

Q.1 Find the cube root of each of the following integers.

(i) -125

(ii) -5832

(iii) -2744000

(iv) -753571

(v) -32768

Solution (i) $\sqrt[3]{-125} = -\sqrt[3]{125}$
 $= -\sqrt[3]{5^3} = -5$

(ii) $\sqrt[3]{-5832} = -\sqrt[3]{5832}$

$\therefore 5832 = 2^3 \times 3^3 \times 3^3$

$\Rightarrow \sqrt[3]{5832} = 2 \times 3 \times 3$
 $= 18$

hence $\sqrt[3]{-5832} = -18$

2	5832
2	2916
2	1458
3	729
3	243
3	81
3	27
3	9
3	3
	1

(iii) $\sqrt[3]{-2744000} = -\sqrt[3]{2744} \times \sqrt[3]{1000}$

2	2744
2	1372
2	686
7	343
7	49
7	7
	1

$\therefore \sqrt[3]{2744} = 2 \times 7 = 14$

$\sqrt[3]{1000} = 10$

hence $\sqrt[3]{-2744000} = -14 \times 10$
 $= -140$

$$(v) \sqrt[3]{-753571} = -\sqrt[3]{753571}$$

$$\therefore \sqrt[3]{753571} = 7 \times 13 = 91$$

Therefore

$$\sqrt[3]{-753571} = -91$$

7	753571
7	107653
7	15379
13	2197
13	169
13	13
	1

$$(v) \sqrt[3]{-32768} = -\sqrt[3]{32768}$$

$$32768 = 2^3 \times 2^3 \times 2^3 \times 2^3 \times 2^3$$

$$\therefore \sqrt[3]{32768} = 2 \times 2 \times 2 \times 2 \times 2 = 32$$

Therefore $\sqrt[3]{-32768} = -32$

2	32768
2	16384
2	8192
2	4096
2	2048
2	1024
2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

Q.2 Show that

$$(i) \sqrt[3]{27} \times \sqrt[3]{64} = \sqrt[3]{27 \times 64} \quad (iv) \sqrt[3]{64 \times 729} = \sqrt[3]{64} \times \sqrt[3]{729}$$

$$(iii) \sqrt[3]{-125 \times 216} = \sqrt[3]{-125} \times \sqrt[3]{216}$$

$$(iv) \sqrt[3]{-125 \times -1000} = \sqrt[3]{-125} \times \sqrt[3]{-1000}$$

Solution: $\sqrt[3]{27} \times \sqrt[3]{64} = \sqrt[3]{27 \times 64}$

LHS $\sqrt[3]{27} \times \sqrt[3]{64} = 3 \times 4 = 12$

RHS $\sqrt[3]{27 \times 64} = \sqrt[3]{1728}$
 $= \sqrt[3]{2^3 \times 2^3 \times 3^3}$
 $= 2 \times 2 \times 3 = 12$

$\therefore LHS = RHS.$

2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
3	3
	1

$$(ii) \sqrt[3]{64 \times 729} = \sqrt[3]{64} \times \sqrt[3]{729}$$

LHS $\sqrt[3]{64 \times 729} = \sqrt[3]{46656}$
 $= \sqrt[3]{2^3 \times 2^3 \times 9^3}$
 $= 2 \times 2 \times 9 = 36$

2	46656
2	23328
2	11664
2	5832
2	2916
2	1458
9	729
9	81
9	9
	1

$$\underline{\text{RHS}} \quad \sqrt[3]{64} \times \sqrt[3]{729}$$

$$= 4 \times 9$$

$$= 36$$

$$\because \sqrt[3]{64} = 4, \sqrt[3]{729} = 9$$

$$\therefore \text{LHS} = \text{RHS}$$

$$(iii) \sqrt[3]{-125 \times 216} = \sqrt[3]{-125} \times \sqrt[3]{216}$$

$$\begin{aligned} \underline{\text{LHS}} \quad \sqrt[3]{-27000} &= \sqrt[3]{-27} \times \sqrt[3]{1000} \\ &= -\sqrt[3]{27} \times \sqrt[3]{1000} \\ &= -3 \times 10 = -30 \end{aligned}$$

$$\begin{aligned} \underline{\text{RHS}} \quad \sqrt[3]{-125} \times \sqrt[3]{216} &= -\sqrt[3]{125} \times \sqrt[3]{216} \quad \left\{ \begin{array}{l} \because \sqrt[3]{125} = 5 \\ \sqrt[3]{216} = 6 \end{array} \right. \\ &= -5 \times 6 \\ &= -30 \end{aligned}$$

$$\therefore \text{LHS} = \text{RHS}$$

$$(iv) \sqrt[3]{-125 \times -1000} = \sqrt[3]{-125} \times \sqrt[3]{-1000}$$

$$\begin{aligned} \underline{\text{LHS}}: \sqrt[3]{-125000} &= +\sqrt[3]{5^3 \times 5^3 \times 2^3} \\ &= +5 \times 5 \times 2 = +50 \end{aligned}$$

$$\underline{\text{RHS}} \quad \sqrt[3]{-125} \times \sqrt[3]{-1000}$$

$$= -\sqrt[3]{125} \times -\sqrt[3]{1000}$$

$$= -5 \times -10 = 50$$

$$\text{Therefore LHS} = \text{RHS}$$

Q.3 Find the cube root of each of the following numbers.

(i) 8×125

(ii) -1728×216

(iii) -27×2744

(iv) -729×-15625

Solution: (i) $\sqrt[3]{8 \times 125} = \sqrt[3]{8} \times \sqrt[3]{125}$
 $= 2 \times 5$
 $= 10$

(ii) $\sqrt[3]{-1728 \times 216}$
 $= \sqrt[3]{-1728} \times \sqrt[3]{216}$
 $= -\sqrt[3]{1728} \times \sqrt[3]{216}$
 $= -\sqrt[3]{2^3 \times 2^3 \times 3^3} \times \sqrt[3]{2^3 \times 3^3}$
 $= -2 \times 2 \times 3 \times 2 \times 3$
 $= -72$

2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
3	3
	1

2	216
2	108
2	54
3	27
3	9
3	3
	1

(iii) $\sqrt[3]{-27 \times 2744}$
 $= \sqrt[3]{-27} \times \sqrt[3]{2744}$
 $= -\sqrt[3]{27} \times \sqrt[3]{2744}$
 $= -\sqrt[3]{3^3} \times \sqrt[3]{2^3 \times 7^3}$
 $= -3 \times 2 \times 7 = -42$

3	27
3	9
3	3
	1

2	2744
2	1372
2	686
7	343
7	49
7	7
	1

$$(iv) \sqrt[3]{-729 \times -15625} = \sqrt[3]{-729} \times \sqrt[3]{-15625} \\ = -\sqrt[3]{729} \times -\sqrt[3]{15625}$$

$$\therefore \sqrt[3]{729} = 9, \sqrt[3]{15625} = 15$$

$$\therefore \sqrt[3]{-729 \times -15625} = -9 \times -15 \\ = 225$$

Q4 Evaluate

$$(i) \sqrt[3]{4^3 \times 6^3}$$

$$(ii) \sqrt[3]{8 \times 17 \times 17 \times 17}$$

$$(iii) \sqrt[3]{700 \times 2 \times 49 \times 5}$$

$$(iv) 125 \sqrt[3]{a^6} - \sqrt[3]{125 a^6}$$

Solution: (i) $\sqrt[3]{4^3 \times 6^3} = 4 \times 6 = 24$

$$(ii) \sqrt[3]{8 \times 17 \times 17 \times 17} = \sqrt[3]{2^3 \times 17^3} = 2 \times 17 = 34$$

$$(iii) \sqrt[3]{700 \times 2 \times 49 \times 5} = \sqrt[3]{7 \times 2^2 \times 5^2 \times 2^1 \times 7^2 \times 5^1} \\ = \sqrt[3]{7^3 \times 2^3 \times 5^3} = 7 \times 2 \times 5 \\ = 70$$

$$(iv) 125 \sqrt[3]{a^6} = 125 \sqrt[3]{a^3 \times a^3} \\ = 125 \times a \times a \\ = 125a^2$$

Q6 Find the cube root of each of the following numbers.

(i) 0.00 1728

(ii) 0.003375

(iii) 0.001

(iv) 1.331

Solutions (i) $\sqrt[3]{0.001728} = \frac{\sqrt[3]{1728}}{\sqrt[3]{1000000}} = \frac{\sqrt[3]{1728}}{\sqrt[3]{10^6}}$

$$= \frac{\sqrt[3]{2^3 \times 2^3 \times 3^3}}{\sqrt[3]{100^3}} = \frac{2 \times 2 \times 3}{100}$$

$$= \frac{12}{100} = 0.12$$

(ii) $\sqrt[3]{0.003375}$

$$= \frac{\sqrt[3]{3375}}{\sqrt[3]{1000000}} = \frac{\sqrt[3]{3^3 \times 5^3}}{\sqrt[3]{100^3}} = \frac{3 \times 5}{100} = \frac{15}{100} = 0.15$$

(iii) $\sqrt[3]{0.001} = \sqrt[3]{\frac{1}{1000}} = \frac{\sqrt[3]{1}}{\sqrt[3]{10^3}} = \frac{1}{10} = 0.1$

(iv) $\sqrt[3]{1.331} = \sqrt[3]{\frac{1331}{1000}} = \frac{\sqrt[3]{1331}}{\sqrt[3]{1000}} = \frac{11}{10} = 1.1$

Q.7 Evaluate each of the following

$$(i) \sqrt[3]{27} + \sqrt[3]{0.008} + \sqrt[3]{0.064}$$

$$(ii) \sqrt[3]{1000} + \sqrt[3]{0.008} - \sqrt[3]{0.125}$$

$$(iii) \sqrt[3]{\frac{729}{216}} \times \frac{6}{9}$$

$$(iv) \sqrt[3]{\frac{0.027}{0.008}} \div \sqrt{\frac{0.09}{0.04}} - 1$$

$$(v) \sqrt[3]{0.1 \times 0.1 \times 0.1 \times 13 \times 13 \times 13}$$

Solution (i) $\sqrt[3]{27} + \sqrt[3]{0.008} + \sqrt[3]{0.064}$

$$= \sqrt[3]{27} + \frac{\sqrt[3]{8}}{\sqrt[3]{1000}} + \frac{\sqrt[3]{64}}{\sqrt[3]{1000}}$$

$$= 3 + \frac{2}{10} + \frac{4}{10} = 3 + 0.2 + 0.4 = 3.6$$

$$(ii) \sqrt[3]{1000} + \sqrt[3]{0.008} - \sqrt[3]{0.125}$$

$$= \sqrt[3]{10^3} + \sqrt[3]{\frac{8}{1000}} - \sqrt[3]{\frac{125}{1000}}$$

$$= 10 + \frac{\sqrt[3]{8}}{\sqrt[3]{1000}} - \frac{\sqrt[3]{125}}{\sqrt[3]{1000}} = 10 + \frac{2}{10} - \frac{5}{10}$$

$$= 10 + 0.2 - 0.5$$

$$= 9.7$$

$$(iii) \sqrt[3]{\frac{729}{216}} \times \frac{6}{9}$$

$$= \frac{\sqrt[3]{9^3}}{\sqrt[3]{6^3}} \times \frac{6}{9}$$

$$= \frac{9}{6} \times \frac{6}{9} = 1$$

$$(iv) \sqrt[3]{\frac{0.027}{0.008}} \div \sqrt{\frac{0.09}{0.04}} - 1$$

$$= \frac{0.3}{0.2} \div \frac{0.3}{0.2} - 1$$

$$= \frac{0.3}{0.2} \times \frac{0.2}{0.3} - 1$$

$$= 1 - 1$$

$$= 0$$

$$(v) \sqrt[3]{0.1 \times 0.1 \times 0.1 \times 13 \times 13 \times 13} = \sqrt[3]{(0.1)^3 \times (13)^3}$$

$$= 0.1 \times 13$$

$$= 1.3$$

Q.8 Show that

$$(i) \frac{\sqrt[3]{729}}{\sqrt[3]{1000}} = \sqrt[3]{\frac{729}{1000}}$$

$$(ii) \frac{\sqrt[3]{-512}}{\sqrt[3]{343}} = \sqrt[3]{\frac{-512}{343}}$$

Solution (i) $\frac{\sqrt[3]{729}}{\sqrt[3]{1000}} = \sqrt[3]{\frac{729}{1000}}$

LHS $\frac{\sqrt[3]{729}}{\sqrt[3]{1000}} = \frac{\sqrt[3]{3^3 \times 3^3}}{\sqrt[3]{2^3 \times 5^3}} = \frac{3 \times 3}{2 \times 5} = \frac{9}{10}$

RHS $\sqrt[3]{\frac{729}{1000}} = \sqrt[3]{\frac{9^3}{10^3}} = \sqrt[3]{\left(\frac{9}{10}\right)^3} = \frac{9}{10}$

$\therefore \text{LHS} = \text{RHS}$

(ii) LHS $= \frac{\sqrt[3]{-512}}{\sqrt[3]{343}} = \frac{\sqrt[3]{(-8)^3}}{\sqrt[3]{(7)^3}} = \frac{-8}{7}$

RHS $\sqrt[3]{\frac{-512}{343}} = \sqrt[3]{\frac{(-8)^3}{(7)^3}} = \sqrt[3]{\left(\frac{-8}{7}\right)^3} = \frac{-8}{7}$

$\therefore \text{LHS} = \text{RHS}$

Q.9 Fill in the blanks:

(i) $\sqrt[3]{125 \times 27} = 3 \times \dots$ (ii) $\sqrt[3]{8 \times \dots} = 8$

(iii) $\sqrt[3]{1728} = 4 \times \dots$ (iv) $\sqrt[3]{480} = \sqrt[3]{3} \times 2 \times \sqrt[3]{\dots}$

(v) $\sqrt[3]{\dots} = \sqrt[3]{7} \times \sqrt[3]{8}$ (vi) $\sqrt[3]{\dots} = \sqrt[3]{4} \times \sqrt[3]{5} \times \sqrt[3]{6}$

(vii) $\sqrt[3]{\frac{27}{125}} = \frac{\dots}{5}$ (viii) $\sqrt[3]{\frac{729}{1331}} = \frac{9}{\dots}$ (ix) $\sqrt[3]{\frac{512}{\dots}} = \frac{8}{13}$

Solution (i) $\sqrt[3]{125 \times 27}$

$$= \sqrt[3]{5 \times 5 \times 5 \times 3 \times 3} = \sqrt[3]{5^3 \times 3^2}$$

$$= 3 \times 5$$

(ii) $\sqrt[3]{8 \times \dots} = 8$

$$\therefore \sqrt[3]{8 \times 8 \times 8} = 8$$

(iii) $\sqrt[3]{1728} = 12 = 4 \times 3$

(iv) $\sqrt[3]{480} = \sqrt[3]{2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 5}$

$$= \sqrt[3]{2^3 \times 2^2 \times 3 \times 5}$$

$$= 2 \times \sqrt[3]{4} \times \sqrt[3]{3} \times \sqrt[3]{5}$$

$$= \sqrt[3]{3} \times 2 \times \sqrt[3]{20}$$

$$\therefore \text{Ans} = \sqrt[3]{20}$$

(v) $\sqrt[3]{} = \sqrt[3]{7} \times \sqrt[3]{8}$

$$\therefore \sqrt[3]{7} \times \sqrt[3]{8} = \sqrt[3]{7 \times 8}$$

$$= \sqrt[3]{56}$$

(vi) $\sqrt[3]{4} \times \sqrt[3]{5} \times \sqrt[3]{6} = \sqrt[3]{4 \times 5 \times 6} = \sqrt[3]{120}$

$$(vi) \sqrt[3]{\frac{27}{125}} = \frac{\sqrt[3]{27}}{\sqrt[3]{125}} = \frac{3}{5}$$

$$(vii) \sqrt[3]{\frac{729}{1331}} = \frac{\sqrt[3]{729}}{\sqrt[3]{1331}} = \frac{9}{11}$$

$$(ix) \sqrt[3]{\frac{512}{2197}} = \frac{8}{13}$$

$$\therefore 13^3 = 2197$$

$$\therefore \sqrt[3]{\frac{512}{2197}} = \frac{8}{13}$$

Q.10 The volume of a cubical box is 474.552 m^3 .

Find the length of each side of the box.

Solution: $\therefore$ Volume of cube = 474.552

$$(\text{side})^3 = 474.552$$

$$\text{side} = \sqrt[3]{474.552} = \sqrt[3]{\frac{474552}{1000}}$$

$$= \frac{\sqrt[3]{474552}}{\sqrt[3]{1000}} = \frac{\sqrt[3]{2^3 \times 3^3 \times 13^3}}{\sqrt[3]{10^3}}$$

$$= \frac{2 \times 3 \times 13}{10} = \frac{78}{10} = 7\frac{4}{5}$$

Q.11 Three numbers are to one another $2:3:4$.

The sum of their cubes is 0.334125 . Find the numbers.

Solution:Let numbers are $2x$, $3x$ and $4x$

$$(2x)^3 + (3x)^3 + (4x)^3 = 0.334125$$

$$\Rightarrow 8x^3 + 27x^3 + 64x^3 = 0.334125$$

$$\Rightarrow 99x^3 = 0.334125$$

$$\Rightarrow x^3 = \frac{0.334125}{99} = \frac{3375}{1000000}$$

$$\Rightarrow x = \frac{\sqrt[3]{3375}}{\sqrt[3]{100^3}} = \frac{\sqrt[3]{15^3}}{\sqrt[3]{100^3}} = \frac{15}{100} = 0.15$$

Q.12 Find the side of a cube whose volume is $\frac{24389}{216} \text{ m}^3$.

Solution

$$\text{Volume} = \frac{24389}{216}$$

$$\Rightarrow (\text{side})^3 = \frac{24389}{216}$$

$$\Rightarrow \text{side} = \sqrt[3]{\frac{24389}{216}} = \frac{\sqrt[3]{24389}}{\sqrt[3]{216}}$$

$$= \frac{\sqrt[3]{29^3}}{\sqrt[3]{2^3 \times 3^3}} = \frac{29}{2 \times 3}$$

$$= \frac{29}{6}$$

$\therefore$ The length of side of cube = $\frac{29}{6} \text{ m}$.

Q.13 Evaluate

(i) $\sqrt[3]{36} \times \sqrt[3]{384}$

(ii) $\sqrt[3]{96} \times \sqrt[3]{144}$

(iii) $\sqrt[3]{100} \times \sqrt[3]{270}$

(iv) $\sqrt[3]{121} \times \sqrt[3]{297}$

Solution (i) $\sqrt[3]{36} \times \sqrt[3]{384}$

$$= \sqrt[3]{36 \times 384}$$

$$= \sqrt[3]{2^2 \times 3^2 \times 2^3 \times 2^3 \times 2^1 \times 3^1}$$

$$= \sqrt[3]{2^3 \times 2^3 \times 2^3 \times 3^3}$$

$$= 2 \times 2 \times 2 \times 3$$

$$= 24$$

2	36
2	18
3	9
3	3
	1

2	384
2	192
2	96
2	48
2	24
2	12
2	6
3	3
	1

(ii) $\sqrt[3]{96} \times \sqrt[3]{144}$

$$= \sqrt[3]{96 \times 144}$$

$$= \sqrt[3]{2^5 \times 3^1 \times 2^4 \times 3^2}$$

$$= \sqrt[3]{2^9 \times 3^3}$$

$$= 2 \times 2 \times 2 \times 3$$

$$= 24$$

2	96
2	48
2	24
2	12
2	6
3	3
	1

2	144
2	72
2	36
2	18
3	9
3	3
	1

$$(ii) \sqrt[3]{100} \times \sqrt[3]{270}$$

$$\therefore \because 100 = 2^2 \times 5^2$$

$$270 = 3^3 \times 2 \times 5$$

$$\therefore \sqrt[3]{2^2 \times 5^2 \times 3^3 \times 2 \times 5} = \sqrt[3]{2^3 \times 3^3 \times 5^3}$$

$$= 2 \times 3 \times 5 = 30$$

$$(iv) \sqrt[3]{121} \times \sqrt[3]{297}$$

$$= \sqrt[3]{121 \times 297}$$

$$= \sqrt[3]{11 \times 11 \times 3 \times 3 \times 3 \times 11}$$

$$= \sqrt[3]{3^3 \times 11^3} = 3 \times 11 = 33$$

Q.14 Find the cube root of the numbers 3048625, 20346417, 210644875, 57066625, using fact that

$$(i) 3048625 = 3375 \times 729$$

$$(ii) 20346417 = 9261 \times 2197$$

$$(iii) 210644875 = 42875 \times 4913$$

$$(iv) 57066625 = 166375 \times 343$$

Solution:

$$(i) \therefore \sqrt[3]{3048625} = \sqrt[3]{3375} \times \sqrt[3]{729}$$

$$= \sqrt[3]{15^3} \times \sqrt[3]{9^3}$$

$$= 15 \times 9 = 135$$

$$\begin{aligned}
 \text{(i)} \quad \sqrt[3]{20346417} &= \sqrt[3]{9261} \times \sqrt[3]{2197} \\
 &= \sqrt[3]{21^3} \times \sqrt[3]{13^3} \\
 &= 21 \times 13 \\
 &= 273
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii)} \quad \sqrt[3]{210644875} &= \sqrt[3]{42875} \times \sqrt[3]{4913} \\
 &= \sqrt[3]{5^3 \times 7^3} \times \sqrt[3]{17^3} \\
 &= 5 \times 7 \times 17 \\
 &= 595
 \end{aligned}$$

$$\begin{aligned}
 \text{(iv)} \quad \sqrt[3]{57066625} &= \sqrt[3]{166375} \times \sqrt[3]{343} \\
 &= \sqrt[3]{5^3 \times 11^3} \times \sqrt[3]{7^3} \\
 &= 5 \times 11 \times 7 \\
 &= 385
 \end{aligned}$$

Q.15 Find the unit digits of the cube root of the following numbers.

(i) 226981

(ii) 13824

(iii) 571787

(iv) 175616

Solution (i) unit digit in $\sqrt[3]{226981}$ is 1

(ii) unit digit in $\sqrt[3]{13824}$ is 4.

(iii) unit digit = 3

(iv) unit digit = 6.