

Q1) A cuboidal water tank is 6 m long, 5 m wide and 4.5 m deep. How many liters of water can it hold?

Solution:

Given data:

Length (l) = 6m

Breadth (b) = 5m

Height (h) = 4.5m

Volume of the tank = $l \times b \times h$

$$= 6 \times 5 \times 4.5$$

$$= 135\text{m}^3$$

It is given that,

$$1\text{m}^3 = 1000 \text{ liters}$$

$$\text{Therefore, } 135\text{m}^3 = (135 \times 1000)\text{liters}$$

$$= 135000 \text{ liters}$$

The tank can hold 1,35,000 liters of water.

Q2) A cuboidal vessel is 10 m long and 8 m wide. How high must it be made to hold 380 cubic meters of a liquid?

Solution:

Given that

Length of the vessel (l) = 10 m

Width of the Cuboidal vessel = 8 m

Let 'h' be the height of the cuboidal vessel.

Volume of the vessel = 380m^3

$$\text{Therefore, } l \times b \times h = 380\text{m}^3$$

$$\Rightarrow 10 \times 8 \times h = 380$$

$$\Rightarrow h = \frac{380}{10 \times 8}$$

$$\Rightarrow h = 4.75\text{m}$$

Therefore, height of the vessel should be 4.75 m.

Q3) Find the cost of digging a cuboidal pit 8 m long, 6 m broad and 3 m deep at the rate of Rs 30 per m^3

Solution:

Given,

Length of the cuboidal pit (l) = 8 m

Breadth of the cuboidal pit (b) = 6 m

Depth of the cuboidal pit (h) = 3 m

Volume of the Cuboidal pit = $l \times b \times h$

$$= 8 \times 6 \times 3$$

$$= 144 \text{ m}^3$$

Cost of digging $1 \text{ m}^3 = \text{Rs. } 30$

Cost of digging $144 \text{ m}^3 = 144 \times 30 = \text{Rs. } 4320$

Q4) If V is the volume of a cuboid of dimensions a, b, c and S is its surface area, then prove that

$$\frac{1}{V} = \frac{2}{S} \left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c} \right)$$

Solution:

Given Data:

Length of the cube (l) = a

Breadth of the cube (b) = b

Height of the cube (h) = c

Volume of the cube (V) = l*b*h

$$= a \times b \times c$$

$$= abc$$

Surface area of the cube (S) = 2 (lb+bh+hl)

$$= 2 (ab+bc+ca)$$

$$\text{Now, } \frac{ab+bc+ca}{abc} \times \frac{2}{2(ab+bc+ca)} = \frac{2}{S} \left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c} \right)$$

$$\text{Therefore, } \frac{1}{abc} = \frac{2}{S} \left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c} \right)$$

$$\text{Therefore, } \frac{1}{V} = \frac{2}{S} \left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c} \right)$$

Hence Proved.

Q5) The areas of three adjacent faces of a cuboid are x, y and z.. If the volume is V, Prove that $V^2 = xyz$.

Solution:

Let a, b and d be the length, breadth, and height of the cuboid.

Then, $x = ab$

$y = bc$

$z = ca$

and $V = abc$ [$V = l \times b \times h$]

$$= xyz = ab \times bc \times ca = (abc)^2$$

and $V = abc$

$$V^2 = (abc)^2$$

Therefore, $V^2 = (xyz)$

Q6) If the areas of three adjacent face of a cuboid are 8 cm^2 , 18 cm^2 and 25 cm^2 . Find the volume of the cuboid.

Solution:

WKT, if x, y, z denote the areas of three adjacent faces of a cuboid.

$$=x = l*b, y = b*h, z = l*h$$

Volume (V) is given by

$$V = l*b*h$$

$$\text{Now, } xyz = lb*bh*hl = V^2$$

$$\text{Here } x = 8$$

$$y = 18$$

$$z = 25$$

$$\text{Therefore, } V^2 = 8 * 18 * 25 = 3600$$

$$\Rightarrow V = 60\text{cm}^3$$

Q7) The breadth of a room is twice its height, one half of its length and the volume of the room is 512 cu. dm. Find its dimensions.

Solution:

Consider l, b and h are the length, breadth and height of the room.

$$\text{So, } b = 2h \text{ and } b = \frac{1}{2}l$$

$$\Rightarrow \frac{1}{2} = 2h$$

$$\Rightarrow l = 4h$$

$$\Rightarrow l = 4h, b = 2h$$

$$\text{Now, Volume} = 512\text{dm}^3$$

$$\Rightarrow 4h * 2h * h = 512$$

$$\Rightarrow h^3 = 64$$

$$\Rightarrow h = 4$$

$$\text{So, Length of the room (l)} = 4h = 4*4 = 16 \text{ dm}$$

$$\text{Breadth of the room (b)} = 2h = 2*4 = 8 \text{ dm}$$

$$\text{And Height of the room (h)} = 4 \text{ dm.}$$

Q8) A river 3m deep and 40m wide is flowing at the rate of 2km per hour. How much water will fall into the sea in a minute?

Solution

$$\text{Radius of the water flow} = 2\text{km per hour} = \left(\frac{2000}{60}\right) \text{ m/min}$$

$$= \left(\frac{100}{3}\right) \text{ m/min}$$

$$\text{Depth of the river (h)} = 3\text{m}$$

$$\text{Width of the river (b)} = 40\text{m}$$

$$\text{Volume of the water flowing in 1 min} = \frac{100}{3} * 40 * 3 = 4000^3$$

Thus, 1 minute $4000\text{m}^3 = 4000000$ litres of water will fall in the sea.

Q9) Water in a canal 30dm wide and 12dm deep, is flowing with a velocity of 100km every hour. What much area will it irrigate in 30 minutes if 8cm of standing water is desired?

Solution:

Given that,

Water in the canal forms a cuboid of Width (b) = 30dm = 3m

Height (h) = 12dm = 1.2m

Cuboid length is equal to the distance traveled in 30 min with the speed of 100 km per hour.

$$\text{Therefore, Length of the cuboid} = 100 * \frac{30}{60} = 60\text{km} = 50000\text{metres}$$

$$\text{So, volume of water to be used for irrigation} = 5000 * 3 * 1.2 \text{ m}^3$$

Water accumulated in the field forms a cuboid of base area equal to the area of the field and height equal to $\frac{8}{100}$ metres

$$\text{Therefore, Area of field} * \frac{8}{100} = 50000 * 3 * 1.2$$

$$\Rightarrow \text{Area of field} = \frac{50000 * 3 * 1.2 * 100}{8}$$

$$\Rightarrow 22, 50, 000\text{metres}.$$

Q10) Three metal cubes with edges 6cm, 8cm, 10cm respectively are melted together and formed into a single cube. Find the volume, surface area and diagonal of the new cube.

Solution:

Let 'a' be the length of each edge of the new cube.

$$\text{Then } a^3 = (6^3 + 8^3 + 10^3)\text{cm}^3$$

$$\Rightarrow a^3 = 1728$$

$$\Rightarrow a = 12$$

$$\text{Therefore, Volume of the new cube} = a^3 = 1728\text{cm}^3$$

$$\text{Surface area of the new cube} = 6a^2 = 6 * (12)^2 = 864\text{cm}^2$$

$$\text{Diagonal of the newly formed cube} = \sqrt{3}a = 12\sqrt{3}\text{cm}$$

Q11) Two cubes, each of volume 512cm^3 are joined end to end. Find the surface area of the resulting cuboid.

Solution:

Given that,

$$\text{Volume of the cube} = 512\text{cm}^3$$

$$\Rightarrow \text{side}^3 = 512$$

$$\Rightarrow \text{side}^3 = 8^3$$

$$\Rightarrow \text{side} = 8\text{cm}$$

Dimensions of the new cuboid formed

$$\text{Length (l)} = 8+8 = 16\text{cm}, \text{Breadth (b)} = 8\text{cm}, \text{Height (h)} = 8\text{cm}$$

$$\text{Surface area} = 2(lb+bh+hl)$$

$$= 2(16*8+8*8+16*8)$$

$$= 640 \text{ cm}^2$$

Therefore, Surface area is 640 cm^2 .

Q12) Half cubic meter of gold-sheet is extended by hammering so as to cover an area of 1 hectare. Find the thickness of the gold-sheet.

Solution:

$$\text{Given that, Volume of gold-sheet} = 0.5\text{m}^3$$

$$\text{Area of the gold-sheet} = 1 \text{ hectare} = 1*10000 = 10000 \text{ m}^2$$

$$\text{Therefore, Thickness of gold sheet} = \frac{\text{Volume of solid}}{\text{Area of gold sheet}}$$

$$\Rightarrow \frac{0.5\text{m}^3}{1\text{Hectare}}$$

$$\Rightarrow \frac{0.5\text{m}^3}{10000\text{m}^2}$$

$$\Rightarrow \frac{100\text{m}}{20000}$$

$$\text{Therefore, Thickness of silver sheet} = \frac{1}{200}\text{cm}$$

Q13) A metal cube of edge 12cm is melted and formed into three smaller cubes. If the edges of the two smaller cubes are 6cm and 8cm, find the edge of the third smaller cube.

Solution:

$$\text{Volume of the large cube} = v_1 + v_2 + v_3$$

Let the edge of the third cube be 'x' cm

$$12^3 = 6^3 + 8^3 + a^3 \quad [\text{Volume of cube} = \text{side}^3]$$

$$1728 = 216 + 512 + x^3$$

$$\Rightarrow x^3 = 1728 - 728 = 1000$$

$$\Rightarrow x = 10\text{cm}$$

Therefore, Side of third side = 10cm

Q14) The dimensions of a cinema hall are 100m, 50m, 18m. How many persons can sit in the hall, if each person requires 150m^3 of air?

Solution:

Given that

$$\text{Volume of cinema hall} = 100*50*18 \text{ m}^3$$

Volume of air required by each person = 150 m^3

$$\begin{aligned}\text{Number of persons who sit in the hall} &= \frac{\text{Volume of cinema hall}}{\text{Volume of air required by each person}} \\ &= \frac{100 \times 50 \times 18}{150} = 600 \quad [\text{Since, } V = l \times b \times h]\end{aligned}$$

Therefore, number of persons who can sit in the hall = 600 members.

Q15) Given that 1 cubic cm of marble weighs 0.25kg, the weight of marble block 28cm in width and 5cm thick is 112kg. Find the length of the block.

Solution:

Let the length of the marble block be 'l'cm

$$\begin{aligned}\text{Volume of the marble block} &= l \times b \times h \text{ cm}^3 \\ &= l \times 28 \times 5 \text{ cm}^3\end{aligned}$$

Therefore, weight of the marble square = $140l \times 0.25 \text{ kg}$

As mentioned in the question, weight of the marble = 112 kgs

Therefore,

$$\begin{aligned}112 &= 140l \times 0.25 \\ \Rightarrow l &= \frac{112}{140 \times 0.25} = 3.2 \text{ cm}.\end{aligned}$$

Q16) A box with lid is made of 2cm thick wood. Its external length, breadth and height are 25cm, 18cm and 15cm respectively. How much cubic cm of a fluid can be placed in it? Also, find the volume of the wood used in it.

Solution:

Given,

The external dimensions of cuboid are as follows

Length (l) = 25 cm

Breadth (b) = 18 cm

Height (h) = 15 cm

$$\begin{aligned}\text{External volume of the case with cover (cuboid)} &= l \times b \times h \text{ cm}^3 \\ &= 25 \times 18 \times 15 \text{ cm}^3 \\ &= 6750 \text{ cm}^3\end{aligned}$$

Now, the internal dimensions of the cuboid is as follows

Length (l) = $25 - (2 \times 2) = 21 \text{ cm}$

Breadth (b) = $18 - (2 \times 2) = 14 \text{ cm}$

Height (h) = $15 - (2 \times 2) = 11 \text{ cm}$

$$\begin{aligned}\text{Now, Internal volume of the case with cover (cuboid)} &= l \times b \times h \text{ cm}^3 \\ &= 21 \times 14 \times 11 \text{ cm}^3 \\ &= 3234 \text{ cm}^3\end{aligned}$$

Therefore, Volume of the fluid that can be placed = 3234 cm^3

Now, volume of the wood utilized = External volume – Internal volume

$$= 3516 \text{ cm}^3$$

Q17) The external dimensions of a closed wooden box are 48cm, 36cm, 30cm. The box is made of 1.5cm thick wood. How many bricks of size 6cm x 3cm x 0.75cm can be put in this box?

Solution:

Given that,

The external dimensions of the wooden box are as follows:

$$\text{Length (l)} = 48\text{cm}, \text{Breadth (b)} = 36\text{cm}, \text{Height (h)} = 30\text{cm}$$

Now, the internal dimensions of the wooden box are as follows:

$$\text{Length (l)} = 48 - (2 \times 1.5) = 45\text{cm}$$

$$\text{Breadth (b)} = 36 - (2 \times 1.5) = 33\text{cm}$$

$$\text{Height (h)} = 30 - (2 \times 1.5) = 27\text{cm}$$

$$\text{Internal volume of the wooden box} = l \times b \times h \text{ cm}^3$$

$$= 45 \times 33 \times 27 \text{ cm}^3$$

$$= 40095 \text{ cm}^3$$

$$\text{Volume of the brick} = 6 \times 3 \times 0.75 = 13.5 \text{ cm}^3$$

$$\text{Therefore, Number of bricks} = \frac{40095}{13.5} = 2970 \text{ bricks}$$

Therefore, 2970 bricks can be kept inside the wooden box.

Q18) How many cubic centimeters of iron are there in an open box whose external dimensions are 36cm, 25cm and 16.5cm, the iron being 1.5cm thick throughout? If 1 cubic cm of iron weighs 15gms. Find the weight of the empty box in kg.

Solution:

Given,

Outer dimensions of iron:

$$\text{Length (l)} = 36\text{cm}$$

$$\text{Breadth (b)} = 25\text{cm}$$

$$\text{Height (h)} = 16.5\text{cm}$$

Inner dimensions of iron:

$$\text{Length (l)} = 36 - (2 \times 1.5) = 33\text{cm}$$

$$\text{Breadth (b)} = 25 - (2 \times 1.5) = 22\text{cm}$$

$$\text{Height (h)} = 16.5 - 1.5 = 15\text{cm}$$

$$\text{Volume of Iron} = \text{Outer volume} - \text{Inner volume}$$

$$= (36 \times 25 \times 16.5) - (33 \times 22 \times 15)$$

$$= 3960 \text{ cm}^3$$

$$\text{Weight of Iron} = 3960 \times 15 = 59400 \text{ grams} = 59.4 \text{ kgs}$$

Q19) A cube of 9cm edge is immersed completely in a rectangular vessel containing water. If the dimensions of the base are 15cm and 12cm, find the rise in water level in the vessel.

Solution:

$$\text{Volume of the cube} = S^3 = 9^3 = 729 \text{ cm}^3$$

$$\text{Area of the base} = l \times b = 15 \times 12 = 180 \text{ cm}^2$$

$$\text{Rise in water level} = \frac{\text{Volume of the cube}}{\text{Area of base of rectangular vessel}}$$

$$= \frac{729}{180}$$

$$= 4.05 \text{ cm}$$

Q20) A rectangular container, whose base is a square of side 5cm, stands on a horizontal table, and holds water up to 1cm from the top. When a solid cube is placed in the water it is completely submerged, the water rises to the top and 2 cubic cm of water overflows. Calculate the volume of the cube and also the length of its edge.

Solution:

Let the length of each edge of the cube be 'x'cm

Then, volume of the cube = Volume of water inside the tank + Volume of water that overflowed

$$x^3 = (5 \times 5 \times 1) + 2$$

$$x^3 = 27$$

$$x = 3 \text{ cm}$$

$$\text{Hence, volume of the cube} = 27 \text{ cm}^3$$

$$\text{And edge of the cube} = 3 \text{ cm}$$

Q21) A field is 200 m long and 150 m broad. There is a plot, 50 m long and 40 m broad, near the field. The plot is dug 7m deep and the earth taken out is spread evenly on the field. By how many meters is the level of the field raised? Give the answer to the second place of decimal.

Solution:

$$\text{Volume of the earth dug out} = 50 \times 40 \times 7 = 14000 \text{ m}^3$$

Let 'h' be the rise in the height of the field

Therefore, volume of the field (cuboidal) = Volume of the earth dug out

$$\Rightarrow 200 \times 150 \times h = 14000$$

$$\Rightarrow h = \frac{14000}{200 \times 150} = 0.47 \text{ m}$$

Q22) A field is in the form of a rectangular length 18m and width 15m. A pit 7.5m long, 6m broad and 0.8m deep, is dug in a corner of the field and the earth taken out is spread over the remaining area of the field. Find out the extent to which the level of the field has been raised.

Solution:

Let 'h' metres be the rise in the level of field

Volume of earth taken out from the pit = $7.5 \times 6 \times 0.8 = 36 \text{ m}^3$

Area of the field on which the earth taken out is to be spread = $18 \times 15 - 7.5 \times 6 = 225 \text{ m}^2$

Now, Area of the field $\times h$ = Volume of the earth taken out from the pit

$$\Rightarrow 225 \times h = 7.5 \times 6 \times 0.8$$

$$\Rightarrow h = \frac{36}{225} = 0.16 \text{ m} = 16 \text{ cm}$$

Q23) A rectangular tank is 80m long and 25m broad. Water flows into it through a pipe whose cross-section is 25 cm^2 , at the rate of 16 km per hour. How much the level of the water rises in the tank in 45 minutes?

Solution:

Consider 'h' be the rise in water level.

Volume of water in rectangular tank = $8000 \times 2500 \times h \text{ cm}^3$

Cross-sectional area of the pipe = 25 cm^2

Water coming out of the pipe forms a cuboid of base area 25 cm^2 and length equal to the distance travelled in 45 minutes with the speed 16 km/hour

$$\text{i.e., length} = \text{Length} = 16000 \times 100 \times \frac{45}{60} \text{ cm}$$

Therefore, The Volume of water coming out pipe in 45 minutes = $25 \times 16000 \times 100 \times (45/60)$

Now, volume of water in the tank = Volume of water coming out of the pipe in 45 minutes

$$\Rightarrow 8000 \times 2500 \times h = 16000 \times 100 \times \frac{45}{60} \times 25$$

$$\Rightarrow h = \frac{25 \times 16000 \times 100 \times 45}{60 \times 8000 \times 2500} = 1.5 \text{ cm}$$

Q24) Water in a rectangular reservoir having base 80 m by 60 m is 6.5m deep. In what time can the water be pumped by a pipe of which the cross-section is a square of side 20cm if the water runs through the pipe at the rate of 15km/hr.

Solution:

Flow of water = 15km/hr

= 15000 m/hr

Volume of water coming out of the pipe in one hour,

$$\Rightarrow \frac{20}{100} \times \frac{20}{100} \times 15000 = 600 \text{ m}^3$$

Volume of the tank = $80 \times 60 \times 6.5$

$$= 31200 \text{ m}^3$$

$$\text{Time taken to empty the tank} = \frac{\text{Volume of tank}}{\text{Volume of water coming out of pipe in one hour}}$$

$$= \frac{31200}{600} = 52 \text{ hours}$$

Q25) A village having a population of 4000 requires 150 liters of water per head per day. It has a tank measuring 20m x 15m x 6m. For how many days will the water of this tank last?

Solution:

Given that,

Length of the cuboidal tank (l) = 20m

Breadth of the cuboidal tank (b) = 15m

Height of the cuboidal tank (h) = 6m

Capacity of the tank = $l \times b \times h = 20 \times 15 \times 6$

= 1800 m^3

= 1800000 litres

Water consumed by the people of village in one day = 4000×150 litres

= 600000 litres

Let water of this tank last for 'n' days

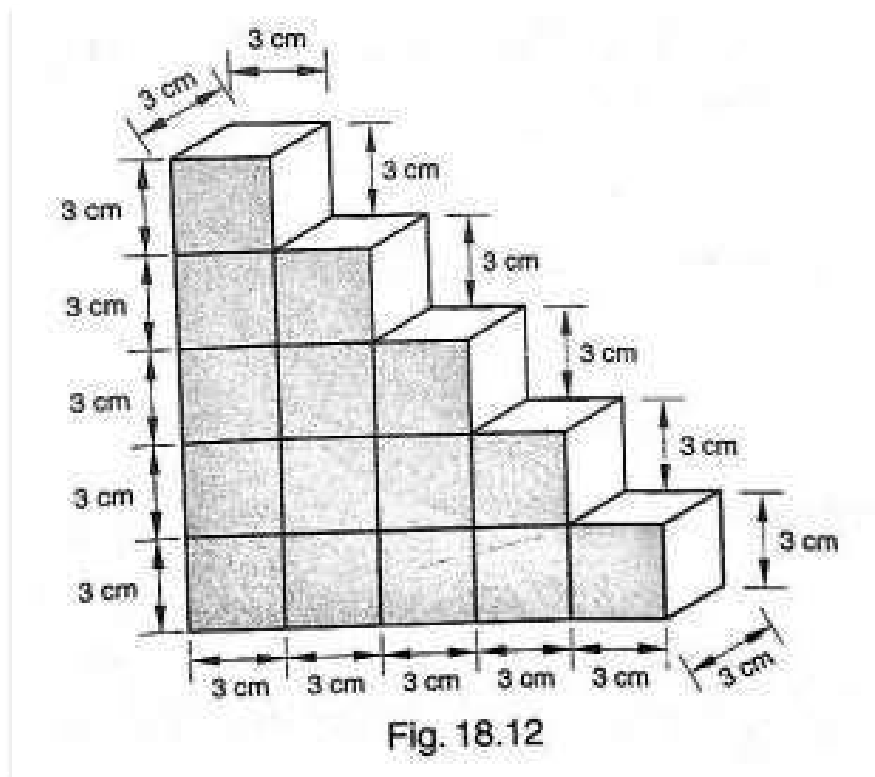
Therefore, water consumed by all people of village in n days = capacity of the tank

= $n \times 600000 = 1800000$

= $n = \frac{1800000}{600000} = 3$

Thus, the water will last for 3 days in the tank.

Q26) A child playing with building blocks, which are of the shape of the cubes, has built a structure as shown in Fig. 18.12. If the edge of each cube is 3cm, find the volume of the structure built by the child.



Solution:

Volume of each cube = edge*edge*edge

= $3 \times 3 \times 3$

= 27 cm^3

Number of cubes in the structure = 15

Therefore, volume of the structure = 27×15

$$= 405 \text{ cm}^3$$

Q27) A godown measures 40m x 25m x 10m. Find the maximum number of wooden crates each measuring 1.5m x 1.25m x 0.5m that can be stored in the godown.

Solution:

Given,

Godown length (l_1) = 40m

Godown breadth (b_1) = 25m

Godown height (h_1) = 10m

Volume of the godown = $l_1 \times b_1 \times h_1 = 40 \times 25 \times 10$

$$= 10000 \text{ m}^3$$

Wooden crate length (l_2) = 1.5m

Wooden crate breadth (b_2) = 1.25m

Wooden crate height (h_2) = 0.5m

Volume of the wooden crate = $l_2 \times b_2 \times h_2 = 1.5 \times 1.25 \times 0.5$

$$= 0.9375 \text{ m}^3$$

The number of wooden crates stored in the godown is taken as 'n'

Volume of 'n' wooden crates = Volume of godown

$$= 0.9375n = 10000$$

$$= n = \frac{10000}{0.9375} = 10666.66$$

Therefore, the number of wooden crates that can be stored in the godown is 10666.66.

Q28) A wall of length 10m was to be built across an open ground. The height of the wall is 4m and thickness of the wall is 24cm. If this wall is to be built up with bricks whose dimensions are 24cm x 12cm x 8cm, how many bricks would be required?

Solution:

Given that,

The wall with all its bricks makes up space occupies by it, we need to find the volume of the wall, which is nothing but cuboid.

Here, length = 10m = 1000cm

Thickness = 24cm

Height = 4m = 400cm

Therefore, volume of the wall = $l \times b \times h$

$$= 1000 \times 24 \times 400 \text{ cm}^3$$

Now, each brick is a cuboid with length = 24cm

Breadth = 12cm

Height = 8cm

So, volume of each brick = $l \times b \times h = 24 \times 12 \times 8 = 2304 \text{ cm}^3$

The number of bricks required is given by,

$$\frac{\text{Volume of the wall}}{\text{Volume of each brick}}$$
$$= \frac{1000 \times 24 \times 400}{2304} = 4166.6 \text{ bricks}$$

So, the wall requires 4167 bricks.