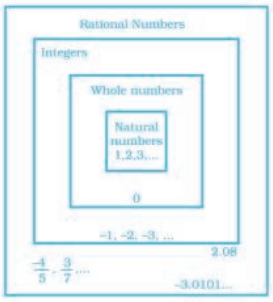
UNIT 1

RATIONAL NUMBERS

(A) Main Concepts and Results

- A number that can be expressed in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$, is called a **rational number**.
- **Lowest form of a rational number –** A rational number $\frac{p}{q}$ is said to be in the lowest form or simplest form if p and q have no common factor other than 1 and $q \neq 0$.



Addition, subtraction, multiplication and division of rational numbers are done in the same way as we do for fractions.

- Rational numbers are closed under the operations of addition, subtraction and multiplication.
- The operations of addition and multiplication for rational numbers are

  (i) commutative, (ii) associative
- The rational number 0 is the additive identity for rational numbers.
- The rational number 1 is the multiplicative identity for rational numbers.

- The additive inverse of the rational number $\frac{a}{b}$ is $\frac{-a}{b}$ and vice-versa.
- The reciprocal or multiplicative inverse of the rational number $\frac{a}{b}$ is $\frac{c}{d}$ if $\frac{a}{b}\times\frac{c}{d}=1$.
- **Distributivity of rational numbers –** For all rational numbers a, b and c

  $a\,(b + c) = ab + ac$

  $a\,(b - c) = ab - ac$
- Rational numbers can be represented on a number line.
- Between any two given rational numbers there are infinitely many rational numbers. The idea of **mean** helps us to find rational numbers between two given rational numbers.

(B) Solved Examples

In examples 1 to 3, there are four options out of which one is correct. Choose the correct answer.

Example 1 : Which of the following is not true?

(a) $\frac{2}{3}+\frac{5}{4}=\frac{5}{4}+\frac{2}{3}$ (b) $\frac{2}{3}-\frac{5}{4}=\frac{5}{4}-\frac{2}{3}$

(c) $\frac{2}{3}\times\frac{5}{4}=\frac{5}{4}\times\frac{2}{3}$ (d) $\frac{2}{3}\div\frac{5}{4}=\frac{2}{3}\times\frac{4}{5}$

Solution : The correct answer is (b).

Example 2 : Multiplicative inverse of $\frac{0}{1}$ is

(a) 1 (b) –1 (c) 0 (d) not defined

Solution : The correct answer is (d).

Example 3 : Three rational numbers lying between $\frac{-3}{4}$ and $\frac{1}{2}$ are

(a) $-\frac{1}{2}, 0, \frac{3}{4}$ (b) $\frac{-1}{4}, \frac{1}{4}, \frac{3}{4}$

(c) $\frac{-1}{4}, 0, \frac{1}{4}$ (d) $\frac{-5}{4}, 0, \frac{1}{4}$

Solution : The correct answer is (c).

In examples 4 and 5, fill in the blanks to make the statements true.

Example 4 : The product of a non-zero rational number and its reciprocal is _____.

Solution : 1

Example 5 : If $x = \frac{1}{3}$ and $y = \frac{6}{7}$ then $xy - \frac{y}{x} =$ _____.

Solution : $\frac{-16}{7}$

In examples 6 and 7, state whether the given statements are true or false.

Example 6 : Every rational number has a reciprocal.

Solution : False

Example 7 : $\frac{-4}{5}$ is larger than $\frac{-5}{4}$.

Solution : True

Example 8 : Find $\frac{4}{7} \times \frac{14}{3} \div \frac{2}{3}$.

Solution : $\frac{4}{7} \times \frac{14}{3} \div \frac{2}{3} = \frac{4}{7} \times \left(\frac{14}{3} \times \frac{3}{2}\right)$

$= \frac{4}{7} \times 7 = 4$

Example 9 : Using appropriate properties, find $\frac{2}{3} \times \frac{-5}{7} + \frac{7}{3} + \frac{2}{3} \times \frac{-2}{7}$.

Solution : $\frac{2}{3} \times \left(\frac{-5}{7}\right) + \frac{7}{3} + \frac{2}{3} \times \left(\frac{-2}{7}\right)$

$= \frac{-5}{7} \times \frac{2}{3} - \frac{2}{7} \times \frac{2}{3} + \frac{7}{3}$

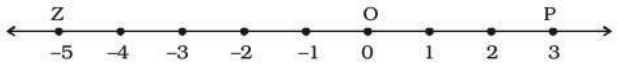
$$= \left(\frac{-5}{7} - \frac{2}{7}\right) \times \frac{2}{3} + \frac{7}{3}$$

$$= -\frac{2}{3} + \frac{7}{3} = \frac{5}{3}$$

Example 10 : Let O, P and Z represent the numbers 0, 3 and -5 respectively on the number line. Points Q, R and S are between O and P such that OQ = QR = RS = SP.

What are the rational numbers represented by the points Q, R and S. Next choose a point T between Z and O so that ZT = TO. Which rational number does T represent?

Solution :



As OQ = QR = RS = SP

and OQ + QR + RS + SP = OP

therefore Q, R and S divide OP into four equal parts.

So, R is the mid-point of OP, i.e. $R = \frac{0+3}{2} = \frac{3}{2}$

Q is the mid-point of OR, i.e. $Q = \frac{1}{2}\left(0 + \frac{3}{2}\right) = \frac{3}{4}$

and S is the mid-point of RP, i.e. $S = \frac{1}{2}\left(\frac{3}{2} + 3\right) = \frac{9}{4}$

therefore, $Q = \frac{3}{4}$, $R = \frac{3}{2}$ and $S = \frac{9}{4}$

Also Z T = TO

So, T is the mid-point of OZ, i.e. $T = \frac{0+(-5)}{2} = \frac{-5}{2}$

Think and Discuss

1. **Explain** the first step in solving an addition equation with fractions having *like* denominators.
2. **Explain** the first step in solving an addition equation with fractions having *unlike* denominators.

Example 11 : A farmer has a field of area $49\frac{4}{5}$ ha. He wants to divide it equally among his one son and two daughters. Find the area of each one's share.
(ha means hectare; 1 hectare = 10,000 m^2)

Solution : $49\frac{4}{5}\text{ ha} = \frac{249}{5}\text{ ha}$

$$\text{Each share} = \frac{1}{3}\times\frac{249}{5}\text{ ha} = \frac{83}{5}\text{ ha} = 16\frac{3}{5}\text{ ha}$$

Example 12 : Let a, b, c be the three rational numbers where $a=\frac{2}{3}, b=\frac{4}{5}$ and $c=-\frac{5}{6}$

Verify:
(i) $a+(b+c)=(a+b)+c$ (Associative property of addition)
(ii) $a\times(b\times c)=(a\times b)\times c$ (Associative property of multiplication)

Solution : (i) L.H.S $= a+(b+c)$

$$= \frac{2}{3}+\left[\frac{4}{5}+\left(\frac{-5}{6}\right)\right]$$

$$= \frac{2}{3}+\left[\frac{24-25}{30}\right]$$

$$= \frac{2}{3}+\left(\frac{-1}{30}\right)$$

$$= \frac{20-1}{30}=\frac{19}{30}$$

R.H.S. of (i) $= (a+b)+c$

$$= \left(\frac{2}{3}+\frac{4}{5}\right)+\left(\frac{-5}{6}\right)$$

$$= \left(\frac{10+12}{15}\right)+\left(\frac{-5}{6}\right)$$

$$= \frac{22}{15}-\frac{5}{6}=\frac{44-25}{30}=\frac{19}{30}$$

So, $\frac{2}{3}+\left[\frac{4}{5}+\left(\frac{-5}{2}\right)\right]=\left(\frac{2}{3}+\frac{4}{5}\right)+\left(\frac{-5}{6}\right)$

Hence verified.

(ii) L.H.S $= a \times (b \times c)$

$$= \frac{2}{3} \times \left[\frac{4}{5} \times \left(\frac{-5}{6}\right)\right]$$

$$= \frac{2}{3} \times \left(\frac{-20}{30}\right) = \frac{2}{3} \times \left(\frac{-2}{3}\right)$$

$$= \frac{2 \times (-2)}{3 \times 3} = \frac{-4}{9}$$

R.H.S. $= (a \times b) \times c$

$$= \left(\frac{2}{3} \times \frac{4}{5}\right) \times \left(\frac{-5}{6}\right)$$

$$= \frac{2 \times 4}{3 \times 5} \times \frac{-5}{6}$$

$$= \frac{8}{15} \times \left(\frac{-5}{6}\right)$$

$$= \frac{8 \times (-5)}{15 \times 6} = \frac{-40}{90} = \frac{-4}{9}$$

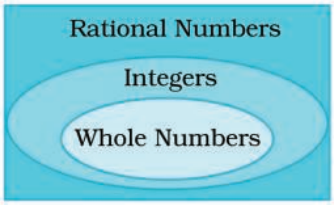
So, $\frac{2}{3} \times \left[\frac{4}{5} \times \left(\frac{-5}{6}\right)\right] = \left[\frac{2}{3} \times \frac{4}{5}\right] \times \left(\frac{-5}{6}\right)$

Example 13 : Solve the following questions and write your observations.

(i) $\frac{5}{3} + 0 = ?$ (ii) $\frac{-2}{5} + 0 = ?$ (iii) $\frac{3}{7} + 0 = ?$

(iv) $\frac{2}{3} \times 1 = ?$ (v) $\frac{-6}{7} \times 1 = ?$ (vi) $\frac{9}{8} \times 1 = ?$

Solution : (i) $\frac{5}{3} + 0 = \frac{5}{3}$ (ii) $\frac{-2}{5} + 0 = \frac{-2}{5}$ (iii) $\frac{3}{7} + 0 = \frac{3}{7}$



(iv) $\frac{2}{3} \times 1 = \frac{2}{3}$ (v) $\frac{-6}{7} \times 1 = \frac{-6}{7}$ (vi) $\frac{9}{8} \times 1 = \frac{9}{8}$

Observation

From (i) to (iii), we observe that: (i) When we add 0 to a rational number we get the same rational number.
From (iv) to (vi), we observe that: (ii) When we multiply a rational number by 1 we get the same rational number.
(iii) Therefore, 0 is the **additive identity** of rational numbers and 1 is the **'multiplicative identity'** of rational numbers.

Example 14 : Write any 5 rational numbers between $\frac{-5}{6}$ and $\frac{7}{8}$.

Solution : $\frac{-5}{6} = \frac{-5 \times 4}{6 \times 4} = \frac{-20}{24}$

and $\frac{7}{8} = \frac{7 \times 3}{8 \times 3} = \frac{21}{24}$

Thus, rational numbers $\frac{-19}{24}, \frac{-18}{24}, \frac{-17}{24}, \ldots\ldots \frac{20}{24}$ lie between $\frac{-5}{6}$ and $\frac{7}{8}$.

Example 15 : Identify the rational number which is different from the other three : $\frac{2}{3}, \frac{-4}{5}, \frac{1}{2}, \frac{1}{3}$. Explain your reasoning.

Solution : $\frac{-4}{5}$ is the rational number which is different from the other three, as it lies on the left side of zero while others lie on the right side of zero on the number line.

Example 16 : **Problem Solving Strategies**

Problem : The product of two rational numbers is –7. If one of the number is –10, find the other.

Solution : Understand and explore

- What information is given in the question?
  One of the two rational numbers
  Product of two rational numbers
- What are you finding?
  The other rational number

Plan a strategy

- Let the unknown rational number be x. Form an equation with the conditions given. Then solve the equation.

Solve

Let the other rational number be x

$-10 \times x = -7$

$x = \frac{-7}{-10}$, $x = \frac{7}{10}$

Check

$-10 \times \frac{7}{10} = -7$. Hence, the result is correct.

Think and Discuss

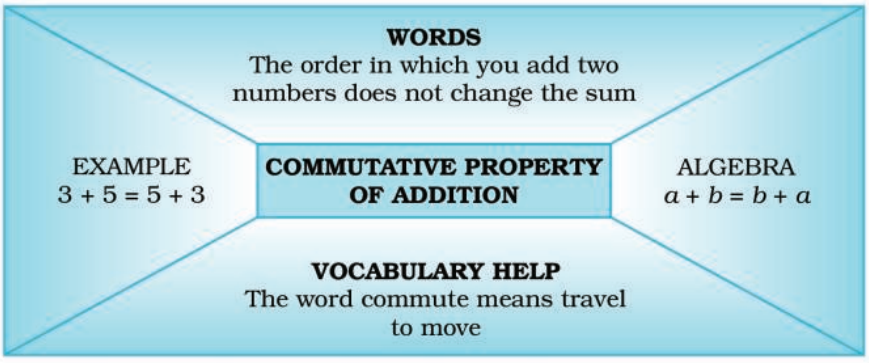
Some other easier ways to find the answer.

Is the product greater than both the rational numb or less than both the rational numbers?

Note taking Skills

Focus on Graphic Organisers

You can use an information frame to organize information about a mathematical concept or property, such as the commutative property of addition.



Make an information frame for the distributive property.

(C) Exercise

In questions 1 to 25, there are four options out of which one is correct. Choose the correct answer.

1. A number which can be expressed as $\frac{p}{q}$ where p and q are integers and $q \neq 0$ is

(a) natural number. (b) whole number.
(c) integer. (d) rational number.

2. A number of the form $\frac{p}{q}$ is said to be a rational number if

(a) p and q are integers.
(b) p and q are integers and $q \neq 0$
(c) p and q are integers and $p \neq 0$
(d) p and q are integers and $p \neq 0$ also $q \neq 0$.

3. The numerical expression $\frac{3}{8}+\frac{(-5)}{7}=\frac{-19}{56}$ shows that

(a) rational numbers are closed under addition.
(b) rational numbers are not closed under addition.
(c) rational numbers are closed under multiplication.
(d) addition of rational numbers is not commutative.

4. Which of the following is not true?

(a) rational numbers are closed under addition.
(b) rational numbers are closed under subtraction.
(c) rational numbers are closed under multiplication.
(d) rational numbers are closed under division.

5. $-\frac{3}{8}+\frac{1}{7}=\frac{1}{7}+\left(\frac{-3}{8}\right)$ is an example to show that

(a) addition of rational numbers is commutative.
(b) rational numbers are closed under addition.
(c) addition of rational number is associative.
(d) rational numbers are distributive under addition.

6. Which of the following expressions shows that rational numbers are associative under multiplication.

(a) $\frac{2}{3}\times\left(\frac{-6}{7}\times\frac{3}{5}\right) = \left(\frac{2}{3}\times\frac{-6}{7}\right)\times\frac{3}{5}$

(b) $\frac{2}{3}\times\left(\frac{-6}{7}\times\frac{3}{5}\right) = \frac{2}{3}\times\left(\frac{3}{5}\times\frac{-6}{7}\right)$

(c) $\frac{2}{3}\times\left(\frac{-6}{7}\times\frac{3}{5}\right) = \left(\frac{3}{5}\times\frac{2}{3}\right)\times\frac{-6}{7}$

(d) $\left(\frac{2}{3}\times\frac{-6}{7}\right)\times\frac{3}{5} = \left(\frac{-6}{7}\times\frac{2}{3}\right)\times\frac{3}{5}$

7. Zero (0) is

(a) the identity for addition of rational numbers.

(b) the identity for subtraction of rational numbers.

(c) the identity for multiplication of rational numbers.

(d) the identity for division of rational numbers.

8. One (1) is

(a) the identity for addition of rational numbers.

(b) the identity for subtraction of rational numbers.

(c) the identity for multiplication of rational numbers.

(d) the identity for division of rational numbers.

9. The additive inverse of $\frac{-7}{19}$ is

(a) $\frac{-7}{19}$ (b) $\frac{7}{19}$ (c) $\frac{19}{7}$ (d) $\frac{-19}{7}$

10. Multiplicative inverse of a negative rational number is

(a) a positive rational number.

(b) a negative rational number.

(c) 0

(d) 1

11. If $x + 0 = 0 + x = x$, which is rational number, then 0 is called

(a) identity for addition of rational numbers.

(b) additive inverse of x.

(c) multiplicative inverse of x.

(d) reciprocal of x.

12. To get the product 1, we should multiply $\frac{8}{21}$ by

(a) $\frac{8}{21}$ (b) $\frac{-8}{21}$ (c) $\frac{21}{8}$ (d) $\frac{-21}{8}$

13. $-(-x)$ is same as

(a) $-x$ (b) x (c) $\frac{1}{x}$ (d) $\frac{-1}{x}$

14. The multiplicative inverse of $-1\frac{1}{7}$ is

(a) $\frac{8}{7}$ (b) $\frac{-8}{7}$ (c) $\frac{7}{8}$ (d) $\frac{7}{-8}$

15. If x be any rational number then $x + 0$ is equal to

(a) x (b) 0 (c) $-x$ (d) Not defined

16. The reciprocal of 1 is

(a) 1 (b) -1 (c) 0 (d) Not defined

17. The reciprocal of -1 is

(a) 1 (b) -1 (c) 0 (d) Not defined

18. The reciprocal of 0 is

(a) 1 (b) -1 (c) 0 (d) Not defined

19. The reciprocal of any rational number $\frac{p}{q}$, where p and q are integers and $q \neq 0$, is

(a) $\frac{p}{q}$ (b) 1 (c) 0 (d) $\frac{q}{p}$

20. If y be the reciprocal of rational number x, then the reciprocal of y will be

(a) x (b) y (c) $\frac{x}{y}$ (d) $\frac{y}{x}$

21. The reciprocal of $\frac{-3}{8}\times\left(\frac{-7}{13}\right)$ is

(a) $\frac{104}{21}$ (b) $\frac{-104}{21}$ (c) $\frac{21}{104}$ (d) $\frac{-21}{104}$

22. Which of the following is an example of distributive property of multiplication over addition for rational numbers.

(a) $-\frac{1}{4}\times\left\{\frac{2}{3}+\left(\frac{-4}{7}\right)\right\}=\left[-\frac{1}{4}\times\frac{2}{3}\right]+\left[-\frac{1}{4}\times\left(\frac{-4}{7}\right)\right]$

(b) $-\frac{1}{4}\times\left\{\frac{2}{3}+\left(\frac{-4}{7}\right)\right\}=\left[\frac{1}{4}\times\frac{2}{3}\right]-\left(\frac{-4}{7}\right)$

(c) $-\frac{1}{4}\times\left\{\frac{2}{3}+\left(\frac{-4}{7}\right)\right\}=\frac{2}{3}+\left(-\frac{1}{4}\right)\times\frac{-4}{7}$

(d) $-\frac{1}{4}\times\left\{\frac{2}{3}+\left(\frac{-4}{7}\right)\right\}=\left\{\frac{2}{3}+\left(\frac{-4}{7}\right)\right\}-\frac{1}{4}$

23. Between two given rational numbers, we can find

(a) one and only one rational number.

(b) only two rational numbers.

(c) only ten rational numbers.

(d) infinitely many rational numbers.



Plan a strategy

- Some problems contain a lot of information. Read the entire problem carefully to be sure you understand all the facts. You may need to read it over several times, perhaps aloud so that you can hear yourself and understand it well.
- Then decide which information is most important (prioritise). Is there any information that is absolutely necessary to solve the problem? This information is most important.
- Finally, put the information in order (sequence). Use comparison words like *before*, *after*, *longer*, *shorter*, and so on to help you. Write down the sequence before you try to solve the problem.

Read the problem given below, and then answer the questions that follow

- Five friends are standing in line for the opening of a show. They are in line according to their arrival. Shreya arrived 3 minutes after Sachin. Roy took his place in line at 9:01 P.M. He was 1 minute behind Reena and 7 minutes ahead of Shreya. The first person arrived at 9:00 P.M. Babu showed up 6 minutes after the first person. List the time of each person's arrival.

  (a) Whose arrival information helped you determine each person's arrival time?

  (b) Can you determine the order without the time?

  (c) List the friends' order of arrival from the earliest to the last.

24. $\frac{x+y}{2}$ is a rational number.

(a) Between x and y

(b) Less than x and y both.

(c) Greater than x and y both.

(d) Less than x but greater than y.

25. Which of the following statements is always true?

(a) $\frac{x-y}{2}$ is a rational number between x and y.

(b) $\frac{x+y}{2}$ is a rational number between x and y.

(c) $\frac{x \times y}{2}$ is a rational number between x and y.

(d) $\frac{x \div y}{2}$ is a rational number between x and y.

In questions 26 to 47, fill in the blanks to make the statements true.

26. The equivalent of $\frac{5}{7}$, whose numerator is 45 is _____.

27. The equivalent rational number of $\frac{7}{9}$, whose denominator is 45 is _____.

28. Between the numbers $\frac{15}{20}$ and $\frac{35}{40}$, the greater number is _____.

29. The reciprocal of a positive rational number is _____.

30. The reciprocal of a negative rational number is _____.

31. Zero has _____ reciprocal.

32. The numbers _____ and _____ are their own reciprocal.

33. If y be the reciprocal of x, then the reciprocal of y^2 in terms of x will be _____.

34. The reciprocal of $\frac{2}{5} \times \left(\frac{-4}{9}\right)$ is _____.

35. $(213 \times 657)^{-1} = 213^{-1} \times$ _____.

36. The negative of 1 is _____.

Writing Strategy:

Write a Convincing Argument

Your ability to write a convincing argument proves that you have understanding of the concept. An effective argument should include the following four parts:

(1) A goal

(2) A response to the goal

(3) Evidence to support the response

(4) A summary statement

Write about it

Compare 10^2 and 2^{10}. For any two numbers, what usually gives the greater number, using the greater number as the base or as the exponent? Give at least one exception.

Step 1 : Identify the goal

For any two numbers, explain whether using the greater number as the base will generally result in a greater number or using it as the exponent. Find one exception.

Step 2 : Provide a response to the goal

Using the greater number as the exponent usually gives the greater number.

Step 3 : Provide evidence to support your response

For the number 10 and 2
Using the greater number, 10, as the exponent will result in a greater number.

$10^2 = 100$

$2^{10} = 1024$

$100 < 1024$

$10^2 < 2^{10}$

Exception for the numbers 2 and 3. Using the greater number, 3, as the exponent will not result in a greater number.

$3^2 = 9$

$2^3 = 8$

$9 > 8$

$3^2 > 2^3$

Step 4 : Summarise your argument

Generally, for any two numbers, using the greater number as the exponent instead of as the base will result in a greater number.

37. For rational numbers $\frac{a}{b}, \frac{c}{d}$ and $\frac{e}{f}$ we have $\frac{a}{b}\times\left(\frac{c}{d}+\frac{e}{f}\right)$ = _____ + _____.

38. $\frac{-5}{7}$ is _____ than –3.

39. There are _____ rational numbers between any two rational numbers.

40. The rational numbers $\frac{1}{3}$ and $\frac{-1}{3}$ are on the _____ sides of zero on the number line.

41. The negative of a negative rational number is always a _____ rational number.

42. Rational numbers can be added or multiplied in any _____.

43. The reciprocal of $\frac{-5}{7}$ is _____.

44. The multiplicative inverse of $\frac{4}{3}$ is _____.

45. The rational number 10.11 in the from $\frac{p}{q}$ is _____.

46. $\frac{1}{5}\times\left[\frac{2}{7}+\frac{3}{8}\right] = \left[\frac{1}{5}\times\frac{2}{7}\right]+$ _____.

47. The two rational numbers lying between –2 and –5 with denominator as 1 are _____ and _____.

In each of the following, state whether the statements are true (T) or false (F).

48. If $\frac{x}{y}$ is a rational number, then y is always a whole number.

49. If $\frac{p}{q}$ is a rational number, then p cannot be equal to zero.

50. If $\frac{r}{s}$ is a rational number, then s cannot be equal to zero.

51. $\frac{5}{6}$ lies between $\frac{2}{3}$ and 1.

52. $\frac{5}{10}$ lies between $\frac{1}{2}$ and 1.

53. $\frac{-7}{2}$ lies between –3 and –4.

54. $\frac{9}{6}$ lies between 1 and 2.

55. If $a \neq 0$, the multiplicative inverse of $\frac{a}{b}$ is $\frac{b}{a}$.

56. The multiplicative inverse of $\frac{-3}{5}$ is $\frac{5}{3}$.

57. The additive inverse of $\frac{1}{2}$ is –2.

58. If $\frac{x}{y}$ is the additive inverse of $\frac{c}{d}$, then $\frac{x}{y}+\frac{c}{d}=0$.

59. For every rational number x, $x + 1 = x$.

60. If $\frac{x}{y}$ is the additive inverse of $\frac{c}{d}$, then $\frac{x}{y}-\frac{c}{d}=0$

61. The reciprocal of a non-zero rational number $\frac{q}{p}$ is the rational number $\frac{q}{p}$.

62. If $x + y = 0$, then $-y$ is known as the negative of x, where x and y are rational numbers.

63. The negative of the negative of any rational number is the number itself.

64. The negative of 0 does not exist.

65. The negative of 1 is 1 itself.

66. For all rational numbers x and y, $x - y = y - x$.

67. For all rational numbers x and y, $x \times y = y \times x$.

68. For every rational number x, $x \times 0 = x$.

69. For every rational numbers x, y and z, $x + (y \times z) = (x + y) \times (x + z)$.

70. For all rational numbers a, b and c, $a(b + c) = ab + bc$.

71. 1 is the only number which is its own reciprocal.

72. –1 is not the reciprocal of any rational number.

73. For any rational number x, $x + (-1) = -x$.

74. For rational numbers x and y, if $x < y$ then $x - y$ is a positive rational number.

75. If x and y are negative rational numbers, then so is $x + y$.

76. Between any two rational numbers there are exactly ten rational numbers.

77. Rational numbers are closed under addition and multiplication but not under subtraction.

78. Subtraction of rational number is commutative.

79. $-\frac{3}{4}$ is smaller than –2.

80. 0 is a rational number.

81. All positive rational numbers lie between 0 and 1000.

82. The population of India in 2004 - 05 is a rational number.

83. There are countless rational numbers between $\frac{5}{6}$ and $\frac{8}{9}$.

84. The reciprocal of x^{-1} is $\frac{1}{x}$.

85. The rational number $\frac{57}{23}$ lies to the left of zero on the number line.

86. The rational number $\frac{7}{-4}$ lies to the right of zero on the number line.

87. The rational number $\frac{-8}{-3}$ lies neither to the right nor to the left of zero on the number line.

88. The rational numbers $\frac{1}{2}$ and -1 are on the opposite sides of zero on the number line.

89. Every fraction is a rational number.

90. Every integer is a rational number.

91. The rational numbers can be represented on the number line.

92. The negative of a negative rational number is a positive rational number.

93. If x and y are two rational numbers such that $x > y$, then $x - y$ is always a positive rational number.

94. 0 is the smallest rational number.

95. Every whole number is an integer.

96. Every whole number is a rational number.

97. 0 is whole number but it is not a rational number.

98. The rational numbers $\frac{1}{2}$ and $-\frac{5}{2}$ are on the opposite sides of 0 on the number line.

99. Rational numbers can be added (or multiplied) in any order

$$\frac{-4}{5}\times\frac{-6}{5}=\frac{-6}{5}\times\frac{-4}{5}$$

100. Solve the following: Select the rational numbers from the list which are also the integers.

$$\frac{9}{4},\frac{8}{4},\frac{7}{4},\frac{6}{4},\frac{9}{3},\frac{8}{3},\frac{7}{3},\frac{6}{3},\frac{5}{2},\frac{4}{2},\frac{3}{1},\frac{3}{2},\frac{1}{1},\frac{0}{1},\frac{-1}{1},\frac{-2}{1},\frac{-3}{2},\frac{-4}{2},\frac{-5}{2},\frac{-6}{2},$$

101. Select those which can be written as a rational number with denominator 4 in their lowest form:

$$\frac{7}{8},\frac{64}{16},\frac{36}{-12},\frac{-16}{17},\frac{5}{-4},\frac{140}{28}$$

102. Using suitable rearrangement and find the sum:

(a) $\frac{4}{7}+\left(\frac{-4}{9}\right)+\frac{3}{7}+\left(\frac{-13}{9}\right)$

(b) $-5+\frac{7}{10}+\frac{3}{7}+(-3)+\frac{5}{14}+\frac{-4}{5}$

103. Verify $-(-x) = x$ for

(i) $x=\frac{3}{5}$ (ii) $x=\frac{-7}{9}$ (iii) $x=\frac{13}{-15}$

104. Give one example each to show that the rational numbers are closed under addition, subtraction and multiplication. Are rational numbers closed under division? Give two examples in support of your answer.

105. Verify the property $x + y = y + x$ of rational numbers by taking

(a) $x=\frac{1}{2},\ y=\frac{1}{2}$ (b) $x=\frac{-2}{3},\ y=\frac{-5}{6}$

(c) $x=\frac{-3}{7},\ y=\frac{20}{21}$ (d) $x=\frac{-2}{5},\ y=\frac{-9}{10}$

106. Simplify each of the following by using suitable property. Also name the property.

(a) $\left[\frac{1}{2}\times\frac{1}{4}\right]+\left[\frac{1}{2}\times 6\right]$ (b) $\left[\frac{1}{5}\times\frac{2}{15}\right]-\left[\frac{1}{5}\times\frac{2}{5}\right]$ (c) $\frac{-3}{5}\times\left\{\frac{3}{7}+\left(\frac{-5}{6}\right)\right\}$

107. Tell which property allows you to compute

$\frac{1}{5}\times\left[\frac{5}{6}\times\frac{7}{9}\right]$ as $\left[\frac{1}{5}\times\frac{5}{6}\right]\times\frac{7}{9}$

108. Verify the property $x \times y = y \times z$ of rational numbers by using

(a) $x = 7$ and $y=\frac{1}{2}$ (b) $x=\frac{2}{3}$ and $y=\frac{9}{4}$

(c) $x=\frac{-5}{7}$ and $y=\frac{14}{15}$ (d) $x=\frac{-3}{8}$ and $y=\frac{-4}{9}$

109. Verify the property $x \times (y \times z) = (x \times y) \times z$ of rational numbers by using

(a) $x = 1,\ y=\frac{-1}{2}$ and $z=\frac{1}{4}$ (b) $x=\frac{2}{3},\ y=\frac{-3}{7}$ and $z=\frac{1}{2}$

(c) $x = \frac{-2}{7}$, $y = \frac{-5}{6}$ and $z = \frac{1}{4}$ (d) $x = 0$, $y = \frac{1}{2}$

and What is the name of this property?

110. Verify the property $x \times (y + z) = x \times y + x \times z$ of rational numbers by taking.

(a) $x = \frac{-1}{2}$, $y = \frac{3}{4}$, $z = \frac{1}{4}$

(b) $x = \frac{-1}{2}$, $y = \frac{2}{3}$, $z = \frac{3}{4}$

(c) $x = \frac{-2}{3}$, $y = \frac{-4}{6}$, $z = \frac{-7}{9}$

(d) $x = \frac{-1}{5}$, $y = \frac{2}{15}$, $z = \frac{-3}{10}$

111. Use the distributivity of multiplication of rational numbers over addition to simplify

(a) $\frac{3}{5} \times \left[\frac{35}{24} + \frac{10}{1}\right]$ (b) $\frac{-5}{4} \times \left[\frac{8}{5} + \frac{16}{15}\right]$

(c) $\frac{2}{7} \times \left[\frac{7}{16} - \frac{21}{4}\right]$ (d) $\frac{3}{4} \times \left[\frac{8}{9} - 40\right]$

112. Simplify

(a) $\frac{32}{5} + \frac{23}{11} \times \frac{22}{15}$ (b) $\frac{3}{7} \times \frac{28}{15} \div \frac{14}{5}$

(c) $\frac{3}{7} + \frac{-2}{21} \times \frac{-5}{6}$ (d) $\frac{7}{8} + \frac{1}{16} - \frac{1}{12}$

113. Identify the rational number that does not belong with the other three. Explain your reasoning

$$\frac{-5}{11}, \frac{-1}{2}, \frac{-4}{9}, \frac{-7}{3}$$

114. The cost of $\frac{19}{4}$ metres of wire is Rs. $\frac{171}{2}$. Find the cost of one metre of the wire.

115. A train travels $\frac{1445}{2}$ km in $\frac{17}{2}$ hours. Find the speed of the train in km/h.

116. If 16 shirts of equal size can be made out of 24m of cloth, how much cloth is needed for making one shirt?

117. $\frac{7}{11}$ of all the money in Hamid's bank account is Rs. 77,000. How much money does Hamid have in his bank account?

118. A $117\frac{1}{3}$ m long rope is cut into equal pieces measuring $7\frac{1}{3}$ m each. How many such small pieces are these?

119. $\frac{1}{6}$ of the class students are above average, $\frac{1}{4}$ are average and rest are below average. If there are 48 students in all, how many students are below average in the class?

120. $\frac{2}{5}$ of total number of students of a school come by car while $\frac{1}{4}$ of students come by bus to school. All the other students walk to school of which $\frac{1}{3}$ walk on their own and the rest are escorted by their parents. If 224 students come to school walking on their own, how many students study in that school?

121. Huma, Hubna and Seema received a total of Rs. 2,016 as monthly allowance from their mother such that Seema gets $\frac{1}{2}$ of what Huma gets and Hubna gets $1\frac{2}{3}$ times Seema's share. How much money do the three sisters get individually?

122. A mother and her two daughters got a room constructed for Rs. 62,000. The elder daughter contributes $\frac{3}{8}$ of her mother's contribution while the younger daughter contributes $\frac{1}{2}$ of her mother's share. How much do the three contribute individually?

123. Tell which property allows you to compare

$$\frac{2}{3}\times\left[\frac{3}{4}\times\frac{5}{7}\right] \text{ and } \left[\frac{2}{3}\times\frac{5}{7}\right]\times\frac{3}{4}$$

124. Name the property used in each of the following.

(i) $-\frac{7}{11}\times\frac{-3}{5}=\frac{-3}{5}\times\frac{-7}{11}$

(ii) $-\frac{2}{3}\times\left[\frac{3}{4}+\frac{-1}{2}\right]=\left[\frac{-2}{3}\times\frac{3}{4}\right]+\left[\frac{-2}{3}\times\frac{-1}{2}\right]$

(iii) $\frac{1}{3}+\left[\frac{4}{9}+\left(\frac{-4}{3}\right)\right]=\left[\frac{1}{3}+\frac{4}{9}\right]+\left[\frac{-4}{3}\right]$

(iv) $\frac{-2}{7}+0=0+\frac{-2}{7}=-\frac{2}{7}$

(v) $\frac{3}{8}\times 1=1\times\frac{3}{8}=\frac{3}{8}$

125. Find the multiplicative inverse of

(i) $-1\frac{1}{8}$ (ii) $3\frac{1}{3}$

126. Arrange the numbers $\frac{1}{4},\frac{13}{16},\frac{5}{8}$ in the descending order.

127. The product of two rational numbers is $\frac{-14}{27}$. If one of the numbers be $\frac{7}{9}$, find the other.

128. By what numbers should we multiply $\frac{-15}{20}$ so that the product may be $\frac{-5}{7}$?

129. By what number should we multiply $\frac{-8}{13}$ so that the product may be 24?

130. The product of two rational numbers is –7. If one of the number is –5, find the other?

131. Can you find a rational number whose multiplicative inverse is –1?

132. Find five rational numbers between 0 and 1.

133. Find two rational numbers whose absolute value is $\frac{1}{5}$.

134. From a rope 40 metres long, pieces of equal size are cut. If the length of one piece is $\frac{10}{3}$ metre, find the number of such pieces.

135. $5\frac{1}{2}$ metres long rope is cut into 12 equal pieces. What is the length of each piece?

136. Write the following rational numbers in the descending order.

$\frac{8}{7}, \frac{-9}{8}, \frac{-3}{2}, 0, \frac{2}{5}$

137. Find (i) $0 \div \frac{2}{3}$ (ii) $\frac{1}{3} \times \frac{-5}{7} \times \frac{-21}{10}$

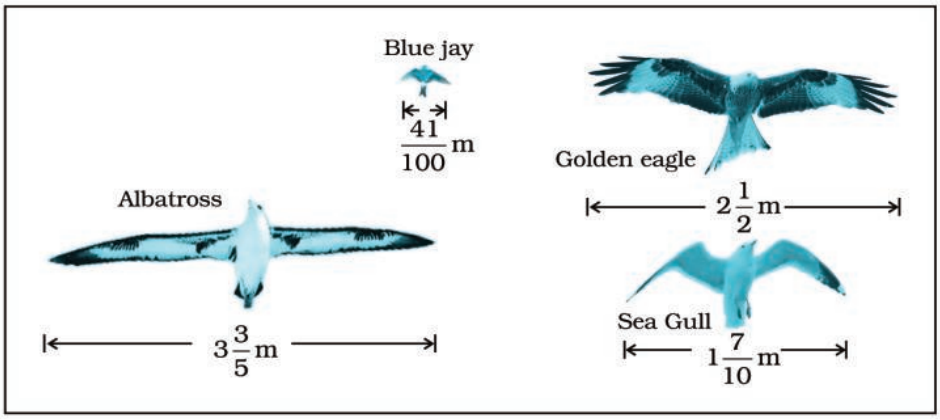
138. On a winter day the temperature at a place in Himachal Pradesh was –16°C. Convert it in degree Fahrenheit (^{0}F) by using the formula.

$\frac{C}{5} = \frac{F-32}{9}$

139. Find the sum of additive inverse and multiplicative inverse of 7.

140. Find the product of additive inverse and multiplicative inverse of $-\frac{1}{3}$.

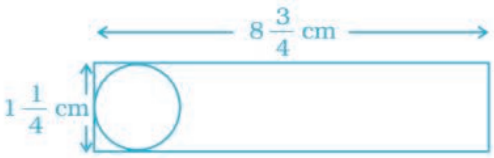
141. The diagram shows the wingspans of different species of birds. Use the diagram to answer the question given below:



(a) How much longer is the wingspan of an Albatross than the wingspan of a Sea gull?

(b) How much longer is the wingspan of a Golden eagle than the wingspan of a Blue jay?

142. Shalini has to cut out circles of diameter $1\frac{1}{4}$ cm from an aluminium strip of dimensions $8\frac{3}{4}$ cm by $1\frac{1}{4}$ cm. How many full circles can Shalini cut? Also calculate the wastage of the aluminium strip.



143. One fruit salad recipe requires $\frac{1}{2}$ cup of sugar. Another recipe for the same fruit salad requires 2 tablespoons of sugar. If 1 tablespoon is equivalent to $\frac{1}{16}$ cup, how much more sugar does the first recipe require?

144. Four friends had a competition to see how far could they hop on one foot. The table given shows the distance covered by each.

Name	**Distance covered (km)**
Seema	$\frac{1}{25}$
Nancy	$\frac{1}{32}$
Megha	$\frac{1}{40}$
Soni	$\frac{1}{20}$

(a) How farther did Soni hop than Nancy?

(b) What is the total distance covered by Seema and Megha?

(c) Who walked farther, Nancy or Megha?

145. The table given below shows the distances, in kilometres, between four villages of a state. To find the distance between two villages,

locate the square where the row for one village and the column for the other village intersect.

	Sonapur	Ramgarh	Himgaon	Rawalpur
Sonapur		$40\frac{2}{3}$	$100\frac{5}{6}$	$16\frac{1}{2}$
Ramgarh	$40\frac{2}{3}$		$210\frac{3}{8}$	$16\frac{2}{3}$
Himgaon	$100\frac{5}{6}$	$210\frac{3}{8}$		$98\frac{3}{4}$
Rawalpur	$16\frac{1}{2}$	$30\frac{2}{3}$	$98\frac{3}{4}$	

(a) Compare the distance between Himgaon and Rawalpur to Sonapur and Ramgarh?

(b) If you drove from Himgaon to Sonapur and then from Sonapur to Rawalpur, how far would you drive?

146. The table shows the portion of some common materials that are recycled.

Material	Recycled
Paper	$\frac{5}{11}$
Aluminium cans	$\frac{5}{8}$
Glass	$\frac{2}{5}$
Scrap	$\frac{3}{4}$

(a) Is the rational number expressing the amount of paper recycled more than $\frac{1}{2}$ or less than $\frac{1}{2}$?

(b) Which items have a recycled amount less than $\frac{1}{2}$?

(c) Is the quantity of aluminium cans recycled more (or less) than half of the quantity of aluminium cans?

(d) Arrange the rate of recycling the materials from the greatest to the smallest.

147. The overall width in cm of several wide-screen televisions are 97.28 cm, $98\frac{4}{9}$ cm, $98\frac{1}{25}$ cm and 97.94 cm. Express these numbers as rational numbers in the form $\frac{p}{q}$ and arrange the widths in ascending order.

148. Roller Coaster at an amusement park is $\frac{2}{3}$m high. If a new roller coaster is built that is $\frac{3}{5}$times the height of the existing coaster, what will be the height of the new roller coaster?

149. Here is a table which gives the information about the total rainfall for several months compared to the average monthly rains of a town. Write each decimal in the form of rational number $\frac{p}{q}$.

Month	**Above/Below normal (in cm)**
May	2.6924
June	0.6096
July	– 6.9088
August	– 8.636

150. The average life expectancies of males for several states are shown in the table. Express each decimal in the form $\frac{p}{q}$ and arrange the states from the least to the greatest male life expectancy.

State-wise data are included below; more indicators can be found in the "FACTFILE" section on the homepage for each state.

State	Male	$\frac{p}{q}$ form	Lowest terms
Andhra Pradesh	61.6		
Assam	57.1		
Bihar	60.7		
Gujarat	61.9		
Haryana	64.1		
Himachal Pradesh	65.1		
Karnataka	62.4		
Kerala	70.6		
Madhya Pradesh	56.5		
Maharashtra	64.5		
Orissa	57.6		
Punjab	66.9		
Rajasthan	59.8		
Tamil Nadu	63.7		
Uttar Pradesh	58.9		
West Bengal	62.8		
India	60.8		

Source: Registrar General of India (2003) SRS Based Abridged Lefe Tables. SRS Analytical Studies, Report No. 3 of 2003, New Delhi: Registrar General of India. The data are for the 1995-99 period; states subsequently divided are therefore included in their pre-partition states (Chhatisgarh in MP, Uttaranchal in UP and Jharkhand in Bihar)

151. A skirt that is $35\frac{7}{8}$ cm long has a hem of $3\frac{1}{8}$ cm. How long will the skirt be if the hem is let down?

152. Manavi and Kuber each receives an equal allowance. The table shows the fraction of their allowance each deposits into his/her saving account and the fraction each spends at the mall. If allowance of each is Rs. 1260 find the amount left with each.

Where money goes	Fraction of allowance	
	Manavi	Kuber
Saving Account	$\frac{1}{2}$	$\frac{1}{3}$
Spend at mall	$\frac{1}{4}$	$\frac{3}{5}$
Left over	?	?

(D) Games and Puzzles

1. Given below is a magic square. Place the numbers $\frac{70}{95}, \frac{-21}{-133}, \frac{25}{95}, \frac{24}{38}$ in the appropriate squares so that sum in each row, column and diagonal is equal.

$\frac{32}{38}$	$\frac{18}{38}$	$\frac{4}{38}$	$\frac{-14}{-38}$
$\frac{-18}{-57}$	?	?	$\frac{104}{152}$
$\frac{22}{38}$	?	?	$\frac{-20}{-95}$
$\frac{1}{19}$	$\frac{-16}{-38}$	$\frac{45}{57}$	$\frac{60}{114}$

Hint: (Rewrite each rational number in its lowest term.)

2. Solve the given crossword filling up the given boxes. Clues are given below for across as well as downward filling. Also, for across and down clues, clue number is written at the corner of the boxes. Answers of clues have to be filled in their respective boxes.

Down 1: $\frac{2}{3}$ and $\frac{-5}{4}$ are _____ numbers.

Down 2: The _____ inverse of $\frac{a}{b}$ is $\frac{-a}{b}$.

Down 3: The addition and multiplication of whole number integers and rational numbers is _____.

Down 4: Since $\frac{1}{0}$ doesn't exist hence 0 has no _____.

Down 5: The number line extends _____ on both the sides.

Down 6: The _____ of two integers may not lead to the formation of another integer.

Down 7: The multiplication of a number by its reciprocal gives_____.

Down 8: Rational numbers can be represented on a _____ line.

Across 1: There are _____ rational numbers between two integers.

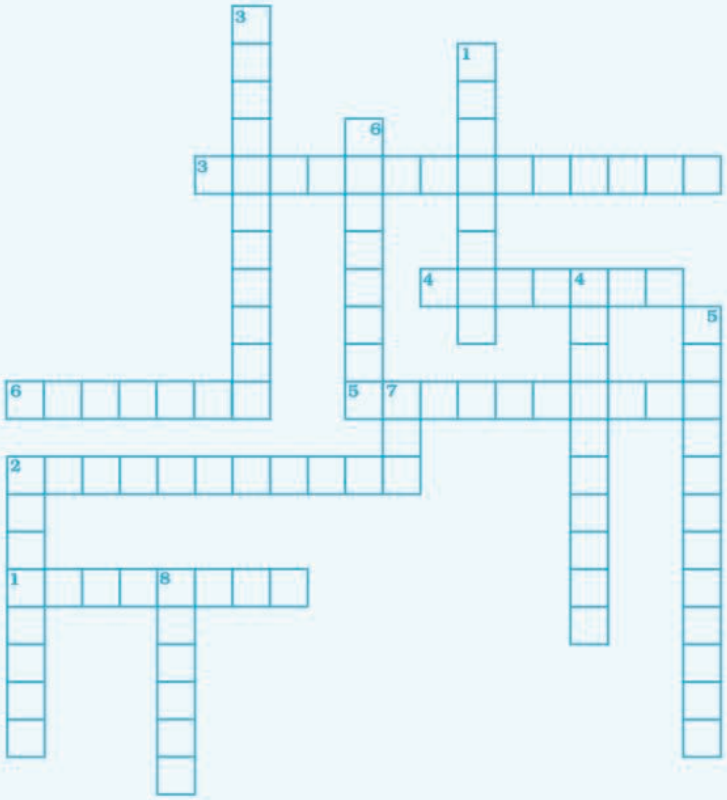
Across 2: The multiplication of rational numbers is commutative and _____.

Across 3: The addition and _____ of two rational numbers lead to the formation of another rational number.

Across 4: All the positive integers excluding 0 are known as _____ numbers.

Across 5: For any rational a ; $a \div 0$ is _____.

Across 6: Reciprocal is also known as the multiplicative _____.

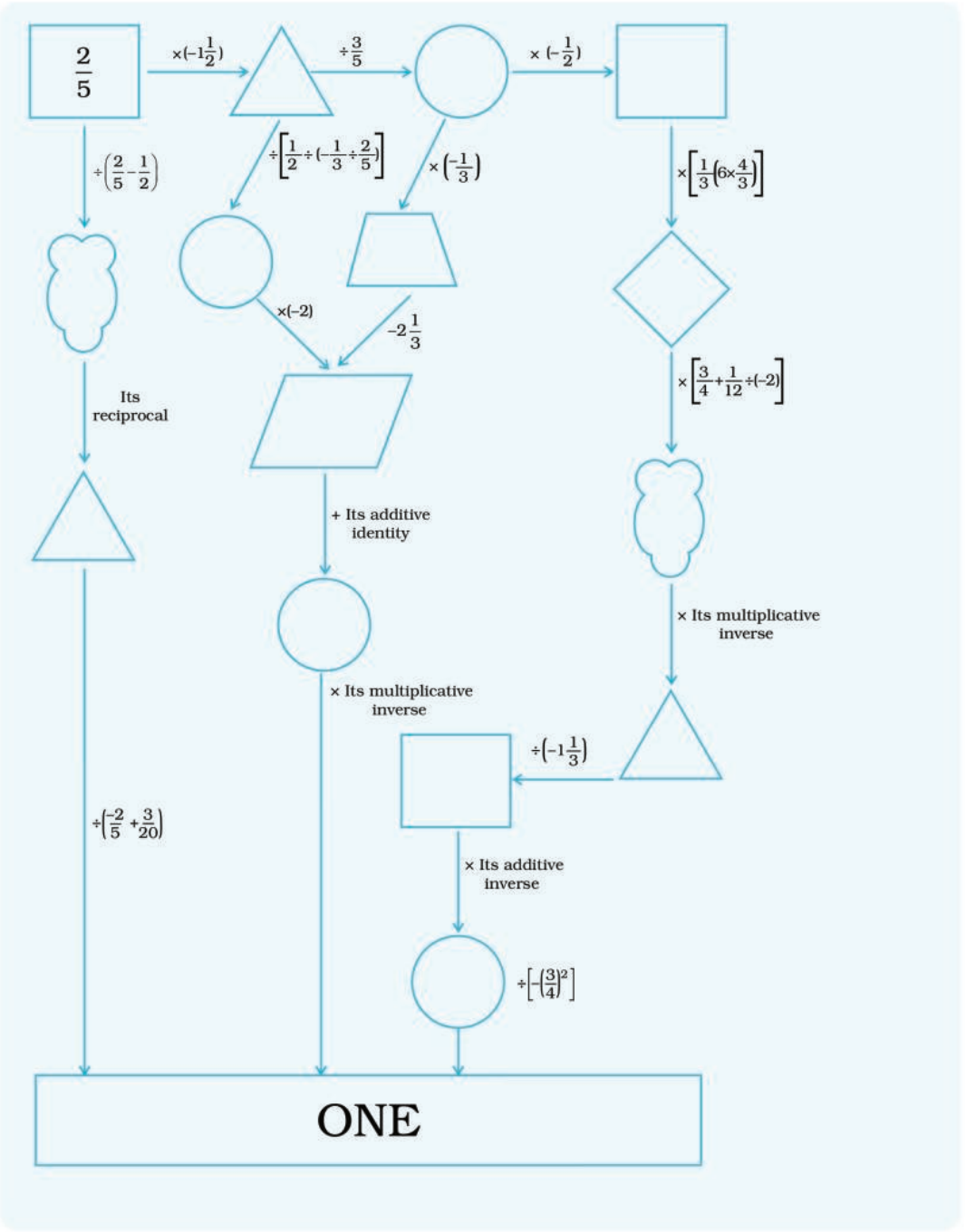


3. Break the Code

Solve this riddle by reducing each rational number to its lowest term. The magnitude of the numerator of rational number so obtained gives you the letter you have to encircle in the word following it. Use the encircled letters to fill in the blanks given below:

S.No.	Rational Number	Lowest Term	Word
(1)	$\frac{-12}{30}$		SPIN
(2)	$\frac{-24}{-36}$		TYPE
(3)	$\frac{39}{52}$		WITH
(4)	$\frac{-48}{144}$		HOST
(5)	$\frac{27}{90}$		SHARP
(6)	$\frac{-34}{-170}$		GAIN
(7)	$\frac{76}{95}$		PROOF
(8)	$\frac{46}{-92}$		RAIN
(9)	$\frac{29}{116}$		AWAY
(10)	$\frac{14}{-42}$		SWEET

1	2	3	4	5	6	7	8	9	10
____	____	____	____	____	____	____	____	____	____


$\frac{2}{5}$
$\times(-1\frac{1}{2})$
$\div\frac{3}{5}$
$\times(-\frac{1}{2})$
$\div\left(\frac{2}{5}-\frac{1}{2}\right)$
$\div\left[\frac{1}{2}\div(-\frac{1}{3}\div\frac{2}{5})\right]$
$\times\left(-\frac{1}{3}\right)$
$\times\left[\frac{1}{3}\left(6\times\frac{4}{3}\right)\right]$
$\times(-2)$
$-2\frac{1}{3}$
$\times\left[\frac{3}{4}+\frac{1}{12}\div(-2)\right]$
Its reciprocal
+ Its additive identity
× Its multiplicative inverse
× Its multiplicative inverse
$\div(-1\frac{1}{3})$
$\div\left(\frac{-2}{5}+\frac{3}{20}\right)$
× Its additive inverse
$\div\left[-\left(\frac{3}{4}\right)^2\right]$
ONE

Rough Work