Chapter-4

Cubes and Cube roots

Exercise 4.1

Q.1 Find the cubes of the following numbers:

(i) 7 (ii) 12 (iii) 16 (iv) 21 (v) 40

(vi) 55 (vii) 100 (viii) 302 (ix) 301

Solution (i) $(7)^3 = 7 \times 7 \times 7 = 343$

(ii) $(12)^3 = 12 \times 12 \times 12 = 1728$

(iii) $(16)^3 = 16 \times 16 \times 16 = 4096$

(iv) $(21)^3 = 21 \times 21 \times 21 = 9261$

(v) $(40)^3 = 40 \times 40 \times 40 = 64000$

(vi) $(55)^3 = 55 \times 55 \times 55 = 166375$

(vii) $(100)^3 = 100 \times 100 \times 100 = 1000000$

(viii) $(302)^3 = 302 \times 302 \times 302 = 27543608$

(ix) $(301)^3 = 301 \times 301 \times 301 = 27270901$

Q.2 Write cubes of all natural numbers between 1 to 10 and verify the following statements.

(i) Cubes of all odd natural numbes are odd.

(ii) Cubes of all even natural numbers are even.

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Solution $(1)^3 = 1$

$(2)^3 = 8$

$(3)^3 = 27$

$(4)^3 = 64$

$(5)^3 = 125$

$6^3 = 216$

$7^3 = 343$

$8^3 = 512$

$9^3 = 729$

$10^3 = 1000$

From the above table, we can say that

(i) Cubes of all odd natural numbers are odd.

(ii) Cubes of all even natural numbers are even.

Q.3 Observe the following pattern

$$1^3 = 1$$

$$1^3 + 2^3 = (1+2)^2$$

$$1^3 + 2^3 + 3^3 = (1+2+3)^2$$

Write the next three rows and Calculate the value of $1^3 + 2^3 + 3^3 + \dots + 9^3 + 10^3$ by the above pattern.

Sol: $1^3 + 2^3 + 3^3 + 4^3 = (1+2+3+4)^2$

$1^3 + 2^3 + 3^3 + 4^3 + 5^3 = (1+2+3+4+5)^2$

$1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 = (1+2+3+4+5+6)^2$

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Also

$$1^3+2^3+3^3+---+8^3+9^3+10^3$$
$$=(1+2+3+---+8+9+10)^2$$
$$=(55)^2=3025$$

Q.4 Write the cubes of 5 natural numbers which are multiples of 3 and verify the followings.

'The cube of a natural number which is a multiple of 3 is a multiple of 27'

Solution: As first five multiples of 3 are 3, 6, 9, 12, 15

$\therefore (3)^3 = 27$

$(6)^3 = 216$

$9^3 = 729$

$12^3 = 1728$

$15^3 = 3375$

Verification: We found that all cubes are divisible by 27.

$\therefore$ The cube of a natural number which is a multiple of 3 is a multiple of 27.

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Q.6 Write the cubes of 5 natural numbers of the form $3n+2$ (i.e 5, 8, 11 ...) and verify the following

The cube of a natural number of the form $3n+2$ is a natural number of the same form i.e. when it is dividend by 3 and the remainder is 2.

Solution: The first 5 natural numbers in the form $(3n+2)$ are 5, 8, 11, 14 and 17. So

$(5)^3 = 125$

$(8)^3 = 512$

$(11)^3 = 1331$

$(14)^3 = 2744$

$(17)^3 = 4913$

also $125 = 41 \times 3 + 2$; $512 = 170 \times 3 + 2$

$1331 = 443 \times 3 + 2$; $2744 = 914 \times 3 + 2$

$4913 = 1637 \times 3 + 2$

∴ cubes of these numbers are also in the form of $3n+2$.

Q.5 Write the cubes of 5 natural numbers which are of the form $3n+1$. is a natural number of the same form i.e when divided by 3, It leaves the remainder 1.

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Solution : five natural numbers in the form of 3n+1 are 4, 7, 10, 13, 16.

$(4)^3 = 64$; $(7)^3 = 343$; $(10)^3 = 1000$

$(13)^3 = 2197$ $(16)^3 = 4096$

Also

$64 = 3 \times 21 + 1$

$343 = 114 \times 3 + 1$

$1000 = 333 \times 3 + 1$

$2197 = 732 \times 3 + 1$

$4096 = 1365 \times 3 + 1$

∴ cubes of above numbers are also in the form of 3n+1.

Q.7 Write the cubes of 5 natural numbers of which are multiples of 7 and verify the following

The cube of a multiple of 7 is a multiple of 7^3.

Sol: The first five multiples of 7 are 7, 14, 21, 28, 35

Cubes of above numbers are

$(7)^3 = 343$, $(14)^3 = 2744$, $(21)^3 = 9261$

$(28)^3 = 21952$ $(35)^3 = 42875$

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Also, we found that all these numbers are divisible by 7^3 i.e 343.

Q8 Which of the following are perfect cubes

(i) 64 (ii) 216 (iii) 343 (iv) 1000.

(v) 1728 (vi) 3087 (vii) 4608 (viii) 106480

(ix) 166375 (x) 458533

Sol: (i) 64

$\because 64 = (2\times2\times2)\times(2\times2\times2)$

here Prime factor 2 is in pairs of triples.

$\therefore$ It is a perfect cube.

2	64
2	32
2	16
2	8
2	4
2	2
	1

(ii) $216 = 2\times2\times2\times3\times3\times3$

$\therefore$ It is a perfect cube.

2	216
2	108
2	54
3	27
3	9
3	3
	1

(iii) $343 = 7\times7\times7$

Yes, It is a perfect cube

7	343
7	49
7	7
	1

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(iv) $1000 = (10)^3 = (2\times5)^3 = 2\times2\times2\times5\times5\times5$

∴ Yes, it is a perfect cube.

(v) $1728 = 2\times2\times2\times2\times2\times2\times3\times3\times3$

2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
3	3
	1

∴ $1728 = (2\times2\times2)\times(2\times2\times2)\times(3\times3\times3)$

Hence, Yes, it is a perfect cube.

(vi) $3087 = 3\times3\times(7\times7\times7)$

3	3087
3	1029
7	343
7	49
7	7
	1

Here, Prime factor 3 is not in a group of triplets.

∴ 3087 is not a perfect cube.

(vii) $4608 = 2\times2\times2\times2\times2\times2\times2\times2\times2\times3\times3\times3$

2	4608
2	2304
2	1152
2	576
2	288
2	144
2	72
2	36
2	18
3	9
3	3

∴ Yes, it is a perfect cube.

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(iii) 106480

∴ 106480

$= 2 \times 2 \times 2 \times 2 \times 5 \times 11 \times 11 \times 11$

here 2 and 5 are not in groups.

hence it is not a perfect cube.

2	106480
2	53240
2	26620
2	13310
5	6655
11	1331
11	121
11	11
	1

(ix) $166375 = 5 \times 5 \times 5 \times 11 \times 11 \times 11$

Yes, it is a perfect cube.

5	166375
5	33275
5	6655
11	1331
11	121
11	11
	1

(x) 456533

7	456533
7	65219
7	9317
11	1331
11	121
M	11
	1

∴ Yes, it is a perfect cube.

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Q9 Which of the following are cubes of even natural numbers?

216, 512, 729, 1000, 3375, 13824

Solution: As we know cube of even number is even.

∴ 216, 512, 1000, 13824 are cubes of even numbers.

Q.10: Which of the following are cubes of odd natural numbers?

125, 343, 1728, 4096, 32768, 6859

Sol: As cube of odd numbers is odd.

∴ 125, 343, 6859 are cubes of odd natural numbers.

Q.11 What is the smallest number by which of the following numbers must be multiplied so that the Products are perfect cubes?

(i) 675 (ii) 8640 (iii) 2560 (iv) 7803

(v) 107811 (vi) 35721

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Solution:

(i) $675 = 5 \times 5 \times 3 \times 3 \times 3$

∴ It should be multiply by 5.

5	675
5	135
3	27
3	9
3	3
	1

(ii)

$8640 = (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times 5 \times (3 \times 3 \times 3)$

To make completi pairs of triplicates, we should Multiply by 5×5 ie 25

2	8640
2	4320
2	2160
2	1080
2	540
2	270
5	135
3	27
3	9
3	3
	1

(iii) $2560 = (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times 5$

It should be multiply by 5×5 ie 25.

2	2560
2	1280
2	640
2	320
2	160
2	80
2	40
2	20
2	10
5	5
	1

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(iv) 7803

By Prime factorisation

$7803 = 3\times3\times3\times17\times17$

∴ To make it perfect cube, it should be Multiply by 17.

(v) 107811

By Prime factorisation we get

$107811 = 3\times3\times3\times3\times11\times11\times11$

∴ It should be multiply by 3 to make perfect cube.

(vi) 35721

By Prime factorisation we get

$35721 = 3\times3\times3\times3\times3\times3\times7\times7$

∴ To make it perfect cube, it should be multiply by 7.

Q.12 By which Smallest number must the following numbers be divided so that the quotient becomes perfect cube?

(i) 675 (ii) 8640 (iii) 1600 (iv) 8788

(v) 7803 (vi) 107811 (vii) 35721 (viii) 243000

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Solution: (i) 675

By Prime factorisation we get

$675 = 3 \times 3 \times 3 \times 5 \times 5$

To make quotient perfect cube, 675 should be divided by 5×5 i.e 25.

(ii) $8640 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5$

$= 2^3 \times 2^3 \times 2^3 \times 5^1$

To make 8640 perfect cube. we divide it by 5.

(iii) $1600 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5$

$= 2^3 \times 2^3 \times 5^2$

To make 8640 perfect cube, it should be divided by 5×5 i.e 25.

(iv) 8788

By Prime factorisation we get

$8788 = 2 \times 2 \times 13 \times 13 \times 13$

$= 2^2 \times 13^3$

To make quotient perfect cube, we divide it by 2×2 i.e 4.

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(v) 7803

By Prime factorisation we get

$7803 = 3 \times 3 \times 3 \times 17 \times 17$

$= 3^3 \times 17^2$

To make quotient perfect cube, it should be divided by 17^2 i.e 289.

(vi) 107811

By Prime factorisation we get

$107811 = 3 \times 3 \times 3 \times 3 \times 11 \times 11 \times 11$

$= 3^3 \times 11^3 \times 3$

To make quotient perfect cube it should be divided by 3.

(vii) 35721

By Prime factorisation we get

$35721 = 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 7$

$= 3^3 \times 3^3 \times 7$

To make quotient perfect cube, it should be divided by 7.

(viii) 243000

By Prime factorisation, we get

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$243000 = 2\times2\times2\times3\times3\times3\times3\times3\times5\times5\times5$

$= 2^3\times3^3\times3^2\times5^3$

To make quotient perfect cube, it should be divided by 3^2 i.e 9.

Q.13 Prove that if a number is trebled then its cube is 27 times the cube of the given number.

Solution: Let the number be x

trebled $= 3x$

Cube of $3x = (3x)^3 = 27x^3$

$\therefore$ $27x^3$ is 27 times the cube of given no.

Q.14 What happens to the cube of a number if the number is multiplied by

(i) 3 ? (ii) 4 ? (iii) 5 ?

Solution: (i) Let the number be x

Its cube $= x^3$

If no. is multiplied by 3, new number $= (3x)$

Cube of new number $= (3x)^3$

$= 27x^3$

$\therefore$ New number is 27 times the given number.

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(ii) If number is Multiplied by 4.

New number $= 4x$

Cube $= (4x)^3 = 64x^3$

∴ New number will be 64 times of the original number.

(iii) If number is multiplied by 5.

New number $= (5x)$

Cube $= (5x)^3 = 125x^3$

∴ New number will be 125 times of the original number.

Q.4 Find the volume of a cube, one face of which has an area of $64\ m^2$.

Solution : Area of one face of cube $= 64\ m^2$

$(side)^2 = 64\ m^2$

$side = \sqrt{64} = 8\ m$

∴ Volume of cube $= (8)^3 = 512\ m^3$

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Q.16 Find the volume of a cube whose surface area is $384\ m^2$.

Solution : Surface area $= 384\ m^2$

$$6 \times (\text{Side})^2 = 384$$

$$(\text{Side})^2 = \frac{384}{6} = 64$$

$$\Rightarrow \text{Side} = \sqrt{64} = 8\ m$$

$\therefore$ Volume of cube $= (\text{Side})^3$

$$= (8)^3 = 512\ m^3$$

Q.17 Evaluate (i) $\{(5^2+12^2)^{1/2}\}^3$

(ii) $\{(6^2+8^2)^{1/2}\}^3$

Solution : (i) $\{(5^2+12^2)^{\frac{1}{2}}\}^3$

$$= \{(25+144)^{\frac{1}{2}}\}^3$$

$$= \{(169)^{\frac{1}{2}}\}^3 = \{(13^2)^{\frac{1}{2}}\}^3$$

$$= (13)^3 = 13 \times 13 \times 13$$

$$= 2197\ m^3$$

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(11) $\{(6^2+8^2)^{1/2}\}^3$

$= \{(36+64)^{\frac{1}{2}}\}^3 = \{(100)^{\frac{1}{2}}\}^3$

$= \{(10^2)^{\frac{1}{2}}\}^3 = 10^3 = 1000$

Q.18 Write the units digits of the cube of each of the following numbers.

31, 109, 388, 833, 4276, 5922, 7774, 44447, 125125125

Solution:

Number (a)	unit digit a^3
31	1
109	9
388	2
833	7
4276	6
5922	8
7774	4
44447	3
125125125	5

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Q.19 Find the cube of following numbers by column method.

(i) 35 (ii) 56 (iii) 72

Solution : (i) 35

here $a=3$, $b=5$

	Column I	Column II	Column III	Column IV
(i)	a^3	$3a^2b$	$3ab^2$	b^3
	$(3)^3=27$	$3\times(3)^2\times(5)$	$3\times3\times(5)^2$	$(5)^3$
	$+15$	135	225	125
	42	$+23$	$+12$	
		158	237	

$\therefore (35)^3 = 42875$

(ii) 56, here $a=5$, $b=6$

Column I	Column II	Column III	Column IV
$(5)^3=125$	$3\times(5)^2\times6$	$3\times5\times(6)^2$	$(6)^3$
$+50$	$=450$	540	216
175	$+56$	$+21$	
	506	561	

$\therefore (56)^3 = 175616$

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(iii) 72 , $a=7$, $b=2$

a^3	$3a^2b$	$3ab^2$	b^3
$(7)^3 = 343$	$3\times(7)^2\times 2$	$3\times 7\times(2)^2$	$(2)^3$
$+30$	$= 294$	84	$= 8$
373	$+8$		
302			

$\therefore (72)^3 = 373248$

Q.20 Which of the following numbers are not perfect cubes.

(i) 64 (ii) 216 (iii) 243 (iv) 1728

Solution (i) $64 = 2\times2\times2\times2\times2\times2$

$= 2^3\times2^3$

2	64
2	32
2	16
2	8
2	4
2	2
	1

$\therefore$ Yes, it is a perfect cube.

(ii) $216 = 2^3\times6^3$

$\therefore$ Yes, it is a perfect cube

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(iii) $243 = 3\times3\times3\times3\times3$

$= 3^2 \times 3^3$

∴ No, it is not a perfect cube.

3	243
3	81
3	27
3	9
3	3
	1

(iv) 1728

$= 2^3 \times 2^3 \times 3^3$

∴ Yes, it is a perfect cube.

2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
3	3
	1

Q.21 For each of the non perfect cubes in Q.No. 20 Find the Smallest number by which it must be

(a) multiplied so that the product is a perfect cube.

(b) divided so that the quotient is a perfect abe.

Sol: (a) 243 is non perfect cube

$\therefore 243 = (3\times3\times3)\times(3\times3)$

∴ It should be multiplied by 3 to get a perfect cube.

3	243
3	81
3	27
3	9
3	3
	1

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(b) $\because 243 = (3 \times 3 \times 3) \times 3 \times 3$

Hence it should be divided by 3×3 i.e 9 to get a perfect cube.

Q.22 By taking three different values of n Verify the truth of the following statements.

(i) If n is even, then n^3 is even.

Proof: Let take $n = 4, n = 6, n = 8$

then If $n = 4$, $n^3 = (4)^3 = 64$ is even.

If $n = 6$, $n^3 = (6)^3 = 216$ is even

If $n = 8$, $n^3 = (8)^3 = 512$ is even

(ii) If n is odd, then n^3 is also odd.

Proof: Let $n = 3, 5, 7$

If $n = 3$, $n^3 = (3)^3 = 27$ is odd.

$n = 5$, $n^3 = (5)^3 = 125$ is odd.

$n = 7$, $n^3 = (7)^3 = 343$ is odd.

(iii) If n leaves remainder 1 when divided by 3, then n^3 also leaves 1 as remainder when divided by 3.

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Solution : let possible values of n are $4, 7, 10$

then $n^3 = (4)^3 = 64 = 3 \times 21 + 1$

$n^3 = (7)^3 = 343 = 3 \times 114 + 1$

$n^3 = (10)^3 = 1000 = 3 \times 333 + 1$

(iv) If a natural number n is of the form $3p+2$ then n^3 also a number of same type.

Sol: let us consider 3 natural number of the form $3p+2$ are 5, 8, and 11.

$(5)^3 = 125 = 3 \times 41 + 2$

$(8)^3 = 512 = 3 \times 170 + 2$

$(11)^3 = 1331 = 3 \times 443 + 2$

Hence, statement is verified.

Q.23 Write true or false (F) for the following statements.

(i) 392 is a perfect cube.

(ii) 8640 is not a perfect cube

(iii) No cube can end with exactly two zeroes.

(iv) There is no perfect cube which ends in 4.

(v) For an integer a, a^3 is always greater than a^2.

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(vi) If a and b are integers such that $a^2 > b^2$, then $a^3 > b^3$

(vii) If a divides b, then a^3 divides b^3.

(viii) If a^2 ends in 9, then a^3 ends in 7.

(ix) If a^2 ends in 5, then a^3 ends in 25.

(x) If a^2 ends in an even number of zeroes, then a^3 ends in an odd number of zeroes.

Sol: (i) False (ii) True (iii) True

(iv) False (v) False (vi) False

(vii) True (viii) False (ix) False

(x) False.

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