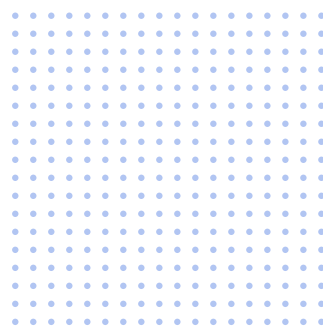




FOR ISI ADMISSION TEST EXAM PREPARATION

ISI Admission Test 2024 Question Paper · JRF Physics, JRF QROR, MS(QMBA), M.Tech(QROR) MMA

EXAM

ISI Admission Test

YEAR

2024

TYPE

Question Paper

SUBJECT

JRF Physics, JRF QROR, MS(QMBA), M.Tech(QROR) MMA

1. Let A_1 and A_2 be two arbitrarily chosen subsets of $\{1, 2, \dots, n\}$.

The number of ways such that $A_1 \cap A_2 = \emptyset$ is

- (A) 2^n (B) $4^n - 3^n$ (C) 3^n (D) $3^n - 1$

2. The limit

$$\lim_{n \rightarrow \infty} \frac{1 + \sqrt{2} + \sqrt[3]{3} + \dots + \sqrt[n]{n}}{n}$$

- (A) is equal to 0 (B) is equal to 1
(C) is equal to 2 (D) does not exist

3. For $x, y \geq 0$, consider the regions $A = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1\}$ and $B = \{(x, y) \in \mathbb{R}^2 : x^4 + y^4 > 1\}$. Then

- (A) $A \subsetneq B$ (B) $B \subsetneq A$
(C) $A \cap B = \emptyset$ (D) $A \cup B = \mathbb{R}^2$

4. The roots of the equation $x^4 + x^3 + x^2 + x + 1 = 0$, when plotted in the complex plane, are

- (A) the vertices of a square
(B) some of the vertices of a regular pentagon
(C) some of the vertices of a regular hexagon
(D) some of the vertices of a regular heptagon

5. Let $f, g : [0, 1] \rightarrow (0, \infty)$ be two continuously differentiable functions such that $f'(x) = g'(x)$ for all $x \in [0, 1]$. Suppose that

$$\frac{f(\frac{1}{2})}{g(\frac{1}{2})} = 1. \text{ Then } \frac{f(\frac{1}{4})}{g(\frac{1}{4})} \text{ is equal to}$$

- (A) 1 (B) 2
(C) $\frac{1}{2}$ (D) either 2 or $\frac{1}{4}$

6. For $w \in \mathbb{C}$, let $\operatorname{Re} w$ and $\operatorname{Im} w$ denote the real and imaginary parts of w , respectively. Suppose $z \in \mathbb{C}$ is such that $|z| = 1$. Which of the following is necessarily true?

- (A) $\operatorname{Re} z$ is rational implies that $\operatorname{Re} z^2$ is rational
(B) $\operatorname{Re} z^2$ is rational implies that $\operatorname{Re} z$ is rational
(C) $\operatorname{Im} z$ is rational implies that $\operatorname{Im} z^2$ is rational
(D) $\operatorname{Im} z^2$ is rational implies that $\operatorname{Im} z$ is rational

7. The maximum value of the function

$$f(x) = \frac{x}{2} - \frac{1}{1+x^2}$$

on the set $\{x \in \mathbb{R} : x(x-1) \leq 12\}$ is

- (A) 0 (B) $\frac{33}{17}$ (C) $-\frac{8}{5}$ (D) -1

8. There are two coins in a box. One of the coins is a fair coin and for the other, the probability of head is $\frac{2}{3}$. A coin is chosen at random from the box and tossed 5 times. Given that at least 4 heads were obtained, what is the conditional probability that the fair coin was chosen?
- (A) $\frac{729}{2521}$ (B) $\frac{1}{2}$ (C) $\frac{2}{3}$ (D) $\frac{243}{275}$
9. Let m be a positive integer such that $2^m + 1$ is a prime. Then which of the following statements is true?
- (A) m must be a prime
(B) m must be an odd integer
(C) m must be a power of 2
(D) m must be a power of 3
10. Let $f : [0, 1] \rightarrow (0, \infty)$ be a continuous function. Define
- $$g(x) = \int_0^x f(t) dt \quad \text{for } 0 \leq x \leq 1.$$
- If $g(1) = 1$, then the equation $g(x) = \frac{1}{2}$
- (A) may not have a solution
(B) may have more than one solution
(C) will have at least two solutions
(D) will have a unique solution

11. It is given that the equation $(\cos p - 1)x^2 + (\cos p)x + \sin p = 0$ has a real root. Then p must lie in the interval

(A) $(0, 2\pi)$ (B) $(-\pi, \pi)$ (C) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (D) $(0, \pi)$

12. Let

$$S = \left\{ \begin{pmatrix} a & b\sqrt{5} \\ -b\sqrt{5} & a \end{pmatrix} : a, b \in \mathbb{Z} \right\}.$$

The number of $A \in S$ such that A^{-1} is also in S is

- (A) 2 (B) 4
(C) more than 4 but finite (D) infinite

13. The equations of three curves are given below:

$$\begin{aligned} y_1(x) &= \frac{1}{2}x + \frac{1}{2}x^5, \\ y_2(x) &= \frac{1}{2}x^3 + \frac{1}{2}x^7, \\ y_3(x) &= 1 + \frac{1}{2}x^3 + \frac{1}{2}x^7. \end{aligned}$$

Which of the following is true?

- (A) y_1 and y_2 meet at 2 points, y_1 and y_3 meet at 1 point
(B) y_1 and y_2 meet at 3 points, y_1 and y_3 meet at 2 points
(C) y_1 and y_2 meet at more than 3 points, y_1 and y_3 meet at 1 point
(D) y_1 and y_2 meet at 3 points, y_1 and y_3 meet at 1 point
14. Let $C = \left\{ \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \end{pmatrix} \right\}$ be a basis of $\mathbb{R}^2$ and $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be defined by $T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x+y \\ x-2y \end{pmatrix}$. Let $T(C)$ represent the matrix of T with respect to the basis C . Which of the following is true?

- (A) $T(C) = \begin{pmatrix} -3 & -2 \\ 3 & 1 \end{pmatrix}$ (B) $T(C) = \begin{pmatrix} 3 & -2 \\ -3 & 1 \end{pmatrix}$
(C) $T(C) = \begin{pmatrix} -3 & -1 \\ 3 & 2 \end{pmatrix}$ (D) $T(C) = \begin{pmatrix} 3 & -1 \\ -3 & 2 \end{pmatrix}$

15. Suppose that A is an $n \times n$ matrix where $n \geq 2$. Then $A^n = 0$ if and only if

- (A) $A = 0$
- (B) all the eigenvalues of A are zero
- (C) A is singular
- (D) $\text{Rank } A \leq 1$

16. For fixed $a_1, a_2, \dots, a_5 \in \mathbb{R}$, the graph of

$$a_1x^2 + a_2x + a_3y^2 + a_4y + a_5 = 0$$

on the xy -plane can never be

- (A) a circle
- (B) an ellipse
- (C) a parabola with axis parallel to either the x -axis or the y -axis
- (D) a parabola with axis parallel to neither the x -axis nor the y -axis

17. If

$$2021^{2024} = 100m + n,$$

for some $m \in \{0, 1, 2, \dots\}$ and $n \in \{0, 1, \dots, 99\}$, then the value of n is

- (A) 21
- (B) 31
- (C) 61
- (D) 81

18. Let A be an $m \times n$ matrix with each entry equal to $+1$, -1 or 0 such that every column has exactly one $+1$ and exactly one -1 . Then

- (A) Rank $A \leq m - 1$ (B) Rank $A = n$
 (C) $m \leq n$ (D) $m - 1 \leq n$

19. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a bounded differentiable function such that

$$f(x) + f'(x) = \sin x \quad \text{for all } x \in \mathbb{R},$$

where f' is the derivative of f . Then the value of $f(0)$ is

- (A) 0 (B) 1 (C) $\frac{1}{2}$ (D) $-\frac{1}{2}$

20. Let $f(\theta) = 2(1 - \cos \theta) - \theta \sin \theta = 4 \sin \frac{\theta}{2} (\sin \frac{\theta}{2} - \frac{\theta}{2} \cos \frac{\theta}{2})$. Then which of the following assertions is true?

- (A) $f(\theta) < 0$ for $\theta \in (0, \pi)$
 (B) $f(\theta) < 0$ for $\theta \in (\pi, 2\pi)$
 (C) $f(\theta) > 0$ for $\theta \in (0, 2\pi)$
 (D) $f(\theta)$ takes both negative and positive values in $(0, 2\pi)$

21. The value of the infinite sum

$$\sum_{n=0}^{\infty} 2^{-n} \cos \left(\frac{n\pi}{3} \right)$$

equals

- (A) 1 (B) $4 + 2\sqrt{3}$ (C) $2 + \sqrt{3}$ (D) $2 - \sqrt{3}$

22. Consider the parabola $y^2 = 4ax$, $a > 0$. From any point $A(x_1, y_1)$ on the parabola a line is drawn which is perpendicular to the line $x + a = 0$. Let B be the foot of the perpendicular. If C is the focus of the parabola, then the triangle ABC is equilateral implies

- (A) $y_1 = \pm 3a$ (B) $y_1 = \pm 2\sqrt{3}a$
 (C) $y_1 = \pm 4a$ (D) $x_1 = 2a$

23. Let $S_n = \sum_{k=1}^n \frac{1}{k}$. Then

- (A) $S_{2^n} \geq \frac{n}{2}$ for every $n \geq 1$
 (B) $\{S_n\}_{n=1}^\infty$ is a bounded sequence
 (C) $|S_{2^n} - S_{2^{n-1}}| \rightarrow 0$ as $n \rightarrow \infty$
 (D) $\frac{S_n}{n} \rightarrow 1$ as $n \rightarrow \infty$

24. Let $p(x)$ be a polynomial in x of degree $k \geq 0$. For $n = 1, 2, \dots$, define $a_n = p(n+1) - p(n)$. If $\{a_n\}_{n=1}^\infty$ is in arithmetic progression, then we must have

- (A) $k \leq 1$ (B) $k \leq 2$
 (C) $k \geq 3$ (D) $k \geq 4$

25. Consider the circular disc on the xy -plane with centre $(0, 1)$ and radius 1 unit. Let $A = (0, 2)$. The disc is rolled on the x -axis so that its centre is now at $(1, 1)$. The point A has now moved to a point, say, B . Then the coordinates of B are

- (A) $(1 + \sin \frac{1}{2\pi}, 1 + \cos \frac{1}{2\pi})$ (B) $(1 + \sin 1, 1 + \cos 1)$
 (C) $(1 + \sin \frac{1}{2\pi}, 1 - \cos \frac{1}{2\pi})$ (D) $(1, 2)$

26. For $x \in \mathbb{R}$, let $[x]$ denote the greatest integer less than or equal to x . Let $e^2 < a < e^3$. Then

$$\int_{1/a}^1 [\log_e x] \frac{dx}{x^2}$$

is equal to

- (A) $e^2 + e + 1 - a$ (B) $e^2 + e + 1 - 2a$
 (C) $e^2 + e + 1 - 3a$ (D) $e^2 + e + 1 - 4a$

27. Let $n \geq 2$. Suppose $\alpha_1, \alpha_2, \dots, \alpha_n$ are real numbers such that $\alpha_1 < \alpha_2 < \dots < \alpha_n$. Let

$$P(x) = (x - \alpha_1)^2(x - \alpha_2) \dots (x - \alpha_n), \quad x \in \mathbb{R}.$$

If P' is the derivative of P , then $P'(x)$ has

- (A) no real root
 (B) n real roots, the smallest of which is α_1
 (C) $n - 1$ real roots, the smallest of which is α_1
 (D) n real roots, the smallest of which is strictly larger than α_1
28. Let A be a real matrix of order 3 such that $AA^T = I_3$. Here A^T denotes the transpose of the matrix A and I_3 is the identity matrix of order 3. Assume that $\det A = -1$ and $A^T \neq A$. Let $\text{Tr } A$ denote the trace of the matrix A . Then the quantity $(\text{Tr } A)^2 - \text{Tr } A^2$ simplifies to

- (A) $\text{Tr } A$ (B) $2(\text{Tr } A)$
 (C) $3(\text{Tr } A)$ (D) $4(\text{Tr } A)$

29. The 26 letters of the English alphabet are randomly permuted. What is the probability that at least two of the three letters I, N, D come next to each other?

(A) $\frac{72}{325}$ (B) $\frac{3}{26}$ (C) $\frac{3}{13}$ (D) $\frac{3}{25 \times 26}$

30. The value of

$$\int_0^1 \int_{x^2}^1 e^{y^{3/2}} dy dx$$

is

(A) $\frac{2}{3}(e - 1)$ (B) $e - 1$ (C) $\frac{2e}{3}$ (D) $\frac{2e}{3} - \frac{3}{2}$