Basic Geometrical Ideas

Chapter 4

4.1 Introduction

Geometry has a long and rich history. The term 'Geometry' is the English equivalent of the Greek word '*Geometron*'. '*Geo*' means Earth and '*metron*' means Measurement. According to historians, the geometrical ideas shaped up in ancient times, probably due to the need in art, architecture and measurement. These include occasions when the boundaries of cultivated lands had to be marked without giving room for complaints. Construction of magnificent palaces, temples, lakes, dams and cities, art and architecture propped up these ideas. Even today geometrical ideas are reflected in all forms of art, measurements, architecture, engineering, cloth designing etc. You observe and use different objects like boxes, tables, books, the tiffin box you carry to your school for lunch, the ball with which you play and so on. All such objects have different shapes. The ruler which you use, the pencil with which you write are straight. The pictures of a bangle, the one rupee coin or a ball appear round.



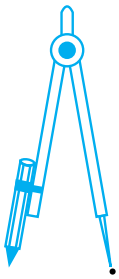
Here, you will learn some interesting facts that will help you know more about the shapes around you.

4.2 Points

By a sharp tip of the pencil, mark a dot on the paper. Sharper the tip, thinner will be the dot. This almost invisible tiny dot will give you an idea of a point.

A point determines a location.

These are some models for a point :



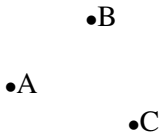
The tip of a compass



The sharpened end of a pencil

The pointed end of a needle

If you mark three points on a paper, you would be required to distinguish them. For this they are denoted by a single capital letter like A,B,C.

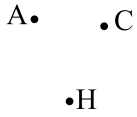


These points will be read as point A, point B and point C.

Of course, the dots have to be invisibly thin.

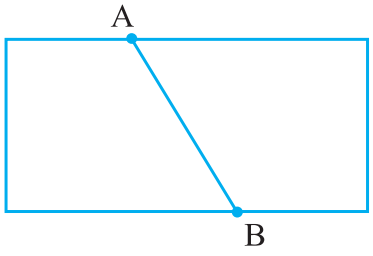
Try These

1. With a sharp tip of the pencil, mark four points on a paper and name them by the letters A,C,P,H. Try to name these points in different ways. One such way could be this



2. A star in the sky also gives us an idea of a point. Identify at least five such situations in your daily life.

4.3 A Line Segment

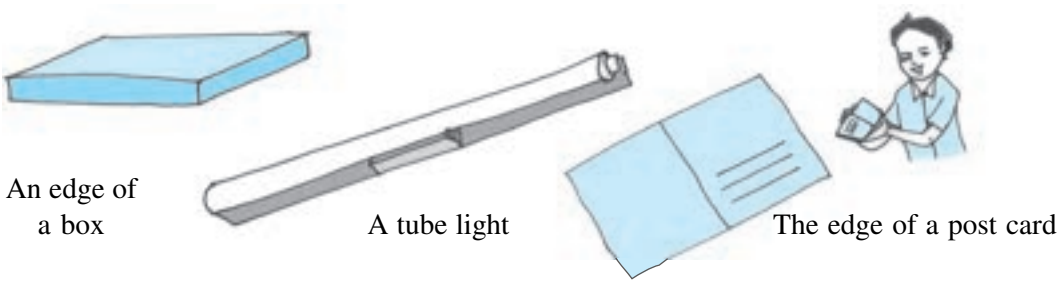


Fold a piece of paper and unfold it. Do you see a fold? This gives the idea of a line segment. It has two end points A and B.

Take a thin thread. Hold its two ends and stretch it without a slack. It represents a line segment. The ends held by hands are the end points of the line segment.



The following are some models for a line segment :



An edge of a box | A tube light | The edge of a post card

Try to find more examples for line segments from your surroundings.

Mark any two points A and B on a sheet of paper. Try to connect A to B by all possible routes. (Fig 4.1)

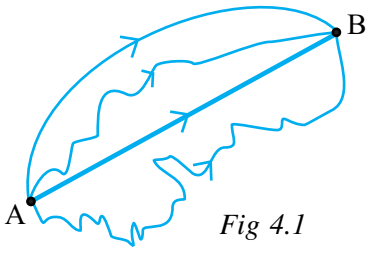


Fig 4.1

What is the shortest route from A to B?

This shortest join of point A to B (including A and B) shown here is a line segment. It is denoted by $\overline{AB}$ or $\overline{BA}$. The points A and B are called the end points of the segment.

Try These

1. Name the line segments in the figure 4.2. Is A, the end point of each line segment?

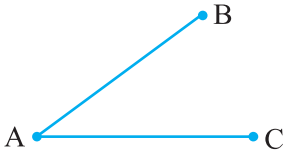


Fig 4.2

4.4 A Line

Imagine that the line segment from A to B (i.e. $\overline{AB}$) is extended beyond A in one direction and beyond B in the other direction without any end (see figure). You now get a **model for a line**.



Do you think you can draw a complete picture of a line? No. (Why?)

A line through two points A and B is written as $\overleftrightarrow{AB}$. It extends indefinitely in both directions. So it contains a countless number of points. (Think about this).

Two points are enough to fix a line. We say 'two points determine a line'.

The adjacent diagram (Fig 4.3) is that of a line PQ written as $\overleftrightarrow{PQ}$. Sometimes a line is denoted by a letter like *l, m.*

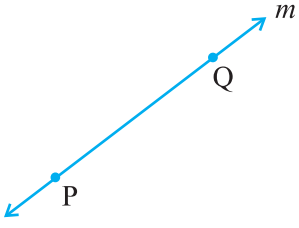


Fig 4.3

4.5 Intersecting Lines

Look at the diagram (Fig 4.4). Two lines l_1 and l_2 are shown. Both the lines pass through point P. We say l_1 and l_2 intersect at P. If two lines have one common point, they are called *intersecting lines*.

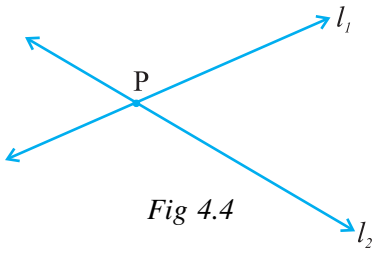
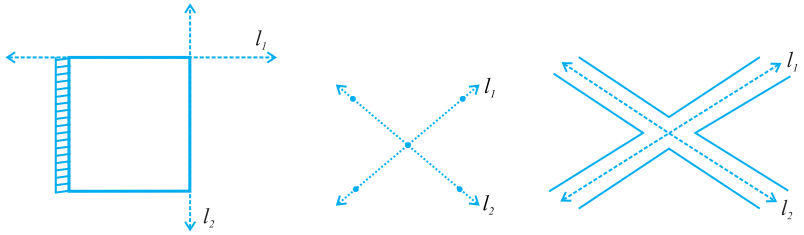


Fig 4.4

The following are some models of a pair of intersecting lines (Fig 4.5) :

Try to find out some more models for a pair of intersecting lines.



Two adjacement edges of your notebook | The letter X of the English alphabet | Crossing-roads

Fig 4.5

Do This

Take a sheet of paper. Make two folds (and crease them) to represent a pair of intersecting lines and discuss :

(a) Can two lines intersect in more than one point?

(b) Can more than two lines intersect in one point?

4.6 Parallel Lines

Let us look at this table (Fig 4.6). The top ABCD is flat. Are you able to see some points and line segments?

Are there intersecting line segments?

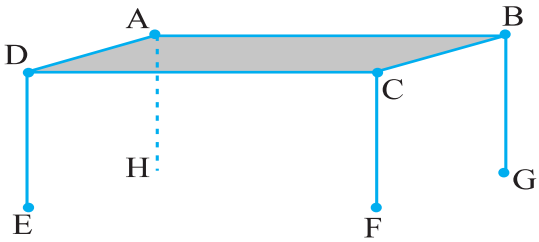


Fig 4.6

Yes, AB and BC intersect at the point B.

Which line segments intersect at A? at C? at D?

Do the lines AD and CD intersect?

Do the lines $\overleftrightarrow{AD}$ and $\overleftrightarrow{BC}$ intersect?

You find that on the table's surface there are line segment which will not meet, however far they are extended. $\overleftrightarrow{AD}$ and $\overleftrightarrow{BC}$ form one such pair. Can you identify one more such pair of lines (which do not meet) on the top of the table?

Lines like these which do not meet are said to be parallel; and are called **parallel lines**.

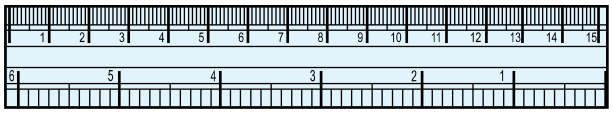
Think, discuss and write

Where else do you see parallel lines? Try to find ten examples.

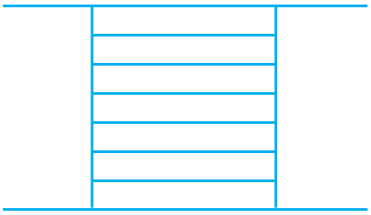
If two lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are parallel, we write $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$.

If two lines l_1 and l_2 are parallel, we write $l_1 \parallel l_2$.

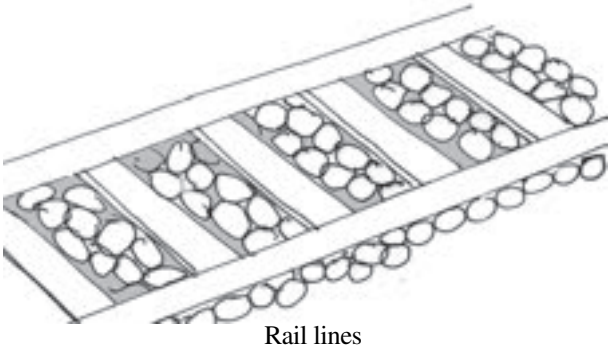
Can you identify parrallel lines in the following figures?



The opposite edges of ruler (scale)

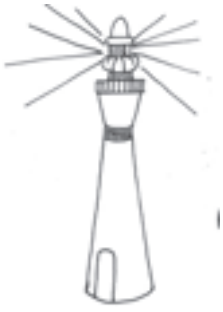


The cross-bars of this window



Rail lines

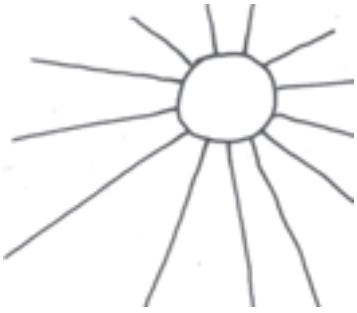
4.7 Ray



Beam of light from a light house



Ray of light from a torch



Sun rays

The following are some models for a ray :

A *ray* is a portion of a line. It starts at one point (called starting point or initial point) and goes endlessly in a direction.

Look at the diagram (Fig 4.7) of ray shown here. Two points are shown on the ray. They are (a) A, the starting point (b) P, a point on the path of the ray.

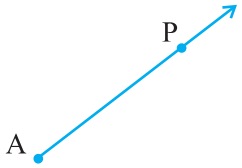


Fig 4.7

We denote it by $\overrightarrow{AP}$.

Think, discuss and write

If $\overrightarrow{PQ}$ is a ray,

(a) What is its starting point?

(b) Where does the point Q lie on the ray?

(c) Can we say that Q is the starting point of this ray?

Try These

1. Name the rays given in this picture (Fig 4.8).
2. Is T a starting point of each of these rays?

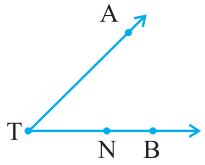


Fig 4.8

Here is a ray $\overrightarrow{OA}$ (Fig 4.9). It starts at O and passes through the point A. It also passes through the point B.

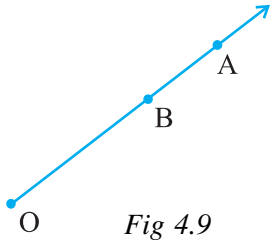


Fig 4.9

Can you also name it as $\overrightarrow{OB}$? Why?

$\overrightarrow{OA}$ and $\overrightarrow{OB}$ are same here.

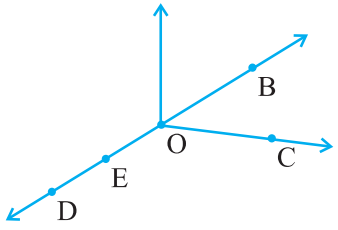
Can we write $\overrightarrow{OA}$ as $\overrightarrow{AO}$? Why or why not?

Draw five rays and write appropriate names for them.

What do the arrows on each of these rays show?

EXERCISE 4.1

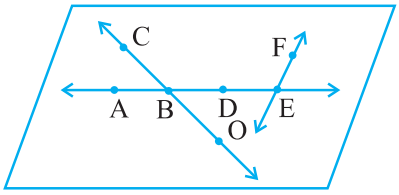
1. Use the figure to name :
   (a) Five points
   (b) A line
   (c) Four rays
   (d) Five line segments



2. Name the line given in all possible (twelve) ways, choosing only two letters at a time from the four given.



3. Use the figure to name :



(a) Line containing point E.
(b) Line passing through A.
(c) Line on which O lies
(d) Two pairs of intersecting lines.

4. How many lines can pass through (a) one given point? (b) two given points?

5. Draw a rough figure and label suitably in each of the following cases:

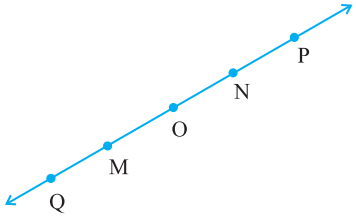
(a) Point P lies on $\overline{AB}$.

(b) $\overleftrightarrow{XY}$ and $\overleftrightarrow{PQ}$ intersect at M.

(c) Line l contains E and F but not D.

(d) $\overleftrightarrow{OP}$ and $\overleftrightarrow{OQ}$ meet at O.

6. Consider the following figure of line $\overleftrightarrow{MN}$. Say whether following statements are true or false in context of the given figure.



(a) Q, M, O, N, P are points on the line $\overleftrightarrow{MN}$.

(b) M, O, N are points on a line segment $\overline{MN}$.

(c) M and N are end points of line segment $\overline{MN}$.

(d) O and N are end points of line segment $\overline{OP}$.

(e) M is one of the end points of line segment $\overline{QO}$.

(f) M is point on ray $\overrightarrow{OP}$.

(g) Ray $\overrightarrow{OP}$ is different from ray $\overrightarrow{QP}$.

(h) Ray $\overrightarrow{OP}$ is same as ray $\overrightarrow{OM}$.

(i) Ray $\overrightarrow{OM}$ is not opposite to ray $\overrightarrow{OP}$.

(j) O is not an initial point of $\overrightarrow{OP}$.

(k) N is the initial point of $\overrightarrow{NP}$ and $\overrightarrow{NM}$.

4.8 Curves

Have you ever taken a piece of paper and just doodled? The pictures that are results of your doodling are called *curves*.

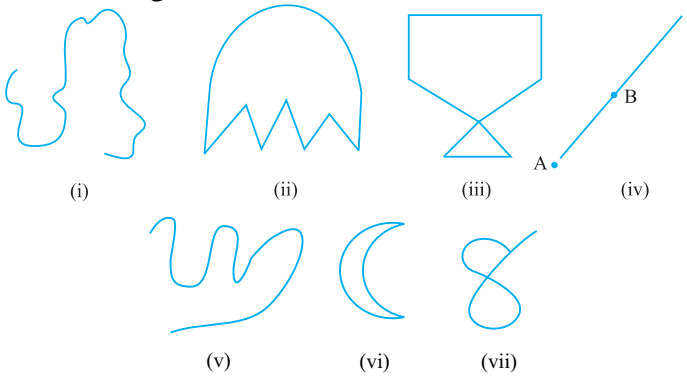


Fig 4.10

You can draw some of these drawings without lifting the pencil from the paper and without the use of a ruler. These are all curves (Fig 4.10).

'Curve' in everyday usage means "not straight". In Mathematics, a curve can be straight like the one shown in fig 4.10 (iv).

Observe that the curves (iii) and (vii) in Fig 4.10 cross themselves, whereas the curves (i), (ii), (v) and (vi) in Fig 4.10 do not. If a curve does not cross itself, then it is called a **simple curve**.

Draw five more simple curves and five curves that are not simple.

Consider these now (Fig 4.11).

What is the difference between these two? The first i.e. Fig 4.11 (i) is an **open curve** and the second i.e. Fig 4.11(ii) is a **closed curve**. Can you identify some closed and open curves from the figures Fig 4.10 (i), (ii), (v), (vi)? Draw five curves each that are open and closed.

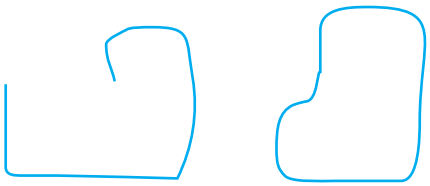


Fig 4.11

Position in a figure

A court line in a tennis court divides it into three parts : inside the line, on the line and outside the line. You cannot enter inside without crossing the line.

A compound wall separates your house from the road. You talk about 'inside' the compound, 'on' the boundary of the compound and 'outside' the compound.

In a closed curve, thus, there are three parts.

(i) interior ('inside') of the curve

(ii) boundary ('on') of the curve and

(iii) exterior ('outside') of the curve.

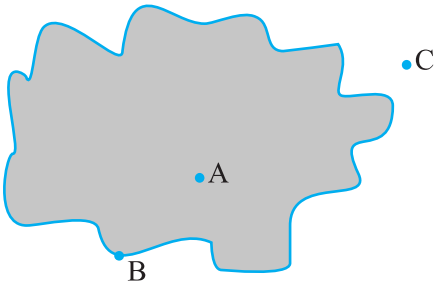


Fig 4.12

In the figure 4.12, A is in the interior, C is in the exterior and B is on the curve.

The interior of a curve together with its boundary is called its "**region**".

4.9 Polygons

Look at these figures 4.13 (i), (ii), (iii), (iv), (v) and (vi).

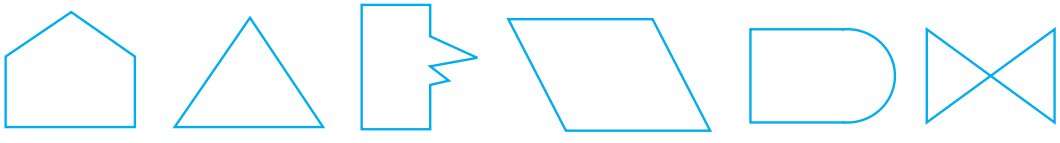


Fig 4.13

What can you say? Are they closed? How does each one of them differ from the other? (i), (ii), (iii), (iv) and (vi) are special because they are made up entirely of line segments. Out of these (i), (ii), (iii) and (iv) are also simple closed curves. They are called **polygons.**

So, a figure is a polygon if it is a simple closed figure made up entirely of line segments. Draw ten differently shaped polygons.



Do This

Try to form a polygon with

1. Five matchsticks.
2. Four matchsticks.
3. Three matchsticks.
4. Two matchsticks.

In which case was it not possible? Why?

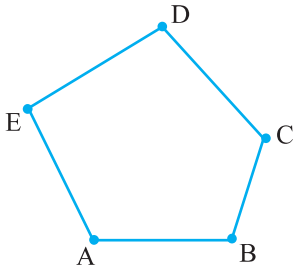


Fig 4.14

Sides, vertices and diagonals

Examine the figure given here (Fig 4.14).

Give justification to call it a polygon.

The line segments forming a polygon are called its *sides*.

What are the sides of polygon ABCDE? (Note how the corners are named in order.)

Sides are $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DE}$ and $\overline{EA}$.

The meeting point of a pair of sides is called its *vertex*.

Sides $\overline{AE}$ and $\overline{ED}$ meet at E, so E is a vertex of the polygon ABCDE. Points B and C are its other vertices. Can you name the sides that meet at these points?

Can you name the other vertices of the above polygon ABCDE?

Any two sides with a common end point are called the *adjacent sides* of the polygon.

Are the sides $\overline{AB}$ and $\overline{BC}$ adjacent? How about $\overline{AE}$ and $\overline{DC}$?

The end points of the same side of a polygon are called the *adjacent vertices*. Vertices E and D are adjacent, whereas vertices A and D are not adjacent vertices. Do you see why?

Consider the pairs of vertices which are not adjacent. The joins of these vertices are called the diagonals of the polygon.

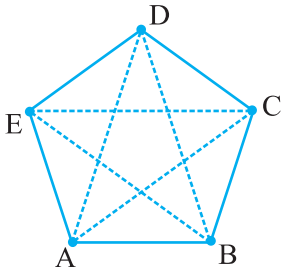


Fig 4.15

In the figure 4.15, $\overline{AC}$, $\overline{AD}$, $\overline{BD}$, $\overline{BE}$ and $\overline{CE}$ are diagonals.

Is $\overline{BC}$ a diagonal, Why or why not?

If you try to join adjacent vertices, will the result be a diagonal?

Name all the sides, adjacent sides, adjacent vertices of the figure ABCDE (Fig 4.15).

Draw a polygon ABCDEFGH and name all the sides, adjacent sides and vertices as well as the diagonals of the polygon.

EXERCISE 4.2

1. Classify the following curves as (i) Open or (ii) Closed.



(a) (b) (c) (d) (e)

2. Draw rough diagrams to illustrate the following :
   (a) Open curve (b) Closed curve.
3. Draw any polygon and shade its interior.
4. Consider the given figure and answer the questions :
   (a) Is it a curve? (b) Is it closed?



5. Illustrate, if possible, each one of the following with a rough diagram:
   (a) A closed curve that is not a polygon.
   (b) An open curve made up entirely of line segments.
   (c) A polygon with two sides.

4.10 Angles

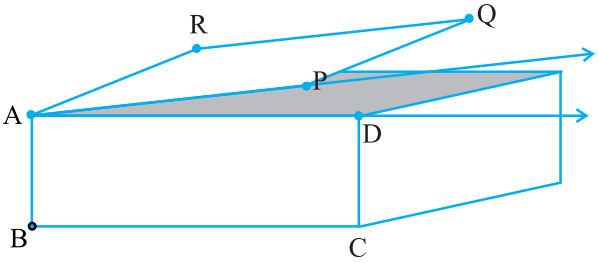


Fig 4.16

Angles are made when corners are formed.

Here is a picture (Fig 4.16) where the top of a box is like a hinged lid. The edges AD of the box and AP of the door can be imagined as two rays $\overrightarrow{AD}$ and $\overrightarrow{AP}$. These two rays have a common initial point A. The two rays here together are said to form an angle.

An angle is made up of two rays starting from a common initial point.

The two rays forming the angle are called the *arms* or *sides* of the angle.

The common initial point is the *vertex* of the angle.

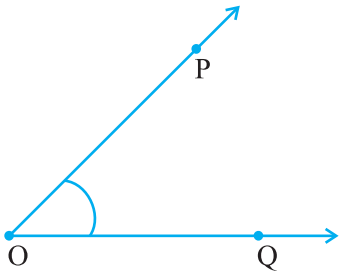


Fig 4.17

This is an angle formed by rays $\overrightarrow{OP}$ and $\overrightarrow{OQ}$ (Fig 4.17). To show this we use a small curve at the vertex. (see Fig 4.17). O is the vertex. What are the sides? Are they not $\overrightarrow{OP}$ and $\overrightarrow{OQ}$?

How can we name this angle? We can simply say that it is an angle at O. To be more specific we identify some two points, one on each side and the vertex to name the angle. Angle POQ is thus a better way of naming the angle. We denote this by $\angle POQ$.

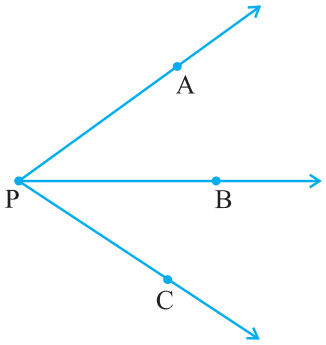


Fig 4.18

Think, discuss and write

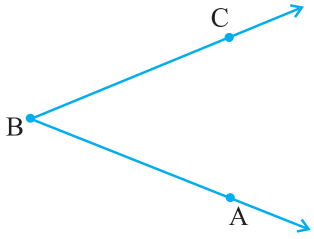
Look at the diagram (Fig 4.18).What is the name of the angle? Shall we say $\angle P$? But then which one do we mean? By $\angle P$ what do we mean? Is naming an angle by vertex helpful here? Why not?

By $\angle P$ we may mean $\angle APB$ or $\angle CPB$ or even $\angle APC$! We need more information.

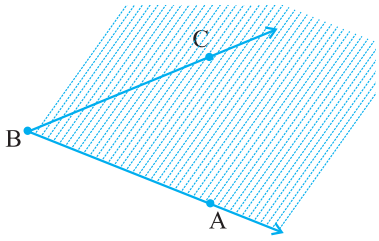
Note that in specifying the angle, the vertex is always written as the middle letter.

Do This

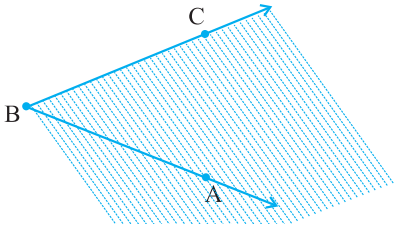
Take any angle, say $\angle ABC$.



Shade that portion of the paper bordering $\overrightarrow{BA}$ and where $\overrightarrow{BC}$ lies.



Now shade in a different colour the portion of the paper bordering $\overrightarrow{BC}$ and where $\overrightarrow{BA}$ lies.



The portion common to both shadings is called the interior of $\angle ABC$ (Fig 4.19). (Note that **the interior** is not a restricted area; it extends indefinitely since the two sides extend indefinitely).

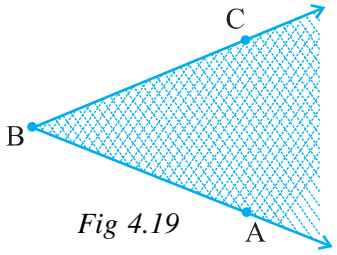


Fig 4.19

In this diagram (Fig 4.20), X is in the interior of the angle, Z is not in the interior but in the exterior of the angle; and S is on the $\angle PQR$. Thus, the angle also has three parts associated with it.

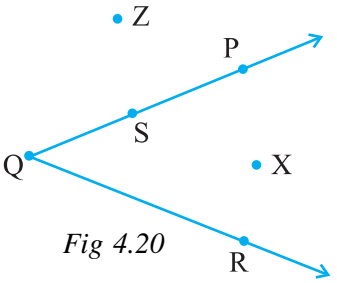
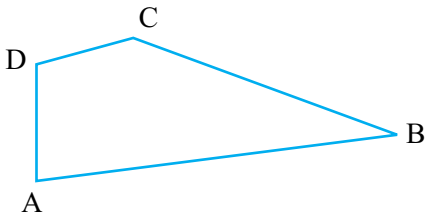


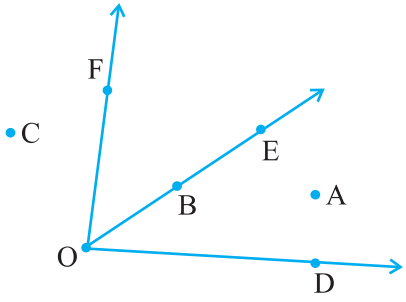
Fig 4.20

EXERCISE 4.3

1. Name the angles in the given figure.



2. In the given diagram, name the point(s)
   (a) In the interior of $\angle DOE$
   (b) In the exterior of $\angle EOF$
   (c) On $\angle EOF$



3. Draw rough diagrams of two angles such that they have
   (a) One point in common.
   (b) Two points in common.
   (c) Three points in common.
   (d) Four points in common.
   (e) One ray in common.

What have we discussed?

1. A point determines a location. It is usually denoted by a capital letter.
2. A line segment corresponds to the shortest distance between two points. The line segment joining points A and B is denoted by $\overline{AB}$.
3. A line is obtained when a line segment like $\overline{AB}$ is extended on both sides indefinitely; it is denoted by $\overleftrightarrow{AB}$ or sometimes by a single small letter like *l*.
4. Two distinct lines meeting at a point are called *intersecting lines*.
5. Two lines in a plane are said to be parallel if they do not meet.
6. A ray is a portion of line starting at a point and going in one direction endlessly.
7. Any drawing (straight or non-straight) done without lifting the pencil may be called a curve. In this sense, a line is also a curve.
8. A simple curve is one that does not cross itself.
9. A curve is said to be closed if its ends are joined; otherwise it is said to be open.
10. A polygon is a simple closed curve made up of line segments. Here,
    (i) The line segments are the sides of the polygon.
    (ii) Any two sides with a common end point are adjacent sides.
    (iii) The meeting point of a pair of sides is called a *vertex*.
    (iv) The end points of the same side are adjacent vertices.
    (v) The join of any two non-adjacent vertices is a diagonal.
11. An angle is made up of two rays starting from a common starting/initial point.

    Two rays $\overrightarrow{OA}$ and $\overrightarrow{OB}$ make $\angle AOB$ (or also called $\angle BOA$).

    An angle leads to three divisions of a region:

    On the angle, the interior of the angle and the exterior of the angle.