21 Three Dimensional Geometry

- **A Line in Space**

  Any two distinct points in space determine a unique line. Lines in space are generally denoted by L, M, N,...

- **Direction of a Line**

  If A and B are two distinct points on a line L in R^3, then $\overrightarrow{AB}$ and $\overrightarrow{BA}$ are two vectors in opposite directions. If $\overrightarrow{AB}$ has direction $\vec{l}$, then the direction of $\overrightarrow{BA}$ is $-\vec{l}$. The direction of $\overleftrightarrow{AB}$ (i.e. line L) is taken as $\pm\vec{l}$.

  If the direction cosines of line L i.e. direction cosines of any vector on line L are l, m, n, then $-l, -m, -n$ are also direction cosines of line L. Here, $l^2+m^2+n^2=1$.

- **The equation of a line passing through $A(\vec{a})$ and having the same direction as a non-zero vector $\vec{l}$:**

  - **The vector equation :** $\vec{r}=\vec{a}+k\vec{l},\ k\in R.$

  - **The parametric equations** : If $\vec{r}=(x, y, z)$, $\vec{a}=(x_1, y_1, z_1)$ and $\hat{l}=(l_1, l_2, l_3)$, then the parametric equations of a line are $x=x_1+kl_1,\ y=y_1+kl_2,\ z=z_1+kl_3\ (k\in R)$.

  - **The cartesian equation (Symmetric form) :**

    $$\frac{x-x_1}{l_1}=\frac{y-y_1}{l_2}=\frac{z-z_1}{l_3}\ (=k)$$

    If $l_1=0,\ l_2\neq 0, l_3\neq 0,$ then the equations of a line are, $x=x_1,\ \frac{y-y_1}{l_2}=\frac{z-z_1}{l_3}$ **or**

    $\frac{x-x_1}{0}=\frac{y-y_1}{l_2}=\frac{z-z_1}{l_3}$ (Here, denominator of $x-x_1$ is 0 means the first component of $\vec{l}$ i.e. $l_1=0$. So, $x=x_1$. Similarly if $l_1=0, l_2=0$, then the equations of line are $\frac{x-x_1}{0}=\frac{y-y_1}{0}=\frac{z-z_1}{l_3}$, i.e. $x=x_1,\ y=y_1,\ z$ arbitrary.)

    If the direction cosines of a line are l, m, n, then the equations of the line are $\frac{x-x_1}{l}=\frac{y-y_1}{m}=\frac{z-z_1}{n}$, where $l^2+m^2+n^2=1.$

  - For some $k\in R$, $(x_1+kl_1,\ y_1+kl_2,\ z_1+kl_3)$ is any point on line.

- The equations of coordinate axes in R^3 :

  The equations of X-axis : $y=0,\ z=0$

  The equations of Y-axis : $z=0,\ x=0$

  The equations of Z-axis : $x=0,\ y=0$

- **The equation of a line passing through two distinct points :**
  - The vector equation of the line passing through two distinct points $A(\vec{a})$ and $B(\vec{b})$ is :

    $\vec{r} = \vec{a} + k(\vec{b} - \vec{a}),\ k \in R$
  - **The parametric equations :** If $\vec{r} = (x, y, z),\ \vec{a} = (x_1, y_1, z_1)$ and $\vec{b} = (x_2, y_2, z_2)$, then the parametric equations are : $\left.\begin{array}{l} x = x_1 + k(x_2 - x_1) \\ y = y_1 + k(y_2 - y_1) \\ z = z_1 + k(z_2 - z_1) \end{array}\right\} k \in R$
  - **The cartesian equation :**

    $$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

    If $x_1 = x_2$, then the equations become $\frac{x - x_1}{0} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$.

    If $x_1 = x_2,\ y_1 = y_2$, then $x = x_1,\ y = y_1,\ z$ is arbitrary..

- **Collinear points :**

  Let $A(\vec{a}),\ B(\vec{b})$ and $C(\vec{c})$ be distinct points in R^3.
  - If A, B, C are collinear, then $[\vec{a}\ \ \vec{b}\ \ \vec{c}] = 0$
  - **A, B, C are collinear** $\Leftrightarrow (\vec{c} - \vec{a}) \times (\vec{b} - \vec{a}) = \vec{0}$
  - **Result :** If $A(\vec{a}),\ B(\vec{b})$ and $C(\vec{c})$ are three distinct points in space, then a necessary and sufficient condition for A, B, C to be collinear is that there exist three non-zero real numbers l, m, n such that $l + m + n = 0$ and $l\vec{a} + m\vec{b} + n\vec{c} = \vec{0}$.

- **The measure of the angle between two lines in space**

  Suppose $L : \vec{r} = \vec{a} + k\vec{l},\ k \in R$ and $M : \vec{r} = \vec{b} + k\vec{m},\ k \in R$ are two lines in space.

  (i) If $\vec{l} = \vec{m}$ or $\vec{l} = -\vec{m}$, then $\vec{l} \times \vec{m} = \vec{0}$. Then the measure of the angle between the lines is defined to be zero. Since direction of lines are same, they are coincident or parallel.

  (ii) If $\vec{l} \perp \vec{m}$ i.e. $\vec{l} \cdot \vec{m} = 0$, then the lines are mutually perpendicular. Then the measure of the angle between them is $\frac{\pi}{2}$.

  (iii) If $\vec{l} \neq \pm \vec{m}$ and $\vec{l} \cdot \vec{m} \neq 0$ and if α is the measure of the acute angle between the lines L and M, then

$$\cos\alpha = \frac{|\vec{l}\cdot\vec{m}|}{|\vec{l}||\vec{m}|},\ 0<\alpha<\frac{\pi}{2}$$

This also holds good for $\alpha = 0$ and $\alpha = \frac{\pi}{2}$. Thus, $\cos\alpha = \frac{|\vec{l}\cdot\vec{m}|}{|\vec{l}||\vec{m}|},\ 0 \le \alpha \le \frac{\pi}{2}$.

- **The angle bisector of two lines :**

Let the point of intersection of two intersecting lines $L : \vec{r} = \vec{a} + k\vec{l},\ k \in R$ and $M : \vec{r} = \vec{a} + k\vec{m},\ k \in R$ be $A(\vec{a})$. The bisector of the angle formed by the intersection of two lines is $\vec{r} = \vec{a} + t\left\{\frac{\vec{l}}{|\vec{l}|} \pm \frac{\vec{m}}{|\vec{m}|}\right\}$.

For the bisector of the acute angle (internal bisector) '+' sign is taken and for the bisector of the obtuse angle (external bisector) '–' sign is taken.

- If two distinct lines $\vec{r} = \vec{a} + k\vec{l},\ k \in R$ and $\vec{r} = \vec{b} + k\vec{m},\ k \in R$ intersect, then

$(\vec{a} - \vec{b}) \bullet (\vec{l} \times \vec{m}) = 0.$

If $\vec{a} = (x_1, y_1, z_1),\ \vec{b} = (x_2, y_2, z_2),\ \vec{l} = (l_1, l_2, l_3)$ and $\vec{m} = (m_1, m_2, m_3)$, then the condition in cartesian form for two lines to intersect is $\begin{vmatrix} x_1 - x_2 & y_1 - y_2 & z_1 - z_2 \\ l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \end{vmatrix} = 0$

- The necessary condition for two distinct lines $\vec{r} = \vec{a} + k\vec{l},\ k \in R$ and $\vec{r} = \vec{b} + k\vec{m},\ k \in R$ to be coplanar is $(\vec{a} - \vec{b}) \bullet (\vec{l} \times \vec{m}) = 0.$
- If there is no plane that contains both the lines L and M, then L and M are called non-coplanar or skew lines.
- If $(\vec{a} - \vec{b}) \cdot (\vec{l} \times \vec{m}) \neq 0$ for two lines $\vec{r} = \vec{a} + k\vec{l},\ k \in R$ and $\vec{r} = \vec{b} + k\vec{m},\ k \in R$, then the two lines are skew lines.

- **Relation between two lines :**

For line $L : \vec{r} = \vec{a} + k\hat{l},\ k \in R$ and $M : \vec{r} = \vec{b} + k\vec{m},\ k \in R$,

(1) If $\vec{l} \times \vec{m} = \vec{0}$ and $(\vec{b} - \vec{a}) \times \vec{l} \neq \vec{0}$, then L||M.

(2) If $\vec{l} \times \vec{m} = \vec{0}$ and $(\vec{b} - \vec{a}) \times \vec{l} = \vec{0}$, then L and M are coincident lines.

(3) If $\vec{l} \times \vec{m} \neq \vec{0}$ and $(\vec{b} - \vec{a}) \cdot (\vec{l} \times \vec{m}) \neq 0$, then lines L and M are skew lines.

(4) If $\vec{l} \times \vec{m} \neq \vec{0}$ and $(\vec{b} - \vec{a}) \cdot (\vec{l} \times \vec{m}) = 0$, then lines L and M are intersecting lines.

- **The perpendicular distance of a point from a line :**

The perpendicular distance of point $P(\vec{p})$ from the line $\vec{r} = \vec{a} + k\vec{l}$, $k \in R$ is $\frac{|(\vec{p} - \vec{a}) \times \vec{l}|}{|\vec{l}|}$ or $|(\vec{p} - \vec{a}) \times \hat{l}|$.

- **The perpendicular distance between two parallel lines :**

Let $L : \vec{r} = \vec{a} + k\vec{l}$, $k \in R$ and $M : \vec{r} = \vec{b} + k\vec{l}$, $k \in R$ be two parallel lines. Perpendicular distance between L and M is $\frac{|(\vec{b} - \vec{a}) \times \vec{l}|}{|\vec{l}|}$.

- **The perpendicular distance between two skew lines (The shortest distance) :**

Let $L : \vec{r} = \vec{a} + k\vec{l}$, $k \in R$ and $\vec{r} = \vec{b} + k\vec{m}$, $k \in R$ be skew lines. The shortest distance between L and M is $\frac{|(\vec{b} - \vec{a}) \cdot (\vec{l} \times \vec{m})|}{|\vec{l} \times \vec{m}|}$.

- **Foot of the perpendicular from a point on a line, perpendicular line and image of line :**

(1) Coordinates of the foot of perpendicular M from point P to line $L : \frac{x - x_1}{l_1} = \frac{y - y_1}{l_2} = \frac{z - z_1}{l_3}$ are $(x_1 + kl_1,\ y_1 + kl_2,\ z_1 + kl_3)$.

Find k to find the coordinates of the foot of perpendicular from $\overrightarrow{PM} \perp \vec{l}$.

(2) The equation of the line perpendicular to L through point P($\vec{p}$) is $\vec{r} = \vec{p} + k\overrightarrow{PM}$, $k \in R$.

(3) If Q is the image of point P in L, then M is the midpoint of $\overline{PQ}$ and coordinates of Q can be found using coordinates of P and M.

- **The point of intersection of intersecting lines :**

Two lines $L : \vec{r} = \vec{a} + k\vec{l}$, $k \in R$ and $M : \vec{r} = \vec{b} + k\vec{m}$, $k \in R$ intersect each other in a unique point.

Take any point $(x_1 + k_1 l_1,\ y_1 + k_1 l_2,\ z_1 + k_1 l_3)$ on line $L : \frac{x - x_1}{l_1} = \frac{y - y_1}{l_2} = \frac{z - z_1}{l_3}$ and take any point $(x_2 + k_2 m_1,\ y_2 + k_2 m_2,\ z_2 + k_2 m_3)$ on line $M : \frac{x - x_2}{m_1} = \frac{y - y_2}{m_2} = \frac{z - z_2}{m_3}$.

By comparing the *x, y* and *z* coordinates of both the points, obtain the values of k_1 and k_2 and then substituting these values in any of two equations for points gives the coordinates of the required point of intersection.

- **Plane**
- **Postulates :**

(1) Three distinct non-collinear points determine a unique plane.

(2) A point and a line not passing through it, determine a unique plane.

(3) There is a unique plane containing two parallel lines.

(4) There is a unique plane containing two intersecting lines.

- **The plane passing through three distinct non-collinear points :**
- The vector equation of the plane passing through three distinct non-collinear points $A(\vec{a})$, $B(\vec{b})$ and $C(\vec{c})$ is $\vec{r} = l\vec{a} + m\vec{b} + n\vec{c}$, $l, m, n \in R$ and $l+m+n=1$.
- $(\vec{r}-\vec{a})\cdot[(\vec{b}-\vec{a})\times(\vec{c}-\vec{a})] = 0$ is the vector equation of a plane.
- **The parametric equations of a plane :** Taking $\vec{r} = (x, y, z)$, $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$ and $C(x_3, y_3, z_3)$,

The parametric equations of a plane are :

$$x = lx_1 + mx_2 + nx_3$$
$$y = ly_1 + my_2 + ny_3 \quad \text{where } l, m, n \in R \text{ and } l+m+n=1.$$
$$z = lz_1 + mz_2 + nz_3$$

- **The cartesian equation :** $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0.$
- The equation of *XY* – plane is $z = 0$

The equation of *YZ* – plane is $x = 0$

The equation of *ZX* – plane is $y = 0$

- **The condition for four distinct points of R^3 to be coplanar :**

Let $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$, $C(x_3, y_3, z_3)$ and $D(x_4, y_4, z_4)$ be points of R^3,

A, B, C, D are coplanar $\Leftrightarrow \begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \\ x_4-x_1 & y_4-y_1 & z_4-z_1 \end{vmatrix} = 0.$ i.e. $\begin{vmatrix} x_1 & x_2 & x_3 & x_4 \\ y_1 & y_2 & y_3 & y_4 \\ z_1 & z_2 & z_3 & z_4 \\ 1 & 1 & 1 & 1 \end{vmatrix} = 0$

- **The equation of a plane making intercepts on the coordinates axes :**

If a plane π intersects three coordinates axes at points $A(a, 0, 0)$, $B(0, b, 0)$ and $C(0, 0, c)$, then *a, b, c* are called the X-intercept, the Y-intercept and the Z-intercept of the plane π respectively.

- Suppose intercepts made by a plane π on X-axis, Y-axis and Z-axis are respectively, *a, b* and *c* $(a \neq 0,\ b \neq 0,\ c \neq 0)$, then the equation of the plane is $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1.$

- **Normal to a plane :**

  There exists a line which is perpendicular to every line in a plane. It is called a normal to the plane. Usually a normal is denoted by $\vec{n}$ or $\vec{n_1}, \vec{n_2}, \vec{n_3}, \ldots$.

- **The equation of a plane having normal $\vec{n}$:**

  - $\vec{r} \cdot \vec{n} = \vec{a} \cdot \vec{n}$ is the vector equation of the plane passing through $A(\vec{a})$ and having normal $\vec{n}$.

    If $\vec{a} \cdot \vec{n} = d$, then the vector equation of the plane is $\vec{r} \cdot \vec{n} = d$.

  - **The cartesian equation :** Taking $\vec{r} = (x, y, z)$, $\vec{n} = (a, b, c)$, $\vec{a} = (x_1, y_1, z_1)$, cartesian equation of plane becomes $ax + by + cz = d$, where $d = ax_1 + by_1 + cz_1$ or $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$, $a^2 + b^2 + c^2 \neq 0$.

  - The equation of the plane parallel to X-axis is $by + cz = d$

    The equation of the plane parallel to Y-axis is $ax + cz = d$

    The equation of the plane parallel to Z-axis is $ax + by = d$

  - $x\cos\alpha + y\cos\beta + z\cos\gamma = p$ is the equation of the plane having α, β, γ as the direction angles of the normal and p is the length of the normal from origin.

  - If the equation of the plane is $ax + by + cz = d$, then to convert such an equation into the form of $x\cos\alpha + y\cos\beta + z\cos\gamma = d$, we divide the given equation by $|\vec{n}|$.

    That is $\frac{a}{|\vec{n}|}x + \frac{b}{|\vec{n}|}y + \frac{c}{|\vec{n}|}z = \frac{d}{|\vec{n}|}$ form of equation obtained.

  - If $d > 0$, then let $\vec{n} = (a, b, c)$ so that $\frac{d}{|\vec{n}|} = p$ is positive,

    $$\hat{n} = \frac{\vec{n}}{|\vec{n}|} = \left(\frac{a}{|\vec{n}|}, \frac{b}{|\vec{n}|}, \frac{c}{|\vec{n}|}\right) = (\cos\alpha, \cos\beta, \cos\gamma)$$

  - If $d < 0$, then let $\vec{n} = (-a, -b, -c)$ and

    $$\hat{n} = \frac{\vec{n}}{|\vec{n}|} = \left(\frac{-a}{|\vec{n}|}, \frac{-b}{|\vec{n}|}, \frac{-c}{|\vec{n}|}\right) = (\cos\alpha, \cos\beta, \cos\gamma) \text{ and } \frac{-d}{|\vec{n}|} = p.$$

- The equation of the plane parallel to plane $ax+by+cz+d=0$ is $ax+by+cz+k=0$. where $k \in R-\{d\}$.

- The equation of the plane perpendicular to line $\frac{x-x_1}{l}=\frac{y-y_1}{m}=\frac{z-z_1}{n}$ is $lx+my+nz+d=0$.

- **Intersection of a line and a plane :**

Let $L: \vec{r}=\vec{a}+k\vec{l}$, $k \in R$ represent a line and $\pi: \vec{r} \cdot \vec{n}=d$ represent a plane. $(\vec{l} \neq \vec{0},\ \vec{n} \neq \vec{0})$.

Taking $\vec{l}=(l_1, l_2, l_3)$, $\vec{n}=(a, b, c)$, $\vec{a}=(x_1, y_1, z_1)$,

(1) If $\vec{l} \cdot \vec{n}=0$ and $ax_1+by_1+cz_1 \neq d$, then the line and the plane do not intersect i.e., $L \cap \pi = \emptyset$.

(2) If $\vec{l} \cdot \vec{n}=0$ and $ax_1+by_1+cz_1=d$, then the line lies in the plane and so, $L \cap \pi = L$

(3) If $\vec{l} \cdot \vec{n} \neq 0$, then the line intersects the plane in exactly one point. In order to obtain the point of intersection, consider any point $P(x_1+kl_1,\ y_1+kl_2,\ z_1+kl_3)$ of the line $\frac{x-x_1}{l_1}=\frac{y-y_1}{l_2}=\frac{z-z_1}{l_3}$ and substitute in the equation of the plane to get the value of *k*. Substitute the value of *k* in the coordinates of the point P and hence point of intersection is obtained.

- **The measure of the angle between two planes :**

Suppose the measure of the angle between the planes $\pi_1: \vec{r} \cdot \vec{n_1}=d_1$ and $\pi_2: \vec{r} \cdot \vec{n_2}=d_2$ is θ.

(1) The measure of the angle between planes π_1 and π_2 is $\frac{\pi}{2} \Leftrightarrow \vec{n_1} \cdot \vec{n_2}=0$.

(2) The measure of the angle between π_1 and π_2 is zero $\Leftrightarrow \vec{n_1} \times \vec{n_2}=\vec{0}$.

(3) $\cos\theta=\frac{|\vec{n_1} \cdot \vec{n_2}|}{|\vec{n_1}||\vec{n_2}|}$, $0 \leq \theta \leq \frac{\pi}{2}$, i.e. $\theta = \cos^{-1}\frac{|\vec{n_1} \cdot \vec{n_2}|}{|\vec{n_1}||\vec{n_2}|}$.

- If $a_1x+b_1y+c_1z=d_1$ and $a_2x+b_2y+c_2z=d_2$, are given planes, then

$$\cos\theta = \frac{|a_1a_2+b_1b_2+c_1c_2|}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}.$$

i.e. $$\theta = \cos^{-1}\frac{|a_1a_2+b_1b_2+c_1c_2|}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}.$$

- **The angle between a line and a plane :**

  Suppose $\vec{r} = \vec{a} + k\vec{l}$, $k \in R$ is the equation of a line and $\vec{r} \bullet \vec{n} = d$ is the equation of a plane.
  If the measure of the angle between the line and the plane is α,

  then $\alpha = sin^{-1} \dfrac{|\vec{l} \bullet \vec{n}|}{|\vec{l}||\vec{n}|}$, $0 < \alpha < \dfrac{\pi}{2}$.

- **The equation of the plane passing through two parallel lines :**

  - The vector equation of the plane passing through two parallel lines $\vec{r} = \vec{a} + k\vec{l}$, $k \in R$ and $\vec{r} = \vec{b} + k\vec{l}$, $k \in R$ is $(\vec{r} - \vec{a}) \bullet [(\vec{b} - \vec{a}) \times \vec{l}\,] = 0$.

  - If $\vec{a} = (x_1, y_1, z_1)$, $\vec{b} = (x_2, y_2, z_2)$ and $\vec{l} = (l_1, l_2, l_3)$, then cartesian equation of the plane is $\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ l_1 & l_2 & l_3 \end{vmatrix} = 0$.

- **The equation of the plane containing two intersecting lines :**

  - Let $\vec{r} = \vec{a} + k\vec{l}$, $k \in R$ and $\vec{r} = \vec{b} + k\vec{m}$, $k \in R$ be two intersecting lines. The vector equation of the plane containing given intersecting lines is $(\vec{r} - \vec{a}) \bullet (\vec{l} \times \vec{m}) = 0$.

  - If $\vec{a} = (x_1, y_1, z_1)$, $\vec{l} = (l_1, l_2, l_3)$ and $\vec{m} = (m_1, m_2, m_3)$, then the cartesian equation of the plane is $\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \end{vmatrix} = 0$.

- **The perpendicular distance from a point outside a plane to the plane :**

  Let $\vec{r} \cdot \vec{n} = d$ be the equation of a given plane and $P(\vec{p})$ be a given point, $p \notin \pi$.

  Perpendicular distance of $P(\vec{p})$ from the plane $\pi = \dfrac{|\vec{p} \cdot \vec{n} - d|}{|\vec{n}|}$.

  If equation of plane is $ax + by + cz + d = 0$ and $\vec{p} = (x_1, y_1, z_1)$, then

  perpendicular distance $= \dfrac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$.

- **Distance Between Two Parallel Planes :**

  Perpendicular distance between two parallel planes $\vec{r} \cdot \vec{n} = d_1$ and $\vec{r} \cdot \vec{n} = d_2$ is $p = \dfrac{|d_1 - d_2|}{|\vec{n}|}$.

  - If planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ are parallel to each other, then $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{c_1}{c_2}$, $d_1 \neq d_2$.

- **Intersection of two planes :**

  - If $\vec{n_1} \times \vec{n_2} \neq \vec{0}$, then the planes $\vec{r} \cdot \vec{n_1} = d_1$ and $\vec{r} \cdot \vec{n_2} = d_2$ intersect each other. Intersection of two planes is a line, whose equation is $\vec{r} = \vec{a} + k\vec{l}$, $k \in R$, where $\vec{l} = \vec{n_1} \times \vec{n_2}$.

    In order to find intersection of two planes i.e. in order to find a point on the line of intersection of two planes, take $z = 0$ (or $x = 0$ or $y = 0$) and solve the equations in x and y to obtain x and y.

  - If planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ intersect each other, then equation of any plane through their line of intersection is

    $$a_1x + b_1y + c_1z + d_1 + \lambda(a_2x + b_2y + c_2z + d_2) = 0,\ \lambda \in R.$$

    or $\lambda(a_1x + b_1y + c_1z + d_1) + a_2x + b_2y + c_2z + d_2 = 0$

- **The image of a point in the plane :**

  Let $Q(x_2, y_2, z_2)$ be the image of point $P(x_1, y_1, z_1)$ in the plane $ax + by + cz + d = 0$.

  Now, $\dfrac{x_2 - x_1}{a} = \dfrac{y_2 - y_1}{b} = \dfrac{z_2 - z_1}{c} = -2\left(\dfrac{ax_1 + by_1 + cz_1 + d}{a^2 + b^2 + c^2}\right) = r.$

  $\therefore$ Coordinates of image Q are $(x_1 + ar,\ y_1 + br,\ z_1 + cr)$.

- **The foot of the perpendicular from a point outside the plane to a plane :**

  Suppose $P(x_1, y_1, z_1)$ is not a point in the plane $ax + by + cz + d = 0$. If $M(x_2, y_2, z_2)$ is the foot of the perpendicular from point P to the plane, then $x_2 = x_1 + ar$, $y_2 = y_1 + br$, $z_2 = z_1 + br$

  where, $r = -\dfrac{ax_1 + by_1 + cz_1 + d}{a^2 + b^2 + c^2}$.

- **The equation of the plane bisecting the angle between given two planes :**

  The equation of the plane bisecting the angle between given two planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ is

  $$\frac{a_1x + b_1y + c_1z + d_1}{\sqrt{a_1^2 + b_1^2 + c_1^2}} = \pm \frac{a_2x + b_2y + c_2z + d_2}{\sqrt{a_2^2 + b_2^2 + c_2^2}}.$$

- **Image of a line in a plane :**

  Line L : $\dfrac{x - x_1}{l_1} = \dfrac{y - y_1}{l_2} = \dfrac{z - z_1}{l_3}$ and plane π : $ax + by + cz + d = 0$ are given.

- Obtain point of the point of intersection P of line and plane.
- Obtain image Q of point (x_1, y_1, z_1) of line in plane.
- Obtain equation of $\overleftrightarrow{PQ}$.

$\overleftrightarrow{PQ}$ is image of the given line in the plane.

: Multiple Choice Questions :

(1) If the direction *cosines* of a line in space are $\frac{1}{c}, \frac{1}{c}, \frac{1}{c}$, then .....

(A) $0 < c < 1.5$ (B) $c > 2$ (C) $c = \frac{1}{\sqrt{3}}$ (D) $c = \sqrt{3}$

Sol. : Taking direction cosines of the line as $cos\alpha, cos\beta, cos\gamma$,

$$cos^2\alpha + cos^2\beta + cos^2\gamma = 1$$

$$\therefore \quad \frac{1}{c^2} + \frac{1}{c^2} + \frac{1}{c^2} = 1$$

$$\therefore \quad 3 = c^2$$

$$\therefore \quad c = \pm\sqrt{3}$$

Ans. : (D)

(2) The equation of the line passing through (3, 2, 1) and perpendicular to $5x - 2y - z - 9 = 0$ is .....

(A) $\frac{x-5}{3} = \frac{y+2}{2} = \frac{z+1}{1}$ (B) $\frac{x+3}{5} = \frac{y+2}{-2} = \frac{z+1}{-1}$

(C) $\frac{x-3}{5} = \frac{2-y}{2} = \frac{1-z}{1}$ (D) $\frac{x+3}{-5} = \frac{y+2}{2} = \frac{z+1}{1}$

Sol. : The normal to the plane $5x - 2y - z - 9 = 0$ is $\vec{n} = 5\hat{i} - 2\hat{j} - \hat{k}$. As the line is perpendicular to the plane, its direction vector is $\vec{n}$. Also the line is passing through (3, 2, 1).

$\therefore$ The equation of the line : $\frac{x-x_1}{l_1} = \frac{y-y_1}{l_2} = \frac{z-z_1}{l_3}$

$$\therefore \quad \frac{x-3}{5} = \frac{y-2}{-2} = \frac{z-1}{-1}$$

$$\therefore \quad \frac{x-3}{5} = \frac{2-y}{2} = \frac{1-z}{1}$$

Ans. : (C)

(3) The perpendicular distance of the point $(1, 2, -3)$ from the line $\frac{x+2}{-1} = \frac{y+1}{2} = \frac{z-3}{3}$ is .....

(A) $\frac{15}{14}$ (B) $\sqrt{\frac{531}{14}}$ (C) $\frac{531}{14}$ (D) $\frac{\sqrt{531}}{14}$

Sol. : The perpendicular distance of the point $P(1, 2, -3)$ from the line $\frac{x+2}{-1} = \frac{y+1}{2} = \frac{z-3}{3}$ is

$|\overrightarrow{AP} \times \hat{l}|$, where $A(-2, -1, 3)$ and $\vec{l} = (-1, 2, 3)$

$$p = \frac{|(3, 3, -6) \times (-1, 2, 3)|}{|(-1, 2, 3)|}$$

$$= \frac{|(21, -3, 9)|}{\sqrt{14}} = \sqrt{\frac{531}{14}}$$ **Ans. : (B)**

(4) If the points $(-3, -1, 2)$, $(1, 4, x)$ and $(5, y, 8)$ are collinear, then x and y are respectively .....

(A) 9 and 5 (B) –5 and 9 (C) 5 and 9 (D) 5 and –9

Sol. : The points $\vec{a} = -3\hat{i} - \hat{j} + 2\hat{k}$, $\vec{b} = \hat{i} + 4\hat{j} + x\hat{k}$ and $\vec{c} = 5\hat{i} + y\hat{j} + 8\hat{k}$ are collinear.

$\therefore \quad (\vec{b} - \vec{a}) \times (\vec{c} - \vec{a}) = \vec{0}$.

$\therefore \quad (4\hat{i} + 5\hat{j} + (x-2)\hat{k}) \times (8\hat{i} + (y+1)\hat{j} + 6\hat{k}) = \vec{0}$

$\therefore \quad (-xy + 2y - x + 32)\hat{i} + (8x - 40)\hat{j} + (4y - 36)\hat{k} = \vec{0}$

$\therefore \quad 8x - 40 = 0$ and $4y - 36 = 0$

$\therefore \quad x = 5$ and $y = 9$

See that $-xy + 2y - x + 32 = 0$ **Ans. : (C)**

Another method :

$\overrightarrow{BC} = k\ \overrightarrow{AB}$

$\therefore \quad (4, 5, x-2)\, k = (4, y-4, 8-x)$

$\therefore \quad 4k = 4$

$\therefore \quad k = 1$

$\therefore \quad (4, 5, x-2) = (4, y-4, 8-x)$

$\therefore \quad y - 4 = 5 \Rightarrow y = 9$

$\therefore \quad x - 2 = 8 - x \Rightarrow x = 5$

$\therefore x = 5, \quad y = 9$

(5) The direction cosines of the line making angles of equal measure with the coordinate axes are ..... .

(A) $\sqrt{3}, \sqrt{3}, \sqrt{3}$ (B) $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$ (C) 1, 1, 1 (D) $-\sqrt{3}, -\sqrt{3}, -\sqrt{3}$

Sol. : If direction cosines of line are l, m, n, then $l = m = n$ and $l^2 + m^2 + n^2 = 1.$

$\therefore \quad 3l^2 = 1$

$\therefore \quad l = \pm \frac{1}{\sqrt{3}} = m = n.$

$\therefore$ Direction cosines of line are $\pm \frac{1}{\sqrt{3}}, \pm \frac{1}{\sqrt{3}}, \pm \frac{1}{\sqrt{3}}.$ **Ans. : (B)**

Note : There is only one unit vector i.e. in (B).

(6) If the line makes angles of measure α, β, γ with the positive direction of axes, then $sin^2\alpha + sin^2\beta + sin^2\gamma = $ .....

(A) 2 (B) 1 (C) 0 (D) 3

Sol. : The direction angles of the line are α, β, γ.

$\therefore \quad cos^2\alpha + cos^2\beta + cos^2\gamma = 1$

$\therefore \quad 1 - sin^2\alpha + 1 - sin^2\beta + 1 - sin^2\gamma = 1$

$\therefore \quad 2 = sin^2\alpha + sin^2\beta + sin^2\gamma$ **Ans. : (A)**

(7) The distance of point $(1, 2, 3)$ from X-axis is .....

(A) 1 (B) $\sqrt{14}$ (C) $\sqrt{13}$ (D) 5

Sol. : The distance of $A(x_1, y_1, z_1)$ from $\frac{x}{1} = \frac{y}{0} = \frac{z}{0}$ is

$|\overrightarrow{OA} \times \hat{i}| = |(x_1, y_1, z_1) \times (1, 0, 0)| = |(0, z_1, -y_1)| = \sqrt{{y_1}^2 + {z_1}^2}$

The distance of the point $(1, 2, 3)$ from X-axis $= \sqrt{y^2 + z^2} = \sqrt{4+9} = \sqrt{13}.$

Ans. : (C)

(8) If a vector of magnitude $6\sqrt{3}$ makes angles of equal measures with the coordinate axes, then the vector is ..... .

(A) $\sqrt{3}(\hat{i} + \hat{j} + \hat{k})$ (B) $-6(\hat{i} + \hat{j} + \hat{k})$ (C) $2\sqrt{3}(\hat{i} + \hat{j} + \hat{k})$ (D) $-2\sqrt{3}(\hat{i} + \hat{j} + \hat{k})$

Sol. : Vector $\vec{r}$ makes angles of equal measure with the coordinate axes. Let direction cosines be l, m, n. Then $l = m = n$ and $l^2 + m^2 + n^2 = 1.$

$\therefore \quad 3l^2 = 1.$ So, $l = \pm \frac{1}{\sqrt{3}}$

As $|\vec{r}| = 6\sqrt{3}$, $\vec{r} = |\vec{r}|(l\hat{i} + m\hat{j} + n\hat{k}) = \pm 6(\hat{i} + \hat{j} + \hat{k})$ **Ans. : (B)**

(9) Direction ratios of a line perpendicular to $\frac{x-6}{1} = \frac{y-7}{2} = \frac{z-17}{-3}$ and $\frac{x-4}{-2} = \frac{y+5}{1} = \frac{z+3}{2}$ are ..... .

(A) 7, –4, –5 (B) –7, 4, 5 (C) 7, 4, 5 (D) 7, –4, 5

Sol. : Suppose direction ratios of a line perpendicular to given lines are *a, b, c.*

Now, a line having direction ratios *a, b, c* is perpendicular to $\frac{x-6}{1} = \frac{y-7}{2} = \frac{z-17}{-3}$.

$\therefore \quad a + 2b - 3c = 0$. Similarly, $-2a + b + 2c = 0$

$$\therefore \quad \frac{a}{4+3} = \frac{b}{-2+6} = \frac{c}{1+4}$$

$\therefore$ Direction ratios : 7, 4, 5.

Second method :

Required direction ratio = (1, 2, –3) × (–2, 1, 2) = (7, 4, 5) **Ans. : (C)**

(10) The measure of the angle between the lines $\vec{r} = (4\hat{j} - \hat{k}) + t(-3\hat{i} + 2\hat{j} + \hat{k}),\ t \in R$ and $\vec{r} = (2\hat{i} + \hat{j} - \hat{k}) + s(2\hat{i} + \hat{j} - 3\hat{k}),\ s \in R$ is .....

(A) $\frac{\pi}{3}$ (B) $\frac{2\pi}{3}$ (C) $\frac{\pi}{6}$ (D) $\frac{5\pi}{6}$

Sol. : $\vec{l} = (-3, 2, 1),\ \vec{m} = (2, 1, -3)$

Now, $\cos\theta = \frac{|\vec{l} \cdot \vec{m}|}{|\vec{l}|\,|\vec{m}|} = \frac{|-6+2-3|}{\sqrt{14} \cdot \sqrt{14}} = \frac{7}{14} = \frac{1}{2}$

$\therefore \quad \theta = \frac{\pi}{3}$. **Ans. : (A)**

(11) The measure of the angle between two diagonals of a cube having side of length *a* units is .....

(A) $\cos^{-1}\frac{1}{\sqrt{3}}$ (B) $\cos^{-1}\frac{1}{3}$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{3}$

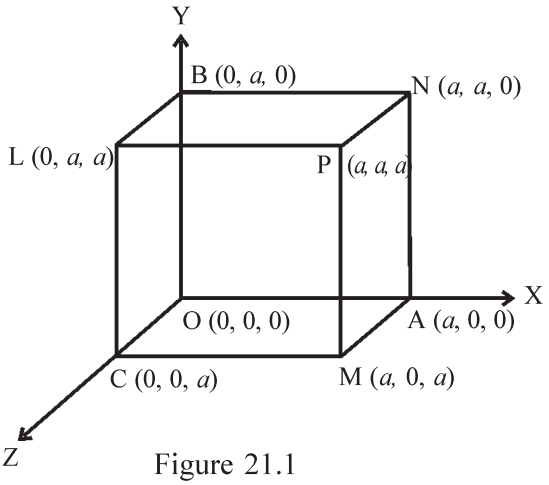


Figure 21.1

Sol. : The cube has 4 digaonals : $\overline{OP}, \overline{AL}, \overline{BM}$ and $\overline{CN}$.

$\vec{l} = (a, a, a)$ for $\overrightarrow{\mathbf{OP}}$. So, $\hat{l} = \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

$\vec{m} = (-a, a, a)$ for $\overrightarrow{AL}$. So, $\hat{m} = \left(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

$cos\,(\overrightarrow{OP}\,\hat{,}\,\overrightarrow{AL}) = |\hat{l} \cdot \hat{m}|$

$= \left|\frac{1}{\sqrt{3}}\left(-\frac{1}{\sqrt{3}}\right) + \frac{1}{\sqrt{3}}\left(\frac{1}{\sqrt{3}}\right) + \frac{1}{\sqrt{3}}\left(\frac{1}{\sqrt{3}}\right)\right|$

$= \frac{1}{3}$

$\therefore \quad (\overrightarrow{OP}\,\hat{,}\,\overrightarrow{AL}) = cos^{-1}\,\frac{1}{3}$.

Similarly the angle between any two diagonals of the cube has measure $cos^{-1}\,\frac{1}{3}$. **Ans. : (B)**

(12) A line passes through $A(1, 3, 2)$ and $B(3, 9, 5)$. The coordinates of a point on this line at a distance of 7 units from the midpoint of $\overline{AB}$ is ..... .

(A) $(4, 12, 13)$ (B) $\left(0, 0, \frac{1}{2}\right)$ (C) $\left(4, 12, \frac{13}{4}\right)$ (D) $\left(0, 0, \frac{1}{4}\right)$

Sol. : The equation of $\overleftrightarrow{AB}$ is $\vec{r} = \vec{a} + k\vec{l}$, $k \in R$

$\therefore \quad \vec{r} = (1, 3, 2) + k(2, 6, 3), \; k \in R$

$\therefore$ Any point on $\overleftrightarrow{AB}$ is $P(1 + 2k,\; 3 + 6k,\; 2 + 3k)$ (i)

The distance of point P from mid-point $M\left(2, 6, \frac{7}{2}\right)$ is 7.

$\therefore \quad PM = 7$. So, $PM^2 = 49$

$\therefore \quad (2k + 1 - 2)^2 + (6k + 3 - 6)^2 + \left(3k + 2 - \frac{7}{2}\right)^2 = 49$

$\therefore \quad (2k - 1)^2 + (6k - 3)^2 + \left(3k - \frac{3}{2}\right)^2 = 49$

$\therefore \quad (2k - 1)^2 \left(1 + 9 + \frac{9}{4}\right) = 49$

$\therefore \quad (2k - 1)^2 = 4$

$\therefore \quad 2k - 1 = 2$ or $2k - 1 = -2$

$\therefore \quad k = \frac{3}{2}$ or $k = -\frac{1}{2}$

$\therefore$ The required points on the line are $\left(4, 12, \frac{13}{2}\right)$ and $\left(0, 0, \frac{1}{2}\right)$. (From (i)) **Ans. : (B)**

(13) Find the shortest distance between $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-1}{2}$ and $\frac{x-4}{4} = \frac{y-5}{5} = \frac{z-2}{3}$.

(A) $\sqrt{6}$ (B) $\frac{1}{6}$ (C) $\frac{1}{\sqrt{6}}$ (D) 6

Sol. : $\vec{a} = (2, 3, 1),\ \vec{l} = (3, 4, 2),\ \vec{b} = (4, 5, 2),\ \vec{m} = (4, 5, 3)$

$\vec{b} - \vec{a} = (2, 2, 1)$ $\qquad$ $\vec{l} \times \vec{m} = (2, -1, -1)$

$\therefore$ The shortest distance $= \dfrac{|(\vec{b} - \vec{a}) \cdot (\vec{l} \times \vec{m})|}{|\vec{l} \times \vec{m}|}$.

$= \dfrac{|2(2) + 2(-1) + 1(-1)|}{\sqrt{6}}$

$= \dfrac{1}{\sqrt{6}}$ **Ans. : (C)**

(14) Lines $\vec{r} = (2, -5, 1) + k(3, 2, 6),\ k \in R$ and $\dfrac{x-7}{1} = \dfrac{y}{2} = \dfrac{z+6}{2}$ are ..... .

(A) intersecting lines (B) parallel lines

(C) coincident lines (D) skew lines

Sol. : $\vec{a} = (2, -5, 1),\ \vec{l} = (3, 2, 6)$ and $\vec{b} = (7, 0, -6),\ \vec{m} = (1, 2, 2)$.

$\vec{l} \times \vec{m} = (-8, 0, 4) \neq \vec{0}$

$\therefore$ Given lines are neither parallel nor coincident lines.

Now, $(\vec{b} - \vec{a}) \cdot (\vec{l} \times \vec{m}) = (5, 5, -7) \cdot (-8, 0, 4) = -40 - 28 \neq 0$

$\therefore$ The lines are skew lines. **Ans. : (D)**

(15) If the line passing through $(4, 1, 2)$ and $(5, y, 0)$ is parallel to the line passing through $(2, 1, 1)$ and $(3, 3, z)$, then (y, z) = ...

(A) $(-1, 3)$ (B) $(3, -1)$ (C) $(-3, 1)$ (D) $(-3, -1)$

Sol. : Direction ratios of given lines are $1,\ y-1,\ -2$ and $1,\ 2,\ z-1$.

Lines are parallel to each other.

$\therefore \quad \dfrac{1}{1} = \dfrac{y-1}{2} = \dfrac{-2}{z-1}$

$\therefore \quad y = 3$ and $z = -1$ **Ans. : (B)**

(16) The foot of the perpendicular from the point $(4, 8, 1)$ to the line joining the points $(-1, -3, 2)$ and $(3, -1, 0)$ is ..... .

(A) $\left(\frac{19}{3}, \frac{2}{3}, \frac{-5}{3}\right)$ (B) $\left(\frac{19}{3}, \frac{2}{3}, \frac{5}{3}\right)$ (C) $\left(\frac{19}{3}, \frac{-2}{3}, \frac{5}{3}\right)$ (D) $\left(\frac{19}{3}, \frac{-2}{3}, \frac{-5}{3}\right)$

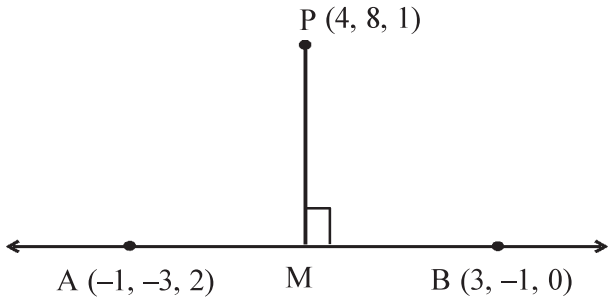


Figure 21.2

Sol. : The equation of $\overleftrightarrow{AB}$ passing through $A(-1,-3,2)$ and $B(3,-1,0)$ is

$\vec{r} = (-1,-3,2) + k'(4,2,-2),\ k' \in R$

$\therefore\quad \vec{r} = (-1,-3,2) + k(2,1,-1),\ k = 2k' \in R$

Suppose the point $M(-1+2k,\ -3+k,\ 2-k)$ is the foot of the perpendicular from $P(4,8,1)$ to $\overleftrightarrow{AB}$.

Now, $\vec{PM} \perp \vec{l}$. So, $\vec{PM} \bullet \vec{l} = 0$

$\therefore\quad (2k-5,\ k-11,\ -k+1) \bullet (2,1,-1) = 0$

$\therefore\quad 4k-10+k-11+k-1 = 0$

$\therefore\quad 6k = 22$. So, $k = \frac{11}{3}$

$\therefore$ The foot of perpendicular is $M\left(\frac{19}{3}, \frac{2}{3}, \frac{-5}{3}\right)$. **Ans. : (A)**

(17) Which of the following points is not on the line $\vec{r} = (2,0,-1) + k(2,1,-3),\ k \in R$?

(A) $(6,2,-7)$ (B) $(2,0,-1)$ (C) $(-2,1,3)$ (D) $(4,1,-4)$

Sol. : The equation of the line is $\vec{r} = (2,0,-1) + k(2,1,-3),\ k \in R$

For the point $(6,2,-7)$, if $(6,2,-7) = (2+2k,\ k,\ -1-3k)$, then

$6 = 2+2k,\quad k = 2,\quad -1-3k = -7$

$k = 2$ satisfies all the three equations.

$\therefore\quad (6,2,-7)$ lies on the line.

$(2,0,-1)$ is also a point on line.

Let $(-2,1,3) = (2+2k,\ k,\ -1-3k)$ for some k, if possible.,

$\therefore\quad -2 = 2+2k,\ 1 = k,\ 3 = -1-3k$

Here, $k = 1$ does not satisfy the remaining equations.

$\therefore\quad (-2,1,3)$ does not lie on the line.

$k = 1$ gives $(4,1,-4)$ lies on the line. **Ans. : (C)**

(18) The two lines $x = ay + b,\ z = cy + d$ and $x = a'y + b',\ z = c'y + d'$ will be perpendicular, if and only if .....

(A) $aa' + bb' + cc' = 0$ (B) $(a+a')(b+b')(c+c') = 0$

(C) $aa' + cc' + 1 = 0$ (D) $aa' + bb' + cc' + 1 = 0$ (AIEEE:2003)

Sol. : $x = ay + b$ and $z = cy + d$ and $x = a'y + b',\ z = c'y + d'$

$\therefore\ \dfrac{x-b}{a} = y$ and $y = \dfrac{z-d}{c}$

$\therefore\ \dfrac{x-b}{a} = \dfrac{y}{1} = \dfrac{z-d}{c}$ Similarly, $\dfrac{x-b'}{a'} = \dfrac{y}{1} = \dfrac{z-d'}{c'}$

$\therefore\ \vec{l} = (a, 1, c)$ and $\vec{m} = (a', 1, c')$

Two lines are mutually perpendicular $\Leftrightarrow\ \vec{l} \cdot \vec{m} = 0\ \Leftrightarrow\ aa'+1+cc' = 0$ **Ans. : (C)**

(19) If the lines $\dfrac{x-2}{1} = \dfrac{y-3}{1} = \dfrac{z-4}{-k}$ and $\dfrac{x-1}{k} = \dfrac{y-4}{2} = \dfrac{z-5}{1}$ are coplanar, then .....

(A) $k = 1$ or -1 (B) $k = 0$ or -3 (C) $k = 3$ or -3 (D) $k = 0$ or -1

(AIEEE : 2003)

Sol. : $\vec{a} = (2, 3, 4),\ \vec{l} = (1, 1, -k)$ and $\vec{b} = (1, 4, 5),\ \vec{m} = (k, 2, 1)$

Lines are coplanar.

$$\therefore\ \begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \end{vmatrix} = 0$$

$$\therefore\ \begin{vmatrix} -1 & 1 & 1 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix} = 0$$

$\therefore\ -1(1+2k) - 1(1+k^2) + 1(2-k) = 0$

$\therefore\ -1 - 2k - 1 - k^2 + 2 - k = 0$

$\therefore\ k^2 + 3k = 0$

$\therefore\ k(k+3) = 0$

$\therefore\ k = 0$ or -3 **Ans. : (B)**

(20) If the three edges of a cube are the coordinate axes, then direction ratios of the diagonal joining the origin and the vertex opposite to it is ..... .

(A) $\langle 2, -2, 2 \rangle$ (B) $\left\langle -\frac{2}{\sqrt{3}}, \frac{2}{\sqrt{3}}, \frac{2}{\sqrt{3}} \right\rangle$ (C) $\langle 1, 1, 1 \rangle$ (D) $\langle 3, 2, 1 \rangle$

Sol. : Suppose a cube having side of length a has edges $\overrightarrow{OA}$, $\overrightarrow{OB}$, $\overrightarrow{OC}$ along X-axis, Y-axis, Z-axis respectively.

One end point is origin $(0, 0, 0)$ and the other endpoint is $P(a, a, a)$.

The direction ratios of $\overrightarrow{OP}$ containing the diagonal $\overline{OP}$ are $\langle a-0, a-0, a-0 \rangle$ i.e. $\langle 1, 1, 1 \rangle$.

Ans. : (C)

(21) The magnitude of the projection of the line segment joining $A(2, 1, -2)$ and $B(3, -4, -1)$ on the line $\frac{x-4}{3} = \frac{y+9}{2} = \frac{z-1}{-6}$ is ..... .

(A) $\frac{26}{7}$ (B) $\frac{19}{7}$ (C) $\frac{16}{7}$ (D) $\frac{13}{7}$

Sol. : The magnitude of the projection of $\overline{AB}$ on line $\vec{r} = \vec{a} + k\vec{l}$, $k \in R$ is

$\left|(x_2 - x_1)l + (y_2 - y_1)m + (z_2 - z_1)n\right|$ where l, m, n are direction cosines of vector $\vec{l}$.

For $\overline{AB}$,

$x_2 - x_1 = 1$, $y_2 - y_1 = -5$, $z_2 - z_1 = 1$ and the direction cosines of the line are $l = \frac{3}{7}$, $m = \frac{2}{7}$, $n = \frac{-6}{7}$.

$\therefore$ The magnitude of projection $= \left|(1)\left(\frac{3}{7}\right) + (-5)\left(\frac{2}{7}\right) + (1)\left(\frac{-6}{7}\right)\right|$

$= \frac{1}{7}\left|3 - 10 - 6\right|$

$= \frac{13}{7}$ **Ans. : (D)**

(22) The perpendicular distance of the line $\{(4 + 3k, 2 + 4k, 4 - 5k) \mid k \in R\}$ from origin is .....

(A) 2 (B) 6 (C) $5\sqrt{2}$ (D) $2\sqrt{5}$

Sol. : Origin $O(0, 0, 0)$ and line $\vec{r} = (4, 2, 4) + k(3, 4, -5)$, $k \in R$ are given.

$\therefore$ $A(\vec{a}) = (4, 2, 4)$ and $\vec{l} = (3, 4, -5)$

$\therefore$ Perpendicular distance $= |\overrightarrow{OA} \times \hat{l}|$

$= \frac{|(-26, 32, 10)|}{5\sqrt{2}} = \frac{30\sqrt{2}}{5\sqrt{2}} = 6$ **Ans. : (B)**

(23) If the lines $x=-1+s,\ y=3-\lambda s,\ z=1+\lambda s$ and $x=\frac{t}{2},\ y=1+t,\ z=2-t,$ ($s,\ t$ are parameters) are coplanar, then $\lambda=$ .....

(A) –2 (B) –1 (C) $-\frac{1}{2}$ (D) 0 (AIEEE : 2004)

Sol. : Cartesian equations of lines :

$$\frac{x+1}{1}=\frac{y-3}{-\lambda}=\frac{z-1}{\lambda} \text{ and } \frac{x}{\frac{1}{2}}=\frac{y-1}{1}=\frac{z-2}{-1}$$

$\therefore\ \vec{a}=(-1,3,1),\ \vec{l}=(1,-\lambda,\lambda)$ and $\vec{b}=(0,1,2),\ \vec{m}=\left(\frac{1}{2},1,-1\right)$

Lines are coplanar.

$$\therefore \begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \end{vmatrix}=0$$

$$\therefore \begin{vmatrix} 1 & -2 & 1 \\ 1 & -\lambda & \lambda \\ \frac{1}{2} & 1 & -1 \end{vmatrix}=0$$

$$\therefore\ 1(\lambda-\lambda)+2\left(-1-\frac{\lambda}{2}\right)+1\left(1+\frac{\lambda}{2}\right)=0$$

$$\therefore\ -2-\lambda+1+\frac{\lambda}{2}=0$$

$$\therefore\ \lambda=-2$$ **Ans. : (A)**

(24) If the lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$ and $\frac{x-3}{1}=\frac{y-k}{2}=\frac{z}{1}$ intersect each other, then $k=$ .....

(A) $\frac{3}{2}$ (B) $\frac{9}{2}$ (C) $\frac{-2}{9}$ (D) $\frac{-3}{2}$ (IIT : 2004)

Sol. : For lines $\vec{r}=(1,-1,1)+\lambda(2,3,4), \lambda\in R$ and $\vec{r}=(3,k,0)+\mu(1,2,1), \mu\in R$,

$\vec{a}=(1,-1,1),\ \vec{l}=(2,3,4)$ and $\vec{b}=(3,k,0),\ \vec{m}=(1,2,1)$.

Given lines are intersecting lines and hence coplanar :

$$\therefore \begin{vmatrix} 2 & k+1 & -1 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{vmatrix}=0$$

$$\therefore\ 2(-5)-(k+1)(-2)-1\,(1)=0$$

$$\therefore\ -10+2k+2-1=0$$

$\therefore\ k=\frac{9}{2}$. But $\vec{l}\times\vec{m}\neq\vec{0}$. Hence lines are intersecting lines for $k=\frac{9}{2}$. **Ans. : (B)**

(25) The measure of the angle between the lines $2x = 3y = -z$ and $6x = -y = -4z$ is .....

(A) $\frac{\pi}{2}$ (B) 0 (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{4}$ (AIEEE: 2005)

Sol. : The lines are $\frac{x}{3} = \frac{y}{2} = \frac{z}{-6}$ and $\frac{x}{2} = \frac{y}{-12} = \frac{z}{-3}$,

$\vec{l} = (3, 2, -6)$; $\vec{m} = (2, -12, -3)$

$$\cos\theta = \frac{|\vec{l} \cdot \vec{m}|}{|\vec{l}||\vec{m}|}$$

$$= \frac{|6 - 24 + 18|}{\sqrt{9 + 4 + 36} \cdot \sqrt{4 + 144 + 9}} = 0$$

$\therefore \vec{l} \perp \vec{m}$

$\therefore$ The lines are mutually perpendicular.

$\therefore$ The measure of the angle between them is $\frac{\pi}{2}$. **Ans. : (A)**

(26) The equation of the line passing through $A(1, 1, 1)$ and perpendicular $\overleftrightarrow{BC}$ joining $B(1, 4, 6)$ and $C(5, 4, 4)$ is .....

(A) $\frac{x-1}{2} = \frac{y-1}{3} = \frac{z-1}{4}$ (B) $\frac{x-1}{3} = \frac{y-4}{4} = \frac{z-6}{5}$

(C) $\frac{x-5}{3} = \frac{y-4}{4} = \frac{z-4}{5}$ (D) $\frac{x-1}{-2} = \frac{y-1}{-3} = \frac{z-1}{4}$

Sol. : The equation of $\overleftrightarrow{BC}$ is $\vec{r} = \vec{a} + k(\vec{b} - \vec{a})$, $k \in R$ $= (1, 4, 6) + k(4, 0 - 2)$ $k \in R$

Let $M(1 + 4k, 4, 6 - 2k)$ be the foot of the perpendicular from A to $\overleftrightarrow{BC}$.

$\therefore \overrightarrow{AM} \perp \vec{l}$

$\therefore (4k, 3, 5 - 2k) \cdot (4, 0, -2) = 0$

$\therefore 16k - 10 + 4k = 0$

$\therefore k = \frac{1}{2}$

$\overrightarrow{AM} = (4k, 3, 5 - 2k) = (2, 3, 4)$

$\therefore$ The equation of the required perpendicular line is $\vec{r} = (1, 1, 1) + k\ (\overrightarrow{AM})$, $k \in R$

$\therefore \vec{r} = (1, 1, 1) + k\ (2, 3, 4)$, $k \in R$

$\therefore \frac{x-1}{2} = \frac{y-1}{3} = \frac{z-1}{4}$

Note : $\overrightarrow{BC} = (4, 0, -2)$ is perpendicular to only the line given in option (A). **Ans. : (A)**

(27) If the projection of $\overrightarrow{AB}$ on $\overleftrightarrow{OX}$, $\overleftrightarrow{OY}$ and $\overleftrightarrow{OZ}$ are 3, 4 and 12 respectively, then $|\overrightarrow{AB}|$ = ...

(A) 169 (B) 144 (C) 19 (D) 13

Sol. : If $\overrightarrow{AB} = x_1\hat{i} + x_2\hat{j} + x_3\hat{k}$, then projections of $\overrightarrow{AB}$ on $\overleftrightarrow{OX}$ $= \overrightarrow{AB} \cdot \hat{i}$.

$\therefore\ x_1 = 3$. Similarly $x_2 = 4,\ x_3 = 12$

$\therefore\ |\overrightarrow{AB}| = \sqrt{x_1^2 + x_2^2 + x_3^2} = 13$ **Ans. : (D)**

(28) If the direction ratios of two lines satisfy the equations $3lm - 4ln + mn = 0$ and $l + 2m + 3n = 0$, then the measure of the angle between those two lines is ..... .

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{6}$

Sol. : Substituting $l = -2m - 3n$ in $3lm - 4ln + mn = 0$,

$3m(-2m-3n) - 4(-2m-3n)n + mn = 0$

$\therefore\ -6m^2 - 9mn + 8mn + 12n^2 + mn = 0$

$\therefore\ m^2 = 2n^2$

$\therefore\ m = \pm\sqrt{2}\,n$

For $m = \sqrt{2}n$, $l = -2\sqrt{2}n - 3n$ and for $m = -\sqrt{2}n$, $l = 2\sqrt{2}n - 3n$

$\therefore$ The direction ratios of both the lines are $-2\sqrt{2}-3,\ \sqrt{2},\ 1$ and $2\sqrt{2}-3,\ -\sqrt{2},\ 1$

Let the angle between the two lines have measure θ.

$(-2\sqrt{2}-3,\ \sqrt{2},\ 1) \cdot (2\sqrt{2}-3,\ -\sqrt{2},\ 1) = -(8-9) - 2 + 1 = 0$

$\therefore\ \theta = \frac{\pi}{2}$ **Ans. : (A)**

(29) If the shortest distance between the lines $\vec{r} = (1, -1, 0) + k(2, 0, 1),\ k \in R$ and $\frac{x-2}{1} = \frac{y+1}{1} = \frac{z-a}{-1}$ is $\frac{1}{\sqrt{14}}$, then a = ...

(A) 0 or –1 (B) 0 or 1 (C) 1 or –1 (D) 2 or –2

Sol. : $\vec{r} = (1, -1, 0) + k\,(2, 0, 1),\ k \in R$ and $\frac{x-2}{1} = \frac{y+1}{1} = \frac{z-a}{-1}$ are given lines.

$\vec{a} = (1, -1, 0),\ \vec{l} = (2, 0, 1)$ and $\vec{b} = (2, -1, a),\ \vec{m} = (1, 1, -1)$

$\vec{l} \times \vec{m} = (2, 0, 1) \times (1, 1, -1) = (-1, 3, 2)$. Thus, $|\vec{l} \times \vec{m}| = \sqrt{14}$

$(\vec{b} - \vec{a}) \cdot (\vec{l} \times \vec{m}) = (1, 0, a) \cdot (-1, 3, 2) = -1 + 2a$

Now, the shortest distance $= \frac{|(\vec{b}-\vec{a})\cdot(\vec{l}\times\vec{m})|}{|\vec{l}\times\vec{m}|}. = \frac{|2a-1|}{\sqrt{14}} = \frac{1}{\sqrt{14}}$

$\therefore\ 2a-1=1$ or $2a-1=-1$

$\therefore\ a=1$ or $a=0$ **Ans. : (B)**

(30) If a line makes an angle of measure $\frac{\pi}{4}$ with the positive direction of X-axis and Y-axis, then the line makes an angle of measure ..... with the positive direction of Z-axis.

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{2}$ (D) $\frac{\pi}{6}$ (AIEEE : 2007)

Sol. : If a line has direction angles α, β, γ, then $cos^2\alpha + cos^2\beta + cos^2\gamma = 1$

$\therefore\ cos^2\frac{\pi}{4} + cos^2\frac{\pi}{4} + cos^2\gamma = 1$

$\therefore\ \frac{1}{2} + \frac{1}{2} + cos^2\gamma = 1$

$\therefore\ cos^2\gamma = 0$. Thus, $cos\gamma = 0$

$\therefore\ \gamma = \frac{\pi}{2}$ **Ans. : (C)**

(31) If lines $\frac{x-1}{k} = \frac{y-2}{2} = \frac{z-3}{3}$ and $\frac{x-2}{3} = \frac{y-3}{k} = \frac{z-1}{2}$ are intersecting lines, then the integral value of k is .....

(A) –2 (B) –5 (C) 5 (D) 2 (AIEEE : 2008)

Sol. : For lines $\frac{x-1}{k} = \frac{y-2}{2} = \frac{z-3}{3}$ and $\frac{x-2}{3} = \frac{y-3}{k} = \frac{z-1}{2}$,

$\vec{a} = (1, 2, 3),\ \vec{l} = (k, 2, 3)$ and $\vec{b} = (2, 3, 1),\ \vec{m} = (3, k, 2)$

The lines are intersecting and hence coplanar.

$\therefore \begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \end{vmatrix} = 0$

$\therefore \begin{vmatrix} 1 & 1 & -2 \\ k & 2 & 3 \\ 3 & k & 2 \end{vmatrix} = 0$

$\therefore\ 1(4-3k) - 1(2k-9) - 2(k^2-6) = 0$

$\therefore\ 2k^2 + 5k - 25 = 0$

$\therefore\ (2k-5)\,(k+5) = 0$

$\therefore\ k = \frac{5}{2}$ or $k = -5$

$\therefore$ Integral value of k is -5. See that $(-5, 2, 3) \times (3, -5, 2) \neq \vec{0}$ **Ans. : (B)**

Note : $\vec{l}\times\vec{m} \neq \vec{0}$. So, lines are neither coincident nor parallel.

(32) A line $\overleftrightarrow{AB}$ in three dimensional space makes angles of measures $\frac{\pi}{4}$ and $\frac{2\pi}{3}$ with the positive direction of X-axis and the positive direction of Y-axis respectively. If $\overleftrightarrow{AB}$ makes an acute angle of measure θ with the positive direction of Z-axis, then θ = ...

(AIEEE: 2010)

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{5\pi}{12}$

Sol. : If direction angles of $\overleftrightarrow{AB}$ are α, β, γ, then $cos^2\alpha + cos^2\beta + cos^2\gamma = 1$

$\therefore\ cos^2\frac{\pi}{4} + cos^2\frac{2\pi}{3} + cos^2\theta = 1$

$\therefore\ \frac{1}{2} + \frac{1}{4} + cos^2\theta = 1$

$\therefore\ cos^2\theta = \frac{1}{4}$

$\therefore\ cos\theta = \frac{1}{2}$ (θ is an acute angle)

$\therefore\ \theta = \frac{\pi}{3}$ **Ans. : (C)**

(33) The perpendicular distance of the point $P(3, -1, 11)$ from the line $\frac{x}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ is .....

(A) $\sqrt{29}$ (B) $\sqrt{33}$ (C) $\sqrt{53}$ (D) $\sqrt{65}$ (AIEEE : 2011)

Sol. : $\overrightarrow{PA} = (0,2,3) - (3,-1,11) = (-3,3,-8)$ (A = (0, 2, 3))

The perpendicular distance of point $P(3,-1,11)$ from the line $\vec{r} = (0, 2, 3) + k(2, 3, 4),\ k \in R$ is

$$= \frac{|\overrightarrow{PA} \times \vec{l}|}{|\vec{l}|}$$

$$= \frac{|(-3,3,-8)\times(2,3,4)|}{\sqrt{4+9+16}}$$

$$= \frac{|(36,-4,-15)|}{\sqrt{29}} = \frac{\sqrt{1296+16+225}}{\sqrt{29}} = \frac{\sqrt{1537}}{\sqrt{29}} = \sqrt{53}$$ **Ans. : (C)**

(34) If $l+m+n=0,\ l^2+m^2-n^2=0$ and l, m, n are direction cosines of two lines, then the angle between them has measure .....

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

Sol. : l, m, n are direction cosines of lines.

$\therefore\ l^2+m^2+n^2=1$ and $l^2+m^2=n^2$ (Given)

$\therefore\ 2n^2=1.$ So, $n=\pm\frac{1}{\sqrt{2}}$

Now from $l+m+n=0$, substitute $n=-l-m$ in $l^2+m^2-n^2=0$,

$l^2+m^2-(l+m)^2=0$

$\therefore\ -2lm=0$

$\therefore\ l=0$ or $m=0$

If $l=0$, then $m+n=0$. So, $m=-n=\mp\frac{1}{\sqrt{2}}$

$\therefore$ Direction cosines of one line : $0, \mp\frac{1}{\sqrt{2}}, \pm\frac{1}{\sqrt{2}}$

and if $m=0$, then $l+n=0$. So, $l=-n=\mp\frac{1}{\sqrt{2}}$

$\therefore$ Direction cosines of the other line : $\mp\frac{1}{\sqrt{2}}, 0, \pm\frac{1}{\sqrt{2}}$

Taking θ as the measure of the angle between these two lines,

$$\cos\theta = \left|(0)\left(\mp\frac{1}{\sqrt{2}}\right)+\left(\mp\frac{1}{\sqrt{2}}\right)(0)+\left(\pm\frac{1}{\sqrt{2}}\right)\left(\pm\frac{1}{\sqrt{2}}\right)\right| = \frac{1}{2}$$

$\therefore\ \theta = \frac{\pi}{3}$ **Ans. : (C)**

(35) The point which is equidistant from distinct pooints $(x, 0, 0), (0, y, 0), (0, 0, z)$ and $(0, 0, 0)$ is .....

(A) (x, y, z) (B) $\left(\frac{x}{2}, \frac{y}{2}, \frac{z}{2}\right)$ (C) $\left(\frac{x}{3}, \frac{y}{3}, \frac{z}{3}\right)$ (D) $(3x, 3y, 3z)$

Sol. : Suppose $P(a, b, c)$ is equidistant from the given points.

$\therefore$ Let $A(x, 0, 0), B(0, y, 0), C(0, 0, z), O(0, 0, 0)$,

$PA^2=PO^2$

$\therefore\ (x-a)^2+b^2+c^2=a^2+b^2+c^2$

$\therefore\ x^2-2ax=0$

$\therefore\ a=\frac{x}{2}$ (as points are distinct, $x\neq 0$)

Similarly, from $PB^2=PO^2$ and $PC^2=PO^2$ respectively, $b=\frac{y}{2}$ and $c=\frac{z}{2}$

$\therefore$ Required point is $\left(\frac{x}{2}, \frac{y}{2}, \frac{z}{2}\right)$. **Ans. : (B)**

(36) If a line makes angles of measure α, β, γ with the positive direction of X-axis, Y-axis and Z-axis respectively, then $cos2\alpha + cos2\beta + cos2\gamma = $ .....

(A) -1 (B) 1 (C) -2 (D) 2

Sol. : Here, $cos^2\alpha + cos^2\beta + cos^2\gamma = 1$

$$\text{Now } \cos 2\alpha + \cos 2\beta + \cos 2\gamma = 2cos^2\alpha - 1 + 2cos^2\beta - 1 + 2cos^2\gamma - 1$$

$$= 2\left(cos^2\alpha + cos^2\beta + cos^2\gamma\right) - 3$$

$$= 2(1) - 3 = -1$$

Ans. : (A)

(37) The equation of the line passing through $(4, 2, 3)$ and parallel to line $\frac{x-4}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$ is ..... .

(A) $\frac{x-4}{4} = \frac{y+1}{2} = \frac{z+10}{3}$ (B) $\frac{x-4}{-4} = \frac{y-2}{1} = \frac{z-3}{10}$

(C) $\frac{x-4}{2} = \frac{y-2}{-3} = \frac{z-3}{8}$ (D) $\frac{x-4}{4} = \frac{y-2}{2} = \frac{z-3}{3}$

Sol. : The direction vector of the given line is $\vec{l} = (2, -3.8)$ = the direction vector of the line parallel to the given line.

$\therefore$ The equation of the line passing through $(4, 2, 3)$ and having direction vector $\vec{l} = (2, -3, 8)$ is

$\frac{x-4}{2} = \frac{y-2}{-3} = \frac{z-3}{8}$.

Ans. : (C)

(38) The image of point $(3, 0, 2)$ relative to the line $\frac{x+4}{3} = \frac{y+3}{5} = \frac{z-1}{2}$ is .....

(A) $(-1, 2, 3)$ (B) $(-3, 0, -2)$ (C) $(-4, 2, 1)$ (D) $(-5, 4, 4)$

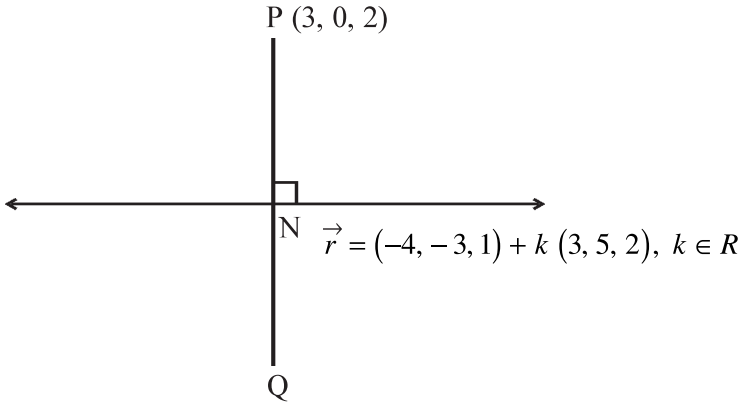


Figure 21.3

Sol. : Let, $(-4 + 3k, -3 + 5k, 1 + 2k)$ be any point on the line $\vec{r} = (-4, -3, 1) + k\,(3, 5, 2),\ k \in R$.

Also, let N be the foot of the perpendicular from $P(3, 0, 2)$,

$\overrightarrow{PN} \perp \vec{l}$

$\therefore \ (3k-4-3, 5k-3-0, 2k+1-2) \cdot (3, 5, 2) = 0$

$\therefore \ 9k-21+25k-15+4k-2=0$

$\therefore \ 38k-38=0$

$\therefore \ k=1$

$\therefore$ The foot of perpendicular is $N(-1, 2, 3)$

If $Q(a, b, c)$ is the image of $P(3, 0, 2)$ relative to the given line, then N is the midpoint of $\overline{PQ}$.

$$(-1, 2, 3) = \left(\frac{a+3}{2}, \frac{b+0}{2}, \frac{c+2}{2}\right)$$

$\therefore \ Q(-5, 4, 4)$ **Ans. : (D)**

(39) The length of the segment cut off by the lines $\frac{x+2}{1} = \frac{y-7}{-1} = \frac{z-5}{3}$ and

$\vec{r} = (6, 3, -3) + k(4, 2, -3), \ k \in R$ by a line having direction vector $-5\hat{i} + 7\hat{j} + 2\hat{k}$ is .....

(A) $\sqrt{86}$ (B) 12 (C) $\sqrt{78}$ (D) $\sqrt{131}$

Sol. : Suppose the line having direction vector $-5\hat{i} + 7\hat{j} + 2\hat{k}$ intersects $\frac{x+2}{1} = \frac{y-7}{-1} = \frac{z-5}{3}$.

i.e. $\vec{r} = (-2, 7, 5) + k(1, -1, 3), \ k \in R$ at point $P(-2+k_1, 7-k_1, 5+3k_1)$ and the line $\vec{r} = (6, 3, -3) + k(4, 2, -3), \ k \in R$ at point $Q(6+4k_2, 3+2k_2, -3-3k_2)$.

$$\overrightarrow{PQ} = (8+4k_2-k_1, \ -4+2k_2+k_1, \ -8-3k_2-3k_1)$$

But $\overleftrightarrow{PQ}$ has direction $(-5, 7, 2)$.

$$\therefore \ \frac{8+4k_2-k_1}{-5} = \frac{-4+2k_2+k_1}{7} = \frac{-8-3k_2-3k_1}{2}$$

$$\therefore \ \frac{8+4k_2-k_1}{-5} = \frac{-4+2k_2+k_1}{7}$$

$$\therefore \ 38k_2 - 2k_1 = -36 \qquad (1)$$

and $\frac{-4+2k_2+k_1}{7} = \frac{-8-3k_2-3k_1}{2}$

$$\therefore \ 25k_2 + 23k_1 = -48 \qquad (2)$$

Solving equations (1) and (2),

$874k_2 - 46k_1 = -828$ and $50k_2 + 46k_1 = -96$

$\therefore\ 924k_2 = -924.$ So, $k_2 = -1$ and $k_1 = -1$

$\therefore\ P(-3, 8, 2)$ and $Q(2, 1, 0)$

$\therefore\ \overline{PQ}$ is cut off on $\overleftrightarrow{PQ}$ by the given two lines.

$\therefore\ PQ = \sqrt{(2+3)^2 + (1-8)^2 + (0-2)^2} = \sqrt{25 + 49 + 4} = \sqrt{78}$ **Ans. : (C)**

(40) The distance between the lines $\vec{r} = 2\hat{i} - 3\hat{j} + 4\hat{k} + \lambda(\hat{i} - 2\hat{j} + \hat{k}),\ \lambda \in R$ and

$\vec{r} = 3\hat{i} + 4\hat{j} - \hat{k} + \mu(-2\hat{i} + 4\hat{j} - 2\hat{k}),\ \mu \in R$ is .....

(A) $3\sqrt{14}$ (B) $\sqrt{21}$ (C) $3\sqrt{2}$ (D) 0

Sol. : For given lines, $\vec{a} = (2, -3, 4)$, $\vec{l} = (1, -2, 1)$ and $\vec{b} = (3, 4, -1)$, $\vec{m} = (-2, 4, -2)$.

Here, $\vec{l} = k\vec{m},\ k = -\frac{1}{2}$

$\therefore$ Given lines are either parallel or coincident.

$(2, -3, 4) = (3, 4, -1) + \mu(-2, 4, -2) \Rightarrow (-1, -7, 5) = \mu(-2, 4, -2)$

which is not possible. So, (2, –3, 4) does not lie on L_2.

$\therefore$ Lines are parallel.

The perpendicular distance between them $= |(\vec{b} - \vec{a}) \times \hat{l}|$

$= \frac{|(1, 7, -5) \times (1, -2, 1)|}{\sqrt{6}}$

$= \frac{|(-3, -6, -9)|}{\sqrt{6}} = \frac{\sqrt{126}}{\sqrt{6}} = \sqrt{21}$ **Ans. : (B)**

(41) The direction cosines of two lines satisfy the equations $al + bm + cn = 0$ and $fmn + gnl + hlm = 0$. If these lines are mutually perpendicular, then ..... $(a \neq 0, b \neq 0, c \neq 0)$

(A) $\frac{f}{a^2} + \frac{g}{b^2} + \frac{c}{h^2} = 0$ (B) $\frac{f}{a} + \frac{g}{b} + \frac{h}{c} = 0$

(C) $\frac{a^2}{f} + \frac{b^2}{g} + \frac{c^2}{h} = 0$ (D) $\frac{a}{f} + \frac{b}{g} + \frac{c}{h} = 0$

Sol. : $al + bm + cn = 0$

$\therefore$ Substituting $n = \frac{-al - bm}{c}$ in $fmn + gnl + hlm = 0$,

$fm\frac{(-al - bm)}{c} + gl\frac{(-al - bm)}{c} + hlm = 0$

$\therefore\ -aflm - bfm^2 - agl^2 - bglm + chlm = 0$

Dividing by $-m^2$, $af\left(\frac{l}{m}\right) + bf + ag\left(\frac{l}{m}\right)^2 + bg\left(\frac{l}{m}\right) - ch\left(\frac{l}{m}\right) = 0$

$\therefore\ ag\left(\frac{l}{m}\right)^2 + (af + bg - ch)\left(\frac{l}{m}\right) + bf = 0$. A quadratic equation in $\frac{l}{m}$ whose roots are $\alpha = \frac{l_1}{m_1}$ and $\beta = \frac{l_2}{m_2}$ is obtained.

The product of roots $\alpha\beta = \frac{l_1 l_2}{m_1 m_2} = \frac{bf}{ag}$. So, $\frac{l_1 l_2}{\left(\frac{f}{a}\right)} = \frac{m_1 m_2}{\left(\frac{g}{b}\right)}$

Similarly if l or m is eliminated, then $\frac{n_1 n_2}{\left(\frac{h}{c}\right)} = \frac{l_1 l_2}{\left(\frac{f}{a}\right)} = \frac{m_1 m_2}{\left(\frac{g}{b}\right)}$.

Let the direction cosines of the line be l_1, m_1, n_1 and l_2, m_2, n_2. If the lines are mutually perpendicular, then

$l_1 l_2 + m_1 m_2 + n_1 n_2 = 0$

$\therefore\ \frac{f}{a} + \frac{g}{b} + \frac{h}{c} = 0$ **Ans. : (B)**

(42) If a line makes angles of measures α, β, γ and δ with the four diagonals of a cube, then $\sin^2\alpha + \sin^2\beta + \sin^2\gamma + \sin^2\delta$ = ...

(A) $\frac{4}{3}$ (B) $\frac{8}{3}$ (C) 1 (D) 2

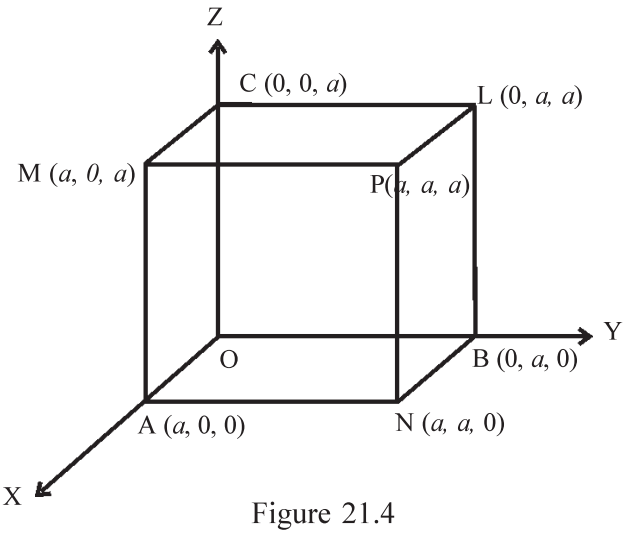


Figure 21.4

Sol. : Suppose the direction cosines of the line are l, m, n. For diagonals $\overline{OP}$, $\overline{AL}$, $\overline{BM}$, $\overline{CN}$ of cube, $\overrightarrow{\mathbf{OP}} = (a, a, a)$, $\overrightarrow{\mathbf{AL}} = (-a, a, a)$, $\overrightarrow{\mathbf{BM}} = (a, -a, a)$ and $\overrightarrow{\mathbf{CN}} = (a, a, -a)$.

$\therefore$ The direction cosines of $\overrightarrow{\mathbf{OP}}$ are $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$.

If the given line makes an angle of measure α with $\overleftrightarrow{\mathrm{OP}}$, then $cos\alpha = \frac{|l+m+n|}{\sqrt{3}}$

Similarly, $cos\beta = \frac{|-l+m+n|}{\sqrt{3}}$, $cos\gamma = \frac{|l-m+n|}{\sqrt{3}}$ and $cos\delta = \frac{|l+m-n|}{\sqrt{3}}$.

Now, $cos^2\alpha + cos^2\beta + cos^2\gamma + cos^2\delta = \frac{4\left(l^2+m^2+n^2\right)}{3} = \frac{4}{3}$

$\therefore \; sin^2\alpha + sin^2\beta + sin^2\gamma + sin^2\delta = 4 - \left(cos^2\alpha + cos^2\beta + cos^2\gamma + cos^2\delta\right)$

$= 4 - \frac{4}{3} = \frac{8}{3}$ **Ans. : (B)**

(43) A line makes an angle of measure θ with X-axis and with Z-axis, and makes an angle of measure β with Y-axis. If $sin^2\beta = 3sin^2\theta$, then $cos^2\theta$ = ...

(A) $\frac{3}{5}$ (B) $\frac{1}{5}$ (C) $\frac{2}{3}$ (D) $\frac{2}{5}$ (AIEEE:2004)

Sol. : The line makes angles of measures θ, β and θ with X-axis, Y-axis and Z-axis respectively.

$\therefore \; cos^2\theta + cos^2\beta + cos^2\theta = 1$

$\therefore \; 2cos^2\theta + 1 - sin^2\beta = 1$

$\therefore \; 2cos^2\theta - 3sin^2\theta = 0$ $\left(sin^2\beta = 3sin^2\theta\right)$

$\therefore \; 2cos^2\theta - 3(1 - cos^2\theta) = 0$

$\therefore \; 5cos^2\theta = 3$

$\therefore \; cos^2\theta = \frac{3}{5}$. **Ans. : (A)**

(44) If the foot of the perpendicular from the origin to a plane is $(-3, 4, -2)$, then the equation of the plane is ..... .

(A) $3x - 4y + 2z - 29 = 0$ (B) $3x - 4y + 2z + 29 = 0$

(C) $4x - 2y - 3z + 29 = 0$ (D) $4x - 2y - 3z - 29 = 0$

Sol. : The foot of the perpendicular from the origin $O(0, 0, 0)$ to the plane is $P(-3, 4, -2)$.

$\therefore$ The vector normal to the plane is $\vec{n} = \overrightarrow{\mathbf{OP}} = (-3, 4, -2) = (a, b, c)$

$\therefore$ The equation of the plane is $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$

$\therefore \; -3(x+3) + 4(y-4) - 2(z+2) = 0$

$\therefore \; -3x - 9 + 4y - 16 - 2z - 4 = 0$

$\therefore \; 3x - 4y + 2z + 29 = 0$ **Ans. : (B)**

(45) The point of intersection of line $\frac{x+3}{2} = \frac{y-1}{1} = \frac{z-2}{2}$ and plane $2x-3y-z+7=0$ is .....

(A) $(-3, 1, 2)$ (B) $(2, -3, -1)$ (C) $(2, 1, 2)$ (D) $(-11, -3, -6)$

Sol. : Any point on line $\vec{r} = (-3, 1, 2) + k(2, 1, 2),\ k \in R$ is $(-3+2k, 1+k, 2+2k)$.

If it is on plane $2x-3y-z+7=0$, then

$$2(-3+2k)-3(1+k)-(2+2k)+7=0$$

$$-4 = k$$

$\therefore$ The point of intersection $= (-11, -3, -6)$ **Ans. : (D)**

(46) The equation of the plane parallel to $\vec{r} \cdot (3\hat{i} + 5\hat{j} + 4\hat{k}) = 0$ and passing through $\hat{i} + 3\hat{j} + 2\hat{k}$ is .....

(A) $3x+4y+4z+26=0$ (B) $3x+5y+4z-26=0$

(C) $x+3y+2z+26=0$ (D) $x+3y+2z-26=0$

Sol. : The equation of the plane parallel to the plane $\vec{r} \bullet (3\hat{i} + 5\hat{j} + 4\hat{k}) = 0$ is

$$\vec{r} \bullet (3\hat{i} + 5\hat{j} + 4\hat{k}) + t = 0$$

It is passing through $(1, 3, 2)$.

$\therefore\ (1, 3, 2) \bullet (3, 5, 4) + t = 0$

$\therefore\ t = -26$

$\therefore$ The equation of the plane is $\vec{r} \bullet (3\hat{i} + 5\hat{j} + 4\hat{k}) - 26 = 0$

$\therefore\ 3x+5y+4z-26=0$ **Ans. : (B)**

(47) The foot of the perpendicular from $(4, 1, 3)$ to the plane $x-y-2z-3=0$ is .....

(A) $(6, -1, -1)$ (B) $(1, -1, -2)$ (C) $(-3, -2, -5)$ (D) $(5, 0, 1)$

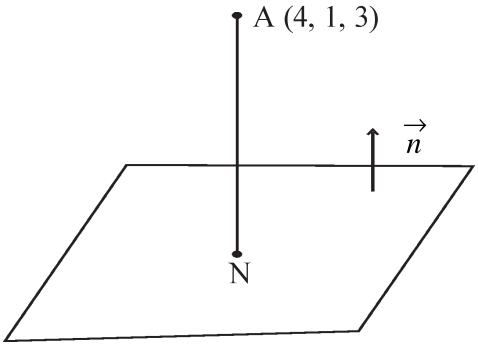


Figure 21.5

Sol. : The vector normal to the plane $x-y-2z-3=0$ is $\vec{n} = (1, -1, -2)$.

Let N be the foot of perpendicular from A to the plane.

The equation of $\overleftrightarrow{AN}$ is $\vec{r} = \vec{a} + k\vec{n}$, $k \in R$

$\therefore \quad \vec{r} = (4, 1, 3) + k\,(1, -1, -2),\ k \in R$

$\therefore$ For some $k \in R$, $N(4+k, 1-k, 3-2k)$ and it lies on the plane $x - y - 2z - 3 = 0$.

$\therefore \quad 4 + k - 1 + k - 6 + 4k - 3 = 0$

$\therefore \quad 6k = 6.$ So, $k = 1$

$\therefore$ The foot of perpendicular is $N(5, 0, 1)$. **Ans. : (D)**

(48) The equation of the line passing through $(2, -1, 3)$ and perpendicular to plane $5x + 4y - 7z - 1 = 0$ is .....

(A) $\vec{r} = 2\hat{i} - \hat{j} + 3\hat{k} + \lambda(5\hat{i} + 4\hat{j} - 7\hat{k}),\ \lambda \in R$

(B) $\vec{r} = 5\hat{i} + 4\hat{j} - 7\hat{k} + \lambda(2\hat{i} - \hat{j} + 3\hat{k}),\ \lambda \in R$

(C) $\vec{r} = 2\hat{i} - \hat{j} + 3\hat{k} + \lambda(3\hat{i} + 5\hat{j} - 10\hat{k}),\ \lambda \in R$

(D) cannot be obtained

Sol. : The vector normal to plane is $\vec{n} = (5, 4, -7)$. This is the direction vector of the perpendicular line.

The line passes through $\vec{a} = (2, -1, 3)$.

$\therefore$ The equation of the perpendicular line is $\vec{r} = \vec{a} + \lambda\vec{n},\ \lambda \in R$

$\therefore \quad \vec{r} = (2, -1, 3) + \lambda\,(5, 4, -7),\ \lambda \in R$

$\therefore \quad \vec{r} = 2\hat{i} - \hat{j} + 3\hat{k} + \lambda(5\hat{i} + 4\hat{j} - 7\hat{k}),\ \lambda \in R$ **Ans. : (A)**

(49) The magnitude of vector normal to the plane is $3\sqrt{3}$. The normal of the plane makes angles of equal measure with all the three coordinate axes. If this plane passes through the point $(3, -1, 2)$, then its equation is .....

(A) $\vec{r} \cdot (3\hat{i} - \hat{j} + 2\hat{k}) = 0$ (B) $\vec{r} \cdot (3\hat{i} - \hat{j} + 2\hat{k}) = 6$

(C) $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 4$ (D) $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 0$

Sol. : The vector normal to the plane makes angles of equal measure with all the three coordinate axes.

Let $\vec{n} = (l, l, l)$

$\therefore \quad l^2 + l^2 + l^2 = 1.$ So, $l = \pm\frac{1}{\sqrt{3}}$

Now, as $|\vec{n}| = 3\sqrt{3}$, $\vec{n} = 3\sqrt{3}\left(\frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}}\right) = 3\hat{i} + 3\hat{j} + 3\hat{k}$

The equation of the plane is $\vec{r} \bullet \vec{n} = \vec{a} \bullet \vec{n}$.

$\therefore \quad \vec{r} \bullet (3\hat{i} + 3\hat{j} + 3\hat{k}) = (3, -1, 2) \cdot (3, 3, 3)$

$\therefore \quad \vec{r} \bullet (\hat{i} + \hat{j} + \hat{k}) = 4$ **Ans. : (C)**

(50) The plane $x - 2y + 3z = 15$ divides the line segment joining $(-2, 4, 7)$ and $(3, -5, 8)$ in ..... ratio.

(A) $-2 : 11$ (B) $2 : 11$ (C) $17 : 26$ (D) $1 : 2$

Sol. : Suppose the plane $x - 2y + 3z = 15$ divides the line segment joining $P(-2, 4, 7)$ and $Q(3, -5, 8)$ in ratio $\lambda : 1$ from P.

$\therefore$ The point of Intersection of $\overleftrightarrow{PQ}$ with the plane $R\left(\frac{3\lambda - 2}{\lambda + 1}, \frac{-5\lambda + 4}{\lambda + 1}, \frac{8\lambda + 7}{\lambda + 1}\right)$ lies on the plane $x - 2y + 3z = 15$.

$\therefore \quad \frac{3\lambda - 2}{\lambda + 1} - 2\left(\frac{-5\lambda + 4}{\lambda + 1}\right) + 3\left(\frac{8\lambda + 7}{\lambda + 1}\right) = 15$

$\therefore \quad 3\lambda - 2 + 10\lambda - 8 + 24\lambda + 21 = 15\lambda + 15$

$\therefore \quad 22\lambda = 4$

$\therefore \quad \lambda = \frac{2}{11}$

$\therefore$ Ratio is $2 : 11$ **Ans. : (B)**

(51) The distance between the planes $2x + y + 2z - 8 = 0$ and $4x + 2y + 4z + 5 = 0$ is ..... .

(A) $\frac{7}{2}$ (B) $\frac{5}{2}$ (C) $\frac{3}{2}$ (D) $\frac{9}{2}$

(AIEEE : 2004, JEE : 2013)

Sol. : The perpendicular distance between parallel planes

$4x + 2y + 4z - 16 = 0$ and $4x + 2y + 4z + 5 = 0$ is

$$= \frac{|d_2 - d_1|}{\sqrt{a^2 + b^2 + c^2}}$$

$$= \frac{|5+16|}{\sqrt{16 + 4 + 16}} = \frac{21}{6} = \frac{7}{2}$$ **Ans. : (A)**

(52) The direction numbers of normal to the plane passing through $(1, 0, 0)$ and $(0, 1, 0)$ and making an angle of measure $\frac{\pi}{4}$ with the plane $x + y = 3$ are .....

(A) $1, \sqrt{2}, 1$ (B) $1, 1, \sqrt{2}$ (C) $1, 1, 2$ (D) $\sqrt{2}, 1, 1$

(AIEEE : 2002)

Sol. : Suppose direction numbers of normal to the plane are a, b, c.

The plane passes through $(1, 0, 0)$ and $(0, 1, 0)$.

$\therefore\ a(x_2-x_1)+b(y_2-y_1)+c(z_2-z_1)=0$

$\therefore\ a(0-1)+b(1-0)+c(0-0)=0$

$\therefore\ -a+b=0$

$\therefore\ a=b$

$\therefore$ Normal vector $\vec{n_1}=(a, a, c)$.

and normal vector to the plane $x+y=3$ is $\vec{n_2}=(1, 1, 0)$.

Now, $\cos\theta=\dfrac{|\vec{n_1}\cdot\vec{n_2}|}{|\vec{n_1}||\vec{n_2}|}$

$\therefore\ \cos\dfrac{\pi}{4}=\dfrac{|a+a|}{\sqrt{2a^2+c^2}\cdot\sqrt{2}}$

$\therefore\ \dfrac{1}{\sqrt{2}}=\dfrac{2|a|}{\sqrt{2a^2+c^2}\cdot\sqrt{2}}$

$\therefore\ 2a^2+c^2=4a^2$

$\therefore\ c=\pm\sqrt{2}\,a$

$\therefore$ Direction numbers are $a, a, \pm\sqrt{2}\,a$ i.e. $1, 1, \pm\sqrt{2}$. **Ans. : (B)**

(53) Two systems of rectangular axes have the same origin. If a plane intersects them at distances a, b, c and a', b', c' from the origin, then .....

(A) $\dfrac{1}{a^2}+\dfrac{1}{b^2}+\dfrac{1}{c^2}+\dfrac{1}{a'^2}+\dfrac{1}{b'^2}+\dfrac{1}{c'^2}=0$ (B) $\dfrac{1}{a^2}-\dfrac{1}{b^2}-\dfrac{1}{c^2}+\dfrac{1}{a'^2}-\dfrac{1}{b'^2}-\dfrac{1}{c'^2}=0$

(C) $\dfrac{1}{a^2}+\dfrac{1}{b^2}+\dfrac{1}{c^2}-\dfrac{1}{a'^2}-\dfrac{1}{b'^2}-\dfrac{1}{c'^2}=0$ (D) $\dfrac{1}{a^2}+\dfrac{1}{b^2}+\dfrac{1}{c^2}+\dfrac{1}{a'^2}+\dfrac{1}{b'^2}+\dfrac{1}{c'^2}=2$

(AIEEE : 2003)

Sol. : The intercepts made by the plane on the axes are a, b, c.

$\therefore$ The equation of the plane is $\pm\dfrac{x}{a}\pm\dfrac{y}{b}\pm\dfrac{z}{c}=1$

Similarly, the equation of the other plane is $\pm\dfrac{x}{a'}\pm\dfrac{y}{b'}\pm\dfrac{z}{c'}=1$.

Perpendicular distance of plane from origin is $d_1=\dfrac{|0+0+0-1|}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}}=\dfrac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}}$.

Similarly, for another system,

$d_2=\dfrac{1}{\sqrt{\frac{1}{(a')^2}+\frac{1}{(b')^2}+\frac{1}{(c')^2}}}$.

Now, $d_1 = d_2$

$$\therefore \quad \frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}} = \frac{1}{\sqrt{\frac{1}{(a')^2}+\frac{1}{(b')^2}+\frac{1}{(c')^2}}}$$

$$\therefore \quad \frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}-\frac{1}{(a')^2}-\frac{1}{(b')^2}-\frac{1}{(c')^2} = 0$$

Ans. : (C)

(54) The equation of the plane passing through $(-1, 0, 1)$ and $(2, 2, 3)$ and parallel to line $\frac{x-2}{3} = \frac{y-1}{-2} = z-1$ is .....

(A) $3x-2y+z+2=0$ (B) $2x-y-4z+2=0$

(C) $2x+y-4z-6=0$ (D) $2x+y-4z+6=0$

Sol. : Suppose normal to the plane is $\vec{n} = (a, b, c)$.

$A(-1, 0, 1)$ lies on the plane.

$\therefore$ The equation of the plane is $a(x+1)+b(y-0)+c(z-1)=0$ (1)

The plane passes through $B(2, 2, 3)$.

$\therefore \quad 3a + 2b + 2c = 0 \quad (a(x_2 - x_1) + b(y_2 - y_1) + c(z_2 - z_1) = 0)$ (2)

The line $\vec{l} = (3, -2, 1)$ is orthogonal to (a, b, c).

$\therefore \quad 3a-2b+c=0 \qquad (\vec{l} \cdot \vec{n} = 0)$ (3)

Solving (2) and (3),

$$\frac{a}{6} = \frac{b}{3} = \frac{c}{-12}$$

$$\therefore \quad \frac{a}{2} = \frac{b}{1} = \frac{c}{-4}$$

$\therefore \quad a : b : c = 2 : 1 : (-4)$

$\therefore$ The equation of the plane is $2(x+1)+1(y-0)-4(z-1)=0$

$\therefore \quad 2x+y-4z+6=0.$

Ans. : (D)

(55) A plane intersects X-axis at A, Y-axis at B and Z-axis at C. If the centroid of ΔABC is (α, β, γ), then the equation of the plane is .....

(A) $\frac{x}{\alpha}+\frac{y}{\beta}+\frac{z}{\gamma}=1$ (B) $\frac{x}{\alpha}+\frac{y}{\beta}+\frac{z}{\gamma}=3$

(C) $\frac{x}{\alpha}+\frac{y}{\beta}+\frac{z}{\gamma}=\frac{1}{3}$ (D) $\frac{x}{\alpha}+\frac{y}{\beta}+\frac{z}{\gamma}=0$

Sol. : The plane intersects axes at $A(a, 0, 0)$, $B(0, b, 0)$ and $C(0, 0, c)$.

The equation of the plane is $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$.

The centroid of ΔABC is $(\alpha, \beta, \gamma) = \left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right)$

$\therefore\ a = 3\alpha,\ b = 3\beta,\ c = 3\gamma$

$\therefore$ The equation of the plane is $\frac{x}{3\alpha} + \frac{y}{3\beta} + \frac{z}{3\gamma} = 1$

$\therefore\ \frac{x}{\alpha} + \frac{y}{\beta} + \frac{z}{\gamma} = 3$ **Ans. : (B)**

(56) The equation of the plane passing through $(3, 2, 0)$ and containing the line $\frac{x-3}{1} = \frac{y-6}{5} = \frac{z-4}{4}$ is .....

(A) $x - y + z = 1$ (B) $x + y + z = 5$ (C) $x + 2y - z = 1$ (D) $2x - y + z = 5$

(AIEEE : 2002)

Sol. : The equation of the plane is $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l_1 & l_2 & l_3 \end{vmatrix} = 0$ where $(x_1, y_1, z_1) = (3, 2, 0)$, $(x_2, y_2, z_2) = (3, 6, 4)$, $(l_1, l_2, l_3) = (1, 5, 4)$

$\therefore\ \begin{vmatrix} x-3 & y-2 & z \\ 0 & 4 & 4 \\ 1 & 5 & 4 \end{vmatrix} = 0$

$\therefore\ -4(x-3) + 4(y-2) - 4z = 0$

$\therefore\ -x + 3 + y - 2 - z = 0$

$\therefore\ x - y + z = 1$ **Ans. : (A)**

Second method : Suppose the equation of the plane is $a(x-3) + b(y-2) + c(z-0) = 0$.

Any point of line $(k + 3, 5k + 6, 4k + 4)$ lies on the plane.

$\therefore\ a(k + 3 - 3) + b(5k + 6 - 2) + c(4k + 4) = 0 \quad \forall\ k \in R$

$\therefore\ (a + 5b + 4c)k + 4b + 4c = 0 \quad \forall\ k \in R$

$\therefore\ a + 5b + 4c = 0$ and $4b + 4c = 0$. i.e. $b + c = 0$

$\frac{a}{1} = \frac{b}{-1} = \frac{c}{1}$

$\therefore$ The equation of the plane is $(x-3) - (y-2) + (z-0) = 0$

$\therefore\ x - y + z = 1.$

Third method : The equation of the plane is $a(x-3)+b(y-2)+c(z-0)=0$

The normal (a, b, c) to the plane is orthogonal to direction vector $(1, 5, 4)$ of the line.

$\therefore \quad a+5b+4c=0 \qquad (1)$

$(3, 6, 4)$ lies on the plane.

$a(3-3)+b(6-2)+c(4)=0$

$\therefore \quad 4b+4c=0 \qquad (2)$

By (1) - (2), $a+b=0$

$\therefore \quad a=-b=c \qquad$ (From (2))

$\therefore \quad x-3-(y-2)+z=0$ is the equation of the plane.

$\therefore \quad x-y+z=1$

(57) A tetrahedron has vertices at $O(0, 0, 0)$, $A(1, 2, 1)$, $B(2, 1, 3)$, $C(-1, 1, 2)$. Then the angle between the planes formed by points O, A, B and A, B, C is ..... (AIEEE:2003)

(A) $cos^{-1}\left(\frac{17}{31}\right)$ (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{2}$ (D) $cos^{-1}\left(\frac{19}{35}\right)$

Sol. : Normal to the plane formed by points O, A, B is

$$\vec{n_1} = \overrightarrow{OA} \times \overrightarrow{OB} = (1, 2, 1) \times (2, 1, 3) = (5, -1, -3)$$

and normal to the plane formed by points A, B, C is

$$\vec{n_2} = \overrightarrow{AB} \times \overrightarrow{AC} = (1, -1, 2) \times (-2, -1, 1) = (1, -5, -3)$$

If the angle between them has measure θ, then $cos\theta = \frac{|\vec{n_1} \cdot \vec{n_2}|}{|\vec{n_1}||\vec{n_2}|} = \frac{|5+5+9|}{\sqrt{35} \cdot \sqrt{35}} = \frac{19}{35}$

$\therefore \quad \theta = cos^{-1}\left(\frac{19}{35}\right)$ **Ans. : (D)**

(58) If the plane $2x-4y+z=7$ contains the line $\frac{x-4}{1}=\frac{y-2}{1}=\frac{z-k}{2}$, then k = ...

(A) 7 (B) –7 (C) 1 (D) Not any real number (IIT : 2003)

Sol. : The line lies in the plane. Point $(4, 2, k)$ of the line is in the plane.

$\therefore \quad 2(4)-4(2)+k=7$

$\therefore \quad k=7.$ **Ans. : (A)**

(59) The sum of the intercepts made by the plane $x-3y+4z+6=0$ on the coordinate axes is .....

(A) $\frac{11}{2}$ (B) $\frac{-11}{2}$ (C) $\frac{-19}{2}$ (D) 8

Sol. : $x-3y+4z=-6$

$\therefore \frac{x}{-6}+\frac{y}{2}+\frac{z}{-\frac{3}{2}}=1$

$\therefore$ The intercepts made by the plane on coordinate axes : $-6, 2, -\frac{3}{2}$

$\therefore$ Their sum $= \frac{-11}{2}$ **Ans. : (B)**

(60) A plane intersects the axes at A, B, C. If $\left(p^2, p, 1\right)$ is the centroid of ΔABC, then the equation of the plane is .....

(A) $p^2x+py+z=3p^2$ (B) $x+py+p^2z=3p^2$

(C) $p^2x+py+z=3$ (D) $x+py+p^2z=3$

Sol. : Suppose the plane intersects the axes at $A(a, 0, 0),\ B(0, b, 0),\ C(0, 0, c)$.

The equation of the plane : $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

The centroid of ΔABC : $\left(p^2, p, 1\right)=\left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right)$

$\therefore\ a=3p^2, b=3p, c=3$

$\therefore$ The equation of the plane $\frac{x}{3p^2}+\frac{y}{3p}+\frac{z}{3}=1$

$\therefore\ x+py+p^2z=3p^2$ **Ans. : (B)**

(61) The perpendicular distance of point $(4, 2, 3)$ from the plane $2x+3y-6z+11=0$ is .....

(A) $\frac{1}{7}$ (B) $\frac{43}{7}$ (C) 1 (D) $\frac{15}{7}$

Sol. : The perpendicular distance of point $\left(x_1, y_1, z_1\right)$ from the plane $ax+by+cz+d=0$ is

$$p=\frac{\left|ax_1+by_1+cz_1+d\right|}{\sqrt{a^2+b^2+c^2}}$$

$$=\frac{|8+6-18+11|}{\sqrt{4+9+36}}=\frac{7}{7}=1.$$ **Ans. : (C)**

(62) The equation of the perpendicular bisector plane of the line-segment joining the points $(1, 4, 7)$ and $(3, 0, 1)$ is .....

(A) $x-2y-3z=0$ (B) $x-2y-3z+14=0$

(C) $x-2y-3z-14=0$ (D) $x+2y+3z-18=0$

Sol. : $\overline{AB}$ joining $A(1, 4, 7)$ and $B(3, 0, 1)$ has midpoint $M(2, 2, 4)$.

Normal to the perpendicular bisector plane is $\vec{n} = \overrightarrow{AB} = (2, -4, -6)$.

$\therefore$ The equation of the perpendicular bisector plane is $\vec{r} \cdot \vec{n} = \vec{a} \cdot \vec{n}$.

$\therefore \quad \vec{r} \cdot (1, -2, -3) = (2, 2, 4) \cdot (1, -2, -3) = -14$

$\therefore \quad x - 2y - 3z + 14 = 0$

Ans. : (B)

(63) The line of intersection of the planes $x + 3y - z = 1$ and $\vec{r} \cdot (-2\hat{i} + \hat{j} + 4\hat{k}) = 2$ is parallel to ..... vector.

(A) $3\hat{i} - 2\hat{j} - 7\hat{k}$ (B) $-13\hat{i} + 2\hat{j} - 7\hat{k}$ (C) $13\hat{i} - 2\hat{j} + 7\hat{k}$ (D) $2\hat{i} + 7\hat{j} - 13\hat{k}$

Sol. : $\vec{n_1} = (1, 3, -1),\ \vec{n_2} = (-2, 1, 4)$

$\vec{l} = \vec{n_1} \times \vec{n_2} = 13\hat{i} - 2\hat{j} + 7\hat{k}$ is the direction vector of the line of intersection.

It is parallel to vector $-13\hat{i} + 2\hat{j} - 7\hat{k}$.

Ans. : (B), (C)

(64) The distance between line $\vec{r} = 2\hat{i} - 2\hat{j} + 3\hat{k} + \lambda(\hat{i} - \hat{j} + 4\hat{k})$ and plane $\vec{r} \cdot (\hat{i} + 5\hat{j} + \hat{k}) = 5$ is .....

(A) $\frac{10}{3\sqrt{3}}$ (B) $\frac{10}{9}$ (C) $\frac{10}{3}$ (D) $\frac{3}{10}$

(AIEEE : 2005)

Sol. : $(1, -1, 4) \cdot (1, 5, 1) = 0$ and (2, –2, 3) does not lie on the plane.

$\therefore$ The line is parallel to the plane.

Now, point $A(2, -2, 3)$ lies on the line and the equation of the plane is $x + 5y + z - 5 = 0$.

The distance between the line and the plane is the distance of any point on the line from the plane.

$\therefore$ The distance between the line and the plane $= \frac{|1(2) + 5(-2) + 1(3) - 5|}{\sqrt{1^2 + 5^2 + 1^2}}$

$= \frac{10}{3\sqrt{3}}$

Ans. : (A)

(65) The image of $(-1, 3, 4)$ in the plane $x-2y=0$ is .....

(A) $\left(\frac{-17}{3}, \frac{-19}{3}, 4\right)$ (B) $(15, 11, 4)$ (C) $\left(\frac{-17}{3}, \frac{-19}{3}, 1\right)$ (D) $\left(\frac{9}{5}, \frac{-13}{5}, 4\right)$

(AIEEE :2006)

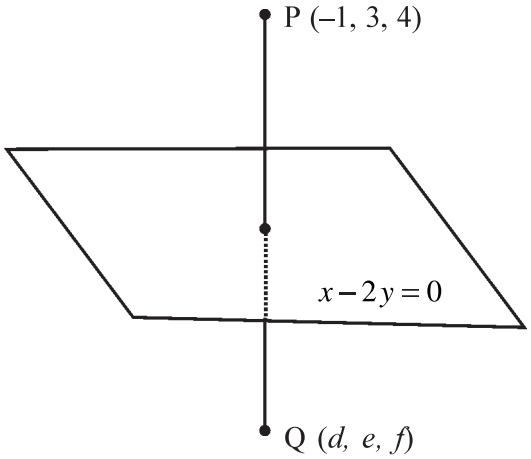


Figure 21.6

Sol. : Suppose the image of $P(-1, 3, 4)$ is $Q(d, e, f)$.

$\therefore$ The midpoint of $\overline{PQ}$ is $M\left(\frac{d-1}{2}, \frac{e+3}{2}, \frac{f+4}{2}\right)$ and it lies in the plane $x-2y=0$.

$\therefore \quad \frac{d-1}{2}-2\left(\frac{e+3}{2}\right)=0$

$\therefore \quad d-2e=7 \qquad (1)$

The equation of the line passing through P and perpendicular to the plane is

$\vec{r}=(-1, 3, 4)+k(1, -2, 0),\ k \in R \qquad (\vec{l}=\vec{n})$

$\therefore \quad \frac{x+1}{1}=\frac{y-3}{-2}=\frac{z-4}{0}$

The perpendicular also passes through Q.

$\therefore \quad \frac{d+1}{1}=\frac{e-3}{-2}=\frac{f-4}{0}$

$\therefore \quad -2d-2=e-3,\ f=4.$

$\therefore \quad 2d+e=1$ and $f=4. \qquad (2)$

Solving equations (1) and (2), $d=\frac{9}{5},\ e=\frac{-13}{5}$

$\therefore$ The image is $\left(\frac{9}{5}, \frac{-13}{5}, 4\right)$ **Ans. : (D)**

(66) The equation of the plane perpendicular to $x+5y+z=4$ and $x+3y+2z=1$ and passing through $(1, 4, -2)$ is .....

(A) $7x+y+2z=7$ (B) $2x+8y+3z=28$ (C) $7x-y-2z=7$ (D) $7x-y+2z=-1$

Sol. : Normals to given planes are $\vec{n_1} = (1, 5, 1)$ and $\vec{n_2} = (1, 3, 2)$

Normal to the plane perpendicular to given planes is

$$\vec{n} = \vec{n_1} \times \vec{n_2} = (1, 5, 1) \times (1, 3, 2) = (7, -1, -2)$$

$\therefore$ The equation of the perpendicular plane is $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$

$\therefore$ $7(x-1) - 1(y-4) - 2(z+2) = 0$

$\therefore$ $7x - y - 2z = 7$ **Ans. : (C)**

(67) A plane passess through $(1, -2, 1)$ and is perpendicular to planes $2x - 2y + z = 0$ and $x - y + 2z = 4$. The perpendicular distance of point $(1, 2, 2)$ from the plane is ..... .

(A) 0 (B) 1 (C) $\sqrt{2}$ (D) $2\sqrt{2}$ (IIT : 2006)

Sol. : Normal to the plane perpendicular to given planes is

$$\vec{n} = \vec{n_1} \times \vec{n_2} = (2, -2, 1) \times (1, -1, 2) = (-3, -3, 0) \text{ i.e. } \vec{n} = (1, 1, 0)$$

The equation of the perpendicular plane is $\vec{r} \cdot \vec{n} = \vec{a} \cdot \vec{n}$ $\left(\vec{a} = (1, -2, 1)\right)$

$\therefore$ $x + y = -1$

$\therefore$ $x + y + 1 = 0$

The distance of plane $x + y + 1 = 0$ from the point $(1, 2, 2)$ is

$$\frac{|1(1) + 1(2) + 0(2) + 1|}{\sqrt{1+1+0}} = \frac{4}{\sqrt{2}} = 2\sqrt{2}$$ **Ans. : (D)**

(68) Let L be the line of intersection of the planes $2x + 3y + z = 1$ and $x + 3y + 2z = 2$. If L makes an angle of measure α with the positive direction of X-axis, then $\cos\alpha =$ .....

(A) $\frac{1}{2}$ (B) 1 (C) $\frac{1}{\sqrt{2}}$ (D) $\frac{1}{\sqrt{3}}$ (AIEEE : 2007)

Sol. : The direction vector of line is $\vec{l} = \vec{n_1} \times \vec{n_2} = (2, 3, 1) \times (1, 3, 2) = (3, -3, 3)$

$\therefore$ $\vec{l} = (1, -1, 1)$

$$\cos\alpha = \frac{|\vec{l} \cdot \hat{i}|}{|\vec{l}||\hat{i}|} = \frac{1}{\sqrt{3}}$$ **Ans. : (D)**

(69) If the line $\frac{x-x_1}{l_1} = \frac{y-y_1}{l_2} = \frac{z-z_1}{l_3}$ is parallel to YZ-plane, then .....

(A) $l_1 = 0$ (B) $l_2 = 0$ (C) $l_3 = 0$ (D) $l_2 = 0, l_3 = 0$

Sol. : The direction vector (l_1, l_2, l_3) of line is orthogonal to (1, 0, 0). (normal to the plane)

$\therefore\ l_1 + 0 + 0 = 0$

$\therefore\ l_1 = 0$ **Ans. : (A)**

(70) The distance of point $(1, 5, 10)$ from the point of intersection of plane $x + y - z - 1 = 0$ and line $\frac{x+2}{3} = \frac{y-1}{4} = \frac{z+2}{12}$ is .....

(A) 15 (B) 11 (C) 13 (D) $\frac{14}{5}$

Sol. : Point $(-2, 1, -2)$ of the line is also a point on the plane $x + y - z - 1 = 0$.

$\therefore\ (-2, 1, -2)$ is the point of intersection of the line and the plane.

$\therefore$ Required distance $= \sqrt{(1+2)^2 + (5-1)^2 + (10+2)^2}$

$= \sqrt{9 + 16 + 144} = 13$ **Ans. : (C)**

(71) If the plane $6x + 4y + 3z - 12 = 0$ intersects the coordinate axes in A, B, C, then the area of ΔABC is ..... sq. units.

(A) $\sqrt{41}$ (B) $\sqrt{61}$ (C) $\sqrt{29}$ (D) $\sqrt{244}$

Sol. : The equation of the plane : $6x + 4y + 3z = 12$

$\therefore\ \frac{x}{2} + \frac{y}{3} + \frac{z}{4} = 1$

$\therefore$ The points of intersection with axes are $A(2, 0, 0),\ B(0, 3, 0),\ C(0, 0, 4)$

Area of ΔABC $= \frac{1}{2} |\overrightarrow{AB} \times \overrightarrow{AC}|$

$= \frac{1}{2} |(-2, 3, 0) \times (-2, 0, 4)|$

$= \frac{1}{2} |(12, 8, 6)|$

$= \frac{1}{2} \sqrt{144 + 64 + 36}$

$= \sqrt{61}$

Note : Area of $\Delta ABC = \frac{1}{2}\sqrt{a^2b^2 + b^2c^2 + c^2a^2} = \frac{1}{2}\sqrt{36 + 144 + 64} = \sqrt{61}$ **Ans. : (B)**

(72) The equation of the plane passing through $P(3, 6, 2)$ and perpendicular to $\overleftrightarrow{OP}$ is .....

(A) $3x+6y+2z=49$ (B) $3x+6y+2z=7$

(C) $2x+6y+3z=49$ (D) $2x+6y+3z=7$

Sol. : Normal to the plane (perpendicular to $\overleftrightarrow{OP}$) is $\vec{n}=\overrightarrow{OP}=(3, 6, 2)$ and the plane passes through $\vec{a}=(3, 6, 2)$.

$\therefore\ 3(x-3)+6(y-6)+2(z-2)=0$

$\therefore\ 3x+6y+2z=49$ **Ans. : (A)**

(73) If the line passing through $(5, 1, a)$ and $(3, b, 1)$ intersects YZ-plane at $\left(0, \frac{17}{2}, \frac{-13}{2}\right)$, then ..... (AIEEE:2008)

(A) $a=8, b=2$ (B) $a=2, b=8$ (C) $a=4, b=6$ (D) $a=6, b=4$

Sol. : The equation of the line passing through $(5, 1, a)$ and $(3, b, 1)$ is

$\vec{r}=(5, 1, a)+k(-2,\ b-1,\ 1-a),\ k\in R$

Let the line intersect the YZ-plane at point $(5-2k,\ 1+k(b-1),\ a+k(1-a))$ for some $k \in$ R

$\therefore\ \left(0, \frac{17}{2}, \frac{-13}{2}\right) = (5-2k,\ 1+k(b-1),\ a+k(1-a))$

$\therefore\ 0=5-2k$. So, $k=\frac{5}{2}$.

$\frac{17}{2}=1+k(b-1)$

$\therefore\ \frac{17}{2}=1+\frac{5}{2}(b-1)$

$\therefore\ b=4$

(If b = 4, a has to be 6 looking at options.)

$\frac{-13}{2}=a+k(1-a)$

$\therefore\ \frac{-18}{2}=\frac{-3a}{2}$

$\therefore\ 6=a$ **Ans. : (D)**

(74) If the line $\frac{x-2}{3}=\frac{y-1}{-5}=\frac{z+2}{2}$ lies in the plane $x+3y-\alpha z+\beta=0$, then $(\alpha,\ \beta)$ = ... (AIEEE : 2009)

(A) $(6, -17)$ (B) $(-6, 7)$ (C) $(5, -15)$ (D) $(-5, 5)$

Sol. : For the line, $\vec{a}=(2, 1, -2)$, $\vec{l}=(3, -5, 2)$ and $\vec{n}=(1, 3, -\alpha)$

$\vec{l}\perp\vec{n}$. So, $\vec{l}\cdot\vec{n}=0$

$\therefore\ 3-15-2\alpha=0$. So, $\alpha=-6$ (So, answer is (B))

and point $(2, 1, -2)$ lies in the plane $x+3y-\alpha z+\beta=0$.

$\therefore\ 2+3+6(-2)+\beta=0$

$\therefore\ \beta=7$

$\therefore\ (\alpha,\ \beta)=(-6, 7)$. **Ans. : (B)**

Note : The answer can be obtained by finding the value of $\alpha=-6$ alone.

(75) $P(3, 2, 6)$ is any point in space. Point Q lies on the line $\vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \mu(-3\hat{i} + \hat{j} + 5\hat{k})$ for some μ_1. If vector $\overrightarrow{PQ}$ is parallel to the plane $x - 4y + 3z = 1$, then μ_1 = ...

(A) $\frac{1}{4}$ (B) $-\frac{1}{4}$ (C) $\frac{1}{8}$ (D) $-\frac{1}{8}$ (IIT:2009)

Sol. : $Q(1-3\mu, -1+\mu, 2+5\mu)$ lies on $\vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \mu(-3\hat{i} + \hat{j} + 5\hat{k})$ for some $\mu \in R$.

$\overrightarrow{PQ} = (1-3\mu-3, -1+\mu-2, 2+5\mu-6)$ is parallel to the plane $x - 4y + 3z = 1$.

$\therefore \overrightarrow{PQ} \perp \vec{n}$, where $\vec{n} = (1, -4, 3)$

$\therefore \overrightarrow{PQ} \cdot \vec{n} = 0$

$\therefore (-3\mu - 2)(1) + (\mu - 3)(-4) + (5\mu - 4)(3) = 0$

$\therefore -3\mu - 2 - 4\mu + 12 + 15\mu - 12 = 0$

$\therefore 8\mu = 2$

$\therefore \mu = \frac{1}{4}$. So $\mu_1 = \frac{1}{4}$ **Ans. : (A)**

(76) The equation of the plane parallel to X-axis and having Y-intercept 2 and Z-intercept 3 is .....

(A) $3y + 2z = 1$ (B) $3y + 2z = 6$ (C) $2y + 3z = 1$ (D) $2y + 3z = 6$

Sol. : The equation of the plane parallel to X-axis is $by + cz + 1 = 0$.

The plane has Y-intercept 2 and Z-intercept 3.

$\therefore 2b + 1 = 0$ and $3c + 1 = 0$

$\therefore b = -\frac{1}{2}$ and $c = -\frac{1}{3}$

The equation of the plane : $-\frac{y}{2} - \frac{z}{3} + 1 = 0$

$\therefore 3y + 2z = 6$

Second method :

The equation of plane parallel fo X-axis is $\frac{y}{b} + \frac{z}{c} = 1$.

Now, Y-intercept = 2 and Z-intercept = 3.

$\therefore$ The equation of the plane is $\frac{y}{2} + \frac{z}{3} = 1$.

$\therefore 3y + 2z = 6$ **Ans. : (B)**

(77) The distance of point $P(1, -2, 1)$ from the plane $x + 2y - 2z = \alpha$, $\alpha > 0$ is 5. The foot of perpendicular from P to the plane is .....

(A) $\left(\frac{8}{3}, \frac{4}{3}, \frac{-7}{3}\right)$ (B) $\left(\frac{4}{3}, \frac{-4}{3}, \frac{1}{3}\right)$ (C) $\left(\frac{1}{3}, \frac{2}{3}, \frac{10}{3}\right)$ (D) $\left(\frac{2}{3}, \frac{-1}{3}, \frac{5}{3}\right)$ (IIT : 2010)

Sol. : The perpendicular distance of point $(1, -2, 1)$ from the plane $x + 2y - 2z - \alpha = 0$.

$$\frac{|1(1) - 2(2) + 1(-2) - \alpha|}{\sqrt{1+4+4}} = 5$$

$\therefore |-5 - \alpha| = 15$

$\therefore 5 + \alpha = 15$ or $5 + \alpha = -15$

$\therefore \alpha = 10$ or $\alpha = -20$

Now, as $\alpha > 0$, $\alpha = 10$.

If M is the foot of the perpendicular from P to the plane, then

The equation of $\overleftrightarrow{PM}$ is $\vec{r} = (1, -2, 1) + k\,(1, 2, -2),\ k \in R$

$\therefore$ Coordinates of M are $(1+k,\ -2+2k,\ 1-2k)$ for some $k \in R$ and M lies on the plane $x+2y-2z-10=0$.

$\therefore\ 1+k-4+4k-2+4k-10 = 0$

$\therefore\ 9k = 15$

$\therefore\ k = \frac{5}{3}$

$\therefore$ The foot of perpendicular is $M\left(\frac{8}{3}, \frac{4}{3}, \frac{-7}{3}\right)$ **Ans. : (A)**

(78) If the planes $3x-6y-2z=7$ and $2x+y-kz=5$ are mutually perpendicular, then k = ...

(A) 0 (B) 1 (C) 2 (D) 3

Sol. : $\vec{n_1} \perp \vec{n_2}$. So, $\vec{n_1} \bullet \vec{n_2} = 0$

$\therefore\ (3, -6, -2) \bullet (2, 1, -k) = 0$

$\therefore\ 6-6+2k=0$. So, $k=0$ **Ans. : (A)**

(79) The direction cosines of the line passing through $P(2, -1, 2)$ are positive. The line makes angles of equal measure with the coordinate axes. The line intersects the plane $2x+y+z=9$ at Q. Length of $\overline{PQ}$ is .....

(A) 1 (B) $\sqrt{2}$ (C) $\sqrt{3}$ (D) 2 (IIT-2009)

Sol. : Let direction cosines of line be l, m, n, and as line makes angles of equal measure with the coordinate axes, $l = m = n$.

$l^2+m^2+n^2=1$

$\therefore\ 3l^2=1$

$\therefore\ l = \pm\frac{1}{\sqrt{3}}$

As direction cosines are positive, direction cosines of the line are $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$.

$\therefore$ The equation of the line pssing through P is $\vec{r} = (2, -1, 2) + k\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right),\ k \in R$

$\therefore$ The point $Q\left(2+\frac{k}{\sqrt{3}},\ -1+\frac{k}{\sqrt{3}},\ 2+\frac{k}{\sqrt{3}}\right)$ of line is at a distance k from point P.

It lies on plane $2x+y+z=9$.

$\therefore\ 2\left(2+\frac{k}{\sqrt{3}}\right)+\left(-1+\frac{k}{\sqrt{3}}\right)+\left(2+\frac{k}{\sqrt{3}}\right) = 9$

$\therefore\ \frac{4k}{\sqrt{3}} = 4$

$\therefore\ k = \sqrt{3}$

$\therefore\ PQ = \sqrt{3}$ **Ans. : (C)**

(80) The equation of the plane passing through the line of intersection of the planes $x + y + z - 1 = 0$ and $2x + 3y - z + 4 = 0$ and which is parallel to X-axis is .....

(A) $y - 3z + 6 = 0$ (B) $3y - z + 6 = 0$ (C) $y + 3z + 6 = 0$ (D) $3y - 2z + 6 = 0$

Sol. : The equation of the plane passing through the line of intersection of the planes $x + y + z - 1 = 0$ and $2x + 3y - z + 4 = 0$ is $x + y + z - 1 + \lambda (2x + 3y - z + 4) = 0$.

$\therefore (1 + 2\lambda)x + (1 + 3\lambda)y + (1 - \lambda)z + (-1 + 4\lambda) = 0$

The plane is parallel to X-axis.

$\therefore 1 + 2\lambda = 0$

$\therefore \lambda = -\frac{1}{2}$

$\therefore$ The equation of the plane is $-y + 3z - 6 = 0$.

i.e. $y - 3z + 6 = 0$ is the equation of the required plane. **Ans. : (A)**

(81) The equation of the plane passing through (1, 2, –3) and perpendicular to the line joining (–1, 3, 4) and (5, 2, –1) is .....

(A) $6x - y - 5z - 19 = 0$ (B) $6x + y - 5z - 23 = 0$

(C) $6x + y + 5z + 7 = 0$ (D) $6x - y + 5z + 19 = 0$

Sol. : Normal to the plane perpendicular to the line joining A(–1, 3, 4) and B(5, 2, –1) is $\vec{n} = \overrightarrow{\mathbf{AB}} = (6, -1, -5)$.

The plane passes thorugh (1, 2, –3).

$\therefore$ The equation of the plane is $6(x - 1) - 1(y - 2) - 5(z + 3) = 0$.

$\therefore 6x - y - 5z - 19 = 0$ is the equation of the required plane. **Ans. : (A)**

(82) If the distance of the plane $x - y - z + a = 0$ from the origin is twice the distance of the point (–1, 1, 1) from the origin, then a = ...

(A) 3 or 9 (B) 3 or –9 (C) –3 or 9 (D) –3 or –9

Sol. : The distance of point (–1, 1, 1) from the origin is $\sqrt{3}$.

The distance of (–1, 1, 1) from $x - y - z + a = 0$ is $\frac{|-1-1-1+a|}{\sqrt{3}} = 2\sqrt{3}$

$\therefore a - 3 = \pm 6$

$\therefore a = 9$ or $a = -3$ **Ans. : (C)**

(83) Plane XOY, divides the line-segment joining A(2, –3, 2) and B(6, 7, –5) in the ratio ..... .

(A) –1 : 3 (B) –5 : 2 (C) 2 : 5 (D) 3 : 7

Sol. : If the plane XOY divides the given line-segment from A in the ratio $\lambda : 1$, then the z-coordinate of the dividing point is 0.

$\therefore \frac{-5\lambda + 2}{\lambda + 1} = 0$

$\therefore \lambda = \frac{2}{5}$

$\therefore$ Ratio = 2 : 5 **Ans. : (C)**

(84) Let $\overline{PA}$ and $\overline{PB}$ be the perpendiculars drawn from point P(1, 4, 5) to the planes XY and YZ respectively. The equation of the plane OAB is ..... .

(A) $20x + 5y + 4z = 0$ (B) $x + 4y + 5z = 0$

(C) $20x + 5y - 4z = 0$ (D) $20x - 5y + 4z = 0$

Sol. : The foot of perpendicular from P(1, 4, 5) to XY-plane is A(1, 4, 0) and to YZ-plane it is B(0, 4, 5). Suppose the equation of the plane OAB is $ax + by + cz = 0$.

It passes through A and B.

$\therefore \quad a + 4b = 0$ and ...(1)

$4b + 5c = 0$ ...(2)

Solving equations (1) and (2),

$a = -4b = 5c$

$\therefore \quad \frac{a}{20} = \frac{b}{-5} = \frac{c}{4} = k$ (Say) $(k \neq 0)$

$\therefore$ The equation of the plane is $20kx - 5ky + 4kz = 0$

$\therefore \quad 20x - 5y + 4z = 0$ **Ans. : (D)**

(85) A variable plane passes through a fixed point (a, b, c) and intersects the axes at P, Q, R. The locus of the point of intersection of planes parallel to the coordinate planes and passing through P, Q, R is .....

(A) $ax + by + cz = 1$ (B) $ax + by + cz = -1$ (C) $\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 1$ (D) $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

Sol. : Suppose the variable plane intersects the axes at P(p, o, o), Q(o, q, o) and R(o, o, r).

$\therefore$ The equation of the plane is $\frac{x}{p} + \frac{y}{q} + \frac{z}{r} = 1$ and it passes through (a, b, c).

$\therefore \quad \frac{a}{p} + \frac{b}{q} + \frac{c}{r} = 1$ ...(1)

The equation of the planes, parallel to coordinate planes and passing through P, Q and R are respectively $x = p$, $y = q$, $z = r$.

$\therefore$ Eliminating p, q, r from (1), the locus of point of intersection of planes is $\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 1$.

Ans. : (C)

(86) The perpendicular distance of point (8, 5, 7) from the plane passing through (0, –1, 3), (1, 2, 2) and (3, 1, 1) is .....

(A) $\sqrt{76}$ (B) $\sqrt{71}$ (C) $\sqrt{66}$ (D) $\sqrt{73}$

Sol. : The equation of the plane passing through given points is

$$\begin{vmatrix} x-0 & y+1 & z-3 \\ 1-0 & 2+1 & 2-3 \\ 3-0 & 1+1 & 1-3 \end{vmatrix} = 0$$

$$\therefore \begin{vmatrix} x & y+1 & z-3 \\ 1 & 3 & -1 \\ 3 & 2 & -2 \end{vmatrix} = 0$$

$\therefore\ x(-4) - (y+1)(1) + (z-3)(-7) = 0$

$\therefore\ -4x - y - 1 - 7z + 21 = 0$

$\therefore\ 4x + y + 7z = 20$

Now, perpendicular distance of point (8, 5, 7) from the plane $4x + y + 7z - 20 = 0$

$$= \frac{|4(8)+1(5)+7(7)-20|}{\sqrt{16+1+49}}$$

$$= \frac{66}{\sqrt{66}} = \sqrt{66}$$ **Ans. : (C)**

(87) The equation of the plane containing the line $\frac{x}{2}=\frac{y}{3}=\frac{z}{6}$ and perpendicular to plane containing the lines $\frac{x}{3}=\frac{y}{4}=\frac{z}{2}$ and $\frac{x}{4}=\frac{y}{2}=\frac{z}{3}$ is ..... [IIT : 2010]

(A) $x + 2y - 2z = 0$ (B) $3x + 2y - 2z = 0$ (C) $x - 2y + z = 0$ (D) $5x + 2y - 4z = 0$

Sol. : Normal to the plane containing the lines $\frac{x}{3}=\frac{y}{4}=\frac{z}{2}$ and $\frac{x}{4}=\frac{y}{2}=\frac{z}{3}$ is

$\vec{a} = \vec{l_1} \times \vec{l_2} = (3, 4, 2) \times (4, 2, 3) = (8, -1, -10)$

Required plane contains the line $\frac{x}{2}=\frac{y}{3}=\frac{z}{4}$.

$\therefore$ Normal to the plane $\vec{n} = \vec{a} \times \vec{l}$

$= (8, -1, -10) \times (2, 3, 4)$

$= (26, -52, 26)$ i.e. $(1, -2, 1)$.

Origin is a point of the plane.

$\therefore$ The equation of the plane is $(x - 0) - 2(y - 0) + (z - 0) = 0$

$\therefore\ x - 2y + z = 0$ **Ans. : (C)**

(88) The measure of the angle between the planes $2x - y + z = 6$ and $x + 2y + 3z = 3$ is .....

(A) $\cos^{-1}\left(\frac{1}{2}\sqrt{\frac{1}{7}}\right)$ (B) $\cos^{-1}\left(\frac{1}{2}\sqrt{\frac{2}{7}}\right)$ (C) $\cos^{-1}\left(\frac{1}{2}\sqrt{\frac{3}{7}}\right)$ (D) $\cos^{-1}\left(\frac{1}{\sqrt{7}}\right)$

Sol. : For planes, $\vec{n_1} = (2, -1, 1)$ and $\vec{n_2} = (1, 2, 3)$

Now, $\cos\theta = \frac{|\vec{n_1} \cdot \vec{n_2}|}{|\vec{n_1}||\vec{n_2}|}$

$$= \frac{|2-2+3|}{\sqrt{4+1+1}\sqrt{1+4+9}} = \frac{3}{2\sqrt{21}} = \frac{1}{2}\sqrt{\frac{3}{7}}$$

$\therefore\ \theta = \cos^{-1}\left(\frac{1}{2}\sqrt{\frac{3}{7}}\right)$ **Ans. : (C)**

(89) The measure of the angle between the line $\frac{x}{2}=\frac{y}{3}=\frac{z}{4}$ and the plane $3x + 2y - 3z = 4$ is .....

(A) $\frac{\pi}{4}$ (B) 0 (C) $cos^{-1}\left(\frac{24}{\sqrt{29}\sqrt{22}}\right)$ (D) $\frac{\pi}{2}$

Sol. : $\vec{l} = (2, 3, 4)$, $\vec{n} = (3, 2, -3)$

If the measure of the angle between the line and the plane is θ, then

$$sin\theta = \frac{|\vec{l}\cdot\vec{n}|}{|\vec{l}||\vec{n}|} = \frac{|2(3)+3(2)+4(-3)|}{\sqrt{29}\cdot\sqrt{22}} = 0$$

$\therefore \theta = 0$ (line is parallel) (Why the line is not in the plane ?) **Ans. : (B)**

(90) If the measure of the angle between the line $x = \frac{y-1}{2} = \frac{z-3}{\lambda}$ and the plane $x + 2y + 3z = 4$ is $cos^{-1}\sqrt{\frac{5}{14}}$, then λ = ... [AIEEE : 2011]

(A) $\frac{2}{3}$ (B) $\frac{3}{2}$ (C) $\frac{2}{5}$ (D) $\frac{5}{3}$

Sol. : $\vec{l} = (1, 2, \lambda)$, $\vec{n} = (1, 2, 3)$

If the measure of the angle between the line and the plane is θ, then

$$sin\theta = \frac{|\vec{l}\cdot\vec{n}|}{|\vec{l}||\vec{n}|} = \frac{|1+4+3\lambda|}{\sqrt{5+\lambda^2}\cdot\sqrt{14}}$$

$$\therefore cos\theta = \sqrt{1-\frac{(5+3\lambda)^2}{(5+\lambda^2)14}}$$

$$\therefore \theta = cos^{-1}\sqrt{\frac{14(5+\lambda^2)-(5+3\lambda)^2}{14(5+\lambda^2)}}$$

$$\therefore cos^{-1}\sqrt{\frac{5}{14}} = cos^{-1}\sqrt{\frac{70+14\lambda^2-25-30\lambda-9\lambda^2}{14(5+\lambda^2)}}$$

$\therefore 5(5 + \lambda^2) = 5\lambda^2 - 30\lambda + 45$

$\therefore 25 = -30\lambda + 45$

$\therefore 30\lambda = 20$

$\lambda = \frac{2}{3}$ **Ans. : (A)**

(91) The distance of point (1, –5, 9) from $x - y + z = 5$ parallel to line $x = y = z$ is ..... [AIEEE : 2011]

(A) $10\sqrt{3}$ (B) $5\sqrt{3}$ (C) $3\sqrt{10}$ (D) $3\sqrt{5}$

Sol. : The equation of the line parallel to $x = y = z$ and passing through A(1, –5, 9) is

$\vec{r} = (1, -5, 9) + k\,(1, 1, 1),\ k \in R$

Point B$(1 + k, -5 + k, 9 + k)$ of this line lies on the plane $x - y + z = 5$ for some $k \in R$.

$\therefore\ 1 + k + 5 - k + 9 + k = 5$

$\therefore\ k = -10.$ So, B = (–9, –15, –1)

$\therefore$ The distance of point B(–9 –15, –1) of the plane from the point A(1, –5, 9) is

$= \sqrt{100 + 100 + 100}$

$= 10\sqrt{3}$ **Ans. : (A)**

(92) The equation of the plane parallel to $x - 2y + 2z - 5 = 0$ and which is at a unit distance from the origin is ..... [AIEEE : 2012]

(A) $x - 2y + 2z - 3 = 0$ (B) $x - 2y + 2z + 1 = 0$

(C) $x - 2y + 2z - 1 = 0$ (D) $x - 2y + 2z + 5 = 0$

Sol. : The equation of the plane parallel to $x - 2y + 2z - 5 = 0$ is $x - 2y + 2z + k = 0$.

Its distance from the origin is 1.

$\therefore\ \frac{|k|}{\sqrt{1+4+4}} = 1$

$\therefore\ k = \pm 3$

Option $k = -3$ can be selected from the given alternatives.

$\therefore$ The equation of the plane is $x - 2y + 2z - 3 = 0$. **Ans. : (A)**

(93) The point of intersection of the line passing through (2, –3, 1) and (3, –4, –5) and the plane $2x + y + z = 7$ is .....

(A) (1, 2, 7) (B) (1, –2, 7) (C) (–1, 2, 7) (D) (1, –2, –7)

Sol. : The equation of the line passing through the points (2, –3, 1) and (3, –4, –5) is $\vec{r} = (2, -3, 1) + k\,(1, -1, -6),\ k \in R$ and point $(2 + k, -3 - k, 1 - 6k)$ of the line lies on the plane $2x + y + z = 7$ for some $k \in R$. So, $2(2 + k) + (-3 - k) + (1 - 6k) = 7$.

$\therefore\ 4 + 2k - 3 - k + 1 - 6k = 7$

$\therefore\ 5k = -5$

$\therefore\ k = -1$

$\therefore$ The point of intersection is $(2 - 1, -3 + 1, 1 + 6) = (1, -2, 7)$. **Ans. : (B)**

(94) The equation of the plane passing through (3, –2, 1) and parallel to lines $\frac{x}{4} = \frac{y-1}{-3} = \frac{z+2}{-2}$ and $\frac{x-2}{-2} = \frac{y}{-2} = \frac{z-1}{3}$ is .....

(A) $13x - 8y + 14z - 37 = 0$ (B) $13x + 8y + 14z + 37 = 0$

(C) $13x - 8y - 14z - 41 = 0$ (D) $13x + 8y + 14z - 37 = 0$

Sol. : The equation of the plane passing through (3, –2, 1) is

$a\,(x-3) + b\,(y+2) + c\,(z-1) = 0$

The plane is parallel to the lines $\frac{x}{4} = \frac{y-1}{-3} = \frac{z+2}{-2}$ and $\frac{x-2}{-2} = \frac{y}{-2} = \frac{z-1}{3}$

$\therefore$ Normal to the plane is $\vec{n} = \vec{l_1} \times \vec{l_2}$ = (4, –3, –2) × (–2, –2, 3) = (–13, –8, –14)

$\therefore$ The equation of the plane is $13\,(x-3) + 8\,(y+2) + 14\,(z-1) = 0$

$\therefore$ $13x + 8y + 14z = 37$ **Ans. : (D)**

(95) The equation of the plane passing through the line of intersection of planes $x + 2y + 3z = 2$ and $x - y + z = 3$ and which is at a distance $\frac{2}{\sqrt{3}}$ from the point (3, 1, –1) is ..... [IIT : 2012]

(A) $5x - 11y + z = 17$ (B) $\sqrt{2}x + y = 3\sqrt{2} - 1$

(C) $x + y + z = \sqrt{3}$ (D) $x - \sqrt{2}y = 1 - \sqrt{2}$

Sol. : The equation of the plane passing through line of intersection of planes $x + 2y + 3z - 2 = 0$ and $x - y + z - 3 = 0$ is $x + 2y + 3z - 2 + \lambda\,(x - y + z - 3) = 0$,

$\therefore$ $(1+\lambda)\,x + (2-\lambda)\,y + (3+\lambda)\,z + (-2-3\lambda) = 0$

The distance of the plane from the point (3, 1, –1) is $\frac{2}{\sqrt{3}}$.

$\therefore$ $\frac{|3 + 3\lambda + 2 - \lambda - 3 - \lambda - 2 - 3\lambda|}{\sqrt{(1+\lambda)^2 + (2-\lambda)^2 + (3+\lambda)^2}} = \frac{2}{\sqrt{3}}$

$\therefore$ $3(4\lambda^2) = 4(3\lambda^2 + 4\lambda + 14)$

$\therefore$ $4\lambda + 14 = 0$

$\therefore$ $\lambda = -\frac{7}{2}$

$\therefore$ The equation of the plane is $\left(1 - \frac{7}{2}\right)x + \left(2 + \frac{7}{2}\right)y + \left(3 - \frac{7}{2}\right)z - \left(2 + 3\left(-\frac{7}{2}\right)\right) = 0$

$\therefore$ $5x - 11y + z = 17$ **Ans. : (A)**

(96) Point P is the point of intersection of the plane $5x - 4y - z = 1$ and the line joining Q(2, 3, 5) and R(1, –1, 4). If S is the foot of the perpendicular from T(2, 1, 4) to $\overleftrightarrow{QR}$, then the length of $\overline{PS}$ is ..... [IIT : 2012]

(A) $\frac{1}{\sqrt{2}}$ (B) $\sqrt{2}$ (C) 2 (D) $2\sqrt{2}$

Sol. : The equation of $\overleftrightarrow{QR}$ is $\vec{r}$ = (2, 3, 5) + k(1, 4, 1), $k \in$ R

If the point $(2 + k, 3 + 4k, 5 + k)$ of the line is the point of intersection P of the line and the plane $5x - 4y - z = 1$, then

$5(2 + k) - 4\,(3 + 4k) - (5 + k) = 1$

$\therefore$ $10 + 5k - 12 - 16k - 5 - k = 1$

$\therefore$ $12k = -8$. So, $k = -\frac{2}{3}$

$\therefore$ The point of intersection is $P\left(2 - \frac{2}{3}, 3 - \frac{8}{3}, 5 - \frac{2}{3}\right) = P\left(\frac{4}{3}, \frac{1}{3}, \frac{13}{3}\right)$

Suppose foot of perpendicular from T to $\overleftrightarrow{\mathbf{QR}}$ is S$(2 + k, 3 + 4k, 5 + k)$ for some k.

Now, T = (2, 1, 4)

$\overrightarrow{\mathbf{TS}} \perp \vec{l}$

$\therefore\ \overrightarrow{\mathbf{TS}} \cdot \vec{l} = 0$

$\therefore\ (k, 2 + 4k, 1 + k) \cdot (1, 4, 1) = 0$

$\therefore\ k + 8 + 16k + 1 + k = 0$

$\therefore\ 18k = -9.$ So, $k = -\frac{1}{2}$

$\therefore\ \text{S}\left(2-\frac{1}{2}, 3-\frac{4}{2}, 5-\frac{1}{2}\right) = \left(\frac{3}{2}, \frac{2}{2}, \frac{9}{2}\right)$

The distance PS $= \sqrt{\left(\frac{4}{3}-\frac{3}{2}\right)^2 + \left(\frac{1}{3}-\frac{2}{2}\right)^2 + \left(\frac{13}{3}-\frac{9}{2}\right)^2}$

$= \frac{1}{6}\sqrt{1+16+1} = \frac{1}{\sqrt{2}}$ **Ans. : (A)**

(97) The line passing through (1, 2, 3) and perpendicular to the plane $x + 2y - 5z + 9 = 0$ is .....

(A) $\frac{x+1}{1} = \frac{y+2}{2} = \frac{z-5}{3}$ (B) $\frac{x+1}{1} = \frac{y+2}{2} = \frac{z+3}{-5}$

(C) $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z+5}{3}$ (D) $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{-5}$

Sol. : The normal to the plane $\vec{n}$ = (1, 2, –5). is the direction vector of line.

$\therefore$ The equation of the line is $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{-5}$ (Check B also.) **Ans. : (D)**

(98) The equation of the plane passing through $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-3}{4}$ and perpendicular to the plane $x + 2y + z = 12$ is .....

(A) $2x - 9y + 5z + 4 = 0$ (B) $3x - 6y + 5z + 4 = 0$

(C) $5x - 2y + 9z + 4 = 0$ (D) $9x - 2y - 5z + 4 = 0$

Sol. : Plane passes through line $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-3}{4}$.

The plane passes through point (1, –1, 3) and its normal vector is orthogonal to $\vec{l} = 2\hat{i} - \hat{j} + 4\hat{k}$.

Also, the required plane is perpendicular to plane $x + 2y + z = 12$ i.e. the normal of required plane is orthogonal to $\vec{n_1}$ = (1, 2, 1).

$\therefore$ The normal of the required plane is $\vec{n} = \vec{n_1} \times \vec{l}$ = (1, 2, 1) × (2, –1, 4) = (9, –2, –5)

$\therefore$ The equation of the plane is $9(x - 1) - 2(y + 1) - 5(z - 3) = 0$

$\therefore\ 9x - 2y - 5z + 4 = 0$ **Ans. : (D)**

(99) The equation of the plane passing through the line of intersection of the planes $3x + 3y - 4z + 7 = 0$ and $2x - 5y + z + 2 = 0$ and which is parallel to line $x = 0, z = 0$ is .....

(A) $21x - 8y + 23 = 0$ (B) $23y + 20z + 23 = 0$

(C) $21x - 17z + 41 = 0$ (D) $20x - 21z + 41 = 0$

Sol. : The equation of the plane passing through the line of intersection of planes $3x + 3y - 4z + 7 = 0$ and $2x - 5y + z + 2 = 0$ is $3x + 3y - 4z + 7 + \lambda (2x - 5y + z + 2) = 0$.

$\therefore$ $(3 + 2\lambda)x + (3 - 5\lambda)y + (-4 + \lambda)z + (7 + 2\lambda) = 0$ is the equation of the required plane.

The plane is parallel to line $x = 0, z = 0$ i.e. $(0, 1, 0)$ is perpendicular to normal vector of plane.

$\therefore$ $3 - 5\lambda = 0$

$\therefore$ $\lambda = \frac{3}{5}$

$\therefore$ The equation of the plane is $21x - 17z + 41 = 0$ **Ans. : (C)**

(100) The equation of the plane passing through the line of intersection of the planes $x + y + z - 6 = 0$ and $2x + 3y + 4z + 5 = 0$ and which is orthogonal to plane $4x + 5y - 3z = 8$ is .....

(A) $x + 7y + 13z - 96 = 0$ (B) $x + 7y + 13z + 96 = 0$

(C) $x + 7y - 13z - 96 = 0$ (D) $x - 7y + 13z + 96 = 0$

Sol. : The equation of the plane passing through the line of intersection of given planes is

$x + y + z - 6 + \lambda (2x + 3y + 4z + 5) = 0$

$\therefore$ $(1 + 2\lambda)x + (1 + 3\lambda)y + (1 + 4\lambda)z + (-6 + 5\lambda) = 0$

It is orthogonal to plane $4x + 5y - 3z = 8$.

$\therefore$ $4(1 + 2\lambda) + 5(1 + 3\lambda) - 3(1 + 4\lambda) = 0$

$\therefore$ $11\lambda = -6$

$\therefore$ $\lambda = -\frac{6}{11}$

$\therefore$ The equation of the plane is $x + 7y + 13z + 96 = 0$ **Ans. : (B)**

(101) The equation of the plane passing through $\frac{x}{3} = \frac{y-1}{2} = \frac{z+1}{-1}$ and $\frac{x+1}{3} = \frac{y}{2} = \frac{z-2}{-1}$ is .....

(A) $5x - 8y + z + 9 = 0$ (B) $5x - 8y + z + 3 = 0$

(C) $5x - 8y - z + 7 = 0$ (D) $5x + 8y - z - 9 = 0$

Sol. : Given lines are parallel to each other. Plane passes through points $(0, 1, -1)$ and $(-1, 0, 2)$ and $\vec{l} = (3, 2, -1)$

$\therefore$ The equation of the plane is $\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ l_1 & l_2 & l_3 \end{vmatrix} = 0$

$\therefore$ $\begin{vmatrix} x & y-1 & z+1 \\ -1 & -1 & 3 \\ 3 & 2 & -1 \end{vmatrix} = 0$

$\therefore$ $-5x + 8(y - 1) + 1 (z + 1) = 0$

$\therefore$ $5x - 8y - z + 7 = 0$ (Are the lines coincident ?) **Ans. : (C)**

: Questions Asked in JEE Main and Advanced :

(102) If the lines $L_1 : x = 5, \frac{y}{3-\alpha} = \frac{z}{-2}$ and $L_2 : x = \alpha, \frac{y}{-1} = \frac{z}{2-\alpha}$ are coplanar, then value(s) of α is (are) ..... [JEE : 2013]

(A) 1 (B) 2 (C) 3 (D) 4

Sol. : For lines $\vec{a} = (5, 0, 0)$ $\vec{l} = (0, 3 - \alpha, -2)$ and $\vec{b} = (\alpha, 0, 0)$ $\vec{m} = (0, -1, 2 - \alpha)$.

Lines are coplanar.

$\therefore (\bar{b} - \bar{a}) \cdot (\bar{l} \times \bar{m}) = 0$

$$\therefore \begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \end{vmatrix} = 0$$

$$\therefore \begin{vmatrix} \alpha - 5 & 0 & 0 \\ 0 & 3 - \alpha & -2 \\ 0 & -1 & 2 - \alpha \end{vmatrix} = 0$$

$\therefore (\alpha - 5)[(3 - \alpha)(2 - \alpha) - 2] = 0$

$\therefore (\alpha - 5)(\alpha^2 - 5\alpha + 4) = 0$

$\therefore \alpha = 5, \alpha = 4, \alpha = 1$ **Ans. : (A), (D)**

(103) Two lines $L_1 : \frac{x-1}{2} = \frac{y}{-1} = \frac{z+3}{1}$, $L_2 : \frac{x-4}{1} = \frac{y+3}{1} = \frac{z+3}{2}$

and two planes $\pi_1 : 7x + y + 2z = 3$ and $\pi_2 : 3x + 5y - 6z = 4$ are given.

Plane $ax + by + cz = d$ is passing through the point of intersection of L_1 and L_2 and it is perpendicular to planes π_1 and π_2.

Match list-I with list-II and select the correct answer using the code given below the lists :

	List-I	**List-II**
P	$a =$	1. 13
Q	$b =$	2. –3
R	$c =$	3. 1
S	$d =$	4. –2

Codes :	P	Q	R	S
(A)	3	2	4	1
(B)	1	3	4	2
(C)	3	2	1	4
(D)	2	4	1	3

[JEE : 2013]

Sol. : M $(2k_1 + 1, -k_1, k_1 - 3)$ is any point on line L_1 and

N $(k_2 + 4, k_2 - 3, 2k_2 - 3)$ is any point on line L_2. $(k_1 \in R, k_2 \in R)$

For the point of intersection of L_1 and L_2, we have

$2k_1 + 1 = k_2 + 4, -k_1 = k_2 - 3; k_1 - 3 = 2k_2 - 3$ for some k_1 and k_2,

$\therefore\ 2k_1 - k_2 = 3$ and $k_1 + k_2 = 3$

$\therefore\ k_1 = 2;\ k_2 = 1$

Also by taking, $k_1 = 2$ and $k_2 = 1$, $k_1 - 3 = 2k_2 - 3$.

$\therefore$ The point of intersection M = N = (5, –2, –1)

Plane $ax + by + cz = d$ is perpendicular to π_1 and π_2.

$(a, b, c) = (7, 1, 2) \times (3, 5, -6) = (-16, 48, 32)$

$\therefore$ We can take $(a, b, c) = (1, -3, -2)$.

$(x - 5) - 3(y + 2) - 2(z + 1) = 0$

$\therefore\ x - 3y - 2z = 13$

$\therefore$ P : $a = 1$, Q : $b = -3$, R : $c = -2$, S : $d = 13$

$\therefore$ P $\longrightarrow$ 3, Q $\longrightarrow$ 2, R $\longrightarrow$ 4, S $\longrightarrow$ 1

$\therefore$ P Q R S is 3 2 4 1 **Ans. : (A)**

(104) If two lines $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{-k}$ and $\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{1}$ are coplanar, then numbers of values of k is ..... [JEE : 2013]

(A) exactly one (B) exactly two (C) exactly three (D) none

Sol. : For lines $\vec{a} = (2, 3, 4)$, $\vec{l} = (1, 1, -k)$ and $\vec{b} = (1, 4, 5)$, $\vec{m} = (k, 2, 1)$. Lines are coplanar.

$$\therefore \begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \end{vmatrix} = 0$$

$$\therefore \begin{vmatrix} -1 & 1 & 1 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix} = 0$$

$\therefore\ -1(1 + 2k) - 1(1 + k^2) + 1\ (2 - k) = 0$

$\therefore\ -1 - 2k - 1 - k^2 + 2 - k = 0$

$\therefore\ k^2 + 3k = 0$

$\therefore\ k(k + 3) = 0$

$\therefore\ k = 0$ or -3

$\therefore$ Two values of k are obtained. **Ans. : (B)**

(105) If the lines $\frac{x+1}{2} = \frac{y-1}{1} = \frac{z+1}{3}$ and $\frac{x+2}{2} = \frac{y-k}{3} = \frac{z}{4}$ are coplanar, then k = ... [JEE : 2013]

(A) $\frac{11}{2}$ (B) $-\frac{11}{2}$ (C) $\frac{9}{2}$ (D) $-\frac{9}{2}$

Sol. : For lines $\vec{a} = (-1, 1, -1)$, $\vec{l} = (2, 1, 3)$ and $\vec{b} = (-2, k, 0)$, $\vec{m} = (2, 3, 4)$.

Lines are coplanar.

$\therefore (\vec{a} - \vec{b}) \cdot (\vec{l} \times \vec{m}) = 0$

$\therefore (1, 1 - k, -1) \cdot (-5, -2, 4) = 0$

$\therefore -5 - 2 + 2k - 4 = 0$

$\therefore k = \frac{11}{2}$ **Ans. : (A)**

(106) The distance of point R(–1, 1, –6) of plane $4x - 3y + z + 13 = 0$ from the foot of the perpendicular drawn from origin to the plane is ..... [JEE 2013]

(A) $\sqrt{14}$ (B) $\sqrt{\frac{19}{2}}$ (C) $3\sqrt{\frac{7}{2}}$ (D) $\frac{3}{\sqrt{2}}$

Sol. : The perpendicular distance from origin to the plane O(0, 0, 0) is OQ,

$$OQ = \frac{|13|}{\sqrt{16 + 9 + 1}} = \frac{13}{\sqrt{26}} = \sqrt{\frac{13}{2}}$$

$$OR = \sqrt{1 + 1 + 36} = \sqrt{38}$$

$$\therefore QR = \sqrt{OR^2 - OQ^2}$$

$$= \sqrt{38 - \frac{13}{2}}$$

$$= \sqrt{\frac{63}{2}}$$

$$= 3\sqrt{\frac{7}{2}}$$

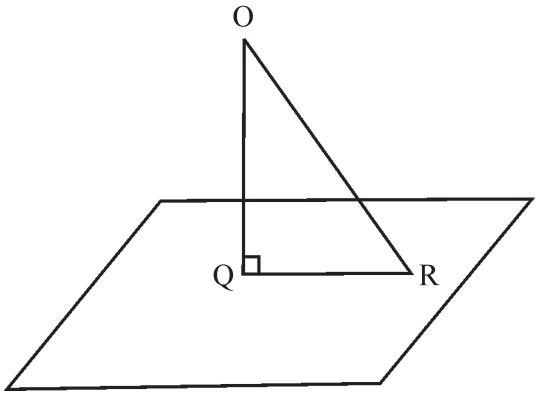


Figure 21.7

Ans. : (C)

(107) If direction cosines l, m and n of two lines satisfy $l + m + n = 0$ and $l^2 + m^2 - n^2 = 0$, then the measure of the angle between the lines is .... [JEE : 2013]

(A) $\frac{\pi}{12}$ (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{4}$

Sol. : Solve the sum as per sum number (34). **Ans. : (C)**

(108) Two lines in space are defined as : $L_1 : \{x = \sqrt{\lambda}\, y + \sqrt{\lambda} - 1, z = (\sqrt{\lambda} - 1)\, y + \sqrt{\lambda}\}$ and $L_2 : \{x = \sqrt{\mu}\, y + (1 - \mu), z = (1 - \sqrt{\mu})\, y + \sqrt{\mu}\}$. If L_1 and L_2 are mutually perpendicular, then .... [JEE : 2013]

(A) $\sqrt{\lambda} + \sqrt{\mu} = 1$ (B) $\lambda \neq \mu$ (C) $\lambda + \mu = 0$ (D) $\lambda = \mu$

Sol. : Line $L_1 : \frac{x - (\sqrt{\lambda} - 1)}{\sqrt{\lambda}} = \frac{y}{1} = \frac{z - \sqrt{\lambda}}{\sqrt{\lambda} - 1}$

and $L_2 : \frac{x - (1 - \mu)}{\sqrt{\mu}} = \frac{y}{1} = \frac{z - \sqrt{\mu}}{1 - \sqrt{\mu}}$

$\therefore \vec{l} = \left(\sqrt{\lambda}, 1, \sqrt{\lambda} - 1\right)$ and $\vec{m} = \left(\sqrt{\mu}, 1, 1 - \sqrt{\mu}\right)$

$L_1 \perp L_2 \Rightarrow \vec{l} \perp \vec{m}$

$\Rightarrow \vec{l} \cdot \vec{m} = 0$

$\Rightarrow \sqrt{\lambda\mu} + 1 + \sqrt{\lambda} - \sqrt{\lambda\mu} - 1 + \sqrt{\mu} = 0$

$\Rightarrow \sqrt{\lambda} + \sqrt{\mu} = 0$ and λ and μ are non-negative

$\Rightarrow \lambda = \mu = 0$ **Ans. : (C), (D)**

(109) The equation of the plane passing through the line of intersection of planes $x + 2y = 3$ and $y - 2z + 1 = 0$ and which is perpendicular to the first plane is .... [JEE : 2013]

(A) $2x - y - 10z = 9$ (B) $2x - y + 7z = 11$ (C) $2x - y + 10z = 11$ (D) $2x - y - 9z = 10$

Sol. : The equation of the plane passing through the line of intersection of given planes is

$x + 2y - 3 + \lambda (y - 2z + 1) = 0$

$\therefore x + (2 + \lambda) y - 2\lambda z - (3 - \lambda) = 0$. $\vec{n_1} = (1, 2 + \lambda, -2\lambda)$ and it is perpendicular the plane $x + 2y = 3$ whose normal is $\vec{n_2} = (1, 2, 0)$.

$\therefore \vec{n_1} \perp \vec{n_2} \Rightarrow \vec{n_1} \cdot \vec{n_2} = 0 \Rightarrow 1 + 4 + 2\lambda = 0 \Rightarrow \lambda = -\frac{5}{2}$

$\therefore$ The equation of the plane is $2x - y + 10z = 11$ **Ans. : (C)**

(110) If direction cosines l, m, n of two lines satisfy $l + m + n = 0$ and $l^2 = m^2 + n^2$, then the measure of the angle between the two lines is ..... . [JEE : 2014]

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{2}$

Sol. : Solve as per solution given in sum number (34). **Ans. : (A)**

(111) The image of the line $\frac{x-1}{3} = \frac{y-3}{1} = \frac{z-4}{-5}$ in the plane $2x - y + z + 3 = 0$ is the line ..... . [JEE : 2014]

(A) $\frac{x+3}{3} = \frac{y-5}{1} = \frac{z-2}{-5}$ (B) $\frac{x+3}{-3} = \frac{y-5}{-1} = \frac{z+2}{5}$

(C) $\frac{x-3}{3} = \frac{y+5}{1} = \frac{z-2}{-5}$ (D) $\frac{x-3}{-3} = \frac{y+5}{-1} = \frac{z-2}{5}$

Sol. : Normal to be plane $2x - y + z + 3 = 0$ is $\vec{n} = (2, -1, 1)$.

$\therefore$ The equation of $\overleftrightarrow{AM}$ through A, perpendicular to plane $2x - y + z + 3 = 0$ is $\frac{x-1}{2} = \frac{y-3}{-1} = \frac{z-4}{1}$.

Suppose B is the image of A in plane, $B \in \overleftrightarrow{AM}$

$\therefore$ For some $k \in R$,

$B(1 + 2k, 3 - k, 4 + k)$

Midpoint of $\overline{AB}$

$M\left(\frac{2+2k}{2}, \frac{6-k}{2}, \frac{8+k}{2}\right)$ is a point of plane.

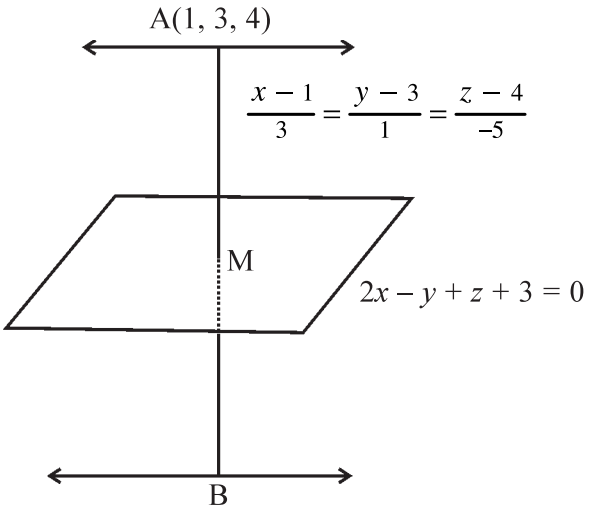


Figure 21.8

$\therefore\ 2\left(\frac{2+2k}{2}\right) - \left(\frac{6-k}{2}\right) + \left(\frac{8+k}{2}\right) + 3 = 0$

$\therefore\ 4 + 4k - 6 + k + 8 + k + 6 = 0$

$\therefore\ 6k = -12$

$\therefore\ k = -2$

$\therefore$ B (–3, 5, 2)

$\therefore$ The image of given line is $\frac{x+3}{3} = \frac{y-5}{1} = \frac{z-2}{-5}$. **Ans. : (A)**

(112) The equation of the plane passing through the point of intersection of the lines $\frac{x-1}{3} = \frac{y-2}{1} = \frac{z-3}{2}$ and $\frac{x-3}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ and which is at the shortest distance from the origin is .... [JEE : 2014]

(A) $7x + 2y + 4z = 54$ (B) $3x + 4y + 5z = 49$

(C) $4x + 3y + 5z = 50$ (D) $5x + 4y + 3z = 57$

Sol. : Suppose point P$(1 + 3k, 2 + k, 3 + 2k)$ of line $\frac{x-1}{3} = \frac{y-2}{1} = \frac{z-3}{2}$ is the point of intersection of given lines.

$\therefore$ P lies on $\frac{x-3}{1} = \frac{y-1}{2} = \frac{z-2}{3}$.

$\therefore\ \frac{1+3k-3}{1} = \frac{2+k-1}{2} = \frac{3+2k-2}{3}$

$\therefore\ 6k - 4 = k + 1$

$\therefore\ 5k = 5$

$\therefore\ k = 1$

(For $k = 1$, $\frac{2+k-1}{2} = \frac{3+2k-2}{3}$ is also true.)

$\therefore$ The distance OP of the point P(4, 3, 5) of intersection of lines is the shortest distance of plane from the origin. Normal to the plane is $\overrightarrow{\mathbf{OP}} = \vec{n} = (4, 3, 5)$

$\therefore$ The equation of the plane which is at the shortest distance from origin is

$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$

$\therefore\ 4(x - 4) + 3(y - 3) + 5(z - 5) = 0$

$\therefore\ 4x + 3y + 5z = 50$ **Ans. : (C)**

(113) If a line of three dimensional space makes an angle of measure θ with X-axis and Y-axis $\left(0 \le \theta \le \frac{\pi}{2}\right)$, then $\theta \in$ ..... .

(A) $\left(\frac{\pi}{3}, \frac{\pi}{2}\right]$ (B) $\left[\frac{\pi}{6}, \frac{\pi}{3}\right]$ (C) $\left(0, \frac{\pi}{4}\right]$ (D) $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$

Sol. : Suppose the line makes an angle of measure α with Z-axis.

$\therefore\ \cos^2\theta + \cos^2\theta + \cos^2\alpha = 1$

$\therefore\ \cos^2\alpha = 1 - 2\cos^2\theta$

$\therefore\ \cos^2\alpha = -\cos 2\theta$

$0 \le cos^2\alpha \le 1 \Rightarrow 0 \le -cos2\theta \le 1$

$\Rightarrow -1 \le cos2\theta \le 0$

$\Rightarrow \frac{\pi}{2} \le 2\theta \le \pi$

$\Rightarrow \frac{\pi}{4} \le \theta \le \frac{\pi}{2}$

$\therefore \theta \in \left[\frac{\pi}{4}, \frac{\pi}{2}\right]$ **Ans. : (D)**

(114) The plane containing the line $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}$ and parallel to $\frac{x}{1} = \frac{y}{1} = \frac{z}{4}$ passes through ..... point. [JEE : 2014]

(A) (1, –2, 5) (B) (1, 0, 5) (C) (0, 3, –5) (D) (–1, –3, 0)

Sol. : The point A(1, 2, 3) of the line lying in the plane is also a point of the plane. The normal to the plane is orthogonal to $\vec{l}$ = (1, 2, 3) and $\vec{m}$ = (1, 1, 4).

$\therefore$ The normal to the plane is $\vec{n} = \vec{l} \times \vec{m}$ = (1, 2, 3) × (1, 1, 4) = (5, –1, –1)

$\therefore$ The equation of the plane is $5(x - 1) - 1(y - 2) - 1(z - 3) = 0$

$\therefore$ $5x - y - z = 0$

It can be checked easily that from the given points, only the point (1, 0, 5) lies on the above plane **Ans. : (B)**

(115) The symmetric form of the line of intersection of the planes $x = ay + b$ and $z = cy + d$ is .... [JEE : 2014]

(A) $\frac{x-b}{a} = \frac{y-1}{1} = \frac{z-d}{c}$ (B) $\frac{x-b-a}{a} = \frac{y-1}{1} = \frac{z-d-c}{c}$

(C) $\frac{x-b}{b} = \frac{y-0}{1} = \frac{z-c}{d}$ (D) $\frac{x-b-a}{b} = \frac{y-1}{0} = \frac{z-d-c}{d}$

Sol. : For the plane $x - ay = b$, normal is $\vec{n_1}$ = (1, –a, 0) and for the plane $z - cy = d$ normal is $\vec{n_2}$ = (0, –c, 1).

$\therefore$ The direction vector of the line of intersection of the planes is $\vec{l} = \vec{n_1} \times \vec{n_2}$

= (1, –a, 0) × (0, –c, 1)

= (–a, –1, –c)

or $\vec{l}$ = (a, 1, c)

To obtain a point on the line of intersection, let $y = 1$. We have $x = a + b$ and $z = c + d$

$\therefore$ Point $(a + b, 1, c + d)$ is on the line of intersection.

$\therefore$ The symmetric form of the line of intersection is $\frac{x-a-b}{a} = \frac{y-1}{1} = \frac{z-c-d}{c}$. **Ans. : (B)**

(116) If the distance between the planes $4x - 2y - 4z + 1 = 0$ and $4x - 2y - 4z + d = 0$ is 7, then d ..... [JEE : 2014]

(A) 41 or –42 (B) 42 or –43 (C) –41 or 43 (D) –42 or 44

Sol. : The distance between two parallel planes is $\frac{|d_1 - d_2|}{|\vec{n}|}$

$\therefore\ 7 = \frac{|d-1|}{\sqrt{16+4+16}}$

$42 = d - 1$ or $-42 = d - 1$

$\therefore\ d = 43$ or $d = -41$ **Ans. : (C)**

(117) If the measure of the angle between the line $2(x + 1) = y = z + 4$ and the plane $2x - y + \sqrt{\lambda}\, z + 4 = 0$ is $\frac{\pi}{6}$, then λ = ... [JEE : 2014]

(A) $\frac{135}{7}$ (B) $\frac{45}{11}$ (C) $\frac{45}{7}$ (D) $\frac{135}{11}$

Sol. : The line $\frac{x+1}{1} = \frac{y}{2} = \frac{z+4}{2}$ has direction vector $\vec{l}$ = (1, 2, 2) and normal to the plane $2x - y + \sqrt{\lambda}\, z + 4 = 0$ is $\vec{n}$ = (2, –1, $\sqrt{\lambda}$).

Let θ be the measure of the angle between the line and the plane. Then $sin\theta = \frac{|\vec{l} \cdot \vec{n}|}{|\vec{l}\,||\vec{n}|}$

$\therefore\ sin\ \frac{\pi}{6} = \frac{|2 - 2 + 2\sqrt{\lambda}|}{\sqrt{1+4+4} \cdot \sqrt{4+1+\lambda}}$

$\therefore\ \frac{1}{2} = \frac{|2\sqrt{\lambda}|}{3\sqrt{5+\lambda}}$

$\therefore\ 45 + 9\lambda = 16\lambda.$ So, $\frac{45}{7} = \lambda$ **Ans. : (C)**

(118) The equation of the line containing the shortest distance between the lines $\frac{x}{1} = \frac{y}{-1} = \frac{z}{1}$ and $\frac{x-1}{0} = \frac{y+1}{-2} = \frac{z}{1}$ is .... [JEE : 2014]

(A) $\frac{x}{1} = \frac{y}{-1} = \frac{z}{-2}$ (B) $\frac{x-1}{1} = \frac{y+1}{-1} = \frac{z}{-2}$ (C) $\frac{x-1}{1} = \frac{y+1}{-1} = \frac{z}{1}$ (D) $\frac{x}{-2} = \frac{y}{1} = \frac{z}{2}$

Sol. : For some $k_1 \in$ R, let the point P(k_1, $-k_1$, k_1) lie on the line $\frac{x}{1} = \frac{y}{-1} = \frac{z}{1}$ and for some $k_2 \in$ R let point Q(1, $-2k_2 - 1$, k_2) lie on the line $\frac{x-1}{0} = \frac{y+1}{-2} = \frac{z}{1}$.

If $\overleftrightarrow{PQ}$ is perpendicular to both the given lines, then PQ becomes the shortest distance.

$\overrightarrow{\mathbf{PQ}} = (-k_1 + 1, -2k_2 + k_1 - 1, k_2 - k_1)$

$\overrightarrow{\mathbf{PQ}} \perp \vec{l}$ and $\overrightarrow{\mathbf{PQ}} \perp \vec{m}$,

where $\vec{l} = (1, -1, 1)$ and $\vec{m} = (0, -2, 1)$

$\overrightarrow{\mathbf{PQ}} = k(\vec{l} \times \vec{m})$

$= k(1, -1, -2)$

$\therefore -k_1 + 1 = 2k_2 - k_1 + 1$ ...(1)

$\therefore 2(-2k_2 + k_1 - 1) = k_2 - k_1$ ...(2)

From (1) $k_2 = 0$

$\therefore 2k_1 - 2 = -k_1$

$\therefore k_1 = \frac{2}{3}$

$\therefore Q(1, -2k_2 - 1, k_2) = (1, -1, 0)$

The direction vector of $\overleftrightarrow{PQ} = \left(1 - \frac{2}{3}, -1 + \frac{2}{3}, 0 - \frac{2}{3}\right)$

$= \left(\frac{1}{3}, -\frac{1}{3}, -\frac{2}{3}\right)$ i.e. $(1, -1, -2)$

$\therefore$ The equation of the line containing the shortest distance is $\frac{x-1}{1} = \frac{y+1}{-1} = \frac{z}{-2}$.

[**Note :** Observe that, $(1, -1, 1) \times (0, -2, 1) = (1, -1, -2)$]

$\therefore$ The only possible answer is (A) or (B). **Ans. : (B)**

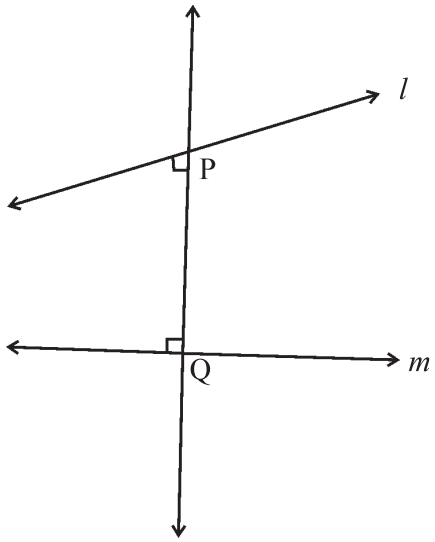


Figure 21.9

(119) Perpendiculars $\overline{PQ}$ and $\overline{PR}$ are drawn from point $P(\lambda, \lambda, \lambda)$ to lines $y = x, z = 1$ and $y = -x, z = -1$. If $\angle QPR$ is a right angle at the point P, then possible value(s) of λ is (are) ....

(A) $\sqrt{2}$ (B) 1 (C) -1 (D) $-\sqrt{2}$

Sol. : For L_1 : $\frac{x}{1} = \frac{y}{1} = \frac{z-1}{0}$, $\vec{l} = (1, 1, 0)$ and let $Q(k_1, k_1, 1)$ be any point on L_1.

For L_2 : $\frac{x}{1} = \frac{y}{-1} = \frac{z+1}{0}$, $\vec{m} = (1, -1, 0)$ and let $R(k_2, -k_2, -1)$ be any point on L_2.

$\overrightarrow{\mathbf{PQ}} = (k_1 - \lambda, k_1 - \lambda, 1 - \lambda)$. Also $\overrightarrow{\mathbf{PQ}} \perp L_1$

$\therefore \overrightarrow{\mathbf{PQ}} \cdot \vec{l} = 0$

$\therefore k_1 - \lambda + k_1 - \lambda = 0.$ So, $k_1 = \lambda$

and $\overrightarrow{\mathbf{PR}} \perp L_2$. So, $\overrightarrow{\mathbf{PR}} \cdot \vec{m} = 0$

$\therefore (k_2 - \lambda, -k_2 - \lambda, -1 - \lambda) \cdot (1, -1, 0) = 0$

$\therefore\ k_2 - \lambda + k_2 + \lambda = 0$

$\therefore\ k_2 = 0$

$\angle$QPR is a right angle.

$\therefore\ \overrightarrow{PQ} \perp \overrightarrow{PR}$. So, $\overrightarrow{PQ} \cdot \overrightarrow{PR} = 0$

$\therefore\ (k_1 - \lambda)(k_2 - \lambda) + (k_1 - \lambda)(-k_2 - \lambda) + (1 - \lambda)(-1 - \lambda) = 0$

$\therefore\ (1 - \lambda)(-1 - \lambda) = 0$ $(k_1 = \lambda)$

$\therefore\ \lambda^2 - 1 = 0$

$\therefore\ \lambda = 1$ or $\lambda = -1$

Taking $\lambda = 1$, P = Q = (1, 1, 1). So, $\lambda = 1$ is not possible.

$\therefore\ \lambda = -1$ **Ans. : (C)**

(120) In R^3, let $\pi_1 : y = 0$ and $\pi_2 : x + z = 1$. There is a plane π_3 other then π_1 and π_2. π_3 passes through the line of intersection of the planes π_1 and π_2. If the distance of the point (0, 1, 0) from π_3 is 1 and distance of the point (α, β, γ) from π_3 is 2, then which of the following holds true ? [JEE : 2015]

(A) $2\alpha + \beta + 2\gamma + 2 = 0$ (B) $2\alpha - \beta + 2\gamma + 4 = 0$

(C) $2\alpha + \beta - 2\gamma - 10 = 0$ (D) $2\alpha - \beta + 2\gamma - 8 = 0$

Sol. : The equation of the plane π_3 passing through the line of intersection of planes π_1 and π_2 is $x + z - 1 + \lambda(y) = 0$.

The distance of (0, 1, 0) from π_3 is $1 = \dfrac{|0 + 0 - 1 + \lambda|}{\sqrt{1 + 1 + \lambda^2}}$

$\therefore\ (\lambda^2 + 2) = \lambda^2 - 2\lambda + 1$

$\therefore\ 2\lambda = -1$

$\therefore\ \lambda = \dfrac{-1}{2}$

The equation of π_3 is $2x - y + 2z - 2 = 0$

The distance of π_3 from (α, β, γ) is $2 = \dfrac{|2\alpha - \beta + 2\gamma - 2|}{\sqrt{4 + 1 + 4}}$

$\therefore\ 2(3) = |2\alpha - \beta + 2\gamma - 2|$

$\therefore\ 2\alpha - \beta + 2\gamma - 2 = 6$ or $2\alpha - \beta + 2\gamma - 2 = -6$

$\therefore\ 2\alpha - \beta + 2\gamma - 8 = 0$ and $2\alpha - \beta + 2\gamma + 4 = 0$ **Ans. : (B), (D)**

(121) Suppose line L of R^3 passes through origin and all points of the line L are at the same distance from planes $\pi_1 : x + 2y - z + 1 = 0$ and $\pi_2 : 2x - y + z - 1 = 0$. Let M be the locus of the feet of the perpendiculars drawn from the points on L to the plane π_1. Which of the following points lie(s) on M ? [JEE : 2015]

(A) $\left(0, \frac{-5}{6}, \frac{-2}{3}\right)$ (B) $\left(\frac{-1}{6}, \frac{-1}{3}, \frac{1}{6}\right)$ (C) $\left(\frac{-5}{6}, 0, \frac{1}{6}\right)$ (D) $\left(\frac{-1}{3}, 0, \frac{2}{3}\right)$

Sol. : All points of line L are at the same distance from planes π_1 and π_2, i.e. line L lies on the plane bisecting the angle between two given planes.

The equation of the plane bisecting the angle between two given planes is

$$\frac{x+2y-z+1}{\sqrt{1+4+1}} = \pm\frac{2x-y+z-1}{\sqrt{4+1+1}}.$$

$\therefore$ $x + 2y - z + 1 = 2x - y + z - 1$ or $x + 2y - z + 1 = -2x + y - z + 1$

$\therefore$ $x - 3y + 2z - 2 = 0$ or $3x + y = 0$

Suppose the equation of the line is $\frac{x-0}{l} = \frac{y-0}{m} = \frac{z-0}{n}$.

If the points lie on the plane bisecting the angle between the given two planes.

$\therefore$ $l - 3m + 2n = 0$ and $3l + m = 0$

$\therefore$ $\frac{l}{-2} = \frac{m}{6} = \frac{n}{10}$

$\therefore$ $\frac{l}{-1} = \frac{m}{3} = \frac{n}{5}$

$\therefore$ The equation of the line is $\frac{x}{-1} = \frac{y}{3} = \frac{z}{5}$

$\therefore$ Any point on line is A$(-k, 3k, 5k)$.

Suppose the foot of perpendicular from A to plane π_1 is B(x, y, z).

$$\therefore \quad \frac{x+k}{1} = \frac{y-3k}{2} = \frac{z-5k}{-1} = -\left(\frac{1(-k)+2(3k)-1(5k)+1}{(1)^2+(2)^2+(-1)^2}\right)$$

$$\therefore \quad \frac{x+k}{1} = \frac{y-3k}{2} = \frac{z-5k}{-1} = \frac{-1}{6}$$

$\therefore$ The foot of perpendicular $(x, y, z) = \left(\frac{-1}{6} - k, \frac{-2}{6} + 3k, \frac{1}{6} + 5k\right)$

The foot of perpendicular for $k = 0$ is $\left(\frac{-1}{6}, \frac{-1}{3}, \frac{1}{6}\right)$.

and, the foot of perpendicular for $k = \frac{-1}{6}$ is $\left(0, \frac{-5}{6}, \frac{-2}{3}\right)$. **Ans. : (A), (B)**

(122) The distance of the point (1, 0, 2) from the point of intersection of the line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$ and the plane $x - y + z = 16$ is ..... . [JEE : 2015]

(A) $2\sqrt{14}$ (B) 8 (C) $3\sqrt{21}$ (D) 13

Sol. : If a point $(3k + 2, 4k - 1, 12k + 2)$ of the line is also a point of plane, then

$3k + 2 - 4k + 1 + 12k + 2 = 16$

$\therefore$ $11k = 11$

$\therefore$ $k = 1$

$\therefore$ The point of intersection of the line and the plane is (5, 3, 14).

Its distance from points (1, 0, 2) is $\sqrt{(5-1)^2+(3-0)^2+(14-2)^2} = \sqrt{16+9+144} = 13$

Ans. : (D)

(123) The equation of the plane passing through the line of intersection of $2x - 5y + z = 3$ and $x + y + 4z = 5$ and parallel to $x + 3y + 6z = 1$ is .....

(A) $2x+6y+12z=13$ (B) $x+3y+6z=-7$ (C) $x+3y+6z=7$ (D) $2x+6y+12z=-13$

Sol. : In order to find a point of intersection, take $z = 0$ in $2x - 5y + z = 3$ and $x + y + 4z = 5$

$\therefore$ $2x - 5y = 3$ and $x + y = 5$.

Solving these equations, we get $x = 4$, $y = 1$

$\therefore$ The equation of the plane passing through (4, 1, 0) and parallel to $x + 3y + 6z = 1$ is

$1(x - 4) + 3(y - 1) + 6(z - 0) = 0$

$\therefore$ $x + 3y + 6z = 7$ **Ans. : (C)**

Second method :

The equation of required plane : $2x - 5y + z - 3 + \lambda(x + y + 4z - 5) = 0$

$\therefore$ $(2 + \lambda)x + (-5 + \lambda)y + (4\lambda + 1)z - 5\lambda - 3 = 0$

It is parallel to $x + 3y + 6z = 1$.

$\therefore$ $\frac{2+\lambda}{1} = \frac{-5+\lambda}{3} = \frac{4\lambda+1}{6}$

$\therefore$ $6 + 3\lambda = -5 + \lambda$

$\therefore$ $\lambda = -\frac{11}{2}$

$\therefore$ $\left(2-\frac{11}{2}\right)x + \left(-5-\frac{11}{2}\right)y + (-22+1)z + \frac{55}{2} - 3 = 0$

$\therefore$ $7x + 21y + 42z - 49 = 0$

$\therefore$ $x + 3y + 6z - 7 = 0$ **Ans. : (C)**

(124) If the shortest distance between the line $\frac{x-1}{\alpha} = \frac{y+1}{-1} = \frac{z}{1}$ $(\alpha \neq 0)$ and the line of intersection of the planes $x + y + z + 1 = 0$ and $2x - y + z + 3 = 0$ is $\frac{1}{\sqrt{3}}$, then α = ... [JEE : 2015]

(A) $-\frac{16}{19}$ (B) $-\frac{19}{16}$ (C) $\frac{32}{19}$ (D) $\frac{19}{32}$

Sol. : For the planes $x + y + z + 1 = 0$ and $2x - y + z + 3 = 0$, normal vectors are respectively,

$\vec{n_1} = (1, 1, 1)$ and $\vec{n_2} = (2, -1, 1)$

$\therefore$ The direction vector of the line of intersection is $\vec{l} = \vec{n_1} \times \vec{n_2} = (1,1,1) \times (2,-1,1) = (2,1,-3)$

Taking $z = 0$, in the equations of the planes, we have $x + y = -1$ and $2x - y = -3$

$\therefore$ $x = \frac{-4}{3}$, $y = \frac{1}{3}$

$\therefore$ A point B$(\vec{b})$ on the line of intersection is $\left(\frac{-4}{3}, \frac{1}{3}, 0\right)$.

For the line $\frac{x-1}{\alpha} = \frac{y+1}{-1} = \frac{z}{1}$, A$(\vec{a})$ = (1, –1, 0) and $\vec{m} = (\alpha - 1, 1)$

$\vec{l} \times \vec{m} = (2, 1, -3) \times (\alpha, -1, 1) = (-2, -2 - 3\alpha, -2 - \alpha)$

or $\vec{l} \times \vec{m} = (2, 2 + 3\alpha, 2 + \alpha)$

The shortest distance $= \dfrac{|(\vec{b} - \vec{a}) \cdot (\vec{l} \times \vec{m})|}{|\vec{l} \times \vec{m}|}$

$\therefore \dfrac{1}{\sqrt{3}} = \dfrac{\left|\left(\frac{-7}{3}, \frac{4}{3}, 0\right) \cdot (2, 2 + 3\alpha, 2 + \alpha)\right|}{\sqrt{4 + (2 + 3\alpha)^2 + (2 + \alpha)^2}}$

$\therefore 10\alpha^2 + 16\alpha + 12 = 3\left(\dfrac{-14}{3} + \dfrac{8}{3} + 4\alpha\right)^2$

$\therefore 10\alpha^2 + 16\alpha + 12 = 12(4\alpha^2 - 4\alpha + 1)$

$\therefore 10\alpha^2 + 16\alpha + 12 = 48\alpha^2 - 48\alpha + 12$

$\therefore 38\alpha^2 - 64\alpha = 0$

$\therefore 2\alpha(19\alpha - 32) = 0$

$\therefore \alpha = 0$ or $\alpha = \dfrac{32}{19}$. But $\alpha \neq 0$

$\therefore \alpha = \dfrac{32}{19}$ **Ans. : (C)**

(125) A plane containing the point (3, 2, 0) and the line $\dfrac{x-1}{1} = \dfrac{y-2}{5} = \dfrac{z-3}{4}$ also contains the point ..... . [JEE : 2015]

(A) (0, 7, –10) (B) (0, 7, 10) (C) (0, 3, 1) (D) (0, –3, 1)

Sol. : For line $\dfrac{x-1}{1} = \dfrac{y-2}{5} = \dfrac{z-3}{4}$, A($\vec{a}$) = (1, 2, 3) and $\vec{l}$ = (1, 5, 4)

A point on the plane is B (3, 2, 0). $\overrightarrow{\mathbf{AB}}$ = (2, 0, –3)

$\therefore$ The normal to plane $\vec{n} = \overrightarrow{\mathbf{AB}} \times \vec{l} = (2, 0, -3) \times (1, 5, 4) = (15, -11, 10)$

$\therefore$ The equation of the plane is $15(x - 3) - 11(y - 2) + 10(z - 0) = 0$

$\therefore 15x - 11y + 10z - 23 = 0$

From the given points, only (0, 7, 10) lies on the plane. **Ans. : (B)**

(126) The shortest distance between Z-axis and the line of intersection of the planes $x + y + 2z - 3 = 0$ and $2x + 3y + 4z - 4 = 0$ is ..... [JEE : 2015]

(A) 1 (B) 2 (C) 3 (D) 4

Sol. : The normals of planes $x + y + 2z - 3 = 0$ and $2x + 3y + 4z - 4 = 0$ are respectively $\vec{n_1} = (1, 1, 2)$ and $\vec{n_2} = (2, 3, 4)$.

The direction vector of the line of intersection is $\vec{l} = \vec{n_1} \times \vec{n_2}$

$= (1, 1, 2) \times (2, 3, 4) = (-2, 0, 1)$

Taking $z = 0$, we get $x + y = 3$ and $2x + 3y = 4$ which on solving give the point $(5, -2, 0)$ on the line of intersection.

The direction vector of Z-axis is $\vec{m} = (0, 0, 1)$

$\therefore \vec{l} \times \vec{m} = (0, 2, 0)$ i.e. we can take $\vec{l} \times \vec{m} = (0, 1, 0)$.

Point $(0, 0, 1)$ can be taken as a point on Z-axis.

$\therefore$ The shortest distance $= \dfrac{|(\vec{b} - \vec{a}) \cdot (\vec{l} \times \vec{m})|}{|\vec{l} \times \vec{m}|} = \dfrac{|(5, -2, -1) \cdot (0, 1, 0)|}{1} = 2$ **Ans. : (B)**

: Questions with More Than One Correct Answers :

(127) Line $\dfrac{x-1}{1} = \dfrac{y}{2} = \dfrac{z+1}{3}$ .....

(A) passes through origin (B) passes thorugh $(1, 0, -1)$

(C) makes a right angle with $x + 2y + 3z - 7 = 0$ (D) is parallel to line $\dfrac{x-1}{1} = \dfrac{y}{2} = \dfrac{1-z}{3}$

Sol. : Point $(1, 0, -1)$ is on the line.

$\therefore$ Option (B) is true.

As direction vector of the line and normal vector of the plane $x + 2y + 3z - 7 = 0$ are the same, the line is perpendicular to the plane.

$\therefore$ Optoin (C) is true.

Direction vector of the given line is $\vec{l} = (1, 2, 3)$ and direction vector of line $\dfrac{x-1}{1} = \dfrac{y}{2} = \dfrac{z-1}{-3}$ is $\vec{m} = (1, 2, -3)$.

$\therefore \vec{l} \times \vec{m} \neq \vec{0}$

$\therefore$ The lines are not parallel to each other.

$\therefore$ Opiton (D) is false.

$\dfrac{0-1}{1} = \dfrac{0}{2} = \dfrac{0+1}{3}$ is not true. **Ans. : (B), (C)**

Option (A) is false.

(128) The line $\dfrac{x-3}{3} = \dfrac{y-1}{1} = \dfrac{z-2}{2}$ .....

(A) lies in the plane $3x + y - 2z = 6$. (B) is coincident with line $\dfrac{x}{3} = \dfrac{y}{1} = \dfrac{z}{2}$

(C) lies in the plane $x + y - 2z = 0$ (D) passes through point $(6, 2, 4)$

Sol. : For the line $\vec{a} = (3, 1, 2)$ and $\vec{l} = (3, 1, 2)$

$3(3k + 3) + k + 1 - 2(2k + 2) = 6 \Rightarrow k = 0$

Plane $3x + y - 2z = 6$ does not contain the point $(3k + 3, k + 1, 2k + 2)$ for all $k \in R$

$\therefore$ The line does not lie in the plane.

$\therefore$ Option (A) is false.

Every point $(3k + 3, k + 1, 2k + 2)$ of the given line satisfies the equations $\frac{x}{3} = \frac{y}{1} = \frac{z}{2}$.

$\therefore$ Both the lines are coincident. Infact $\frac{x-3}{3} = \frac{y-1}{1} = \frac{z-2}{2} \Leftrightarrow \frac{x}{3} = \frac{y}{1} = \frac{z}{2}$.

$\therefore$ Option (B) is true.

Point $(3k + 3, k + 1, 2k + 2)$ $\forall k \in$ R, lies in the plane $x + y - 2z = 0$.

$\therefore$ Option (C) is true.

Point (6, 2, 4) satisfies the equations the of line. (For $k = 1$)

$\therefore$ Option (D) is true. (Consider all *w.r.t.* $\frac{x}{3} = \frac{y}{1} = \frac{z}{2}$. Easier.) **Ans. : (B), (C), (D)**

(129) The line $\frac{x-3}{3} = \frac{y-3}{-4} = \frac{z-5}{2}$ .....

(A) is parallel to $\frac{x}{6} = \frac{y-5}{-8} = \frac{z-2}{4}$ (B) lies in the plane $8x + 3y - 6z = 3$

(C) passes through (3, –3, 5) (D) is perpendicular to $3x - 4y + 2z = 7$

Sol. : For the line L : $\frac{x-3}{3} = \frac{y-3}{-4} = \frac{z-5}{2}$, $\vec{l}$ = (3, –4, 2) and P$(3k + 3, -4k + 3, 2k + 5)$ is any point on the line. $k \in$ R

For line, M : = $\frac{x}{6} = \frac{y-5}{-8} = \frac{z-2}{4}$, $\vec{m}$ = (6, –8, 4).

$\vec{l} \times \vec{m} = \vec{0}$ and point (3, 3, 5) $\notin$ M as $\frac{3}{6} \neq \frac{3-5}{-8} \neq \frac{5-2}{4}$

$\therefore$ Two lines are parallel to each other.

$\therefore$ Option (A) is true.

Point P lies in the plane $8x + 3y - 6z = 3$.

as $8(3k + 3) + 3(-4k + 3) - 6(2k + 5) = 3$, $\forall k \in$ R

$\therefore$ Option (B) is true.

Point (3, –3, 5) does not satisfy equations of line L, because $\frac{3-3}{3} \neq \frac{-3-3}{4}$

$\therefore$ Option (C) is false.

For the plane $3x - 4y + 2z = 7$, the normal is $\vec{n}$ = (3, –4, 2) = $\vec{l}$

$\therefore$ The line is perpendicular to plane.

$\therefore$ Option (D) is true. **Ans. : (A), (B), (D)**

(130) For lines L : $\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-1}{5}$ and M : $\frac{x+2}{4} = \frac{y-1}{3} = \frac{z+1}{-2}$, ...

(A) L || M (B) L and M are skew lines

(C) Line M passes through the point (2, 4, 1) (D) Shortest distance between L and M is $\frac{107}{\sqrt{1038}}$.

Sol. : For line L, $\vec{a} = (1, -1, 1)$, $\vec{l} = (3, 2, 5)$

For line M $\vec{b} = (-2, 1, -1)$, $\vec{m} = (4, 3, -2)$

Now, $\vec{l} \times \vec{m} = (3, 2, 5) \times (4, 3, -2) = (-19, 26, 1) \neq \vec{0}$

$\therefore$ L and M are neither parallel nor coincident. Option (A) is false.

Point $(2, 4, 1) \notin$ M becasue $\frac{2+2}{4} = \frac{4-1}{3} = \frac{1+1}{-2}$ is not true.

$\therefore$ Option (C) is false.

Also, $(\vec{b} - \vec{a}) \cdot (\vec{l} \times \vec{m}) = (-3, 2, -2) \cdot (-19, 26, 1) = 57 + 52 - 2 = 107 \neq 0$

$\therefore$ L and M are skew. Thus, option (B) is true.

$\therefore$ The shortest distance between L and M $= \frac{|(\vec{b} - \vec{a}) \cdot (\vec{l} \times \vec{m})|}{|\vec{l} \times \vec{m}|}$

$= \frac{107}{\sqrt{(-19)^2 + (26)^2 + (1)^2}} = \frac{107}{\sqrt{1038}}$

$\therefore$ Option (D) is true. **Ans. : (B), (D)**

(131) Line L : $\frac{x-1}{2} = \frac{2-y}{3} = \frac{z+3}{4}$ and plane $\pi : 2x + 4y - z = 1$ .....

(A) Line L and plane π are parallel to each other

(B) L $\perp \pi$

(C) The point of intersection of L and π is $(3, -1, 1)$

(D) The measure of the angle between L and π is $sin^{-1}\frac{12}{\sqrt{609}}$

Sol. : For line L, $\vec{a} = (1, 2, -3)$, $\vec{l} = (2, -3, 4)$, P$(2k + 1, -3k + 2, 4k - 3)$ is any point on line.

The normal to the plane π is $\vec{n} = (2, 4, -1)$.

$\vec{l} \cdot \vec{n} = 4 - 12 - 4 \neq 0$. Thus, L and π are not parallel. π does not contain L.

$\therefore$ Option (A) is false.

Also, $\vec{l} \neq k\vec{n}$, $k \in$ R

$\therefore$ L and π are not perpendicular to each other.

$\therefore$ Option (B) is false.

If P $\in$ L is the point of intersection with the plane, π, then P $\in \pi$

$\therefore$ $2(2k + 1) + 4(-3k + 2) - (4k - 3) = 1$

$\therefore$ $12k = 12$. So, $k = 1$

$\therefore$ The point of intersection of the plane and the line is P$(3, -1, 1)$.

$\therefore$ Option (C) is true.

$\therefore$ Let the angle between L and π have measure θ,

$$\therefore \sin\theta = \frac{|\vec{l} \cdot \vec{n}|}{|\vec{l}||\vec{n}|}$$

$$= \frac{|4 - 12 - 4|}{\sqrt{4+9+16}\sqrt{4+16+1}} = \frac{12}{\sqrt{609}}$$

$$\therefore \theta = \sin^{-1}\frac{12}{\sqrt{609}}$$

$\therefore$ Option (D) is true. **Ans. : (C), (D)**

(132) For the plane $3x + 4y - 6z = 12$ .....

(A) The intercepts on the axes are 4, 3 and –2 respectively.

(B) The line $\frac{x-4}{2} = \frac{y-3}{-3} = \frac{z+1}{-1}$ is parallel to it.

(C) (1, 5, 3) is a point on the plane.

(D) The line $\frac{x-0}{3} = \frac{y+1}{4} = \frac{3-z}{6}$ is perpendicular to it.

Sol. : The plane is $3x + 4y - 6z = 12$

$$\therefore \frac{x}{4} + \frac{y}{3} + \frac{z}{-2} = 1$$

$\therefore$ The intercepts on the axes are 4, 3 and –2 respectively.

$\therefore$ Option (A) is true.

$\therefore$ The normal to the plane $\vec{n} = (3, 4, -6)$

For line $\frac{x-4}{2} = \frac{y-3}{-3} = \frac{z+1}{-1}$, direction vector is $\vec{l} = (2, -3, -1)$

$\vec{n} \cdot \vec{l} = 6 - 12 + 6 = 0$

$\therefore \vec{n} \perp \vec{l}$ and for the point (4, 3, –1) of the line, $3x + 4y - 6z \neq 12$

$\therefore$ The point (4, 3, –1) of the line is not on $3x + 4y - 6z = 12$.

$\therefore$ The line is parallel to given plane.

$\therefore$ Option (B) is true.

For the point (1, 5, 3), $3(1) + 4(5) - 6(3) = 3 + 20 - 18 = 5 \neq 12$

$\therefore$ The point (1, 5, 3) is not on plane.

$\therefore$ Option (C) is false.

The line $\frac{x-0}{3} = \frac{y+1}{4} = \frac{3-z}{6}$ has direction vector $\vec{m} = (3, 4, -6) = \vec{n}$

$\therefore$ The line is perpendicular to the plane.

$\therefore$ Option (D) is true. **Ans. : (A), (B), (D)**

(133) If the magnitudes of the projections of a line in space on the coordinate axes are respectively 3, 4, 12, then its direction cosines are .....

(A) $\frac{-3}{13}, \frac{-4}{13}, \frac{-12}{13}$ (B) $\frac{3}{13}, \frac{-4}{13}, \frac{12}{13}$ (C) $\frac{3}{13}, \frac{4}{13}, \frac{12}{13}$ (D) $\frac{3}{13}, \frac{-4}{13}, \frac{-12}{13}$

Sol. : The magnitudes of the projections are 3, 4, 12 respectively.

The magnitudes of the projections on the axes are $|\vec{l} \cdot \hat{i}|, |\vec{l} \cdot \hat{j}|, |\vec{l} \cdot \hat{k}| = (|l_1|, |l_2|, |l_3|)$.

$\therefore$ If $\vec{l} = (l_1, l_2, l_3)$, then $(|l_1|, |l_2|, |l_3|) = (3, 4, 12)$

$\therefore$ $l_1 = \pm 3$, $l_2 = \pm 4$, $l_3 = \pm 12$. So, $|\vec{l}| = 13$

$\therefore$ Direction cosines : $\frac{-3}{13}, \frac{-4}{13}, \frac{-12}{13}$ or $\frac{3}{13}, \frac{4}{13}, \frac{12}{13}$ or $\frac{3}{13}, \frac{-4}{13}, \frac{12}{13}$, $\frac{3}{13}, \frac{-4}{13}, \frac{-12}{13}$

Ans. : (A), (B), (C), (D)

(134) The direction cosines of the normal to the plane $2x + y - 3z + 4 = 0$ are respectively .....

(A) $\frac{-1}{\sqrt{14}}, \frac{3}{\sqrt{14}}, \frac{-2}{\sqrt{14}}$ (B) $\frac{2}{\sqrt{14}}, \frac{1}{\sqrt{14}}, \frac{-3}{\sqrt{14}}$ (C) $\frac{-2}{\sqrt{14}}, \frac{-1}{\sqrt{14}}, \frac{3}{\sqrt{14}}$ (D) $\frac{2}{\sqrt{14}}, \frac{1}{\sqrt{14}}, \frac{3}{\sqrt{14}}$

Sol. : For the plane, normal is $\vec{n} = (2, 1, -3)$

$\therefore$ $|\vec{n}| = \sqrt{4+1+9} = \sqrt{14}$

$\therefore$ The direction cosines of $\vec{n}$ are $\frac{2}{\sqrt{14}}, \frac{1}{\sqrt{14}}, \frac{-3}{\sqrt{14}}$ or $\frac{-2}{\sqrt{14}}, \frac{-1}{\sqrt{14}}, \frac{3}{\sqrt{14}}$

(The equation of the plane can also be written as $-2x - y + 3z = 4$)

Ans. : (B), (C)

(135) The lines $\frac{x+1}{0} = \frac{y-1}{3} = \frac{z-1}{-1}$ and $\frac{x+1}{3} = \frac{y-4}{2} = \frac{z}{0}$ .....

(A) do not intersect each other (B) intersect each other

(C) intersect at point $(-1, 4, 0)$ (D) intersect at point $(-1, 1, 1)$

Sol. : For given lines, $\vec{a} = (-1, 1, 1)$, $\vec{l} = (0, 3, -1)$

and $\vec{b} = (-1, 4, 0)$, $\vec{m} = (3, 2, 0)$

Now, $[\vec{b} - \vec{a} \quad \vec{l} \quad \vec{m}] = \begin{vmatrix} 0 & 3 & -1 \\ 0 & 3 & -1 \\ 3 & 2 & 0 \end{vmatrix} = 0$

Also, $\vec{l} \times \vec{m} \neq \vec{0}$

$\therefore$ The lines intersect each other.

Option (A) is false and Option (B) is true.

$(-1, 3k_1 + 1, -k_1 + 1)$ is any point on the first line and $(3k_2 - 1, 2k_2 + 4, 0)$ is any point on the second line.

The lines intersect each other.

For some k_2 and k_1, $-1 = 3k_2 - 1$, $3k_1 + 1 = 2k_2 + 4$, $-k_1 + 1 = 0$

$\therefore\ k_2 = 0$ and $k_1 = 1$

These values satisfy the equation $3k_1 + 1 = 2k_2 + 4$ also and so the point of intersection is $(-1, 4, 0)$.

Option (C) is true and (D) is false. **Ans. : (B), (C)**

(136) If a line makes angles of measures α, β, γ with the coordinate axes, then .....

(A) $cos^2\alpha = cos^2\beta + cos^2\gamma$ (B) $sin^2\alpha + sin^2\beta + sin^2\gamma = 2$

(C) $sin^2\beta + sin^2\gamma = 1 + cos^2\alpha$ (D) $cos^2\alpha + cos^2\beta + cos^2\gamma = 2$

Sol. : Direction cosines of line are $cos\alpha$, $cos\beta$, $cos\gamma$

$\therefore\ cos^2\alpha + cos^2\beta + cos^2\gamma = 1$

Option (A) and option (D) are false.

Now, $cos^2\alpha + cos^2\beta + cos^2\gamma = 1$

$\therefore\ 1 - sin^2\alpha + 1 - sin^2\beta + 1 - sin^2\gamma = 1$

$\therefore\ sin^2\alpha + sin^2\beta + sin^2\gamma = 2$

Option (B) is true.

and $sin^2\beta + sin^2\gamma = 1 + 1 - sin^2\alpha$

$\therefore\ sin^2\beta + sin^2\gamma = 1 + cos^2\alpha$

Option (C) is true. **Ans. : (B), (C)**

(137) The equation of the line passing through origin and intersecting the line $\frac{x}{1} = \frac{y-3}{1} = \frac{z-3}{2}$ and making an angle of measure $\frac{\pi}{3}$ with it, is .....

(A) $\frac{x}{-1} = \frac{y}{2} = \frac{z}{1}$ (B) $\frac{x}{1} = \frac{y}{-2} = \frac{z}{1}$ (C) $\frac{x}{-2} = \frac{y}{1} = \frac{z}{-1}$ (D) $\frac{x}{2} = \frac{y}{1} = \frac{z}{1}$

Sol. : For line $\frac{x}{1} = \frac{y-3}{1} = \frac{z-3}{2}$, $\vec{a} = (0, 3, 3)$, $\vec{l} = (1, 1, 2)$ and any point on the line is of form $P(k, k + 3, 2k + 3)$.

The direction vector of $\overleftrightarrow{OP}$ is $\overrightarrow{\mathbf{OP}} = (k, k + 3, 2k + 3)$

$\overleftrightarrow{OP}$ makes an angle of measure $\frac{\pi}{3}$ with the given line.

$$\therefore\ cos\frac{\pi}{3} = \frac{|1(k) + 1(k+3) + 2(2k+3)|}{\sqrt{1+1+4}\cdot\sqrt{k^2 + (k+3)^2 + (2k+3)^2}}$$

$$\therefore\ \frac{1}{2} = \frac{|6k+9|}{\sqrt{6}\cdot\sqrt{6k^2+18k+18}} = \frac{3|2k+3|}{6\sqrt{k^2+3k+3}}$$

$\therefore\ k^2 + 3k + 3 = 4k^2 + 12k + 9$

$\therefore\ 3k^2 + 9k + 6 = 0$

$\therefore\ k^2 + 3k + 2 = 0$

$\therefore$ $(k + 2)\ (k + 1) = 0$

$\therefore$ $k = -2$ or $k = -1$

The direction vector of $\overleftrightarrow{OP}$ is $\overrightarrow{\mathbf{OP}} = (-2, 1, -1)$ or $\overrightarrow{\mathbf{OP}} = (-1, 2, 1)$ and $\overleftrightarrow{OP}$ passes through origin O(0, 0, 0).

The equation of $\overleftrightarrow{OP}$ is $\frac{x}{-2} = \frac{y}{1} = \frac{z}{-1}$ and $\frac{x}{-1} = \frac{y}{2} = \frac{z}{1}$ **Ans. : (A), (C)**

(138) The symmetric form of the equations of the line of intersection of two planes $x + y + z - 1 = 0$ and $x - 2y + 4z + 2 = 0$ are .....

(A) $\frac{x-2}{-2} = \frac{y-1}{1} = \frac{z+1}{1}$ (B) $\frac{x}{-2} = \frac{y-1}{1} = \frac{z}{1}$

(C) $\frac{1-x}{2} = \frac{2y-1}{2} = \frac{2z+1}{2}$ (D) $\frac{x+2}{-1} = \frac{y-2}{2} = \frac{z-1}{2}$

Sol. : For planes, normal vectors are $\vec{n_1} = (1, 1, 1)$, $\vec{n_2} = (1, -2, 4)$.

The direction vector of the line of intersection : $\vec{l} = \vec{n_1} \times \vec{n_2} = (6, -3, -3)$ i.e. $\vec{l} = (-2, 1, 1)$

Taking $z = 0$, in the equations of the planes, we get $x + y = 1$ and $x - 2y = -2$ which on solving give (0, 1, 0) as a point on the line of intersection.

$\therefore$ The equations of the line are $\frac{x}{-2} = \frac{y-1}{1} = \frac{z}{1}$.

$\therefore$ Option (B) is true.

Taking $z = -1$, point (2, 0, –1) is obtained on solving the equations

$x + y = 2$ and $x - 2y = 2$.

$\therefore$ The equations of the line are $\frac{x-2}{-2} = \frac{y}{1} = \frac{z+1}{1}$.

$\therefore$ Option (A) is false.

or (2, 1, –1) is does not lie on the plane $x + y + z = 1$

$\therefore$ Option (A) is false.

In option (C), the equation of the line is $\frac{x-1}{-2} = \frac{y-\frac{1}{2}}{1} = \frac{z+\frac{1}{2}}{1}$.

Point $\left(1, \frac{1}{2}, -\frac{1}{2}\right)$ lies on both the planes $x + y + z - 1 = 0$ and $x - 2y + 4z + 2 = 0$.

Also the direction vector is (–2, 1, 1).

$\therefore$ Option (C) is true.

The direction vector of the line is (–1, 2, 2) and not (–2, 1, 1).

$\therefore$ Option (D) is false. **Ans. : (B), (C)**

(139) Plane $2x + 3y + 5z = 0$ .....

(A) is orthogonal to $2x - 3y + z - 7 = 0$ (B) does not pass through the point (2, 2, –2)

(C) passes through point (–2, –2, 2) (D) is perpendicular to line $\frac{x-3}{2} = \frac{y}{3} = \frac{z+7}{5}$

Sol. : Plane $2x + 3y + 5z = 0$ has normal $\vec{n_1} = (2, 3, 5)$

Plane $2x - 3y + z - 7 = 0$ has normal $\vec{n_2} = (2, -3, 1)$

Now, $\vec{n_1} \cdot \vec{n_2} = 4 - 9 + 5 = 0$. So, $\vec{n_1} \perp \vec{n_2}$

$\therefore$ The plane $2x - 3y + z - 7 = 0$ is perpendicular to given plane.

$\therefore$ Option (A) is true.

Substituting the coordinates of point $(2, 2, -2)$ in $2x + 3y + 5z = 0$,

$2(2) + 3(2) + 5(-2) = 4 + 6 - 10 = 0.$

$\therefore$ The point $(2, 2, -2)$ belongs to plane.

$\therefore$ Option (B) is false.

For the point $(-2, -2, 2)$, $2(-2) + 3(-2) + 5(2) = -4 - 6 + 10 = 0$

$\therefore$ The point $(-2, -2, 2)$ also belongs to the plane.

$\therefore$ Option (C) is true.

The line $\frac{x-3}{2} = \frac{y}{3} = \frac{z+7}{5}$ has the direction vector $\vec{l} = (2, 3, 5) = \vec{n_1}$

$\therefore$ The line is perpendicular to the plane.

$\therefore$ Option (D) is true. **Ans. : (A), (C), (D)**

(140) The coordinates of the point lying on the line $x = \frac{y-1}{2} = \frac{z+1}{-3}$ and which is at a distance of $2\sqrt{14}$ from the point $(0, 1, -1)$ is ..... .

(A) $(4, 9, -13)$ (B) $(-4, -7, 13)$ (C) $(-2, -3, 5)$ (D) $(2, 5, -7)$

Sol. : $P(k, 2k + 1, -3k - 1)$ is any point on the given line.

Let the distance of P from the point $(0, 1, -1)$ be $2\sqrt{14}$.

$\therefore k^2 + (2k + 1 - 1)^2 + (-3k - 1 + 1)^2 = 4(14)$

$\therefore 14k^2 = 4(14).$ So, $k^2 = 4$

$\therefore k = 2$ or $k = -2$

$\therefore$ The coordinates of the point on the given line at a distance of $2\sqrt{14}$ from the point $(0, 1, -1)$ are $(2, 5, -7)$ and $(-2, -3, 5)$. **Ans. : (C), (D)**

(141) The equation of the plane bisecting the angle between two given planes $6x - 2y + 3z + 18 = 0$ and $2x - y + 2z + 13 = 0$ is .....

(A) $4x + y - 5z - 37 = 0$ (B) $4x + y - 5z + 37 = 0$

(C) $32x - 13y + 23z - 145 = 0$ (D) $32x - 13y + 23z + 145 = 0$

Sol. : The equation of the plane bisecting the angle between two given planes is

$$\frac{6x - 2y + 3z + 18}{\sqrt{36 + 4 + 9}} = \pm \frac{2x - y + 2z + 13}{\sqrt{4 + 1 + 4}}$$

$\therefore 3(6x - 2y + 3z + 18) = \pm 7(2x - y + 2z + 13)$

$\therefore 18x - 6y + 9z + 54 = \pm (14x - 7y + 14z + 91)$

$\therefore 4x + y - 5z - 37 = 0$ and $32x - 13y + 23z + 145 = 0$ **Ans. : (A), (D)**

: Assertion and Reason Type Questions :

Explanation : Each of these questions contain statement 1 (Assertion) and statement 2 (Reason). Each question has four choices, option (A), (B), (C) and (D), which are as follows. You have to select the correct choice.

(A) Statement 1 is true, statement 2 is true. Statement 2 is the correct explanation for statement 1.

(B) Statement 1 is true, statement 2 is true. Statement 2 is not a correct explanation for statement 1.

(C) Statement 1 is true, statement 2 is false.

(D) Statement 1 is false, statement 2 is true.

(142) **Statement 1 :** The line L : $\frac{x}{4}=\frac{y}{-5}=\frac{z}{3}$ is perpendicular to the plane $4x - 5y + 3z = 20$.

Statement 2 : The direction cosines of the line L are $\frac{4}{5\sqrt{2}}, \frac{-5}{5\sqrt{2}}, \frac{3}{5\sqrt{2}}$.

Sol. : The plane $4x - 5y + 3z = 20$ has normal $\vec{n} = (4, -5, 3)$. If the line L is to be perpendicular to the plane then direction vector of L is $\vec{l} = k\vec{n}$, $k \in$ R. This is true with $\vec{l} = \vec{n}$.

The statement 1 is true. The direction cosines of line L are $\frac{4}{|\vec{l}|}, \frac{-5}{|\vec{l}|}, \frac{3}{|\vec{l}|}$

where $|\vec{l}| = \sqrt{16 + 25 + 9} = 5\sqrt{2}$

$\therefore$ The direction cosines of line L : $\frac{4}{5\sqrt{2}}, \frac{-1}{\sqrt{2}}, \frac{3}{5\sqrt{2}}$.

$\therefore$ Statement 1 and statement 2 both are true and statement 2 is the correct explanation for statement 1 **Ans. : (A)**

(143) **Statement 1 :** The distance between two planes $5x - 12y + 13z = 40$ and $5x - 12y + 13z = 20$ is $\frac{10\sqrt{2}}{13}$.

Statement 2 : The perpendicular distance between two parallel planes $ax + by + cz + d_1 = 0$ and $ax + by + cz = d_2$ is $\frac{|d_1 - d_2|}{\sqrt{a^2 + b^2 + c^2}}$.

Sol. : For the planes $5x - 12y + 13z - 40 = 0$ and $5x - 12y + 13z - 20 = 0$, the planes are parallel as $\vec{n_1} = \vec{n_2}$.

The perpendicular distance between two parallel planes

$$= \frac{|d_1 - d_2|}{\sqrt{a^2 + b^2 + c^2}} = \frac{|-40 - (-20)|}{\sqrt{25 + 144 + 169}} = \frac{20}{13\sqrt{2}} = \frac{10\sqrt{2}}{13}$$

$\therefore$ Statement 1 is true.

Now, perpendicular distance between parallel planes $ax + by + cz + d_1 = 0$ and $ax + by + cz - d_2 = 0$ is $\frac{|d_1 - (-d_2)|}{\sqrt{a^2 + b^2 + c^2}} = \frac{|d_1 + d_2|}{\sqrt{a^2 + b^2 + c^2}}$.

$\therefore$ Statement 2 is false. **Ans. : (C)**

(144) A line : $\frac{x-3}{-1} = \frac{y+1}{3} = \frac{z+1}{2}$ and a plane $\pi : x - y + 2z = 0$ are given.

Statement 1 : L lies in the plane π.

Statement 2 : L is parallel to π.

Sol. : For the line L, $\vec{a} = (3, -1, -1)$, $\vec{l} = (-1, 3, 2)$ and $(-k + 3, 3k - 1, 2k - 1)$ is any point of line L for $k \in R$.

For plane π, $\vec{n} = (1, -1, 2)$

Now, $\vec{l} \cdot \vec{n} = (-1, 3, 2) \cdot (1, -1, 2) = -1 - 3 + 4 = 0$

$\therefore \vec{l} \perp \vec{n}$

$\therefore$ L lies in the plane or L is parallel to π.

Now, substituting coordinates $(-k + 3, 3k - 1, 2k - 1)$ of any point of line in the equation of the plane, $(-k + 3) - (3k - 1) + 2(2k - 1) = 2 \neq 0$

$\therefore$ L does not lie in the plane π. L is parallel to π.

$\therefore$ Statement 1 is false and Statement 2 is true. **Ans. : (D)**

(145) The equations of two lines are L : $\frac{x+3}{1} = \frac{y-2}{-3} = \frac{z-1}{2}$ and M : $\frac{x-1}{-3} = \frac{y+3}{2} = \frac{z-2}{1}$.

Statement 1 : L and M are coplanar.

Statement 2 : The equations $x_1 + 3y_1 = 4$, $3x_1 + 2y_1 = 5$ and $2x_1 - y_1 = 1$ are consistent.

Sol. : For the line L, $\vec{a} = (-3, 2, 1)$, $\vec{l} = (1, -3, 2)$ and for the line M, $\vec{b} = (1, -3, 2)$, $\vec{m} = (-3, 2, 1)$

$$(\vec{b} - \vec{a}) \cdot (\vec{l} \times \vec{m}) = \begin{vmatrix} 4 & -5 & 1 \\ 1 & -3 & 2 \\ -3 & 2 & 1 \end{vmatrix} = 4(-7) + 5(7) + 1\,(-7) = 0$$

$\therefore$ L and M are coplanar.

$\therefore$ Statement 1 is true. Also, $\vec{l} \times \vec{m} \neq \vec{0}$

As the lines are not parallel, they intersect.

Any point on the line L is $(x_1 - 3, -3x_1 + 2, 2x_1 + 1)$

and any point on the line M is $(-3y_1 + 1, 2y_1 - 3, y_1 + 2)$

Now, for the point of intersection,

$x_1 - 3 = -3y_1 + 1$, $\quad -3x_1 + 2 = 2y_1 - 3$, $\quad 2x_1 + 1 = y_1 + 2$

$\therefore \quad x_1 + 3y_1 = 4 \quad 3x_1 + 2y_1 = 5 \quad 2x_1 - y_1 = 1$

Given equations are consistent. $x_1 = 1, y_1 = 1$ is their solution.

$\therefore$ Statement 2 is true.

Statement 2 is a correct explanation for statement 1. **Ans. : (A)**

(146) **Statement 1 :** The image of point P(7, 1, 0) in the line L : $\frac{x-2}{3} = \frac{y}{1} = \frac{z-1}{2}$ is Q(3, 1, 6).

Statement 2 : The line $\frac{x-2}{3} = \frac{y}{1} = \frac{z-1}{2}$ bisects the line-segment joining (7, 1, 0) and (3, 1, 6).

Sol. : M(5, 1, 3) is the midpoint of $\overline{PQ}$.

The point $(3k + 2, k, 2k + 1)$ is any point on line L. $k \in R$

Now, if $3k + 2 = 5$, $k = 1$, $2k + 1 = 3$,

then $k = 1$ satisfies all the equations.

$\therefore$ M $\in$ L and the direction vector of $\overleftrightarrow{PQ}$ is $\vec{m} = (-4, 0, 6)$.

and the direction vector of line L is $\vec{l} = (3, 1, 2)$

$\vec{l} \cdot \vec{m} = 0$

$\therefore$ $\overleftrightarrow{PQ} \perp$ L

$\therefore$ P has image Q in line L.

$\therefore$ Statement 1 is true.

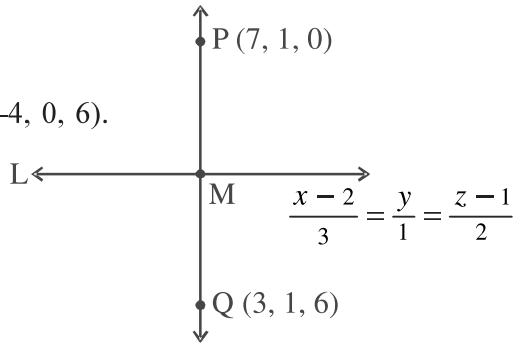


Figure 21.10

Also, the segment joining (7, 1, 0) and (3, 1, 6) is bisected by the given line.

$\therefore$ Statement 2 is true.

But if any line bisects the line-segment at right angle, then only the end-points of line-segment are images of each other.

$\therefore$ Statement 2 is not a correct explanation for statement 1. **Ans. : (B)**

(147) **Statement 1 :** The image of point (4, 3, 1) in the plane $x - y + z = 5$ is (6, 1, 3).

Statement 2 : Plane $x - y + z = 5$ bisects the line-segment joining (4, 3, 1) and (6, 1, 3).

Sol. : In order to obtain image of (4, 3, 1) in plane $x - y + z - 5 = 0$,

$$\frac{x-4}{1} = \frac{y-3}{-1} = \frac{z-1}{1} = -2\,\frac{1(4)-1(3)+1(1)-5}{(1)(1)+(-1)(-1)+(1)(1)}$$

$$= -2\,\frac{(4-3+1-5)}{3} = 2$$

$\therefore$ $x = 2 + 4 = 6$, $y = -2 + 3 = 1$, $z = 2 + 1 = 3$

$\therefore$ The image of (4, 3, 1) in the plane $x - y + z = 5$ is (6, 1, 3).

$\therefore$ Statement 1 is true.

Using statement 1, statement 2 seems to be true. Midpoint (5, 2, 2) lies on the plane $x - y + z = 5$.

$\therefore$ Statement 2 is true. But plane bisects the line-segment only. This plane is to be a perpendicular bisector plane, which is not given. Hence statement 2 is not a correct explanation for statement 1. **Ans. : (B)**

(148) Suppose $\pi_1 : x - y + z = 1$, $\pi_2 : x + y - z = -1$ and $\pi_3 : x - 3y + 3z = 2$ are three planes. L_1 is the line of intersection of planes π_2 and π_3, L_2 is the line of intersection of planes π_3 and π_1 and L_3 is the line of intersection of planes π_1 and π_2. [IIT : 2008]

Statement 1 : Atleast two of the lines L_1, L_2 and L_3 are non-parallel.

Statement 2 : The three planes do not have a common point.

Sol. : For $\pi_1 : x - y + z = 1$, $\vec{n_1} = (1, -1, 1)$

For $\pi_2 : x + y - z = -1$, $\vec{n_2} = (1, 1, -1)$

For $\pi_3 : x - 3y + 3z = 2$, $\vec{n_3} = (1, -3, 3)$

Now, the direction vector of L_1 is $\vec{l} = \vec{n_2} \times \vec{n_3} = (1, 1, -1) \times (1, -3, 3) = (0, -4, -4)$

i.e. $\vec{l} = (0, 1, 1)$

The direction vector of L_2 is $\vec{m} = \vec{n_3} \times \vec{n_1} = (1, -3, 3) \times (1, -1, 1) = (0, 2, 2)$ i.e. $\vec{m} = (0, 1, 1)$

The direction vector of L_3 is $\vec{n} = \vec{n_1} \times \vec{n_2} = (1, -1, 1) \times (1, 1, -1) = (0, 2, 2)$ i.e. $\vec{n} = (0, 1, 1)$

$\therefore \vec{l} = \vec{m} = \vec{n}$

The direction of all the three lines is same.

$L_1 = \pi_2 \cap \pi_3$ which contains $\left(-\frac{1}{4}, -\frac{3}{4}, 0\right)$ which is not in π_1.

$L_2 = \pi_1 \cap \pi_3$ which contains $\left(\frac{1}{2}, -\frac{1}{2}, 0\right)$ which is not in π_2.

$L_3 = \pi_1 \cap \pi_2$ which contains $(0, -1, 0)$ which is not in π_3.

$\therefore$ All the three lines L_1, L_2, L_3 are parallel.

$\therefore$ All the three planes do not have a common point.

$\therefore$ **Statement 1** is false and **Statement 2** is true. **Ans. : (D)**

(149) Suppose $3x - 6y - 2z = 15$ and $2x + y - 2z = 5$ are equations of two planes.

Statement 1 : Parametric equations of the line of intersection of given two planes are $x = 3 + 14t$, $y = 1 + 2t$, $z = 15t$. $t \in R$

Statement 2 : The vector $14\hat{i} + 2\hat{j} + 15\hat{k}$ is parallel to the line of intersection of given planes.

[IIT : 2007]

Sol. : Normals to planes $3x - 6y - 2z = 15$ and $2x + y - 2z = 5$ are respectively,

$\vec{n_1} = (3, -6, -2)$ and $\vec{n_2} = (2, 1, -2)$.

$\therefore$ The direction vector of the line of intersection of planes is

$\vec{n} = \vec{n_1} \times \vec{n_2} = (3, -6, -2) \times (2, 1, -2) = (14, 2, 15)$

Substituting $z = 0$, in the equations of planes, we get $3x - 6y = 15$, $2x + y = 5$

$\therefore$ $x = 3$, $y = -1$. So, $(3, -1, 0)$ is a point of intersection of planes.

$\therefore$ The equation of the line of intersection of planes : $\vec{r} = \vec{a} + t\vec{n}$, $t \in R$

$\therefore$ The parametric equations are $x = 3 + 14t$, $y = -1 + 2t$, $z = 15t$

$\therefore$ **Statement 1** is false.

The vector $14\hat{i} + 2\hat{j} + 15\hat{k} = \vec{n}$ is the direction vector of the line of intersection of planes.

$\therefore$ **Statement 2** is true.

Short method : Taking $t = 0$, we get (3, 1, 0) which is not on any of the planes.

Statement 1 is fasle.

We are left with option (D) only. **Ans. : (D)**

(150) **Statement 1 :** The line joining (1, 2, 3) and (2, 3, 5) is parallel to the line joining (–2, 1, –1) and (1, 4, 5).

Statement 2 : For two lines having the direction ratios a_1, b_1, c_1 and a_2, b_2, c_2, if $\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$, then the lines are parallel or coincident.

Sol. : The direction ratios of line joining (1, 2, 3) and (2, 3, 5) are 1, 1, 2 and the direction ratios of line joining (–2, 1, –1) and (1, 4, 5) are 3, 3, 6.

Here, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow \frac{1}{3} = \frac{1}{3} = \frac{2}{6}$ i.e. $\frac{1}{3} = \frac{1}{3} = \frac{1}{3}$

Also, point (1, 2, 3) does not lie on the second line $\frac{x+2}{1} = \frac{y-1}{1} = \frac{z+1}{2}$.

$\therefore$ Given lines are parallel lines.

$\therefore$ Statement 1 is true.

Statement 2 is also true but statement 2 is not a correct explanation for statement 1 as the lines can also be coincident. **Ans. : (B)**

(151) **Statement 1 :** Points (4, 0, 4), (3, 2, 1) and (2, 4, –2) are collinear.

Statement 2 : For distinct points A, B and C, if AB + BC = AC, then A, B, C are collinear.

Sol. : Suppose P(4, 0, 4), Q (3, 2, 1), R (2, 4, –2)

PQ = $\sqrt{(3-4)^2 + (2-0)^2 + (1-4)^2} = \sqrt{1+4+9} = \sqrt{14}$

QR = $\sqrt{1+4+9} = \sqrt{14}$

PR = $\sqrt{4+16+36} = \sqrt{56} = 2\sqrt{14}$

$\therefore$ PQ + QR = PR

$\therefore$ P – Q – R

$\therefore$ P, Q, R are collinear.

$\therefore$ **Statement 1** is true.

Statement 2 is also true and statement 2 gives the correct explanation for statement 1.

Ans. : (A)

(152) **Statement 1 :** The equation of the plane passing through the line of intersection of planes $3x + 2y - z - 4 = 0$ and $x + y + z - 2 = 0$ and also passing through the point (2, 1, 2) is $7x + 4y - 5z - 8 = 0$.

Statement 2 : If $\pi_2 = 0$ is not the equation of the plane containing the line of intersection of the planes $\pi_1 = 0$ and $\pi_2 = 0$, then the equation of the plane containing the line of intersection of planes is $\pi_1 + \lambda \pi_2 = 0$, $\lambda \in$ R.

Sol. : (2, 1, 2) does not lie on $x + y + z - 2 = 0$.

The equation of the plane passing through the line of intersection of the planes $3x + 2y - z - 4 = 0$ and $x + y + z - 2 = 0$ is $3x + 2y - z - 4 + \lambda (x + y + z - 2) = 0$. $\lambda \in$ R.

The plane passes through (2, 1, 2).

$\therefore$ $6 + 2 - 2 - 4 + \lambda (2 + 1 + 2 - 2) = 0$

$\therefore$ $2 + 3\lambda = 0$. So, $\lambda = \frac{-2}{3}$

$\therefore$ The equation of the plane is $3x + 2y - z - 4 - \frac{2}{3}(x + y + z - 2) = 0$

$\therefore$ $7x + 4y - 5z - 8 = 0$

$\therefore$ Statement 1 is true.

Statement 2 is also true and statement 2 gives the correct explanation of statement 1.

Ans. : (A)

(153) **Statement 1 :** The shortest distance between skew lines $\vec{r} = (1, 1, 0) + k(2, -1, 1)$, $k \in$ R and $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+1}{2}$ is $\frac{10}{\sqrt{59}}$.

Statement 2 : If a plane can not pass through two given lines, then the lines are skew.

Sol. : For given lines, $\vec{a} = (1, 1, 0)$, $\vec{l} = (2, -1, 1)$ and $\vec{b} = (2, 1, -1)$, $\vec{m} = (3, -5, 2)$, $\vec{l} \times \vec{m} = (3, -1, -7)$. Also $(\vec{b} - \vec{a}) \cdot (\vec{l} \times \vec{m}) = (1, 0, -1) \cdot (3, -1, -7) = 10 \neq 0$.

So, lines are skew.

The shortest distance between skew lines is $= \frac{|(\vec{b} - \vec{a}) \cdot (\vec{l} \times \vec{m})|}{|\vec{l} \times \vec{m}|}$

$= \frac{|(1, 0, -1) \cdot (3, -1, -7)|}{\sqrt{9 + 1 + 49}} = \frac{10}{\sqrt{59}}$

$\therefore$ Statement 1 is true.

Statement 2 is also true, but statement 2 is not a correct explanation for statement 1.

Ans. : (B)

(154) **Statement 1 :** The line $\frac{x+3}{13} = \frac{y-3}{-4} = \frac{z-4}{-7}$ lies in the plane $x + 5y - z - 8 = 0$.

Statement 2 : If point (x_1, y_1, z_1) of line $\frac{x - x_1}{l} = \frac{y - y_1}{m} = \frac{z - z_1}{n}$ satisfies the equation of the plane $ax + by + cz + d = 0$ then the line lies in the plane.

Sol. : For the line, $\vec{a} = (-3, 3, 4)$, $\vec{l} = (13, -4, -7)$ and $(-3 + 13k, 3 - 4k, 4 - 7k)$ is any point on the line for $k \in$ R.

The normal to the plane : $\vec{n} = (1, 5, -1)$. Also, $\vec{l} \cdot \vec{n} = 13 - 20 + 7 = 0$

$\therefore$ $\vec{l} \perp \vec{n}$ and for any point on the line, $-3 + 13k + 5(3 - 4k) - (4 - 7k) - 8 = 0$

$\therefore$ All the points of the line lie in the plane. So, the line lies in the plane.

$\therefore$ Statement 1 is true.

Statement 2 is false, because the point (x_1, y_1, z_1) can also be the point of intersection of the line and the plane. **Ans. : (C)**

(155) **Statement 1 :** The lines $\frac{x+1}{1}=\frac{y}{-1}=\frac{z-1}{1}$ and $\frac{x}{3}=\frac{y+1}{2}=\frac{z-2}{1}$ are coplanar and the equation of the plane containing both the lines is $3x - 2y - 5z + 8 = 0$.

Statement 2 : The line $\frac{x}{3}=\frac{y+1}{2}=\frac{z-2}{1}$ is perpendicular to the plane $9x + 6y + 3z - 8 = 0$ and parallel to the plane $x - y - z = 0$.

Sol. : For the line $\frac{x+1}{1}=\frac{y}{-1}=\frac{z-1}{1}$, $\vec{a}$ = (–1, 0, 1) and $\vec{l}$ = (1, –1, 1) and for the line $\frac{x}{3}=\frac{y+1}{2}=\frac{z-2}{1}$, $\vec{b}$ = (0 –1, 2) and $\vec{m}$ = (3 2, 1).

$$(\vec{b}-\vec{a})\cdot(\vec{l}\times\vec{m})=\begin{vmatrix}1 & -1 & 1\\ 1 & -1 & 1\\ 3 & 2 & 1\end{vmatrix}=0$$

$\therefore$ The lines are coplanar.

The equation of the plane containing them is

$$\begin{vmatrix}x-x_1 & y-y_1 & z-z_1\\ l_1 & l_2 & l_3\\ m_1 & m_2 & m_3\end{vmatrix}=0.$$

$$\begin{vmatrix}x+1 & y & z-1\\ 1 & -1 & 1\\ 3 & 2 & 1\end{vmatrix}=0$$

$\therefore$ $-3(x + 1) - y(-2) + (z - 1)(5) = 0$

$\therefore$ $-3x + 2y + 5z - 8 = 0$

$\therefore$ $3x - 2y - 5z + 8 = 0$ is the equation of the plane containing given coplanar lines.

$\therefore$ Statement 1 is true.

For the line $\frac{x}{3}=\frac{y+1}{2}=\frac{z-2}{1}$, $\vec{l}$ = (3, 2, 1) and for the plane $9x + 6y + 3z - 8 = 0$, $\vec{n}$ = (9, 6, 3).

Now, $\vec{l}\times\vec{n}=\vec{0}$. So, the line is perpendicular to the plane $9x + 6y + 3z - 8 = 0$.

For plane $x - y - z = 0$, $\vec{n_1}$ = (1, –1, –1)

$\vec{l}\cdot\vec{n_1}$ = 3(1) + 2(–1) + 1(–1) = 0

$\vec{l}\perp\vec{n_1}$

Now, the point (0, –1, 2) of the line does not belong to plane $x - y - z = 0$.

$\therefore$ The line is parallel to the plane $x - y - z = 0$.

Statement 2 is true but statement 2 is not a correct explanation for statement 1.

Ans. : (B)

(156) **Statement 1 :** The plane $5x + 2z - 8 = 0$ passes through the line of intersection of planes $2x - y + z - 3 = 0$ and $3x + y + z = 5$ and is perpendicular to the plane $2x - 2y - 5z - 9 = 0$.

Statement 2 : The plane $x + y + 3z = 5$ intersects the line $x - 1 = y + 1 = z - 1$ at point (1, 1, 1).

Sol. : The equation of the plane passing through the line of intersection of the planes $2x - y + z - 3 = 0$ and $3x + y + z = 5$ is $2x - y + z - 3 + \lambda (3x + y + z - 5) = 0$, $\lambda \in R$.

Taking $\lambda = 1$, the equation of the plane becomes $5x + 2z - 8 = 0$.

The normal to the plane $5x + 2z - 8 = 0$ is $\vec{n_1} = (5, 0, 2)$ and the normal to the plane $2x - 2y - 5z - 9 = 0$ is $\vec{n_2} = (2, -2, -5)$.

Now, $\vec{n_1} \cdot \vec{n_2} = 5(2) + 0(-2) + 2(-5) = 0$

$\therefore$ The plane $5x + 2z - 8 = 0$ is perpendicular to $2x - 2y - 5z - 9 = 0$.

$\therefore$ Statement 1 is true.

Point (1, 1, 1) does not lie on the line $x - 1 = y + 1 = z - 1$.

$\therefore$ The plane will not intersect the line at (1, 1, 1).

$\therefore$ Statement 2 is false. **Ans. : (C)**

(157) Suppose $L_1 : \frac{x+2}{1} = \frac{y+1}{2} = \frac{z+1}{3}$ and $L_2 : \frac{x+2}{2} = \frac{y-3}{3} = \frac{z-2}{1}$ are given lines.

Statement 1 : The perpendicular distance of the plane, passing through (–2, –1, –1) and whose normal is perpendicular to both the lines L_1 and L_2 from point (1, 1, 1) is $\frac{13}{5\sqrt{3}}$.

Statement 2 : Unit vector perpendicular to both L_1 and L_2 is $\frac{7\hat{i} - 5\hat{j} - \hat{k}}{5\sqrt{3}}$.

Sol. : The direction vectors of lines : $\vec{l} = (1, 2, 3)$ and $\vec{m} = (2, 3, 1)$

The vector orthogonal to both $\vec{l}$ and $\vec{m}$ is $\pm(\vec{l} \times \vec{m}) = \pm [(1, 2, 3) \times (2, 3, 1)] = \pm (-7, 5, -1)$

The normal to the plane is $-7\hat{i} + 5\hat{j} - \hat{k}$

$\therefore$ The equation of the plane having the above normal and which passes through (–2, –1, –1) is

$-7(x + 2) + 5(y + 1) - 1(z + 1) = 0$

$\therefore$ $-7x + 5y - z - 10 = 0$ or $7x - 5y + z + 10 = 0$

The distance of this plane from point (1, 1, 1) $= \frac{|7(1) - 5(1) + 1(1) + 10|}{\sqrt{49 + 25 + 1}} = \frac{13}{5\sqrt{3}}$

$\therefore$ Statement 1 is true.

Unit vector perpendicular to both L_1 and $L_2 = \pm \dfrac{(\vec{l} \times \vec{m})}{|\vec{l} \times \vec{m}|}$

$$= \pm \frac{\left(-7\hat{i} + 5\hat{j} - \hat{k}\right)}{5\sqrt{3}}$$

$$= \frac{-7\hat{i} + 5\hat{j} - \hat{k}}{5\sqrt{3}} \text{ or } \frac{7\hat{i} - 5\hat{j} + \hat{k}}{5\sqrt{3}}$$

$\therefore$ Statement 2 is false. **Ans. : (C)**

: 'Match the Following' Type Questions :

(158) Match the statements in column I with the appropriate answer in column II :

Column I	**Column II**
(P) If lines $x = 1 + k$, $y = 1 + \lambda k$, $z = -3 - \lambda k$, $k \in$ R and $\{(\frac{t}{2}, 2 - t, 1 + t) / t \in \text{R}\}$ are coplanar, then the value of λ is	(1) 1
(Q) The volume of the tetrahedron whose vertices are the points (3, 3, 2), (0, 4, 3), (6, 4, 1), (6, 2, 3) is	(2) 2
(R) If the image of point (–5, –3, –4) in the plane $x + y + z = -3$ is (x_1, y_1, z_1), then y_1 is	(3) –2
(S) If the image of point (5, 8, 15) in the line $x = 6 + 3k$, $y = 7 + 2k$, $z = 7 - 2k$, $k \in$ R is (x_1, y_1, z_1), then x_1 is	(4) 3

(A) (P) $\rightarrow$ (3) (Q) $\rightarrow$ (2) (R) $\rightarrow$ (1) (S) $\rightarrow$ (4)

(B) (P) $\rightarrow$ (3) (Q) $\rightarrow$ (1) (R) $\rightarrow$ (4) (S) $\rightarrow$ (2)

(C) (P) $\rightarrow$ (3) (Q) $\rightarrow$ (2) (R) $\rightarrow$ (4) (S) $\rightarrow$ (1)

(D) (P) $\rightarrow$ (1) (Q) $\rightarrow$ (2) (R) $\rightarrow$ (4) (S) $\rightarrow$ (3)

Sol. : (P) For lines $\vec{a} = (1, 1, -3)$, $\vec{l} = (1, \lambda, -\lambda)$

and $\vec{b} = (0, 2, 1)$, $\vec{m} = \left(\frac{1}{2}, -1, 1\right)$.

Lines are coplanar.

$$\therefore \begin{vmatrix} -1 & 1 & 4 \\ 1 & \lambda & -\lambda \\ \frac{1}{2} & -1 & 1 \end{vmatrix} = 0$$

$$\therefore -1(\lambda - \lambda) - 1\left(1 + \frac{\lambda}{2}\right) + 4\left(-1 - \frac{\lambda}{2}\right) = 0$$

$$\therefore -1 - \frac{\lambda}{2} - 4 - \frac{4\lambda}{2} = 0$$

$\therefore\ -5 = \frac{5\lambda}{2}$

$\therefore\ \lambda = -2$

(P) $\rightarrow$ (3)

(Q) The volume of the tetrahedron V – ABC, whose vertices are V(3, 3, 2), A(0, 4, 3), B(6, 4, 1), C(6, 2, 3) and edges $\overrightarrow{\mathbf{VA}} = (-3, 1, 1)$, $\overrightarrow{\mathbf{VB}} = (3, 1, -1)$, $\overrightarrow{\mathbf{VC}} = (3, -1, 1)$ is required.

$$\Delta = \begin{vmatrix} -3 & 1 & 1 \\ 3 & 1 & -1 \\ 3 & -1 & 1 \end{vmatrix} = -3(1-1) - 1\,(3+3) + 1(-3-3) = -12$$

The volume of the tetrahedron $= \frac{1}{6}\ |\Delta| = 2$

(Q) $\rightarrow$ (2)

(R) The equation of the perpendicular drawn from point (–5, –3, –4) to the plane $x + y + z = -3$ is $\frac{x+5}{1} = \frac{y+3}{1} = \frac{z+4}{1}$. For the image of (–5, –3, –4) in the plane as (x_1, y_1, z_1)

$$\frac{x_1+5}{1} = \frac{y_1+3}{1} = \frac{z_1+4}{1} = \frac{-2\,[(1\,(-5)+1\,(-3)+1\,(-4)+3)]}{1(1)+1(1)+1(1)} = 6$$

$\therefore\ x_1 = 1,\ y_1 = 3,\ z_1 = 2$

$\therefore\ y_1 = 3$

(R) $\rightarrow$ (4)

(S) Let M$(6 + 3k,\ 7 + 2k,\ 7 - 2k)$ be the foot of the perpendicular drawn from point P(5, 8, 15) to the line.

$\therefore\ \overrightarrow{\mathbf{PM}} \perp \vec{l}$ where $\vec{l} = (3, 2, -2)$

$\therefore\ (3k + 1,\ 2k - 1,\ -2k - 8) \cdot (3, 2, -2) = 0$

$\therefore\ 9k + 3 + 4k - 2 + 4k + 16 = 0$

$\therefore\ k = -1$

$\therefore$ The foot of perpendicular is M (3, 5, 9)

Suppose, the image of P is $Q(x_1, y_1, z_1)$. So, M is the midpoint of $\overline{PQ}$.

$\therefore\ 3 = \frac{x_1+5}{2},\ 5 = \frac{y_1+8}{2},\ 9 = \frac{z_1+15}{2}$

$\therefore\ x_1 = 1,\ y_1 = 2,\ z_1 = 3.$ So, $x_1 = 1$

$\therefore$ (S) $\rightarrow$ (1)

Short method : In S, all the alternatives are different. So by just calculating for (S), we get (S) $\rightarrow$ (1), which is in option (C). **Ans. : (C)**

(159)

Column I	Column II
(1) The measure of the angle between the lines $2x = 3y = -z$ and $6x = -y = -4z$ is	(P) 0
(2) The measure of the angle between the planes $x - y + 2z + 8 = 0$ and $2x + y + z - 5 = 0$ is	(Q) $\frac{\pi}{6}$
(3) The measure of the angle between the line $\frac{x}{2} = \frac{y}{3} = \frac{z}{4}$ and the plane $3x + 2y - 3z = 4$ is	(R) $\frac{\pi}{2}$
(4) The measure of the angle between the plane $x - 4y - 3z + 9 = 0$ and the line of intersection of the planes $2x + 3y + z - 11 = 0$ and $x + y - z - 8 = 0$ is	(S) $\frac{\pi}{3}$

(A) (1) $\rightarrow$ (R) (2) $\rightarrow$ (Q) (3) $\rightarrow$ (P) (4) $\rightarrow$ (S)

(B) (1) $\rightarrow$ (R) (2) $\rightarrow$ (S) (3) $\rightarrow$ (P) (4) $\rightarrow$ (Q)

(C) (1) $\rightarrow$ (R) (2) $\rightarrow$ (P) (3) $\rightarrow$ (Q) (4) $\rightarrow$ (S)

(D) (1) $\rightarrow$ (R) (2) $\rightarrow$ (S) (3) $\rightarrow$ (Q) (4) $\rightarrow$ (P)

Sol. : (1) The lines $2x = 3y = -z$ and $6x = -y = -4z$ are respectively $\frac{x}{3} = \frac{y}{2} = \frac{z}{-6}$ and $\frac{x}{2} = \frac{y}{-12} = \frac{z}{-3}$

$\therefore \vec{l} = (3, 2, -6)$ and $\vec{m} = (2, -12, -3)$

If the measure of the angle between the lines is θ, then $cos\theta = \frac{|(3, 2, -6) \cdot (2, -12, -3)|}{\sqrt{9 + 4 + 36}\sqrt{4 + 144 + 9}} = 0$

$\therefore \theta = \frac{\pi}{2}$

$\therefore$ (1) $\rightarrow$ (R)

(2) Normal $\vec{n_1} = (1, -1, 2)$, $\vec{n_2} = (2, 1, 1)$

If the measure of the angle between the planes is θ, then $cos\theta = \frac{|(1, -1, 2) \cdot (2, 1, 1)|}{\sqrt{1 + 1 + 4}\sqrt{4 + 1 + 1}} = \frac{3}{6} = \frac{1}{2}$

$\therefore \theta = \frac{\pi}{3}$

$\therefore$ (2) $\rightarrow$ (S)

(3) $\vec{l} = (2, 3, 4)$ $\vec{n} = (3, 2, -3)$

If the measure of the angle between the line and the plane is θ, then

$$sin\theta = \frac{|(2, 3, 4) \cdot (3, 2, -3)|}{\sqrt{4 + 9 + 16}\sqrt{9 + 4 + 9}} = 0$$

$\therefore \theta = 0$

$\therefore$ (3) $\rightarrow$ (P)

(4) The direction vector of the line of intersection of given planes is,

$$\vec{l} = \vec{n_1} \times \vec{n_2} = (2, 3, 1) \times (1, 1, -1) = (-4, 3, -1)$$

The normal to the plane $x - 4y - 3z + 9 = 0$ is $\vec{n} = (1, -4, -3)$

If the measure of the angle between the line and the plane is θ, then

$$\sin\theta = \frac{|(-4, 3, -1) \cdot (1, -4, -3)|}{\sqrt{16+9+1}\sqrt{1+16+9}}$$

$$= \frac{13}{26} = \frac{1}{2}$$

$\therefore \theta = \frac{\pi}{6}$

$\therefore$ (4) $\rightarrow$ (Q) **Ans. : (B)**

Short method :

(1) has all the alternatives (R) and so we don't consider (1).

(2) $\rightarrow$ (S) is obtained, so any of (B) and (D) can be the answer.

(3) $\rightarrow$ (P) is obtained, so only option (B) is to be considered.

(4) No need to calculate further. **Ans. : (B)**

(160) Consider the following linear equations : $ax + by + cz = 0$, $bx + cy + az = 0$ and $cx + ay + bz = 0$. Match the conditions / expressions in column I with statements in column II and indicate your answer by darkening the appropriate bubbles in the 4×4 matrix given in the ORS. [IIT : 2007]

Column I	Column II
(A) $a + b + c \neq 0$ and $a^2 + b^2 + c^2 = ab + bc + ca$	(P) The equations represent planes intersecting only at a single point.
(B) $a + b + c = 0$ and $a^2 \neq bc$, $b^2 \neq ca$, $c^2 \neq ab$	(Q) The equations represent the line $x = y = z$.
(C) $a + b + c \neq 0$ and $a^2 \neq bc$, $b^2 \neq ca$, $c^2 \neq ab$ $a^2 + b^2 + c^2 \neq ab + bc + ca$	(R) The equations represent identical planes.
(D) $a + b + c = 0$ and $a^2 + b^2 + c^2 = ab + bc + ca$	(S) The equations represent whole of the three dimensional space.

	P	Q	R	S
A	○	○	○	○
B	○	○	○	○
C	○	○	○	○
D	○	○	○	○

Sol. : $ax + by + cz = 0$ (1)

$bx + cy + az = 0$ (2)

$cx + ay + bz = 0$ (3)

$\therefore$ Determinant $D = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

$= a(bc - a^2) - b(b^2 - ac) + c(ab - c^2)$

$= 3abc - a^3 - b^3 - c^3$

$= -(a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

$= -\frac{1}{2}(a + b + c)[(a - b)^2 + (b - c)^2 + (c - a)^2]$

(A) $a + b + c \neq 0$, $a^2 + b^2 + c^2 = ab + bc + ca$

$\therefore (a - b)^2 + (b - c)^2 + (c - a)^2 = 0$

$\therefore a = b = c$ and $a + b + c \neq 0$

$\therefore a, b, c$ are non-zero.

$\therefore$ All the equations are of form $x + y + z = 0$.

$\therefore$ Equations represent identical planes.

$\therefore$ (A) $\rightarrow$ (R)

(B) $a + b + c = 0$ and $a^2 \neq bc$, $b^2 \neq ca$, $c^2 \neq ab$

Taking $a = -(b + c)$ in equation (1)

$$by + cz = (b + c)x \quad (4)$$

and taking $b = -(a + c)$ in equation (2)

$$cy + az = (c + a)x \quad (5)$$

By equations (4) and (5), we have

$(c^2 - ab)z = (c^2 - ab)x$

$\therefore z = x$

Similarly, $x = y = z$

$\therefore$ Given equations represent the line $x = y = z$.

$\therefore$ (B) $\rightarrow$ (Q)

(C) As $a + b + c \neq 0$ and $a^2 \neq bc$, $b^2 \neq ca$, $c^2 \neq ab$, $D \neq 0$

$\therefore$ Equations have only trivial solution $x = 0$, $y = 0$, $z = 0$

$\therefore$ The equations represent the three planes intersecting at a single point namely origin.

$\therefore$ (C) $\rightarrow$ (P)

(D) $a + b + c = 0$ and $a^2 + b^2 + c^2 - ab - bc - ca = 0$

$\therefore$ $a + b + c = 0$ and $a = b = c$ ...[From (A)]

$\therefore$ $a = b = c = 0$

$\therefore$ The equations represent the whole of the three dimensional space.

$\therefore$ (D) $\rightarrow$ (S)

Ans. :

	P	Q	R	S
A	○	○	●	○
B	○	●	○	○
C	●	○	○	○
D	○	○	○	●

(161) In column I, equations of line are given and in column II, type of equations are given. Match lines in column I with their types in column II.

Column I	Column II
(1) $\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$ and $\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$	(p) Parallel
(2) $\frac{x-4}{5} = \frac{y-1}{2} = \frac{z}{1}$ and $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$	(q) Skew
(3) $\frac{x}{1} = \frac{2y-4}{1} = \frac{3-z}{3}$ and $\{2k - 1, k + 1, -6k + 1 \mid k \in R\}$	(r) Coincident
(4) $x = -k$, $y = -4 - 2k$, $z = 4 + k$, $k \in R$ and $\vec{r} = (1, -2, 3) + k(-1, -2, 1)$, $k \in R$	(s) Intersecting

(A) (1) $\rightarrow$ (q) (2) $\rightarrow$ (s) (3) $\rightarrow$ (r) (4) $\rightarrow$ (p)

(B) (1) $\rightarrow$ (q) (2) $\rightarrow$ (r) (3) $\rightarrow$ (p) (4) $\rightarrow$ (s)

(C) (1) $\rightarrow$ (q) (2) $\rightarrow$ (p) (3) $\rightarrow$ (r) (4) $\rightarrow$ (s)

(D) (1) $\rightarrow$ (q) (2) $\rightarrow$ (s) (3) $\rightarrow$ (p) (4) $\rightarrow$ (r)

Sol. : (1) $\vec{a} = (3, 8, 3)$, $\vec{l} = (3, -1, 1)$

$\vec{b} = (-3, -7, 6)$, $\vec{m} = (-3, 2, 4)$

Now, $\vec{l} \times \vec{m} = (-6, -15, 3) \neq \vec{0}$

and $(\vec{b} - \vec{a}) \cdot (\vec{l} \times \vec{m}) = (-6, -15, 3) \cdot (-6, -15, 3) = 36 + 225 + 9 = 270 \neq 0$

$\therefore$ The lines are skew.

$\therefore$ (1) $\rightarrow$ (q)

(2) $\vec{a} = (4, 1, 0)$, $\vec{l} = (5, 2, 1)$

$\vec{b} = (1, 2, 3)$, $\vec{m} = (2, 3, 4)$

Now, $\vec{l} \times \vec{m} = (5, -18, 11) \neq \vec{0}$

and $(\vec{b} - \vec{a}) \cdot (\vec{l} \times \vec{m}) = (-3, 1, 3) \cdot (5, -18, 11) = -15 - 18 + 33 = 0$

$\therefore$ The lines are intersecting lines. $(\vec{l} \times \vec{m} \neq \vec{0})$

$\therefore$ (2) $\rightarrow$ (s)

(3) $\vec{a} = (0, 2, 3)$, $\vec{l} = \left(1, \frac{1}{2}, -3\right)$

$\vec{b} = (-1, 1, 1)$, $\vec{m} = (2, 1, -6)$

Now, $\vec{l} \times \vec{m} = (0, 0, 0) = \vec{0}$

and $(\vec{b} - \vec{a}) \times \vec{m} = (-1, -1, -2) \times (2, 1, -6) = (8, -10, 1) \neq \vec{0}$

$\therefore$ The lines are parallel.

$\therefore$ (3) $\rightarrow$ (*p*)

(4) $\vec{a} = (0, -4, 4)$, $\vec{l} = (-1, -2, 1)$

$\vec{b} = (1, -2, 3)$, $\vec{m} = (-1, -2, 1)$

Now, $\vec{l} \times \vec{m} = \vec{0}$ $(\vec{l} = \vec{m})$

and $(\vec{b} - \vec{a}) \times \vec{l} = (1, 2, -1) \times (-1, -2, 1) = -(1, 2, -1) \times (1, 2, -1) = \vec{0}$

$\therefore$ The lines are coincident lines.

$\therefore$ (4) $\rightarrow$ (*r*) **Ans. : (D)**

Short method :

(1) has all the alternatives same, so we shall not consider (1).

(2) $\rightarrow$ (*s*)

(3) $\rightarrow$ (*p*)

Now, only option (D) remains.

(4) No need to calculate. **Ans. : (D)**

Shorter method :

Firstly calculate (4), we obtain (4) $\rightarrow$ (r)

which is present in option (D) only. **Ans. : (D)**

(162) Match the conditions in column I with statements in column II : [IIT : 2006]

Column I	Column II
(P) The perpendicular distance of the line passing through (0, 1, 0) and perpendicular to plane $x + 2y + 2z = 0$ from origin is	(1) $2\sqrt{2}$
(Q) The distance of the plane passing through (1, –2, 1) and which is perpendicular to both $2x - 2y + z = 0$ and $x - y + 2z = 4$ from point (1, 2, 2) is	(2) $\frac{2\sqrt{3}}{\sqrt{26}}$
(R) The perpendicular distance of the line $\frac{x}{4} = \frac{y-1}{-3} = \frac{z-1}{-1}$ from the line of intersection of the planes $2x + 3y - z - 4 = 0$ and $x + y + z - 2 = 0$ is	(3) $\sqrt{29}$
(S) The perpendicular distance between the planes $2x + 3y - 4z + 44 = 0$ and $2x + 3y - 4z + 15 = 0$ is	(4) $\frac{\sqrt{5}}{3}$

(A) (P) $\rightarrow$ (4) (Q) $\rightarrow$ (1) (R) $\rightarrow$ (2) (S) $\rightarrow$ (3)

(B) (P) $\rightarrow$ (4) (Q) $\rightarrow$ (1) (R) $\rightarrow$ (3) (S) $\rightarrow$ (2)

(C) (P) $\rightarrow$ (4) (Q) $\rightarrow$ (2) (R) $\rightarrow$ (3) (S) $\rightarrow$ (1)

(D) (P) $\rightarrow$ (4) (Q) $\rightarrow$ (2) (R) $\rightarrow$ (1) (S) $\rightarrow$ (3)

Sol. : (P) The line is perpendicular to the plane $x + 2y + 2z = 0$.

$\therefore \vec{l} = \vec{n} = (1, 2, 2)$ and it passes through $\vec{a} = (0, 1, 0)$.

$\therefore$ The equation of the line : $\frac{x}{1} = \frac{y-1}{2} = \frac{z}{2}$.

The perpendicular distance of the line from point P($\vec{p}$) = (0, 0, 0) is

$$\frac{|(\vec{p} - \vec{a}) \times \vec{l}|}{|\vec{l}|} = \frac{|(0, -1, 0) \times (1, 2, 2)|}{\sqrt{1+4+4}} = \frac{|(-2, 0, 1)|}{3} = \frac{\sqrt{5}}{3}$$

$\therefore$ (P) $\rightarrow$ (4)

(Q) The normal to the plane $2x - 2y + z = 0$ is $\vec{n_1} = (2, -2, 1)$ and the normal to the plane $x - y + 2z = 4$ is $\vec{n_2} = (1, -1, 2)$.

The normal to the plane perpendicular to both the planes in $\vec{n} = \vec{n_1} \times \vec{n_2}$

$= (-3, -3, 0)$

The equation of the plane is $-3(x - 1) - 3(y + 2) = 0$. So, $x + y + 1 = 0$

$\therefore$ The distance of $x + y + 1 = 0$ from the point (1, 2, 2) is

$$\frac{|(1) + 2(1) + 1|}{\sqrt{1+1}} = \frac{4}{\sqrt{2}} = 2\sqrt{2}$$

$\therefore$ (Q) $\rightarrow$ (1)

(R) The direction vector of the line of intersection of planes $2x + 3y - z - 4 = 0$ and $x + y + z - 2 = 0$ is $\vec{l} = \vec{n_1} \times \vec{n_2}$

$= (2, 3, -1) \times (1, 1, 1)$

$= (4, -3, -1)$

Taking $z = 0$ in the equations of the plane,

$2x + 3y = 4$ and $x + y = 2$

Solving the equations, $x = 2$, $y = 0$

$\therefore$ The line of intersection passes through $\vec{a} = (2, 0, 0)$.

The line is $\frac{x-2}{4} = \frac{y}{-3} = \frac{z}{-1}$. They are parallel. (Why not coincident ?)

$\therefore$ For line $\frac{x}{4} = \frac{y-1}{-3} = \frac{z-1}{-1}$, $\vec{b} = (0, 1, 1)$ and $\vec{m} = (4, -3, -1)$

$\therefore$ The perpendicular distance between the lines $= \frac{|(\vec{b} - \vec{a}) \times \vec{l}|}{|\vec{l}|}$

$$= \frac{|(-2, 1, 1) \times (4, -3, -1)|}{\sqrt{16 + 9 + 1}} = \frac{|(2, 2, 2)|}{\sqrt{26}} = \frac{2\sqrt{3}}{\sqrt{26}}$$

$\therefore$ (R) $\rightarrow$ (2)

(S) Perpendicular distance between the planes $2x + 3y - 4z + 44 = 0$ and $2x + 3y - 4z + 15 = 0$ is

$$= \frac{|d_1 - d_2|}{|\vec{n}|}$$

$$= \frac{|44 - 15|}{\sqrt{4 + 9 + 16}} = \frac{29}{\sqrt{29}} = \sqrt{29}$$

$\therefore$ (S) $\rightarrow$ (3) **Ans. : (A)**

Short method :

(P) All options of (P) are same. So, we don't consider (P).

(S) Calculation of (S) is very simple and hence on calculating we get (S) $\rightarrow$ (3)

There are two possibilities, either (A) or (D). Now, as calculation of (Q) is simpler, on calculating we get, (Q) $\rightarrow$ (1)

Only option (A) is possible. **Ans. : (A)**

: Linked Comprehension Type Questions :

(163) Let A(2, 3, –2) and B(–3, 13, 13) be two points in the plane. Line L passes through A and plane π passes through point B. From this information, answer the following questions :

(1) The equation of the line L perpendicular to $\overline{AB}$

(A) $\frac{x-2}{-5} = \frac{y-3}{10} = \frac{x+2}{15}$ (B) $\frac{x-2}{13} = \frac{y-3}{2} = \frac{z+2}{3}$

(C) $\frac{x+2}{13} = \frac{y+3}{2} = \frac{z-2}{3}$ (D) $\frac{x-2}{15} = \frac{y-3}{-5} = \frac{z+2}{10}$

(2) The equation of the plane perpendicular to $\overline{AB}$

(A) $-5x + 10y + 15z + 10 = 0$ (B) $x + 2y - 3z + 16 = 0$

(C) $x - 2y - 3z + 68 = 0$ (D) $x - 2y - 3z - 68 = 0$

Sol. (1) The direction ratios of $\overleftrightarrow{AB}$: –5, 10, 15 i.e. –1, 2, 3.

$\therefore$ Let the direction vector of the line L perpendicular to $\overleftrightarrow{AB}$ be $\vec{l} = l_1\hat{i} + l_2\hat{j} + l_3\hat{k}$,

The equation of the line L : $\frac{x-2}{l_1} = \frac{y-3}{l_2} = \frac{z+2}{l_3}$. $\overleftrightarrow{AB} \perp$ L

$\therefore$ $-l_1 + 2l_2 + 3l_3 = 0$

Option (B) satisfies this equation.

(Option (C) also satisfies the above equation, but the line does not pass through point A)

Ans. : (B)

(2) Direction ratios of $\overleftrightarrow{AB}$: –1, 2, 3.

$\therefore$ The normal of the plane perpendicular to $\overleftrightarrow{AB}$ is $\vec{n} = (-1, 2, 3)$

$\therefore$ The equation of the plane π is $-1(x + 3) + 2(y - 13) + 3(z - 13) = 0$

$\therefore$ $-x + 2y + 3z - 68 = 0$

$\therefore$ $x - 2y - 3z + 68 = 0$

(The normal of the plane given in option (A) is also (–1, 2, 3) but it does not pass through (B).)

Ans. : (C)

(164) Suppose L : $\frac{x-5}{2} = \frac{y-3}{-4} = \frac{z-3}{3}$ and M : $\frac{x-2}{4} = \frac{y-5}{-8} = \frac{z}{6}$ are given lines. From this information, answer the following questions :

(1) The perpendicular distance between L and M is

(A) $\frac{109}{29}$ (B) $\sqrt{\frac{109}{29}}$ (C) $\frac{\sqrt{109}}{29}$ (D) $\frac{109}{\sqrt{29}}$

(2) The equation of the plane passing through L and M is

(A) $6x - 3y - 8z + 3 = 0$ (B) $6x - 3y - 8z - 3 = 0$

(C) $8x + 3y - 6z - 31 = 0$ (D) $8x - 3y - 6z - 13 = 0$

(3) The distance of the plane passing through the lines L and M from point (1, –1, 1) is

(A) $\frac{4}{109}$ (B) $\frac{2}{\sqrt{109}}$ (C) $\frac{4}{\sqrt{109}}$ (D) $\frac{2}{109}$

Sol. (1) For the line L, $\vec{a}$ = (5, 3, 3), $\vec{l}$ = (2, –4, 3)

for line M, $\vec{b}$ = (2, 5, 0), $\vec{m}$ = (4, –8, 6) or (2, –4, 3)

Here, $\vec{l} \times \vec{m} = \vec{0}$. Also for (5, 3, 3), $\frac{5-2}{4} = \frac{3-5}{-8} = \frac{3}{6}$ is not true.

∴ The point (5, 3, 3) of line L does not belong to M

∴ L ∥ M

The perpendicular distance between two parallel lines

$$= \frac{|(\vec{a} - \vec{b}) \times \vec{l}|}{|\vec{l}|}$$

$$= \frac{|(3, -2, 3) \times (2, -4, 3)|}{\sqrt{4+16+9}} = \frac{|(6, -3, -8)|}{\sqrt{29}} = \sqrt{\frac{109}{29}}$$ **Ans. : (B)**

(2) The equation of the plane passing through lines L and M : $(\bar{r} - \bar{a}) \cdot [(\bar{b} - \bar{a}) \times \bar{l}] = 0$

$$\begin{vmatrix} x-5 & y-3 & z-3 \\ 2-5 & 5-3 & 0-3 \\ 2 & -4 & 3 \end{vmatrix} = 0$$

$$\therefore \begin{vmatrix} x-5 & y-3 & z-3 \\ -3 & 2 & -3 \\ 2 & -4 & 3 \end{vmatrix} = 0$$

∴ $(x - 5)(-6) - (y - 3)(-3) + (z - 3)(8) = 0$

∴ $-6x + 30 + 3y - 9 + 8z - 24 = 0$

$6x - 3y - 8z + 3 = 0$ **Ans. : (A)**

(3) Distance of the plane $6x - 3y - 8z + 3 = 0$ from point (1, –1, 1)

$$= \frac{|6(1) - 3(-1) - 8(1) + 3|}{\sqrt{36+9+64}} = \frac{4}{\sqrt{109}}$$ **Ans. : (C)**

(165) Let direction cosines of two lines be l, m, n and a, b, c, p, q, r be constants. Direction cosines satisfy $pl + qm + rn = 0$ and $al^2 + bm^2 + cn^2 = 0$.

From the above information, answer the following questions.

(1) Which of the following relations are satisfied by the lines, if $p = q = r = 1$.

(A) $(a + b)\left(\frac{m}{n}\right)^2 + 2a\left(\frac{m}{n}\right) + (c + a) = 0$ (B) $(b + c)\left(\frac{n}{l}\right)^2 + b\left(\frac{n}{l}\right) + (a + b) = 0$

(C) $(c + a)\left(\frac{l}{m}\right)^2 + 2c\left(\frac{l}{m}\right) + (b + c) = 0$ (D) $(c + a)\left(\frac{l}{m}\right)^2 + c\left(\frac{l}{m}\right) + (b + c) = 0$

(2) If $p = q = r = 1$ and $\left(\frac{a+b}{b+c}\right) = \frac{n_1 n_2}{l_1 l_2}$, then

(A) $\frac{m_1 m_2}{l_1 l_2} = \frac{b+c}{c+a}$ (B) $\frac{m_1 m_2}{l_1 l_2} = \frac{c+a}{a+b}$ (C) $\frac{m_1 m_2}{l_1 l_2} = \frac{a+b}{c+a}$ (D) $\frac{c+a}{b+c} = \frac{m_1 m_2}{l_1 l_2}$

(3) If the lines are parallel to each other, then

(A) $\sum\frac{p^2}{a}=0$ (B) $\sum\frac{(b+c)}{p^2}=0$ (C) $\sum p^2\,(b+c)=0$ (D) $\sum\frac{a^2}{p}=0$

(4) If the lines are perpendicular to each other, then

(A) $\Sigma p^2(a+b)=0$ (B) $\Sigma p^2\,(a-b)=0$ (C) $\Sigma p^2\,(b+c)=0$ (D) $\Sigma p^2\,(b-c)=0$

(5) If $p=q=r=1$ and if the lines are mutually perpendicular, then

(A) $a+b+c=0$ (B) $ab+bc+ca=3abc$

(C) $ab+bc+ca=0$ (D) $ab+bc+ca=abc$

(6) If $p=q=r=1$ and if the lines are parallel to each other, then

(A) $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=1$ (B) $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=1$ (C) $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=0$ (D) $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=0$

Sol. Direction cosines of the lines satisfy the equations $pl+qm+rn=0$ and $al^2+bm^2+cn^2=0$.

Now, $pl+qm+rn=0$

$\therefore$ Substituting the value of $l=-\frac{(qm+rn)}{p}$ in $al^2+bm^2+cn^2=0$,

$$a\left\{-\frac{qm+rn}{p}\right\}^2+bm^2+cn^2=0$$

$\therefore$ $(aq^2+bp^2)\,m^2+2aqrmn+(ar^2+cp^2)\,n^2=0$

$\therefore$ $(aq^2+bp^2)\left(\frac{m}{n}\right)^2+2aqr\left(\frac{m}{n}\right)+(ar^2+cp^2)=0$

(1) Taking $p=q=r=1$,

$$(a+b)\left(\frac{m}{n}\right)^2+2a\left(\frac{m}{n}\right)+(a+c)=0 \qquad \text{(i)}$$

Similarly, $(c+a)\left(\frac{l}{m}\right)^2+2c\left(\frac{l}{m}\right)+(b+c)=0$ (ii)

and $(b+c)\left(\frac{n}{l}\right)^2+2b\left(\frac{n}{l}\right)+(a+b)=0$ (iii)

Ans. : (A), (C)

(2) If the roots of equation (ii) are $\frac{l_1}{m_1}$ and $\frac{l_2}{m_2}$, then $\frac{l_1l_2}{m_1m_2}=\frac{b+c}{c+a}$

$\therefore$ $\frac{m_1m_2}{l_1l_2}=\frac{c+a}{b+c}$ **Ans. : (D)**

(3) The lines are parallel to each other. So, they have the same direction cosines.

$\therefore$ The roots of equation $(aq^2+bp^2)\left(\frac{m}{n}\right)^2+2aqr\left(\frac{m}{n}\right)+(ar^2+cp^2)=0$ will be identical.

$\therefore$ Discriminant $\Delta=B^2-4AC=0$ i.e. $B^2=4AC$

$\therefore$ $(2aqr)^2=4(aq^2+bp^2)(ar^2+cp^2)$

$\therefore$ $a^2q^2r^2=a^2q^2r^2+acp^2q^2+abp^2r^2+bcp^4$

$\therefore \quad acq^2 + abr^2 + bcp^2 = 0$

$\therefore \quad \frac{p^2}{a} + \frac{q^2}{b} + \frac{r^2}{c} = 0$ (iv)

$\therefore \quad \sum \frac{p^2}{a} = 0$ **Ans. : (A)**

(4) If two roots of equation $(aq^2 + bp^2)\left(\frac{m}{n}\right)^2 + 2aqr\left(\frac{m}{n}\right) + (ar^2 + cp^2) = 0$ are $\frac{m_1}{n_1}$ and $\frac{m_2}{n_2}$,

then $\frac{m_1 m_2}{n_1 n_2} = \frac{ar^2 + cp^2}{aq^2 + bp^2}$

$\therefore \quad \frac{l_1 l_2}{br^2 + cq^2} = \frac{m_1 m_2}{ar^2 + cp^2} = \frac{n_1 n_2}{aq^2 + bp^2}$ (as per symmetry)

The lines are perpendicular to each other.

$\therefore \quad l_1 l_2 + m_1 m_2 + n_1 n_2 = 0$

$\therefore \quad (br^2 + cq^2) + (ar^2 + cp^2) + (aq^2 + bp^2) = 0$

$\therefore \quad p^2(b + c) + q^2(c + a) + r^2(a + b) = 0$ (v)

$\therefore \quad \sum p^2 (b + c) = 0$ **Ans. : (C)**

(5) The lines are mutually perpendicular and $p = q = r = 1$

$\therefore$ Substituting $p = q = r = 1$ in equation (v),

$2a + 2b + 2c = 0$

$\therefore \quad a + b + c = 0$ **Ans. : (A)**

(6) The lines are parallel and $p = q = r = 1$

$\therefore$ By taking $p = q = r = 1$, in equation (iv)

$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0$ **Ans. : (D)**

(166) Let A(3, 2, 1) be a point of R^3. Line L : $\frac{x-7}{2} = \frac{y-12}{-2} = \frac{z+1}{1}$ and plane $\pi : x + y + z = 11$ are given. From the above information, answer the following questions :

(1) The line passing through A and parallel to L meets the plane π at point B. Coordinates of B are .....

(A) (–13, 8, 6) (B) (13, 8, –6) (C) (13, 8, 6) (D) (13, –8, 6)

(2) The distance of point A from the plane π parallel to the line L is .....

(A) $\sqrt{30}$ (B) 15 (C) 30 (D) $2\sqrt{30}$

(3) The coordinates of the foot of perpendicular M from A to the plane π are .....

(A) $\left(\frac{8}{3}, \frac{11}{3}, \frac{14}{3}\right)$ (B) $\left(\frac{14}{3}, \frac{-11}{3}, \frac{8}{3}\right)$ (C) $\left(\frac{-14}{3}, \frac{-11}{3}, \frac{8}{3}\right)$ (D) $\left(\frac{14}{3}, \frac{11}{3}, \frac{8}{3}\right)$

(4) If the line parallel to L through A meets the plane π at B and if M is the foot of perpendicular drawn from A to the plane π, then BM = ...

(A) $\frac{5}{3}\sqrt{78}$ (B) $5\sqrt{78}$ (C) $\sqrt{78}$ (D) $3\sqrt{78}$

(5) The distance of point A from the point of intersection of line L and plane π is .....

(A) $\sqrt{641}$ (B) $\sqrt{849}$ (C) $\sqrt{757}$ (D) $\sqrt{477}$

Sol. (1) The equation of the line parallel to line L : $\frac{x-7}{2} = \frac{y-12}{-2} = \frac{z-1}{1}$ and passing through A(3, 2, 1) is $\frac{x-3}{2} = \frac{y-2}{-2} = \frac{z-1}{1}$.

A point $(2k + 3, -2k + 2, k + 1)$ of this line lies in the plane $\pi : x + y + z = 11$.

$\therefore\ 2k + 3 - 2k + 2 + k + 1 = 11$

$\therefore\ k = 5$

$\therefore$ The coordinates of point B are (13, –8, 6).

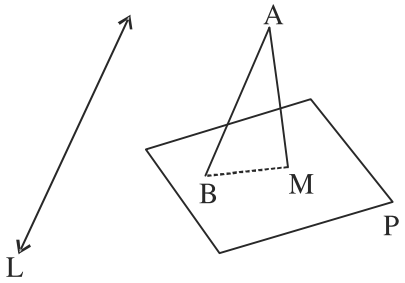


Figure 21.11

Ans. : (D)

(2) The distance of point A measured parallel to line L from the plane π is AB.

$\therefore\ AB = \sqrt{(13-3)^2 + (-8-2)^2 + (6-1)^2} = \sqrt{100 + 100 + 25} = 15$ **Ans. : (B)**

(3) The foot of the perpendicular to the plane π from A(3, 2, 1) is

$\frac{x-3}{1} = \frac{y-2}{1} = \frac{z-1}{1} = r$

where, $r = -\frac{ax_1 + by_1 + cz_1 + d}{al + bm + cn} = -\frac{3 + 2 + 1 - 11}{1(1) + 1(1) + 1(1)} = \frac{5}{3}$

$\therefore$ The foot of perpendicular M$(x, y, z) = \left(\frac{14}{3}, \frac{11}{3}, \frac{8}{3}\right)$ **Ans. : (D)**

(4) Point B(13, –8, 6) and M$\left(\frac{14}{3}, \frac{11}{3}, \frac{8}{3}\right)$

$\therefore$ The distance BM $= \sqrt{\left(13 - \frac{14}{3}\right)^2 + \left(-8 - \frac{11}{3}\right)^2 + \left(6 - \frac{8}{3}\right)^2}$

$= \sqrt{\left(\frac{25}{3}\right)^2 + \left(\frac{-35}{3}\right)^2 + \left(\frac{10}{3}\right)^2} = \frac{5}{3}\sqrt{25 + 49 + 4} = \frac{5}{3}\sqrt{78}$ **Ans. : (A)**

(5) $(2k + 7, -2k + 12, k - 1)$ for some $k \in R$ of the line L is on the plane $\pi : x + y + z = 11$.

$\therefore\ 2k + 7 - 2k + 12 + k - 1 = 11$

$\therefore\ k = -7$

$\therefore$ The point of intersection of the line L and the plane π is C(–7, 26, –8).

$\therefore$ The distance AC $= \sqrt{(-7-3)^2 + (26-2)^2 + (-8-1)^2} = \sqrt{100 + 576 + 81} = \sqrt{757}$

Ans. : (C)

(167) Let a, b, c be three real numbers satisfying $[a\ \ b\ \ c]\begin{bmatrix} 1 & 9 & 7 \\ 8 & 2 & 7 \\ 7 & 3 & 7 \end{bmatrix} = [0\ \ 0\ \ 0]$ ...(i). [IIT : 2011]

From this, answer the following questions :

(1) If the point P(a, b, c), with reference to (i), lies on the plane $2x + y + z = 1$, then the value of $7a + b + c$ is .....

(A) 0 (B) 12 (C) 7 (D) 6

(2) Let ω be a solution of $x^3 - 1 = 0$ with $Im(\omega) > 0$. If $a = 2$ and b and c satisfy (i), then the value of $\frac{3}{\omega^a} + \frac{1}{\omega^b} + \frac{3}{\omega^c}$ is equal to .....

(A) –2 (B) 2 (C) 3 (D) –3

(3) Let $b = 6$, with a and c satisfying (i). If α and β are the roots of the quadratic equation $ax^2 + bx + c = 0$, then $\sum_{n=0}^{\infty}\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)^n$ = ...

(A) 6 (B) 7 (C) $\frac{6}{7}$ (D) ∞

Sol. $[a \quad b \quad c]\begin{bmatrix}1 & 9 & 7\\ 8 & 2 & 7\\ 7 & 3 & 7\end{bmatrix} = [0 \quad 0 \quad 0]$

$\therefore$ $a + 8b + 7c = 0$ (1)

$9a + 2b + 3c = 0$ (2)

$7a + 7b + 7c = 0$

i.e. $a + b + c = 0$ (3)

$\therefore$ $\frac{a}{10} = \frac{b}{60} = \frac{c}{-70} = \frac{k}{10}$ where $k \in R$

$\therefore$ $(a, b, c) = (k, 6k, -7k)$

(1) Point P(a, b, c) is on the plane $2x + y + z = 1$.

$\therefore$ $2k + 6k - 7k = 1$

$\therefore$ $k = 1$

$\therefore$ $a = 1, \quad b = 6, \quad c = -7$

Now, $7a + b + c = 7 + 6 - 7 = 6$ **Ans. : (D)**

(2) For $a = 2$, $k = 2$. So, $b = 12$, $c = -14$

Now, $\frac{3}{\omega^a} + \frac{1}{\omega^b} + \frac{3}{\omega^c} = \frac{3}{\omega^2} + \frac{1}{\omega^{12}} + 3\omega^{14}$

$= \frac{3\omega}{\omega^3} + \frac{1}{(\omega^3)^4} + 3\omega^2 \cdot (\omega^3)^4$

$= 3\omega + 1 + 3\omega^2$ $(\omega^3 = 1)$

$= 3(\omega + \omega^2) + 1 = 3(-1) + 1 = -2$ $(1 + \omega + \omega^2 = 0)$

Ans. : (A)

(3) For $b = 6$, $k = 1$. So, $a = 1$ and $c = -7$

Now, $ax^2 + bx + c = 0$

$\therefore$ $x^2 + 6x - 7 = 0$

$\therefore$ $\alpha = -7$ or $\beta = 1$

$$\sum_{n=0}^{\infty}\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)^n = \sum_{n=0}^{\infty}\left(\frac{1}{-7} + 1\right)^n = \sum_{n=0}^{\infty}\left(\frac{6}{7}\right)^n = 1 + \frac{6}{7} + \left(\frac{6}{7}\right)^2 + \ldots\infty = \frac{1}{1 - \frac{6}{7}} = 7$$

Ans. : (B)

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