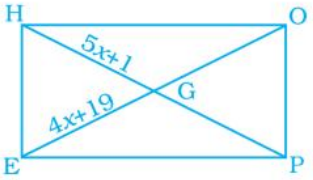
and

∠P = ∠1 + ∠2 = 60° + 60° = 120° = ∠R

Hence ∠S = ∠Q = 60° and ∠P = ∠R = 120°

Example 36 : In the figure, HOPE is a rectangle. Its diagonals meet at G. If HG = $5x + 1$ and EG = $4x + 19$, find x.

Solution :



Since diagonals of a rectangle bisect each other,

HP = 2HG = 2 (5x + 1) = 10x +2

and $OE = 2EG = 2(4x + 19) = 8x + 38$

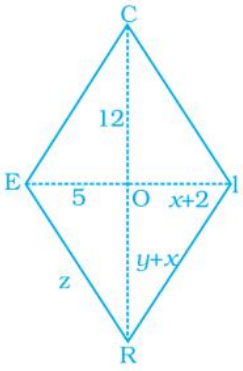
Diagonals of a rectangle are equal. So HP = OE

or $10x + 2 = 8x + 38$

or $2x = 36$ or $x = 18$

Example 37 : Application on the problem strategy

RICE is a rhombus. Find x, y, z. Justify your findings. Hence, find the perimeter of the rhombus.



Solution : Understand and explore the problem

We have to find the values of x, y, z.

i.e. OE, OY and side IR of the rhombus and perimeter of the rhombus.

What do we know?

RICE is a rhombus and

OC = 12, OE = 5, OI = $x + 2$, OR = $x + y$

Plan a strategy

(1) We have to find the parts of the diagonal. Use diagonals of a rhombus bisect each other.

(2) We have to find the side of the rhombus. We use diagonals intersect at right angles and apply pythagoras theorem.

(3) Since all sides of a rhombus are equal, perimeter of the rhombus = 4 × side.

Solve

Step 1. $OI = OE \Rightarrow x + 2 = 5$ or $x = 5 - 2 = 3$.

$OC = OR \Rightarrow 12 = y + x$ or $y = 12 - x$

$12 - 3 = 9$

Step 2. EOR is a right triangle

$ER^2 = OE^2 + OR^2$

$= 5^2 + 12^2$

$= 25 + 14^4 = 169$

ER $= \sqrt{169} = 13$cm

Step 3. Since all sides of a rhombus are equal.

$\therefore$ RE = RI = IC = CE = 13 cm.

Perimeter of RICE = 4 × RE = 4 × 13 cm

= 52 cm

Revise

We have been asked to find x, y and z and we have found that.

Checking

$x + 2 = 5$ and $x = 3 \Rightarrow 3 + 2 = 5$

Hence value of x is correct.

$x + y = 12$ $\quad \because\ x = 3$ and $y = 9$

and $3 + 9 = 12 \Rightarrow$ value of y is correct.

Perimeter of rhombus $= 2\sqrt{d1^2 + d2^2}$ (where d1 and d2 are diagonals)

$= 2\sqrt{24^2 + 10^2}$

$= 2\sqrt{576 + 100}$

$= 2\sqrt{676} = 52$ cm

Think and Discuss

(i) If RICE is a parallelogram, not a rhombus can you find x, y and z?

(ii) If RICE is a rhombus with EC = 20 cm and OC = 12 cm, can you find x, y, z?

Example 38 : Application on the problem solution strategy

Construct a rhombus with side 4.5cm and diagonal 6cm.

Solution : Understand and explore the problem

What do you know?

Here, side of rhombus = 4.5 cm.

Diagonal of rhombus = 6 cm.

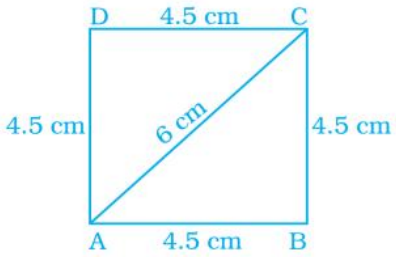
What do we need to make rhombus?

4 sides and its one diagonal

Plan a strategy

(1) Use property of rhombus— all sides are equal.

(2) Make a free hand rough sketch and name it ABCD.



Solve

Step-1. Draw AB = 4.5 cm.



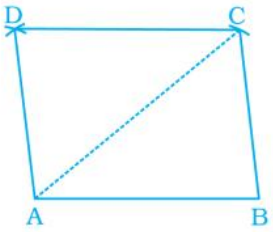
Step-2. With A as centre and radius 6 cm draw an arc above AB.



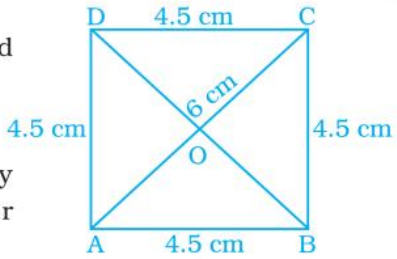
Step-3. With B as centre draw an arc to cut the arc drawn in step 2 at pt C.

Step-4. Join AC and BC.

Step-5. With A and C as centre and radius 4.5 cm draw arcs to intersect each other at D.



Step-6. ABCD is required rhombus.



Checking:

Verify your figure by adopting some other property of rhombus.

Step 1. Join BD to intersect AC as O.

Step 2. Measure ∠AOB. Is it 90°?

Step 3. Measure OA and OC. Are they equal?

Step 4. Measure OB and OD. Are they equal?

If your answer to 2, 3, 4 is yes it means what you have constructed is a right angle.

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Think and Discuss

1. Can you draw this rhombus by using some other property?
2. Can you draw a parallelogram with given measurement?
3. How will you construct this rhombus if instead of side 4.5 cm diagonal 4.5 cm is given?

(C) Exercises

In questions 1 to 52, there are four options, out of which one is correct. Write the correct answer.

1. If three angles of a quadrilateral are each equal to 75°, the fourth angle is

(a) 150° (b) 135° (c) 45° (d) 75°

2. For which of the following, diagonals bisect each other?

(a) Square (b) Kite

(c) Trapezium (d) Quadrilateral

3. For which of the following figures, all angles are equal?

(a) Rectangle (b) Kite

(c) Trapezium (d) Rhombus

4. For which of the following figures, diagonals are perpendicular to each other?

(a) Parallelogram (b) Kite

(c) Trapezium (d) Rectangle

5. For which of the following figures, diagonals are equal?

(a) Trapezium (b) Rhombus

(c) Parallelogram (d) Rectangle

6. Which of the following figures satisfy the following properties?

- All sides are congruent.
- All angles are right angles.
- Opposite sides are parallel.

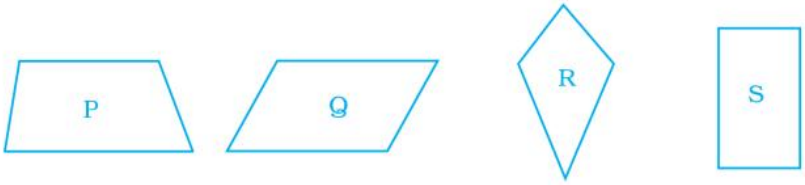
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(a) P (b) Q (c) R (d) S

7. Which of the following figures satisfy the following property?

- Has two pairs of congruent adjacent sides.



(a) P (b) Q (c) R (d) S

8. Which of the following figures satisfy the following property?

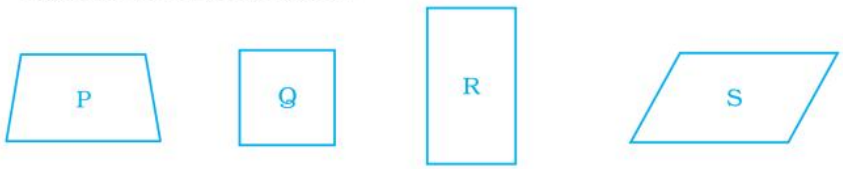
- Only one pair of sides are parallel.



(a) P (b) Q (c) R (d) S

9. Which of the following figures do not satisfy any of the following properties?

- All sides are equal.
- All angles are right angles.
- Opposite sides are parallel.



(a) P (b) Q (c) R (d) S

10. Which of the following properties describe a trapezium?

(a) A pair of opposite sides is parallel.

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(b) The diagonals bisect each other.

(c) The diagonals are perpendicular to each other.

(d) The diagonals are equal.

11. Which of the following is a property of a parallelogram?

(a) Opposite sides are parallel.

(b) The diagonals bisect each other at right angles.

(c) The diagonals are perpendicular to each other.

(d) All angles are equal.

12. What is the maximum number of obtuse angles that a quadrilateral can have ?

(a) 1 (b) 2 (c) 3 (d) 4

13. How many non-overlapping triangles can we make in a n-gon (polygon having n sides), by joining the vertices?

(a) $n-1$ (b) $n-2$ (c) $n-3$ (d) $n-4$

14. What is the sum of all the angles of a pentagon?

(a) 180° (b) 360° (c) 540° (d) 720°

15. What is the sum of all angles of a hexagon?

(a) 180° (b) 360° (c) 540° (d) 720°

16. If two adjacent angles of a parallelogram are $(5x - 5)°$ and $(10x + 35)°$, then the ratio of these angles is

(a) 1 : 3 (b) 2 : 3 (c) 1 : 4 (d) 1 : 2

17. A quadrilateral whose all sides are equal, opposite angles are equal and the diagonals bisect each other at right angles is a _____.

(a) rhombus (b) parallelogram (c) square (d) rectangle

18. A quadrialateral whose opposite sides and all the angles are equal is a

(a) rectangle (b) parallelogram (c) square (d) rhombus

19. A quadrilateral whose all sides, diagonals and angles are equal is a

(a) square (b) trapezium (c) rectangle (d) rhombus

20. How many diagonals does a hexagon have?

(a) 9 (b) 8 (c) 2 (d) 6

21. If the adjacent sides of a parallelogram are equal then parallelogram is a

(a) rectangle (b) trapezium (c) rhombus (d) square

22. If the diagonals of a quadrilateral are equal and bisect each other, then the quadrilateral is a

(a) rhombus (b) rectangle (c) square (d) parallelogram

23. The sum of all exterior angles of a triangle is

(a) 180° (b) 360° (c) 540° (d) 720°

24. Which of the following is an equiangular and equilateral polygon?

(a) Square (b) Rectangle (c) Rhombus (d) Right triangle

25. Which one has all the properties of a kite and a parallelogram?

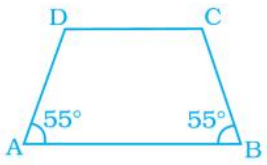
(a) Trapezium (b) Rhombus (c) Rectangle (d) Parallelogram

26. The angles of a quadrilateral are in the ratio 1 : 2 : 3 : 4. The smallest angle is

(a) 72° (b) 144° (c) 36° (d) 18°

27. In the trapezium ABCD, the measure of ∠D is

(a) 55° (b) 115° (c) 135° (d) 125°



28. A quadrilateral has three acute angles. If each measures 80°, then the measure of the fourth angle is

(a) 150° (b) 120° (c) 105° (d) 140°

29. The number of sides of a regular polygon where each exterior angle has a measure of 45° is

(a) 8 (b) 10 (c) 4 (d) 6

30. In a parallelogram PQRS, if $\angle P = 60°$, then other three angles are

(a) 45°, 135°, 120° (b) 60°, 120°, 120°

(c) 60°, 135°, 135° (d) 45°, 135°, 135°

31. If two adjacent angles of a parallelogram are in the ratio 2 : 3, then the measure of angles are

(a) 72°, 108° (b) 36°, 54° (c) 80°, 120° (d) 96°, 144°

32. If PQRS is a parallelogram, then $\angle P - \angle R$ is equal to

(a) 60° (b) 90° (c) 80° (d) 0°

33. The sum of adjacent angles of a parallelogram is

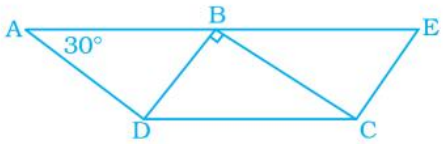
(a) 180° (b) 120° (c) 360° (d) 90°

34. The angle between the two altitudes of a parallelogram through the same vertex of an obtuse angle of the parallelogram is 30°. The measure of the obtuse angle is

(a) 100° (b) 150° (c) 105° (d) 120°

35. In the given figure, ABCD and BDCE are parallelograms with common base DC. If BC $\perp$ BD, then $\angle BEC$ =

(a) 60° (b) 30° (c) 150° (d) 120°



36. Length of one of the diagonals of a rectangle whose sides are 10 cm and 24 cm is

(a) 25 cm (b) 20 cm (c) 26 cm (d) 3.5 cm

37. If the adjacent angles of a parallelogram are equal, then the parallelogram is a

(a) rectangle (b) trapezium

(c) rhombus (d) any of the three

38. Which of the following can be four interior angles of a quadrilateral?

(a) 140°, 40°, 20°, 160° (b) 270°, 150°, 30°, 20°

(c) 40°, 70°, 90°, 60° (d) 110°, 40°, 30°, 180°

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39. The sum of angles of a concave quadrilateral is

(a) more than 360° (b) less than 360°

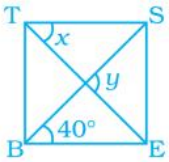
(c) equal to 360° (d) twice of 360°

40. Which of the following can never be the measure of exterior angle of a regular polygon?

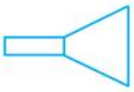
(a) 22° (b) 36° (c) 45° (d) 30°

41. In the figure, BEST is a rhombus, Then the value of $y - x$ is

(a) 40° (b) 50° (c) 20° (d) 10°



42. The closed curve which is also a polygon is

(a)  (b)  (c)  (d) 

43. Which of the following is not true for an exterior angle of a regular polygon with n sides?

(a) Each exterior angle = $\frac{360°}{n}$

(b) Exterior angle = 180° – interior angle

(c) $n = \frac{360°}{\text{exterior angle}}$

(d) Each exterior angle = $\frac{(n-2)\times 180°}{n}$

44. PQRS is a square. PR and SQ intersect at O. Then ∠POQ is a

(a) Right angle (b) Straight angle

(c) Reflex angle (d) Complete angle

45. Two adjacent angles of a parallelogram are in the ratio 1:5. Then all the angles of the parallelogram are

(a) 30°, 150°, 30°, 150° (b) 85°, 95°, 85°, 95°

(c) 45°, 135°, 45°, 135° (d) 30°, 180°, 30°, 180°

46. A parallelogram PQRS is constructed with sides QR = 6 cm, PQ = 4 cm and ∠PQR = 90°. Then PQRS is a

(a) square (b) rectangle (c) rhombus (d) trapezium

47. The angles P, Q, R and S of a quadrilateral are in the ratio 1:3:7:9. Then PQRS is a

(a) parallelogram (b) trapezium with PQ || RS

(c) trapezium with QR||PS (d) kite

48. PQRS is a trapezium in which PQ||SR and ∠P=130°, ∠Q=110°. Then ∠R is equal to:

(a) 70° (b) 50° (c) 65° (d) 55°

49. The number of sides of a regular polygon whose each interior angle is of 135° is

(a) 6 (b) 7 (c) 8 (d) 9

50. If a diagonal of a quadrilateral bisects both the angles, then it is a

(a) kite (b) parallelogram

(c) rhombus (d) rectangle

51. To construct a unique parallelogram, the minimum number of measurements required is

(a) 2 (b) 3 (c) 4 (d) 5

52. To construct a unique rectangle, the minimum number of measurements required is

(a) 4 (b) 3 (c) 2 (d) 1

In questions 53 to 91, fill in the blanks to make the statements true.

53. In quadrilateral HOPE, the pairs of opposite sides are _____.

54. In quadrilateral ROPE, the pairs of adjacent angles are _____.

55. In quadrilateral WXYZ, the pairs of opposite angles are _____.

56. The diagonals of the quadrilateral DEFG are _____ and _____.

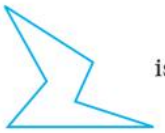
57. The sum of all _____ of a quadrilateral is 360°.

58. The measure of each exterior angle of a regular pentagon is _____.

59. Sum of the angles of a hexagon is _____.

60. The measure of each exterior angle of a regular polygon of 18 sides is _____.

61. The number of sides of a regular polygon, where each exterior angle has a measure of 36°, is _____.

62.  is a closed curve entirely made up of line segments. The another name for this shape is _____.

63. A quadrilateral that is not a parallelogram but has exactly two opposite angles of equal measure is _____.

64. The measure of each angle of a regular pentagon is _____.

65. The name of three-sided regular polygon is _____.

66. The number of diagonals in a hexagon is _____.

67. A polygon is a simple closed curve made up of only _____.

68. A regular polygon is a polygon whose all sides are equal and all _____ are equal.

69. The sum of interior angles of a polygon of n sides is _____right angles.

70. The sum of all exterior angles of a polygon is _____.

71. _____ is a regular quadrilateral.

72. A quadrilateral in which a pair of opposite sides is parallel is _____.

73. If all sides of a quadrilateral are equal, it is a _____.

74. In a rhombus diagonals intersect at _____ angles.

75. _____ measurements can determine a quadrilateral uniquely.

76. A quadrilateral can be constructed uniquely if its three sides and _____ angles are given.

77. A rhombus is a parallelogram in which _____ sides are equal.

78. The measure of _____ angle of concave quadrilateral is more than 180°.

79. A diagonal of a quadrilateral is a line segment that joins two _____ vertices of the quadrilateral.

80. The number of sides in a regular polygon having measure of an exterior angle as 72° is _____.

81. If the diagonals of a quadrilateral bisect each other, it is a _____.

82. The adjacent sides of a parallelogram are 5 cm and 9 cm. Its perimeter is _____.

83. A nonagon has _____ sides.

84. Diagonals of a rectangle are _____.

85. A polygon having 10 sides is known as _____.

86. A rectangle whose adjacent sides are equal becomes a _____.

87. If one diagonal of a rectangle is 6 cm long, length of the other diagonal is _____.

88. Adjacent angles of a parallelogram are _____.

89. If only one diagonal of a quadrilateral bisects the other, then the quadrilateral is known as _____.

90. In trapezium ABCD with AB || CD, if $\angle A = 100°$, then $\angle D =$ _____.

91. The polygon in which sum of all exterior angles is equal to the sum of interior angles is called _____.

In questions 92 to 131 state whether the statements are true (T) or (F) false.

92. All angles of a trapezium are equal.

93. All squares are rectangles.

94. All kites are squares.

95. All rectangles are parallelograms.

96. All rhombuses are squares.

97. Sum of all the angles of a quadrilateral is 180°.

98. A quadrilateral has two diagonals.

99. Triangle is a polygon whose sum of exterior angles is double the sum of interior angles.

100.  is a polygon.

101. A kite is not a convex quadrilateral.

102. The sum of interior angles and the sum of exterior angles taken in an order are equal in case of quadrilaterals only.

103. If the sum of interior angles is double the sum of exterior angles taken in an order of a polygon, then it is a hexagon.

104. A polygon is regular if all of its sides are equal.

105. Rectangle is a regular quadrilateral.

106. If diagonals of a quadrilateral are equal, it must be a rectangle.

107. If opposite angles of a quadrilateral are equal, it must be a parallelogram.

108. The interior angles of a triangle are in the ratio 1:2:3, then the ratio of its exterior angles is 3:2:1.

109.  is a concave pentagon.

110. Diagonals of a rhombus are equal and perpendicular to each other.

111. Diagonals of a rectangle are equal.

112. Diagonals of rectangle bisect each other at right angles.

113. Every kite is a parallelogram.

114. Every trapezium is a parallelogram.

115. Every parallelogram is a rectangle.

116. Every trapezium is a rectangle.

117. Every rectangle is a trapezium.

118. Every square is a rhombus.

119. Every square is a parallelogram.

120. Every square is a trapezium.

121. Every rhombus is a trapezium.

122. A quadrilateral can be drawn if only measures of four sides are given.

123. A quadrilateral can have all four angles as obtuse.

124. A quadrilateral can be drawn if all four sides and one diagonal is known.

125. A quadrilateral can be drawn when all the four angles and one side is given.

126. A quadrilateral can be drawn if all four sides and one angle is known.

127. A quadrilateral can be drawn if three sides and two diagonals are given.

128. If diagonals of a quadrilateral bisect each other, it must be a parallelogram.

129. A quadrilateral can be constructed uniquely if three angles and any two sides are given.

130. A parallelogram can be constructed uniquely if both diagonals and the angle between them is given.

131. A rhombus can be constructed uniquely if both diagonals are given.

Solve the following :

132. The diagonals of a rhombus are 8 cm and 15 cm. Find its side.

133. Two adjacent angles of a parallelogram are in the ratio 1:3. Find its angles.

134. Of the four quadrilaterals— square, rectangle, rhombus and trapezium— one is somewhat different from the others because of its design. Find it and give justification.

135. In a rectangle ABCD, AB = 25 cm and BC = 15. In what ratio does the bisector of ∠C divide AB?

136. PQRS is a rectangle. The perpendicular ST from S on PR divides ∠S in the ratio 2:3. Find ∠TPQ.

137. A photo frame is in the shape of a quadrilateral. With one diagonal longer than the other. Is it a rectangle? Why or why not?

138. The adjacent angles of a parallelogram are $(2x - 4)°$ and $(3x - 1)°$. Find the measures of all angles of the parallelogram.

139. The point of intersection of diagonals of a quadrilateral divides one diagonal in the ratio 1:2. Can it be a parallelogram? Why or why not?

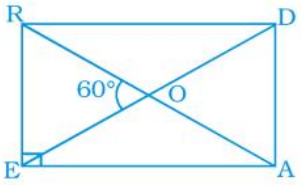
140. The ratio between exterior angle and interior angle of a regular polygon is 1:5. Find the number of sides of the polygon.

141. Two sticks each of length 5 cm are crossing each other such that they bisect each other. What shape is formed by joining their end points? Give reason.

142. Two sticks each of length 7 cm are crossing each other such that they bisect each other at right angles. What shape is formed by joining their end points? Give reason.

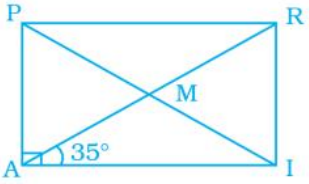
143. A playground in the town is in the form of a kite. The perimeter is 106 metres. If one of its sides is 23 metres, what are the lengths of other three sides?

144. In rectangle READ, find ∠EAR, ∠RAD and ∠ROD



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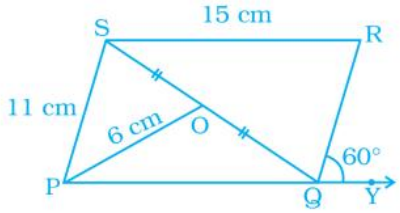
145. In rectangle PAIR, find ∠ARI, ∠RMI and ∠PMA.



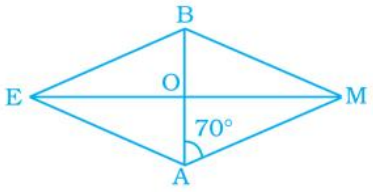
146. In parallelogram ABCD, find ∠B, ∠C and ∠D.



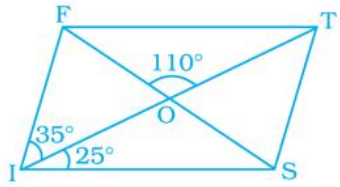
147. In parallelogram PQRS, O is the mid point of SQ. Find ∠S, ∠R, PQ, QR and diagonal PR.



148. In rhombus BEAM, find ∠AME and ∠AEM.

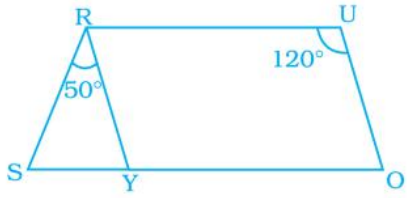


149. In parallelogram FIST, find ∠SFT, ∠OST and ∠STO.

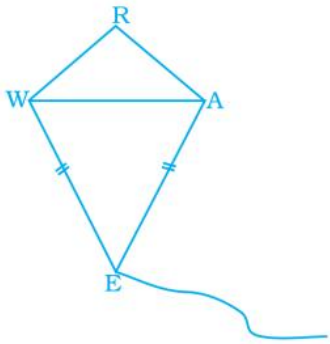


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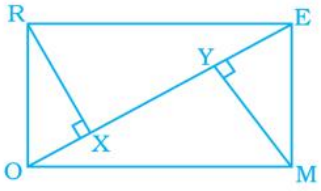
150. In the given parallelogram YOUR, $\angle RUO = 120°$ and OY is extended to point S such that $\angle SRY = 50°$. Find $\angle YSR$.



151. In kite WEAR, $\angle WEA = 70°$ and $\angle ARW = 80°$. Find the remaining two angles.



152. A rectangular MORE is shown below:



Answer the following questions by giving appropriate reason.

(i) Is RE = OM?

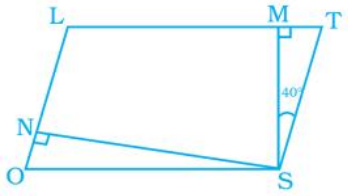
(ii) Is $\angle MYO = \angle RXE$?

(iii) Is $\angle MOY = \angle REX$?

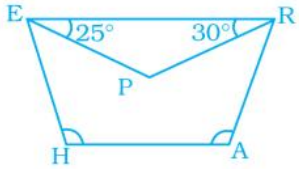
(iv) Is $\Delta MYO \cong \Delta RXE$?

(v) Is MY = RX?

153. In parallelogram LOST, SN⊥OL and SM⊥LT. Find ∠STM, ∠SON and ∠NSM.

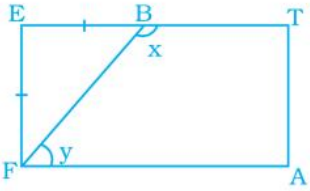


154. In trapezium HARE, EP and RP are bisectors of ∠E and ∠R respectively. Find ∠HAR and ∠EHA.

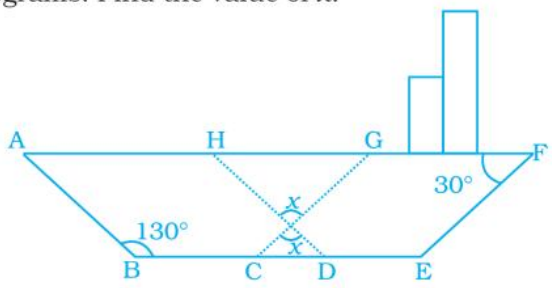


155. In parallelogram MODE, the bisector of ∠M and ∠O meet at Q, find the measure of ∠MQO.

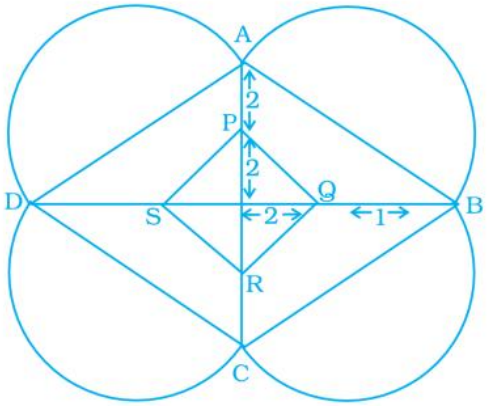
156. A playground is in the form of a rectangle ATEF. Two players are standing at the points F and B where EF = EB. Find the values of x and y.



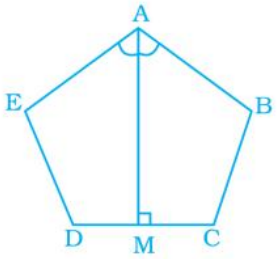
157. In the following figure of a ship, ABDH and CEFG are two parallelograms. Find the value of x.



158. A Rangoli has been drawn on a flor of a house. ABCD and PQRS both are in the shape of a rhombus. Find the radius of semicircle drawn on each side of rhombus ABCD.

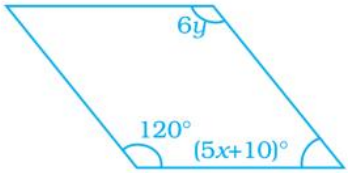


159. ABCDE is a regular pentagon. The bisector of angle A meets the side CD at M. Find ∠AMC

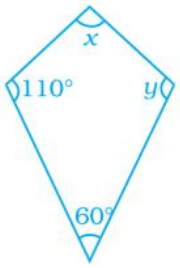


160. Quadrilateral EFGH is a rectangle in which J is the point of intersection of the diagonals. Find the value of x if JF = $8x + 4$ and EG = $24x - 8$.

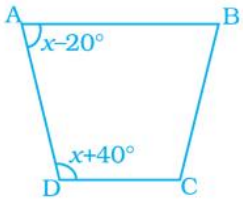
161. Find the values of x and y in the following parallelogram.



162. Find the values of x and y in the following kite.



163. Find the value of x in the trapezium ABCD given below.



164. Two angles of a quadrilateral are each of measure 75° and the other two angles are equal. What is the measure of these two angles? Name the possible figures so formed.

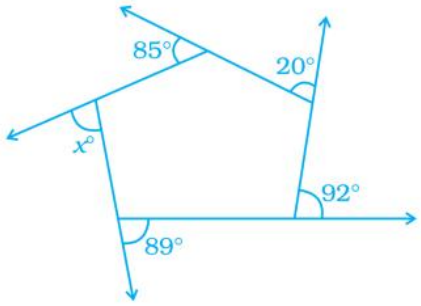
165. In a quadrilateral PQRS, $\angle P = 50°$, $\angle Q = 50°$, $\angle R = 60°$. Find $\angle S$. Is this quadrilateral convex or concave?

166. Both the pairs of opposite angles of a quadrilateral are equal and supplementary. Find the measure of each angle.

167. Find the measure of each angle of a regular octagon.

168. Find the measure of an are exterior angle of a regular pentagon and an exterior angle of a regular decagon. What is the ratio between these two angles?

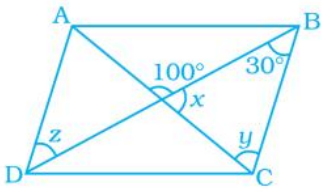
169. In the figure, find the value of x.



170. Three angles of a quadrilateral are equal. Fourth angle is of measure 120°. What is the measure of equal angles?

171. In a quadrilateral HOPE, PS and ES are bisectors of ∠P and ∠E respectively. Give reason.

172. ABCD is a parallelogram. Find the value of x, y and z.



173. Diagonals of a quadrilateral are perpendicular to each other. Is such a quadrilateral always a rhombus? Give a figure to justify your answer.

174. ABCD is a trapezium such that AB || CD, ∠A : ∠D = 2 :1, ∠B : ∠C = 7 : 5. Find the angles of the trapezium.

175. A line l is parallel to line m and a transversal p interesects them at X, Y respectively. Bisectors of interior angles at X and Y interesct at P and Q. Is PXQY a rectangle? Given reason.

176. ABCD is a parallelogram. The bisector of angle A intersects CD at X and bisector of angle C intersects AB at Y. Is AXCY a parallelogram? Give reason.

177. A diagonal of a parallelogram bisects an angle. Will it also bisect the other angle? Give reason.

178. The angle between the two altitudes of a parallelogram through the vertex of an obtuse angle of the parallelogram is 45°. Find the angles of the parallelogram.

179. ABCD is a rhombus such that the perpendicular bisector of AB passes through D. Find the angles of the rhombus.

Hint: Join BD. Then Δ ABD is equilateral.

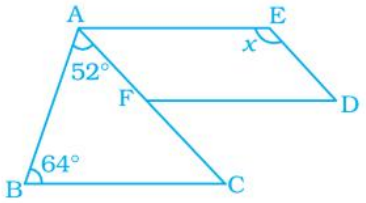
180. ABCD is a parallelogram. Points P and Q are taken on the sides AB and AD respectively and the parallelogram PRQA is formed. If ∠C = 45°, find ∠R.

181. In parallelogram ABCD, the angle bisector of ∠A bisects BC. Will angle bisector of B also bisect AD? Give reason.

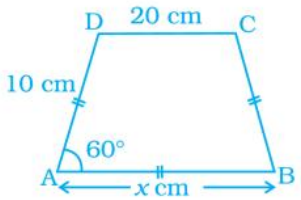
182. A regular pentagon ABCDE and a square ABFG are formed on opposite sides of AB. Find ∠BCF.

183. Find maximum number of acute angles which a convex, a quadrilateral, a pentagon and a hexagon can have. Observe the pattern and generalise the result for any polygon.

184. In the following figure, FD||BC||AE and AC||ED. Find the value of x.



185. In the following figure, AB||DC and AD = BC. Find the value of x.



186. Construct a trapezium ABCD in which AB||DC, ∠A = 105°, AD = 3 cm, AB = 4 cm and CD = 8 cm.

187. Construct a parallelogram ABCD in which AB = 4 cm, BC = 5 cm and ∠B = 60°.

188. Construct a rhombus whose side is 5 cm and one angle is of 60°.

189. Construct a rectangle whose one side is 3 cm and a diagonal equal to 5 cm.

190. Construct a square of side 4 cm.

191. Construct a rhombus CLUE in which CL = 7.5 cm and LE = 6 cm.

192. Construct a quadrilateral BEAR in which BE = 6 cm, EA = 7 cm, RB = RE = 5 cm and BA = 9 cm. Measure its fourth side.

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193. Construct a parallelogram POUR in which, PO=5.5 cm, OU = 7.2 cm and $\angle O = 70°$.

194. Draw a circle of radius 3 cm and draw its diameter and label it as AC. Construct its perpendicular bisector and let it intersect the circle at B and D. What type of quadrilateral is ABCD? Justify your answer.

195. Construct a parallelogram HOME with HO = 6 cm, HE = 4 cm and OE = 3 cm.

196. Is it possible to construct a quadrilateral ABCD in which AB = 3 cm, BC = 4 cm, CD = 5.4 cm, DA = 5.9 cm and diagonal AC = 8 cm? If not, why?

197. Is it possible to construct a quadrilateral ROAM in which RO=4 cm, OA = 5 cm, $\angle O = 120°$, $\angle R = 105°$ and $\angle A = 135°$? If not, why?

198. Construct a square in which each diagonal is 5cm long.

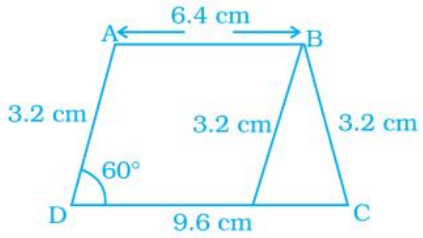
199. Construct a quadrilateral NEWS in which NE = 7cm, EW = 6 cm, $\angle N = 60°$, $\angle E = 110°$ and $\angle S = 85°$.

200. Construct a parallelogram when one of its side is 4cm and its two diagonals are 5.6 cm and 7cm. Measure the other side.

201. Find the measure of each angle of a regular polygon of 20 sides?

202. Construct a trapezium RISK in which RI $\|$ KS, RI = 7 cm, IS = 5 cm, RK=6.5 cm and $\angle I = 60°$.

203. Construct a trapezium ABCD where AB $\|$ CD, AD = BC = 3.2cm, AB = 6.4 cm and CD = 9.6 cm. Measure $\angle B$ and $\angle A$.



[Hint : Difference of two parallel sides gives an equilateral triangle.]

(D) Applications, Games and Puzzles

1 : Constructing a Tessellation

Tessellation: A tessellation is created when a shape is repeated over and over again covering a plane surface without any gaps or overlaps.

Regular Tesselations : It means a tessellation made up of congruent regular polygons. For example:



A tessellation of triangles

This arrangement can be extended to complete tiling of a floor (or tessellation).

Rules for Regular Tessellation:

(i) In tessellation there should be no overlappings/gaps between tiles.

(ii) The tiles must be regular polygons.

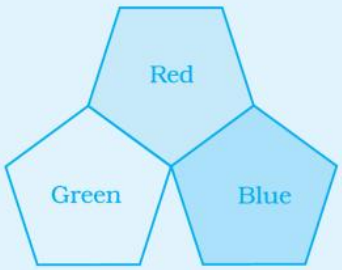
(iii) Design at each vertex must look the same.

Caution

Will pentagons work?

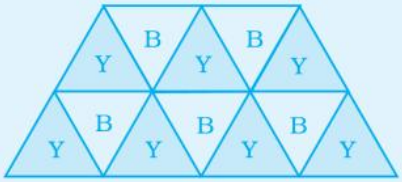
The interior angle of a pentagon is 108^0 . . .

$180^0 + 108^0 + 108^0 = 324^0$ degrees . . . No!



Thus, since the regular polygons must fill the plane at each vertex, the interior angle must be an exact divisor of 360°.

Now, find the regular polygon that can tessellate by trying a sample in table below.

Polygon	**Tessellation**
1. Triangle	
2. Square	
3. Regular Pentagon	
4. Regular Hexagon	
5. Regular Heptagon	
6. Regular Octagon	

Conclusion

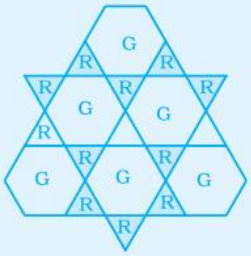
Thus, only regular polygons that can tessellate are

1. _____
2. _____
3. _____

Assignment

1. You can construct a tessellation on computer using following steps:
   - Hold down a basic images and copy it to paintbrush.
   - Keep on moving and pasting by positioning each to see a tessellation.

2. Semi Regular Tessellation : These are made by using two or more different regular polygons. Every vertex must have the same configuration, e.g.:



Y - yellow

B - Blue

G - Green

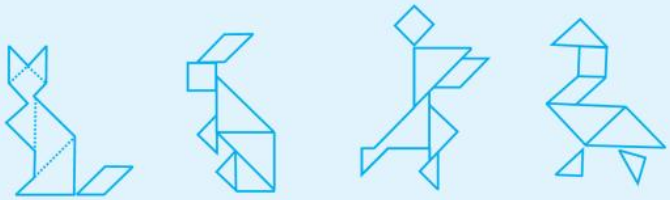
R - Red

Now discover same more tessellation of this type .

2 Constructing a TANGRAM

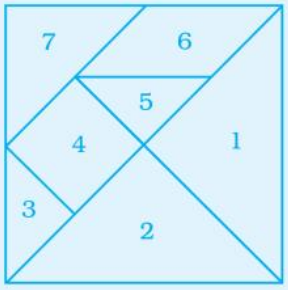
Cut the pieces of given square as shown on next page and make different shapes as shown below.

Different shapes can be made of Tangram Pieces



Try to form a story using different shapes of animals.

Required Square



3 Motivate the students to participate

Read the following description of a square before the students and let them draw what you have described.

Descriptions: My quadrilateral has opposite sides equal.

Let students compare their drawings with each other and with your square. Let students discuss what all their drawings have in common (they are all parallelograms) and what additional information is necessary to guarantee that they all would draw a square.

(e.g. All 4 sides equal and one right angle.)

4: Place '✓' or '×' in the appropriate spaces according to the property of different quadrilaterals.

	Parallelogram	**Rectangle**	**Rhombus**	**Square**	**Trapezium with non parallel sides equal**	**Trapezium**	**Kite**
Opposite sides parallel	✓	✓	✓	✓	×	×	×
Opposite sides equal							
Opposite angles equal							
Diagonal forms congruent triangles							
Diagonals bisect each other							
Diagonals are perpen-dicular							
Diagonals are equal							
Diagonals bisect opposite angles							
All angles are right							
All sides are equal							

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Use the quadrilateral chart at Page 167 to do the following activity and answer the following questions.

(a) How can you use the properties shown in the quadrilateral chart to make a statement that you believe is true about all parallelograms?

(b) How can you use the properties shown in the quadrilateral chart to make a statement that you believe is true about all rhombuses?

(c) How can you use the properties shown in the quadrilateral chart to make a statement that you believe is true about all rhombuses, but not parallelograms?

(d) How can you use the properties shown in the quadrilateral chart to make a statement that you believe is true about only rhombuses?

(e) How are the properties of rhombuses like the properties of parallelograms in general?

(f) How are the properties of rhombuses different from the properties of parallelograms?

(g) Which quadrilaterals have exactly one line of symmetry? Exactly two? Exactly three? Exactly four?

(h) Make a 'Family Tree' to show the relationship among the quadrilaterals you have been investigating.

5: Have students take each of the quadrilateral named below, join, in order, the mid points of the sides and describe the special kind of quadrilaterals they get each time:

(a) Rhombus.

(b) Rectangle.

(c) Trapezium with non-parallel sides equal.

(d) Trapezium with non-parallel sides unequal.

(e) Kite.

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6: Crossword Puzzle

Solve the given crossword and then fill up the given boxes (on the next page). Clues are given below for across as well as downward filling. Also, for across and down clues, clue number is written at the corner of the boxes. Answers of clues have to be filled up in their respective boxes.

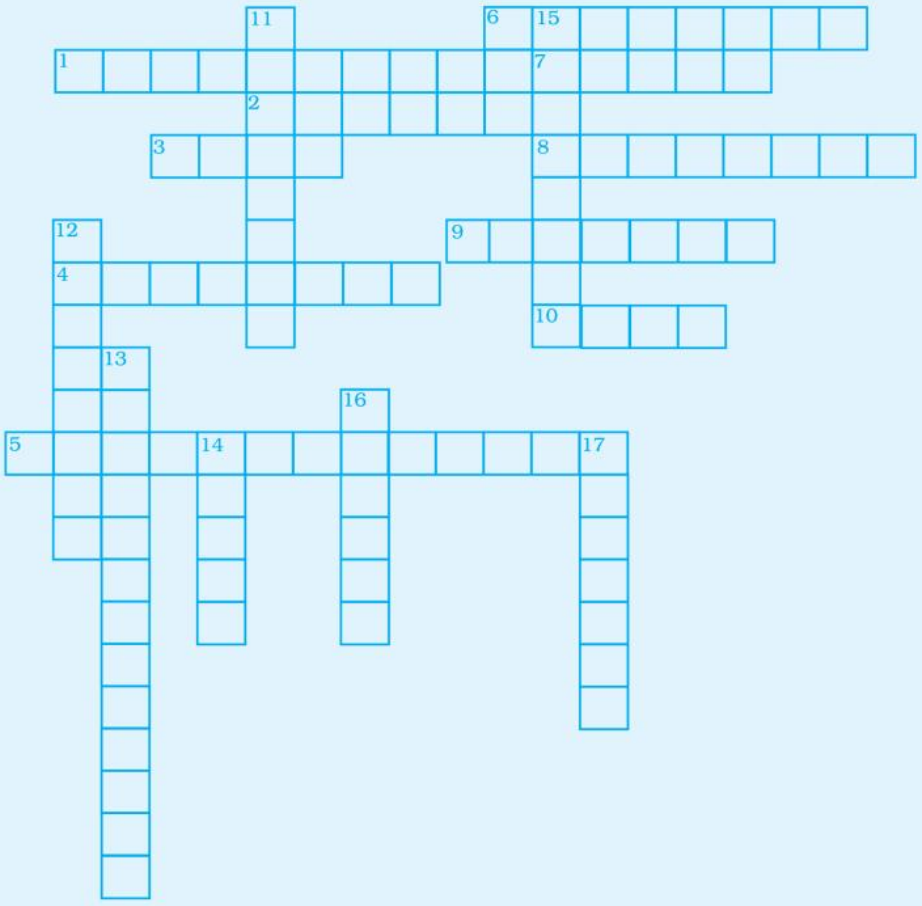
Clues

Across

1. A quadrilateral with pair of parallel sides.
2. A simple closed curve made up of only line segments.
3. A quadrilateral which has exactly two distinct consecutive pairs of sides of equal length.
4. A line segment connecting two non-consecutive vertices of a polygon.
5. The diagonals of a rhombus are _____ bisectors of one another.
6. The _____ sides of a parallelogram are of equal length.
7. The number of sides of a regular polygon whose each exterior angle has a measure of 45^0.
8. The sum of measure of the three angles of a _____ is 180^0.
9. A polygon which is both equiangular and equilateral is called a _____ polygon.
10. Number of sides of a nonagon.

Down

11. Name of the figure 
12. The _____ angles of a parallelogram are supplementary.
13. A _____ is a quadrilateral whose pair of opposite sides are parallel.
14. The diagonals of a rectangle are of _____ length.
15. A five sided polygon.
16. The diagonals of a parallelogram _____ each other.
17. A quadrilateral having all the properties of a parallelogram and also that of a kite.


11
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