

Solution of Practice Paper
Term -II (2021-22)
Class – XII
Mathematics (Code: 041)

Q. No.	VALUE POINTS
	SECTION – A
1	Let $I = \int \left(\frac{1+x}{1+x^2} \right) dx = \int \left(\frac{1}{1+x^2} \right) dx + \int \left(\frac{x}{1+x^2} \right) dx \dots\dots\dots(1)$ $\int \left(\frac{x}{1+x^2} \right) dx = \tan^{-1} x$ [formula] to evaluate $\int \left(\frac{x}{1+x^2} \right) dx$ put $1+x^2=t$ on differentiating both sides we get $2x dx = dt$ or $x dx = \frac{1}{2} dt$ $\int \left(\frac{x}{1+x^2} \right) dx = \int \left(\frac{1}{2} \frac{dt}{t} \right) = \frac{1}{2} \log t = \frac{1}{2} \log 1+x^2$ $= \frac{1}{2} \log 1+x^2$ Putting these values in equation (1) we have $I = \tan^{-1} x + \frac{1}{2} \log 1+x^2 + c$ OR SOL- $\int \left(\frac{3x-6+6-1}{(x-2)^2} \right) dx = \int \left(\frac{3(x-2)+5}{(x-2)^2} \right) dx = 3 \log x-2 + 5 \int (x-2)^{-2} dx$ $= 3 \log x-2 + 5 \frac{(x-2)^{-1}}{-1} + c = 3 \log x-2 - \frac{5}{x-2} + c$
2	Given differential equation is $\frac{d}{dx} \left\{ \left(\frac{dy}{dx} \right)^3 \right\} = 0$ $\Rightarrow 3 \left(\frac{dy}{dx} \right)^{3-1} \frac{d}{dx} \left(\frac{dy}{dx} \right) = 0$ $\Rightarrow 3 \left(\frac{dy}{dx} \right)^2 \frac{d^2 y}{dx^2} = 0$ clearly the highest order derivative occurring in the differential equation is $\frac{d^2 y}{dx^2}$ so its order is 2. Also it is a polynomial equation in derivative and the highest power raised to is $\frac{d^2 y}{dx^2}$ one so its degree is one . Hence the sum of the order and degree of the above given differential equation is $2+1=3$

3	Sol-Let $\hat{a}$ and $\hat{b}$ be unit vectors such that $\hat{a}+\hat{b}$ is also a unit vector $\hat{a} = \hat{b} = \hat{a}+\hat{b}$------(1) we know that $\hat{a}+\hat{b} ^2=(\hat{a}+\hat{b})^2=(\hat{a})^2+(\hat{b})^2+2\hat{a}\hat{b}$------(2) $\hat{a}-\hat{b} ^2=(\hat{a}-\hat{b})^2=(\hat{a})^2+(\hat{b})^2-2\hat{a}\hat{b}$------(3) adding equations (2) and (3) we have $\hat{a}+\hat{b} ^2+ \hat{a}-\hat{b} ^2=2(\hat{a})^2+2(\hat{b})^2= \hat{a} ^2+ \hat{b} ^2$ putting the values of $\hat{a}$ $\hat{b}$ $\hat{a}+\hat{b}$ (each=1) $1^2+ \hat{a}-\hat{b} ^2=2 \hat{a} ^2+2 \hat{b} ^2=2(1)+2(1)$ $\hat{a}-\hat{b} ^2=3 \Rightarrow \hat{a}-\hat{b} =\sqrt{3}$										
4	$\frac{8}{\sqrt{89}}, \frac{-4}{\sqrt{89}}, \frac{3}{\sqrt{89}}$										
5	Sol- Three balls are drawn one by one without replacement from a bag containing 5 white and 4 green balls . Let X denote number of green balls (out of three green balls drawn) $\Rightarrow X=0,1,2$ and 3 only (and not upto 4 , the number of green balls in the bag) $P(X=0)$= Probability of getting green no green balls in the three draws i.e., all the three white balls $=P(WWW)=\frac{5}{9} \times \frac{4}{8} \times \frac{3}{7} = \frac{60}{504} = \frac{5}{42}$ $P(X=1)$= Probability of getting one green balls (and hence two white balls) in three draws $=P(GWW)+P(WGW)+P(WWG)=\frac{4}{9} \times \frac{5}{8} \times \frac{4}{7} + \frac{5}{9} \times \frac{4}{8} \times \frac{4}{7} + \frac{5}{9} \times \frac{4}{8} \times \frac{4}{7}$ $=\frac{240}{504} = \frac{20}{42}$ similarly $P(X=2)$= Probability of getting two green balls (and hence one white ball) in three draws $=\frac{15}{42}$ and similarly $P(X=3)$= Probability of getting all the three green balls = $P(GGG)=\frac{4}{9} \times \frac{3}{8} \times \frac{2}{7} = \frac{2}{42}$ Hence probability distribution of X is <table><tr><td>X</td><td>0</td><td>1</td><td>2</td><td>3</td></tr><tr><td>P(X)</td><td>$\frac{5}{42}$</td><td>$\frac{20}{42}$</td><td>$\frac{15}{42}$</td><td>$\frac{15}{42}$</td></tr></table>	X	0	1	2	3	P(X)	$\frac{5}{42}$	$\frac{20}{42}$	$\frac{15}{42}$	$\frac{15}{42}$
X	0	1	2	3							
P(X)	$\frac{5}{42}$	$\frac{20}{42}$	$\frac{15}{42}$	$\frac{15}{42}$							
6	Solution- There are 26 red cards and 26 black cards in a pack of 52 playing cards . Let R and B denote the event of drawing red card and black card respectively Now required probability=$P(\text{first card is red and second one is black})+P(\text{first card is black and second one is red})$ $=P(R)+P\left(\frac{B}{R}\right)+P(B)P\left(\frac{R}{B}\right)$ $=\frac{26}{52} \times \frac{26}{51} + \frac{26}{52} \times \frac{26}{51}$ $=\frac{26}{51}$										

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SECTION – B

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$$\text{Let } I = \int \left(\frac{dx}{1+x+x^2+x^3} \right) = \int \left(\frac{dx}{(1+x)+x^2(1+x)} \right) = \int \left(\frac{dx}{(1+x)(1+x^2)} \right)$$

$$\text{let } \left(\frac{dx}{(1+x)(1+x^2)} \right) = \frac{A}{1+X} + \frac{BX+C}{1+x^2} \text{-----(1)}$$

Multiply both sides by L.C.M $(1+x)(1+x^2)$ we get

$$1 = A(1+x^2) + (BX+C)(1+x)$$

$$\text{OR } 1 = A + Ax^2 + Bx + Bx^2 + C + Cx$$

Equating coefficients of x^2, x and constant term, we get

$$A+B=0$$

$$B+C=0$$

Putting values

$$A+C=1$$

$$\text{Solving A,B,C we get } A = \frac{1}{2}, C = \frac{1}{2}, B = -A = -\frac{1}{2}$$

Putting values of A,B,C in (1) we get

$$\left(\frac{1}{(1+x)(1+x^2)} \right) = \frac{\frac{1}{2}}{1+X} + \frac{\left(-\frac{1}{2}x + \frac{1}{2} \right)}{1+X^2}$$

$$= \frac{1}{2} \frac{1}{1+X} - \frac{1}{2} \frac{x}{1+x^2} + \frac{1}{2} \frac{1}{1+x^2}$$

$$\text{Hence } I = \frac{1}{2} \log|(1+x)| - \frac{1}{4} \log(1+x^2) + \frac{1}{2} \tan^{-1} x + c$$

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The given differential equation is

$$x dy - y dx = \sqrt{x^2 + y^2} dx$$

dividing by dx

$$x \frac{dy}{dx} - y = \sqrt{x^2 + y^2}$$

$$\Rightarrow x \frac{dy}{dx} = y + \sqrt{x^2 + y^2} \Rightarrow \frac{dy}{dx} = \frac{y}{x} + \frac{\sqrt{x^2 + y^2}}{x}$$

$$\text{Put } y = vx \Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$$

Therefore

$$v + x \frac{dv}{dx} - \frac{vx}{x} = \frac{\sqrt{x^2 + v^2 x^2}}{x} \Rightarrow v + x \frac{dv}{dx} - v = \sqrt{1 + v^2}$$

$$\Rightarrow \int \left(\frac{dv}{\sqrt{1+v^2}} \right) = \int \left(\frac{dx}{x} \right) \Rightarrow \log |v + \sqrt{1+v^2}| = \log|x| + \log|c| \Rightarrow \log \log \left| \frac{y}{x} + \sqrt{1 + \frac{y^2}{x^2}} \right| = \log|Cx| \Rightarrow \frac{y}{x} + \sqrt{\frac{x^2 + y^2}{x^2}}$$

$$= Cx \Rightarrow y + \sqrt{x^2 + y^2} = Cx^2$$

OR

The given differential equation is

$$\frac{dY}{dx} - 3y \cot x = \sin 2x \text{ given that } y=2 \text{ when } x = \frac{\pi}{2}$$

$$\text{Comparing with } \frac{dY}{dx} + Py = Q \text{ we have } P = 3 \cot x \text{ and } Q = \sin 2x$$

$$\int P dx$$

$$= -3 \int \cot x dx = -3 \log \sin x = \log (\sin x)^{-3}$$

$$IF = \int e^{pdx} = e^{\log (\sin x)^{-3}} = (\sin x)^{-3} = \frac{1}{\sin^3 x}$$

The general solution is $y(IF) = \int Q(IF) dx + C$

or $y \frac{1}{\sin^3 x} = \int \sin 2x \frac{1}{\sin^3 x} dx + C$

$$\frac{y}{\sin^3 x} = \int \frac{(2 \sin x \cos x)}{\sin^3 x} dx + C$$

$$= 2 \int \frac{\cos x}{\sin^2 x} dx + C = 2 \int \frac{\cos x}{\sin x \cos x} dx + C = \int 2 \operatorname{cosec} x \cot x dx = -2 \operatorname{cosec} x + C$$

or

$$\frac{y}{\sin^3 x} = \frac{-2}{\sin x} + C$$

$$y = \sin^3 x$$

multiplying every term by LCM = $\sin^3 x$

$$y = -2 \sin^2 x + c \sin^3 x$$

To find C putting $y = 2$ when $x = \frac{\pi}{2}$ (given in (1))

$$2 = -2 \sin^2 \frac{\pi}{2} + c \sin^3 \frac{\pi}{2}$$

or $2 = -2 + c$ or $c = 4$ putting $c = 4$ the required particular solution is

$$y = -2 \sin^2 x + 4 \sin^3 x$$

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$$\vec{a} \times \vec{b} = \vec{c} \Rightarrow \vec{c} \perp \vec{a} \text{ and } \vec{c} \perp \vec{b}$$

(By def of cross product) -----(1)

$$\text{similarly } \vec{b} \times \vec{c} = \vec{a} \Rightarrow \vec{a} \perp \vec{b} \text{ and } \vec{a} \perp \vec{c} \text{ -----(2)}$$

from (1) and (2) we have $\vec{a}, \vec{b}, \vec{c}$ are mutually at right angles.

Now $\vec{a} \times \vec{b} = \vec{c}$ (given) $\Rightarrow |\vec{a} \times \vec{b}| = |\vec{c}|$

Using (2) $|\vec{a}| |\vec{b}| \sin 90^\circ = |\vec{c}|$

$$|\vec{a}| |\vec{b}| = |\vec{c}| \text{ -----(3)}$$

similarly $\vec{b} \times \vec{c} = \vec{a} \Rightarrow |\vec{b} \times \vec{c}| = |\vec{a}|$

$$\Rightarrow |\vec{b}| |\vec{c}| = |\vec{a}| \text{ ----(4)}$$

Dividing (3) by (4) (to eliminate $|\vec{b}|$) we have

$$\frac{|\vec{a}|}{|\vec{c}|} = \frac{|\vec{c}|}{|\vec{a}|} \Rightarrow |\vec{a}|^2 = |\vec{c}|^2 \text{ therefore } |\vec{a}| = |\vec{c}| \text{ -----(5)}$$

dividing (3) by (5) $|\vec{b}| = 1$ putting it in (4) we have

$$|\vec{c}| = |\vec{a}|$$

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Solution-Here $\vec{b}_1 = \hat{i} + 2\hat{j} - 2\hat{k}$ and $\vec{b}_2 = 2\hat{i} + 4\hat{j} - 4\hat{k} = 2(\hat{i} + 2\hat{j} - 2\hat{k})$

$$= 2\vec{b}_1$$

Therefore, the lines are parallel now using the formula for distance between parallel lines =

$$\frac{|(\vec{a}_2 - \vec{a}_1) \times \vec{b}|}{|\vec{b}|}$$

Shortest distance is $2\sqrt{5}$

OR

Sol-

Given point is $\vec{a}_1 = (1, 2, -4) = \hat{i} + 2\hat{j} - 4\hat{k}$ we know that vector along the line $\vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} = \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$ is $\vec{b}_1 = (2\hat{i} + 3\hat{j} + 6\hat{k})$ and a vector along the line $\vec{b}_2 = (\hat{i} + \hat{j} - \hat{k})$

we know vector equation of the plane passing through point $\vec{a}$ and parallel to the two given lines is $(\vec{r} - \vec{a}_1) \cdot \vec{n} = 0$

where $\vec{n} = \vec{b}_1 \times \vec{b}_2$ -----(1)

now $\vec{n} = \vec{b}_1 \times \vec{b}_2$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 6 \\ 1 & 1 & -1 \end{vmatrix}$$

$$= \hat{i}(-3-6) - \hat{j}(-2-6) + \hat{k}(2-3) = -9\hat{i} + 8\hat{j} - \hat{k}$$

$$\text{from equation (1)} \vec{r} \cdot \vec{n} - \vec{a}_1 \cdot \vec{n} = 0 \Rightarrow \vec{r} \cdot \vec{n} = \vec{a}_1 \cdot \vec{n} \text{-----(2)}$$

putting values of $\vec{a}_1$ and $\vec{n}$ in (2) equation of required plane is

$$\text{i.e., } \vec{r} \cdot (-9\hat{i} + 8\hat{j} - \hat{k}) = (\hat{i} + 2\hat{j} - 4\hat{k}) \cdot (-9\hat{i} + 8\hat{j} - \hat{k}) = -9 + 16 + 4$$

$$\text{or } \vec{r} \cdot (-9\hat{i} + 8\hat{j} - \hat{k}) = 11 \quad \text{(Vector form of the plane)}$$

$$\text{or } (x\hat{i} + y\hat{j} + z\hat{k}) \cdot (-9\hat{i} + 8\hat{j} - \hat{k}) = 11$$

$$\text{or } 9x + 8y - z = 11$$

$$\text{or } 9x + 8y - z - 11 = 0 \quad \text{(Cartesian form of the plane)}$$

SECTION – C

11

$$\text{Let } I = \int_{-1}^2 |x^3 - x| dx$$

$$\text{Again let } f(x) = x^3 - x = x(x^2 - 1) = x(x - 1)(x + 1)$$

Now break the limit at $x=0, 1$ (because on putting $f(x)=0$ we get $x=0, 1, -1$)

It is clear that $x^3 - x \geq 0$ on $[-1, 0]$

$$x^3 - x \leq 0 \text{ on } [0, 1]$$

$$x^3 - x \geq 0 \text{ on } [1, 2]$$

Hence the interval of the integral can be subdivided as

$$\begin{aligned} \int_{-1}^2 |x^3 - x| dx &= \int_{-1}^0 (x^3 - x) dx - \int_0^1 (x^3 - x) dx + \int_1^2 (x^3 - x) dx \\ &= \int_{-1}^0 (x^3 - x) dx + \int_0^1 (x - x^3) dx + \int_1^2 (x^3 - x) dx = \frac{11}{4} \end{aligned}$$

12

Given equation of the circle is $x^2 + y^2 = 16$ and $\sqrt{3}y = x$, represents a line through origin.

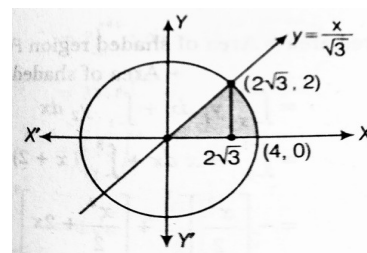
The line $y = \frac{1}{\sqrt{3}}x$ intersect the circle.

$$\text{Therefore } x^2 + \frac{x^2}{3} = 16$$

$$\frac{3x^2 + x^2}{3} = 16 \Rightarrow 4x^2 = 48$$

$$\Rightarrow x^2 = 12 \Rightarrow x = \pm 2\sqrt{3}$$

$$\text{When } x = 2\sqrt{3} \text{ then } y = \frac{2\sqrt{3}}{\sqrt{3}} = 2$$



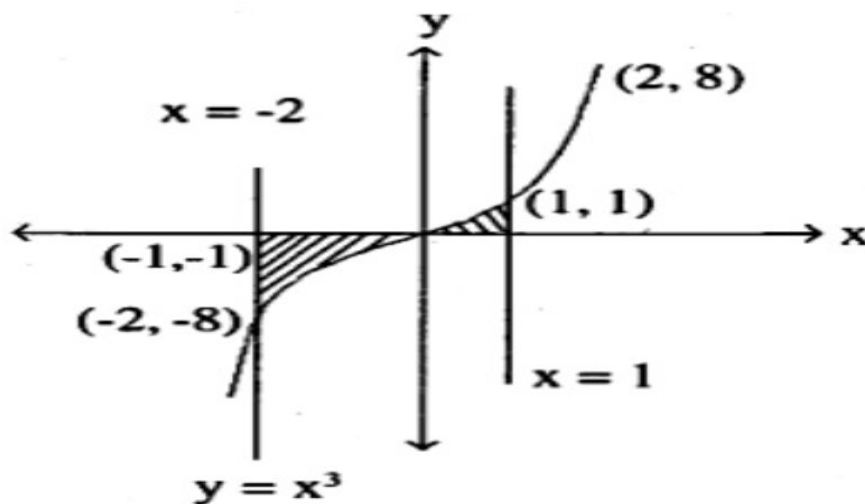
Required area shaded in first quadrant = (Area under the line $y = \frac{1}{\sqrt{3}}x$ from $x=0$ to $2\sqrt{3}$) +

(Area under the circle from $x=2\sqrt{3}$ to $x=4$)

$$= \int_0^{2\sqrt{3}} \frac{1}{\sqrt{3}}x dx + \int_{2\sqrt{3}}^4 \sqrt{16 - x^2} dx$$

after solving it we get required area = $\frac{4\pi}{3}$ sq units

OR



Required area (shaded)

= Area under curve $y = x^3$ with respect to 'x' axis from $x = -2$ to $x = 1$

$$= \int_{-2}^1 |x^3| dx = \left| \int_{-2}^0 x^3 dx \right| + \int_0^1 x^3 dx$$

$$= \left| \int_{-2}^0 -x^3 dx \right| + \int_0^1 x^3 dx$$

As cube a value below 0 is negative

$$= \left[\frac{-x^4}{4} \right]_{-2}^0 + \left[\frac{x^4}{4} \right]_0^1 = - \left[0 - \frac{(-2)^4}{4} \right] + \left[\frac{1}{4} - 0 \right]$$

$$= - \left[\frac{-16}{4} \right] + \frac{1}{4} = \frac{16}{4} + \frac{1}{4} = \frac{17}{4} \text{ sq.units}$$

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We know that d.r.'s of the line $\frac{x}{2} = \frac{y}{3} = \frac{z}{-6}$ are is denominators 2, 3, -6

therefore d.r.'s of any line parallel to it are also 2, 3, -6 therefore equation of the line through P

(1, -2, 3) and parallel to the given line $\frac{x}{2} = \frac{y}{3} = \frac{z}{-6}$ are

$$\frac{x}{2} = \frac{y}{3} = \frac{z}{-6} \quad \frac{x-1}{2} = \frac{y+2}{3} = \frac{z-3}{-6} (= \lambda) \text{-----(1)}$$

let this line meet the given plane $x-y+z=5$ in the point Q say) from equation (1) $x-1=2\lambda$ $y+2=3\lambda$ $z-3=-6\lambda \Rightarrow x=2\lambda+1$ $y=3\lambda-2$

$$z=-6\lambda+3$$

therefore point Q is Q($2\lambda+1$, $3\lambda-2$, $-6\lambda+3$) for some real λ

But this point Q lies on the plane $x-y+z=5$ therefore

$$(2\lambda+1)-(3\lambda-2)+(-6\lambda+3)=5$$

$$2\lambda+1-3\lambda+2-6\lambda+3=5 \Rightarrow -7\lambda=-1 \Rightarrow \lambda=\frac{1}{7}$$

putting $\lambda=\frac{1}{7}$ in (3) coordinates of point Q are $(\frac{9}{7}, \frac{-11}{7}, \frac{15}{7})$

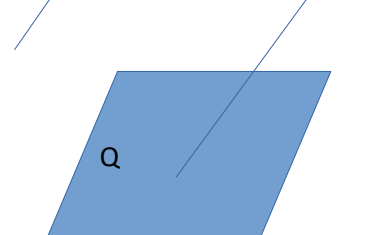
Required distance is PQ

$$\sqrt{\left(\frac{9}{7}-1\right)^2 + \left(\frac{-11}{7}+2\right)^2 + \left(\frac{15}{7}-3\right)^2} = 1$$

$$\sqrt{\left(\frac{2}{7}\right)^2 + \left(\frac{3}{7}\right)^2 + \left(\frac{-6}{7}\right)^2}$$

$$\frac{x}{2} = \frac{y}{3} = \frac{z}{-6}$$

P (1, -2, 3)



$$\sqrt{\frac{4}{49} + \frac{9}{49} + \frac{36}{49}} = \sqrt{\frac{49}{49}} = \mathbf{1}$$

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(a) 0.9 2.

(b) 0.083