



FOR JEST EXAM PREPARATION

JEST 2017 Question Paper · Physics

EXAM	YEAR	TYPE	SUBJECT
JEST	2017	Question Paper	Physics

JEST 2017

Part-A: 1-Mark Questions

Q1. A thin air film of thickness d is formed in a glass medium. For normal incidence, the condition for constructive interference in the reflected beam is (in terms of wavelength λ and integer $m = 0, 1, 2, \dots$)

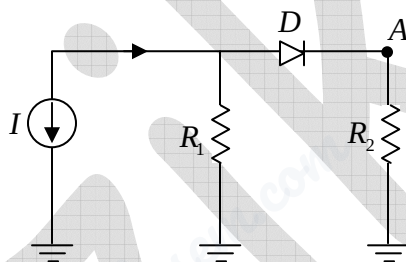
(a) $2d = (m - 1/2)\lambda$

(b) $2d = m\lambda$

(c) $2d = (m - 1)\lambda$

(d) $2\lambda = (m - 1/2)d$

Q2. Consider the circuit shown in the figure where $R_1 = 2.07k\Omega$ and $R_2 = 1.93k\Omega$. Current source I delivers $10mA$ current. The potential across the diode D is $0.7V$. What is the potential at A ?



(a) $10.35V$

(b) $9.65V$

(c) $19.30V$

(d) $4.83V$

Q3. $\int_{-\infty}^{+\infty} (x^2 + 1) \delta(x^2 - 3x + 2) dx = ?$

(a) 1

(b) 2

(c) 5

(d) 7

Q4. A bead of mass M slides along a parabolic wire described by $z = 2(x^2 + y^2)$. The wire rotates with angular velocity Ω about the z -axis. At what value of Ω does the bead maintain a constant nonzero height under the action of gravity along $-\hat{z}$?

(a) $\sqrt{3g}$

(b) $\sqrt{g}$

(c) $\sqrt{2g}$

(d) $\sqrt{4g}$

Q5. Which one is the image of the complex domain $\{z | xy \geq 1, x + y > 0\}$ under the mapping $f(z) = z^2$, if $z = x + iy$?

(a) $\{z | xy \geq 1, x + y > 0\}$

(b) $\{z | x \geq 2, x + y > 0\}$

(c) $\{z | y \geq 2 \forall x\}$

(d) $\{z | y \geq 1 \forall x\}$

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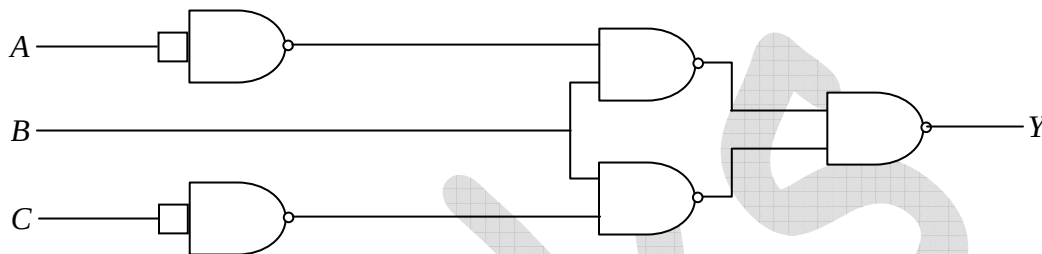
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- Q6. After the detonation of an atom bomb, the spherical ball of gas was found to be of 15 meter radius at a temperature of $3 \times 10^5 K$. Given the adiabatic expansion coefficient $\gamma = 5/3$, what will be the radius of the ball when its temperature reduces to $3 \times 10^3 K$?
- (a) 156 m (b) 50 m (c) 150 m (d) 100 m

- Q7. What is Y for the circuit shown below?



- (a) $Y = \overline{(A + \bar{B})}(\bar{B} + C)$ (b) $Y = \overline{(A + \bar{B})}(B + C)$
- (c) $Y = \overline{(\bar{A} + B)}(\bar{B} + C)$ (d) $Y = \overline{(A + B)}(\bar{B} + C)$
- Q8. What is the dimension of $\frac{\hbar \partial \psi}{i \partial x}$, where ψ is a wavefunction in two dimensions?
- (a) $kg m^{-1} s^{-2}$ (b) $kg s^{-2}$ (c) $kg m^2 s^{-2}$ (d) $kg s^{-1}$
- Q9. A plane electromagnetic wave propagating in air with $\vec{E} = (8\hat{i} + 6\hat{j} + 5\hat{k})e^{i(\omega t + 3x - 4y)}$ is incident on a perfectly conducting slab positioned at $x = 0$. $\vec{E}$ field of the reflected wave is
- (a) $(-8\hat{i} - 6\hat{j} - 5\hat{k})e^{i(\omega t + 3x + 4y)}$ (b) $(-8\hat{i} + 6\hat{j} - 5\hat{k})e^{i(\omega t + 3x + 4y)}$
- (c) $(-8\hat{i} + 6\hat{j} - 5\hat{k})e^{i(\omega t - 3x - 4y)}$ (d) $(-8\hat{i} - 6\hat{j} - 5\hat{k})e^{i(\omega t - 3x - 4y)}$
- Q10. Let $\Lambda = \begin{pmatrix} 1 & 0 \\ 0 & 11 \end{pmatrix}$ and $M = \begin{pmatrix} 10 & 3i \\ -3i & 2 \end{pmatrix}$. Similarly, transformation of M to Λ can be performed by
- (a) $\frac{1}{\sqrt{10}} \begin{pmatrix} 1 & 3i \\ 3i & 1 \end{pmatrix}$ (b) $\frac{1}{\sqrt{9}} \begin{pmatrix} 1 & -3i \\ 3i & 11 \end{pmatrix}$

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(c) $\frac{1}{\sqrt{10}} \begin{pmatrix} 1 & 3i \\ -3i & 11 \end{pmatrix}$

(d) $\frac{1}{\sqrt{9}} \begin{pmatrix} 1 & 3i \\ -3i & 1 \end{pmatrix}$

Q11. (Q_1, Q_2, P_1, P_2) and (q_1, q_2, p_1, p_2) are two sets of canonical coordinates, where Q_i and q_i are the coordinates and P_i and p_i are the corresponding conjugate momenta. If $P_1 = q_2$ and $P_2 = p_1$, then which of the following relations is true?

(a) $Q_1 = q_1, Q_2 = p_2$

(b) $Q_1 = p_2, Q_2 = q_1$

(c) $Q_1 = -p_2, Q_2 = q_1$

(d) $Q_1 = q_1, Q_2 = -p_2$

Q12. Consider magnetic vector potential $\vec{A}$ and scalar potential Φ which define the magnetic field $\vec{B}$ and electric field $\vec{E}$. If one adds $-\vec{\nabla}\lambda$ to $\vec{A}$ for a well-defined λ , then what should be added to Φ so that $\vec{E}$ remains unchanged up to an arbitrary function of time, $f(t)$?

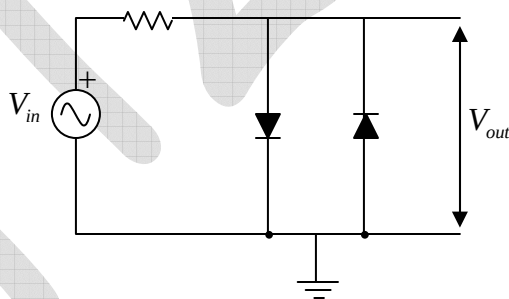
(a) $\frac{\partial \lambda}{\partial t}$

(b) $-\frac{\partial \lambda}{\partial t}$

(c) $\frac{1}{2} \frac{\partial \lambda}{\partial t}$

(d) $-\frac{1}{2} \frac{\partial \lambda}{\partial t}$

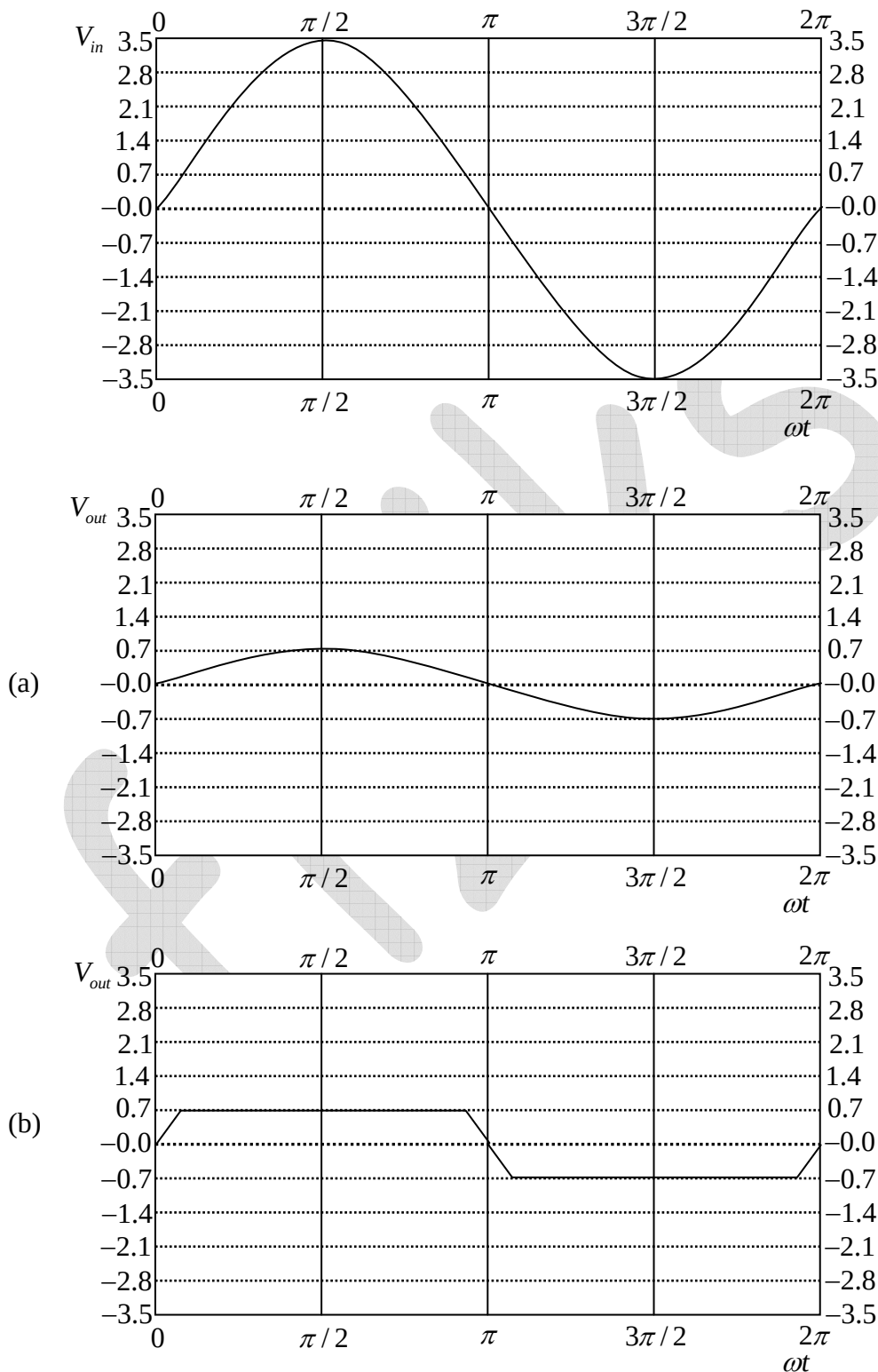
Q13. In the following silicon diode circuit ($V_B = 0.7V$), determine the output voltage waveform (V_{out}) for the given input wave.

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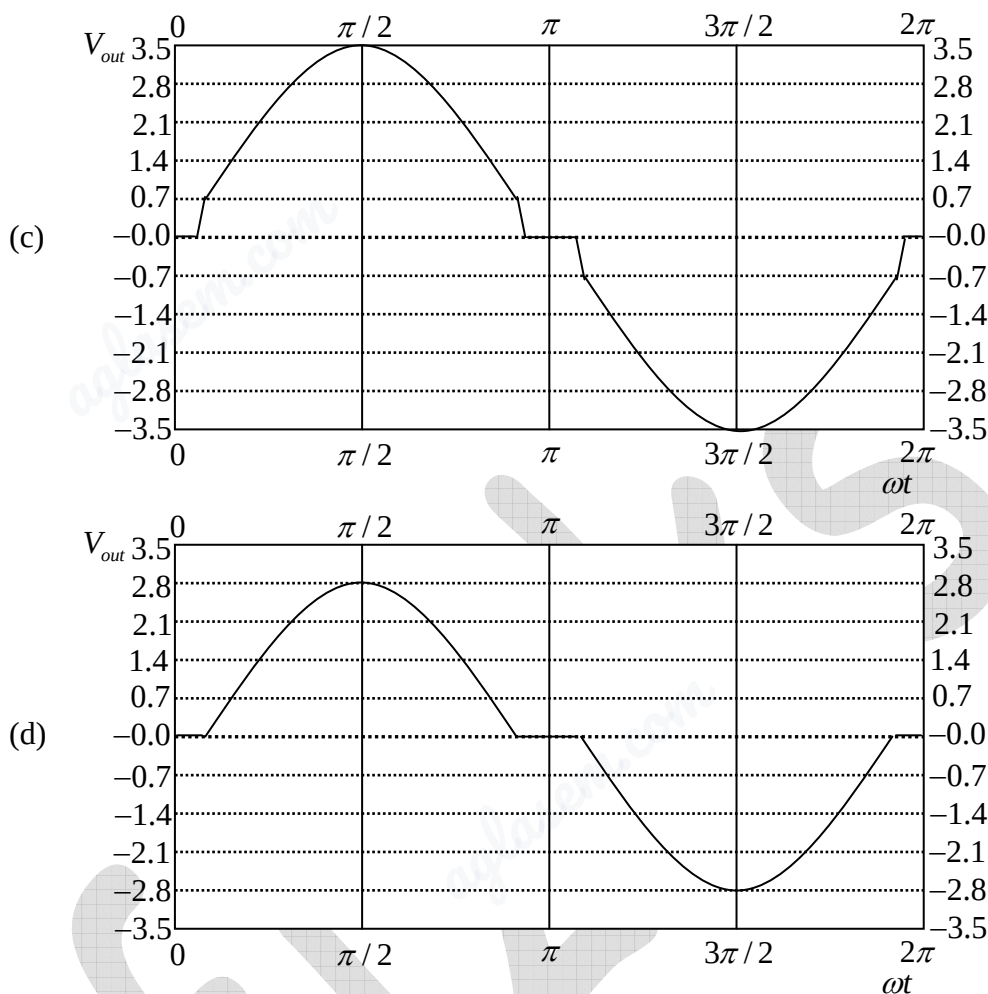
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Q14. $\phi_0(x)$ and $\phi_1(x)$ are respectively are orthonormal wavefunctions of the ground and first excited states of a one dimensional simple harmonic oscillator. Consider the normalised wave function $\psi(x) = c_0\phi_0(x) + c_1\phi_1(x)$, where c_0 and c_1 are real. For what values of c_0 and c_1 will $\langle \psi(x) | x | \psi(x) \rangle$ be maximized?

(a) $c_0 = c_1 = +1/\sqrt{2}$

(b) $c_0 = -c_1 = +1/\sqrt{2}$

(c) $c_0 = +\sqrt{3}/2, c_1 = +1/2$

(d) $c_0 = +\sqrt{3}/2, c_1 = -1/2$

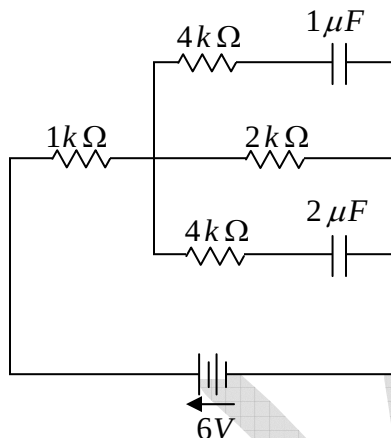
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- Q15. Consider the following circuit in steady state condition. Calculate the amount of charge stored in $1\mu F$ and $2\mu F$ capacitors respectively.



- (a) $4\mu C$ and $8\mu C$ (b) $8\mu C$ and $4\mu C$
 (c) $3\mu C$ and $6\mu C$ (d) $6\mu C$ and $3\mu C$
- Q16. If the mean square fluctuations in energy of a system in equilibrium at temperature T is proportional to T^α , then the energy of the system is proportional to
- (a) $T^{\alpha-2}$ (b) $T^{\frac{\alpha}{2}}$ (c) $T^{\alpha-1}$ (d) T^α
- Q17. Suppose the spin degrees of freedom of a 2 - particle system can be described by a 21-dimensional Hilbert subspace. Which among the following could be the spin of one of the particles?
- (a) $\frac{1}{2}$ (b) 3 (c) $\frac{3}{2}$ (d) 2
- Q18. Water is poured at a rate of $Rm^3/hour$ from the top into a cylindrical vessel of diameter D . The vessel has a small opening of area a ($\sqrt{a} \ll D$) at the bottom. What should be the minimum height of the vessel so that water does not overflow?
- (a) ∞ (b) $\frac{R^2}{2ga^2}$ (c) $\frac{R^2}{2gaD^2}$ (d) $\frac{8R^2}{\pi D^2 g^2}$

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Q19. Suppose that we toss two fair coins hundred times each. The probability that the same number of heads occur for both coins at the end of the experiment is

(a) $\left(\frac{1}{4}\right)^{100} \sum_{n=0}^{100} \binom{100}{n}$

(b) $2\left(\frac{1}{4}\right)^{100} \sum_{n=0}^{100} \binom{100}{n}^2$

(c) $\frac{1}{2}\left(\frac{1}{4}\right)^{100} \sum_{n=0}^{100} \binom{100}{n}^2$

(d) $\left(\frac{1}{4}\right)^{100} \sum_{n=0}^{100} \binom{100}{n}^2$

Q20. What is the equation of the plane which is tangent to the surface $xyz = 4$ at the point $(1, 2, 2)$?

(a) $x + 2y + 4z = 12$

(b) $4x + 2y + z = 12$

(c) $x + 4y + z = 0$

(d) $2x + y + z = 6$

Q21. If the ground state wavefunction of a particle moving in a one dimensional potential is proportional to $\exp(-x^2/2)\cosh(\sqrt{2}x)$, then the potential in suitable units such that $\hbar = 1$, is proportional to

(a) x^2

(b) $x^2 - 2\sqrt{2}x \tanh(\sqrt{2}x)$

(c) $x^2 - 2\sqrt{2}x \tan(\sqrt{2}x)$

(d) $x^2 - 2\sqrt{2}x \coth(\sqrt{2}x)$

Q22. A possible Lagrangian for a free particle is

(a) $L = \dot{q}^2 - q^2$

(b) $L = \dot{q}^2 - q\dot{q}$

(c) $L = \dot{q}^2 - q$

(d) $L = \dot{q}^2 - \frac{1}{q}$

Q23. A rod of mass m and length l is suspended from two massless vertical springs with a spring constants k_1 and k_2 . What is the Lagrangian for the system, if x_1 and x_2 be the displacements from equilibrium position of the two ends of the rod?

(a) $\frac{m}{8}(\dot{x}_1^2 + 2\dot{x}_1\dot{x}_2 + \dot{x}_2^2) - \frac{1}{2}k_1x_1^2 - \frac{1}{2}k_2x_2^2$

(b) $\frac{m}{2}(\dot{x}_1^2 + \dot{x}_1\dot{x}_2 + \dot{x}_2^2) - \frac{1}{4}(k_1 + k_2)(x_1^2 + x_2^2)$

(c) $\frac{m}{6}(\dot{x}_1^2 + \dot{x}_1\dot{x}_2 + \dot{x}_2^2) - \frac{1}{2}k_1x_1^2 - \frac{1}{2}k_2x_2^2$

(d) $\frac{m}{2}(\dot{x}_1^2 - 2\dot{x}_1\dot{x}_2 + \dot{x}_2^2) - \frac{1}{4}(k_1 - k_2)(x_1^2 + x_2^2)$

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Q24. Two equal positive charges of magnitude $+q$ separated by a distance d are surrounded by a uniformly charged thin spherical shell of radius $2d$ bearing a total charge $-2q$ and centred at the midpoint between the two positive charges. The net electric field at distance r from the midpoint ($r > d$) is

- (a) zero (b) proportional to d
(c) proportional to $1/r^3$ (d) proportional to $1/r^4$

Q25. If the Hamiltonian of a classical particles is $H = \frac{p_x^2 + p_y^2}{2m} + xy$, then $\langle x^2 + xy + y^2 \rangle$ at temperature T is equal to

- (a) $k_B T$ (b) $\frac{1}{2} k_B T$ (c) $2k_B T$ (d) $\frac{3}{2} k_B T$

Part-B: 3-Mark Questions

Q1. A solid, insulating sphere of radius 1cm has charge 10^{-7}C distributed uniformly over its volume. It is surrounded concentrically by a conducting thick spherical shell of inner radius 2cm , outer radius 2.5cm and is charged with $-2 \times 10^{-7}\text{C}$. What is the electrostatic potential in Volts on the surface of the sphere?

Q2. A particle is described by the following Hamiltonian

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2} m \omega^2 \hat{x}^2 + \lambda \hat{x}^4$$

where the quartic term can be treated perturbatively. If ΔE_0 and ΔE_1 denote the energy correction of $O(\lambda)$ to the ground state and the first excited state respectively, what is the fraction $\Delta E_1 / \Delta E_0$?

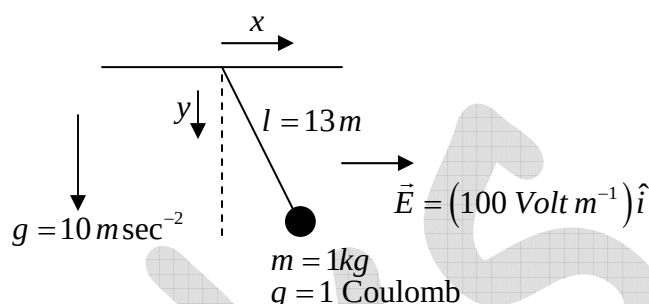
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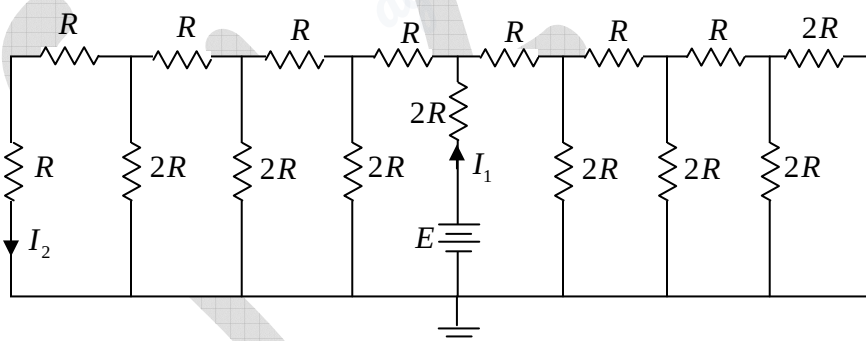
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- Q3. A simple pendulum has a bob of mass 1 kg and charge 1 Coulomb . It is suspended by a massless string of length 13 m . The time period of small oscillations of this pendulum is T_0 . If an electric field $\vec{E} = 100\hat{x}\text{ V/m}$ is applied, the time period becomes T . What is the value of $(T_0/T)^4$?



- Q4. Let a particle of mass $1 \times 10^{-9}\text{ kg}$, constrained to have one dimensional motion, be initially at the origin ($x = 0\text{ m}$). The particle is in equilibrium with a thermal bath ($k_B T = 10^{-8}\text{ J}$). What is $\langle x^2 \rangle$ of the particle after a time $t = 5\text{ s}$?
- Q5. For the circuit shown below, what is the ratio $\frac{I_1}{I_2}$?



- Q6. A ball of mass 0.1 kg and density 2000 kg/m^3 is suspended by a massless string of length 0.5 m under water having density 1000 kg/m^3 . The ball experiences a drag force, $\vec{F}_d = -0.2(\vec{v}_b - \vec{v}_w)$, where $\vec{v}_b$ and $\vec{v}_w$ are the velocities of the ball and water respectively. What will be the frequency of small oscillations for the motion of pendulum, if the water is at rest?

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- Q7. Suppose that the number of microstates available to a system of N particles depends on N and the combined variable UV^2 , where U is the internal energy and V is the volume of the system. The system initially has volume $2m^3$ and energy $200 J$. It undergoes an isentropic expansion to volume $4m^3$. What is the final pressure of the system in SI units?
- Q8. The temperature in a rectangular plate bounded by the lines, $x = 0, y = 0, x = 3$ and $y = 5$ is $T = xy^2 - x^2y + 100$. What is the maximum temperature difference between two points on the plate?
- Q9. A sphere of inner radius $1cm$ and outer radius $2cm$, centered at origin has a volume charge density $\rho_0 = \frac{K}{4\pi r}$, where K is a nonzero constant and r is the radial distance. A point charge of magnitude $10^{-3} C$ is placed at the origin. For what value of K in units of C/m^2 the electric field inside shell is constant?
- Q10. If $\hat{x}(t)$ be the position operator at a time t in the Heisenberg picture for a particle described by the Hamiltonian, $\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2\hat{x}^2$ what is $e^{i\omega t} \langle 0 | \hat{x}(t) \hat{x}(0) | 0 \rangle$ in units of $\frac{\hbar}{2m\omega}$ where $|0\rangle$ is the ground state?

Part-C: 3-Mark Questions

- Q1. Consider a grounded conducting plane which is infinitely extended perpendicular to the y -axis at $y = 0$. If an infinite line of charge per unit length λ runs parallel to x -axis at $y = d$, then surface charge density on the conducting plane is
- (a) $\frac{-\lambda d}{(x^2 + d^2 + z^2)}$ (b) $\frac{-\lambda d}{(x^2 + d^2 + z^2)}$
- (c) $\frac{-\lambda d}{\pi(x^2 + d^2 + z^2)}$ (d) $\frac{-\lambda d}{2\pi(x^2 + d^2 + z^2)}$

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- Q2. A system of particles on N lattice sites is in equilibrium at temperature T and chemical potential μ . Multiple occupancy of the sites is forbidden. The binding energy of a particle at each site is $-\epsilon$. The probability of no site being occupied is

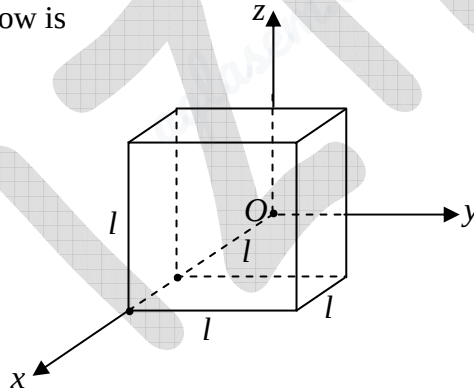
(a) $\frac{1 - e^{\beta(\mu + \epsilon)}}{1 - e^{(N+1)\beta(\mu + \epsilon)}}$ (b) $\frac{1}{[1 + e^{\beta(\mu + \epsilon)}]^N}$

(c) $\frac{1}{[1 + e^{-\beta(\mu + \epsilon)}]^N}$ (d) $\frac{1 - e^{\beta(\mu + \epsilon)}}{1 - e^{-(N+1)\beta(\mu + \epsilon)}}$

- Q3. The integral $I = \int_1^\infty \frac{\sqrt{x-1}}{(1+x)^2} dx$ is

(a) $\frac{\pi}{\sqrt{2}}$ (b) $\frac{\pi}{2\sqrt{2}}$ (c) $\frac{\sqrt{\pi}}{2}$ (d) $\sqrt{\frac{\pi}{2}}$

- Q4. For an electric field $\vec{E} = k\sqrt{x}\hat{x}$ where k is a non-zero constant, total charge enclosed by the cube as shown below is



(a) 0 (b) $k \epsilon_0 l^{5/2} (\sqrt{3} - 1)$

(c) $k \epsilon_0 l^{5/2} (\sqrt{5} - 1)$ (d) $k \epsilon_0 l^{5/2} (\sqrt{2} - 1)$

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Q5. Consider a point particle A of mass m_A colliding elastically with another point particle

B of mass m_B at rest, where $\frac{m_B}{m_A} = \gamma$. After collision, the ratio of the kinetic energy of

particle B to the initial kinetic energy of particle A is given by

(a) $\frac{4}{\gamma + 2 + \frac{1}{\gamma}}$

(b) $\frac{2}{\gamma + \frac{1}{\gamma}}$

(c) $\frac{2}{\gamma + 2 - \frac{1}{\gamma}}$

(d) $\frac{1}{\gamma}$

Q6. Two classical particles are distributed among $N (> 2)$ sites on a ring. Each site can accommodate only one particle. If two particles occupy two nearest neighbour sites, then the energy of the system is increased by ϵ . The average energy of the system at temperature T is

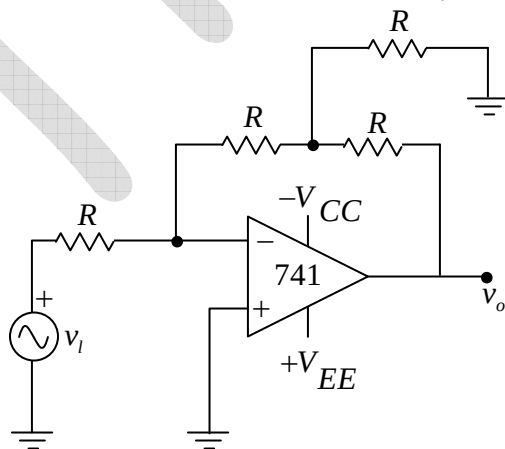
(a) $\frac{2\epsilon e^{-\beta\epsilon}}{(N-3) + 2e^{-\beta\epsilon}}$

(b) $\frac{2N\epsilon e^{-\beta\epsilon}}{(N-3) + 2e^{-\beta\epsilon}}$

(c) $\frac{\epsilon}{N}$

(d) $\frac{2\epsilon e^{-\beta\epsilon}}{(N-2) + 2e^{-\beta\epsilon}}$

Q7. Consider a 741 operational amplifier circuit as shown below, where $V_{CC} = V_{EE} = +15V$ and $R = 2.2 k\Omega$. If $v_i = 2mV$, what is the value of v_o with respect to the ground?



(a) $-1mV$

(b) $-2mV$

(c) $-3mV$

(d) $-4mV$

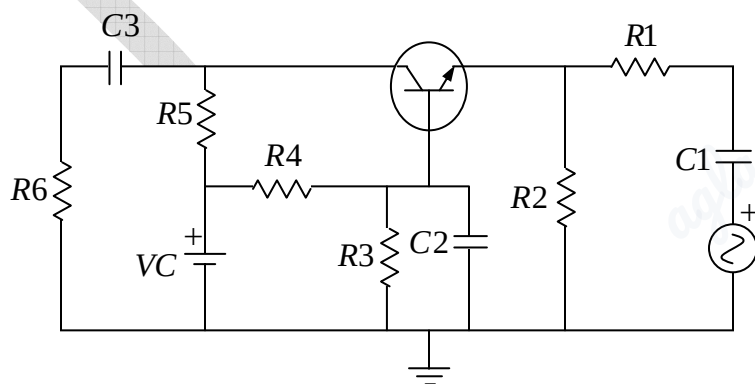
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- Q8. The Fourier transform of the function $\frac{1}{x^4 + 3x^2 + 2}$ up to proportionality constant is
- (a) $\sqrt{2} \exp(-k^2) - \exp(-2k^2)$ (b) $\sqrt{2} \exp(-|k|) - \exp(-\sqrt{2}|k|)$
- (c) $\sqrt{2} \exp(-\sqrt{|k|}) - \exp(-\sqrt{2|k|})$ (d) $\sqrt{2} \exp(-\sqrt{2}k^2) - \exp(-2k^2)$
- Q9. If $\rho = \frac{\left[I + \frac{1}{\sqrt{3}}(\sigma_x + \sigma_y + \sigma_z) \right]}{2}$, where σ 's are the Pauli matrices and I is the identity matrix, then the trace of σ^{2017} is
- (a) 2^{2017} (b) 2^{-2017} (c) 1 (d) $\frac{1}{2}$
- Q10. A cylinder at temperature $T = 0$ is separated into two compartments A and B by a free sliding piston. Compartments A and B are filled by Fermi gases made of spin 1/2 and 3/2 particles respectively. If particles in both the compartments have same mass, the ratio of equilibrium density of the gas in compartment A to that of gas in compartment B is
- (a) 1 (b) $\frac{1}{3^{2/5}}$ (c) $\frac{1}{2^{2/5}}$ (d) $\frac{1}{2^{2/3}}$
- Q11. What is the DC base current (approximated to nearest integer value in μA) for the following $n - p - n$ silicon transistor circuit, given $R_1 = 75 \Omega$, $R_2 = 4.0 k\Omega$, $R_3 = 2.1 k\Omega$, $R_4 = 2.6 k\Omega$, $R_5 = 6.0 k\Omega$, $R_6 = 6.8 k\Omega$, $C_1 = 1 \mu F$, $C_2 = 2 \mu F$, $V_C = +15V$ $\beta_{dc} = 75$?



- (a) 20 (b) 24 (c) 16 (d) 32

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Q12. Consider a particle confined by a potential $V(x) = k|x|$, where k is a positive constant.

The spectrum E_n of the system, within the WKB approximation is proportional to

- (a) $\left(n + \frac{1}{2}\right)^{3/2}$ (b) $\left(n + \frac{1}{2}\right)^{2/3}$ (c) $\left(n + \frac{1}{2}\right)^{1/2}$ (d) $\left(n + \frac{1}{2}\right)^{4/3}$

Q13. Consider the Hamiltonian

$$H(t) = \alpha \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix} + \beta t \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & -2 \end{pmatrix}$$

The time dependent function $\beta(t) = \alpha$ for $t \leq 0$ and zero for $t > 0$. Find

$\left| \langle \Psi(t < 0) | \Psi(t > 0) \rangle \right|^2$, where $|\Psi(t < 0)\rangle$ is the normalised ground state of the system at a time $t < 0$ and $|\Psi(t > 0)\rangle$ is the state of the system at $t > 0$.

- (a) $\frac{1}{2}(1 + \cos(2\alpha t))$ (b) $\frac{1}{2}(1 + \cos(\alpha t))$
(c) $\frac{1}{2}(1 + \sin(2\alpha t))$ (d) $\frac{1}{2}(1 + \sin(\alpha t))$

Q14. The function $f(x) = \cosh x$ which exists in the range $-\pi \leq x \leq \pi$ is periodically repeated between $x = (2m-1)\pi$ and $(2m+1)\pi$, where $m = -\infty$ to ∞ . Using Fourier series, indicate the correct relation at $x = 0$

- (a) $\sum_{n=-\infty}^{\infty} \frac{(-1)^n}{1-n^2} = \frac{1}{2} \left(\frac{\pi}{\cosh \pi} - 1 \right)$ (b) $\sum_{n=-\infty}^{\infty} \frac{(-1)^n}{1-n^2} = 2 \frac{\pi}{\cosh \pi}$
(c) $\sum_{n=-\infty}^{\infty} \frac{(-1)^{-n}}{1+n^2} = 2 \frac{\pi}{\sinh \pi}$ (d) $\sum_{n=1}^{\infty} \frac{(-1)^n}{1+n^2} = \frac{1}{2} \left(\frac{\pi}{\sinh \pi} - 1 \right)$

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Q15. A toy car is made from a rectangular block of mass M and four disk wheels of mass m and radii r . The car is attached to a vertical wall by a massless horizontal spring with spring constant k and constrained to move perpendicular to the wall. The coefficient of static friction between the wheel of the car and the floor is μ . The maximum amplitude of oscillations of the car above which the wheels start slipping is

(a) $\frac{\mu g (M + 2m)(M + 4m)}{mk}$

(b) $\frac{\mu g (M^2 - m^2)}{Mk}$

(c) $\frac{\mu g (M + m)^2}{2mk}$

(d) $\frac{\mu g (M + 4m)(M + 6m)}{2mk}$

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