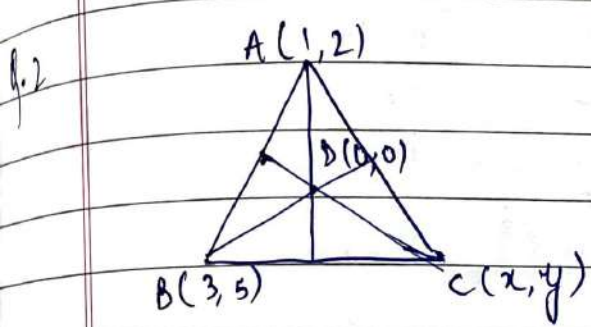


Ch-6  
Exercise 6.4

Q.1 (ii)  $(-2, 3), (2, -1), (4, 0)$   
Coordinates of centroid of  $\Delta$  =  $\left( \frac{-2+2+4}{3}, \frac{3-1+0}{3} \right)$   
=  $\left( \frac{4}{3}, \frac{2}{3} \right)$



$A(1, 2)$  and  $B(3, 5)$   
Centroid  $D$  is at origin  $(0, 0)$ .  
Let the third vertex  $C$  be  $(x, y)$

Coordinates of centroid  $D$  =  $\left( \frac{x+1+3}{3}, \frac{y+2+5}{3} \right)$

|                                   |                                   |
|-----------------------------------|-----------------------------------|
| $\frac{x+1+3}{3} = 0$<br>$x = -4$ | $\frac{y+2+5}{3} = 0$<br>$y = -7$ |
|-----------------------------------|-----------------------------------|

Hence, the coordinates of the third vertex  $C$  are  $(-4, -7)$ .

Q.4  
 $A(3, 2)$  and  $B(-2, 1)$   
Coordinates of centroid  $G = \left( \frac{5}{3}, -\frac{1}{3} \right)$   
Let the third vertex  $C$  be  $(x, y)$

Coordinates of centroid  $= \left( \frac{x+3-2}{3}, \frac{y+2+1}{3} \right)$

$$\frac{x+3-2}{3} = \frac{5}{3}$$

$$x = 5 - 1$$

$$\boxed{x = 4}$$

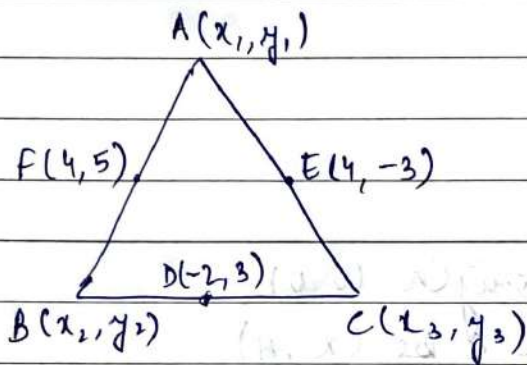
$$\frac{y+2+1}{3} = \frac{-1}{3}$$

$$y = -3 - 1$$

$$\boxed{y = -4}$$

Hence, the coordinates of the third vertex C are  $(4, -4)$

Q. 5



Let

$A(x_1, y_1)$ ,  $B(x_2, y_2)$ ,  $C(x_3, y_3)$  be the vertices of  $\triangle ABC$ .  
 $D(-2, 3)$ ,  $E(4, -3)$ ,  $F(4, 5)$  are the midpoints of sides  $BC$ ,  $CA$  and  $AB$  respectively.

As  $D$  is the midpoint of  $BC$ ,

$$\therefore (-2, 3) = \left( \frac{x_2 + x_3}{2}, \frac{y_2 + y_3}{2} \right)$$

$$\frac{x_2 + x_3}{2} = -2 \quad \text{and} \quad \frac{y_2 + y_3}{2} = 3$$

$$x_2 + x_3 = -4 \quad \text{and} \quad y_2 + y_3 = 6 \quad \text{--- ①}$$

As  $E$  is the midpoint of  $AC$ ,

$$\therefore (4, -3) = \left( \frac{x_1 + x_3}{2}, \frac{y_1 + y_3}{2} \right)$$

$$\frac{x_1 + x_3}{2} = 4 \quad \text{and} \quad \frac{y_1 + y_3}{2} = -3$$

$$x_1 + x_3 = 8 \quad \text{and} \quad y_1 + y_3 = -6 \quad \text{--- (2)}$$

As F is the midpoint of AB,

$$\therefore (4, 5) = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$\frac{x_1 + x_2}{2} = 4 \quad \text{and} \quad \frac{y_1 + y_2}{2} = 5$$

$$x_1 + x_2 = 8 \quad \text{and} \quad y_1 + y_2 = 10 \quad \text{--- (3)}$$

From (1), (2) & (3),

$$(x_2 + x_3) + (x_1 + x_3) + (x_1 + x_2) = -4 + 8 + 8$$

$$2(x_1 + x_2 + x_3) = 12$$

$$x_1 + x_2 + x_3 = 6 \quad \text{--- (4)}$$

$$(y_2 + y_3) + (y_1 + y_3) + (y_1 + y_2) = 6 - 6 + 10$$

$$2(y_1 + y_2 + y_3) = 10$$

$$y_1 + y_2 + y_3 = 5 \quad \text{--- (5)}$$

From (1), (4) & (5),

$$x_1 + x_2 + x_3 = 6$$

$$x_1 - 4 = 6$$

$$\boxed{x_1 = 10}$$

$$y_1 + y_2 + y_3 = 5$$

$$y_1 + 6 = 5$$

$$\boxed{y_1 = -1}$$

From (2), (4) & (5),

$$x_1 + x_2 + x_3 = 6$$

$$x_2 + 8 = 6$$

$$\boxed{x_2 = -2}$$

$$y_1 + y_2 + y_3 = 5$$

$$y_2 - 6 = 5$$

$$\boxed{y_2 = 11}$$

From ③, ④ & ⑤,

$$\begin{array}{l|l} x_1 + x_2 + x_3 = 6 & y_1 + y_2 + y_3 = 5 \\ x_3 + 8 = 6 & y_3 + 10 = 5 \\ \hline x_3 = -2 & y_3 = -5 \end{array}$$

∴ The vertices of  $\Delta$  are  $A(10, -1)$ ,  $B(-2, 11)$ ,  $C(-2, -5)$

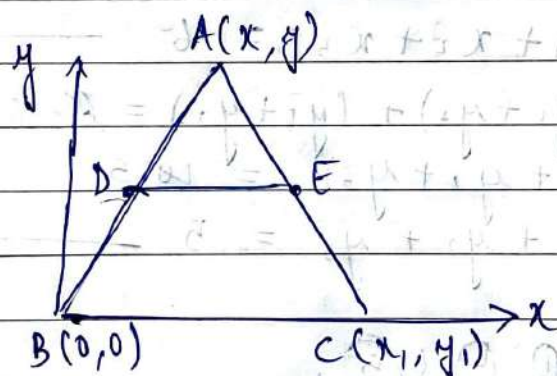
Coordinates of centroid of  $\Delta ABC$

$$= \left( \frac{10 + (-2) + (-2)}{3}, \frac{-1 + 11 + (-5)}{3} \right)$$

$$= \left( \frac{6}{3}, \frac{5}{3} \right)$$

$$= \left( 2, \frac{5}{3} \right)$$

Q.6



To prove -  $DE = \frac{1}{2} BC$

Proof - Let  $ABC$  be a  $\Delta$  such that  $BC$  is along  $x$  axis.  
 ∴ Coordinates of  $\Delta ABC$  are  $A(x, y)$ ,  $B(0, 0)$ ,  $C(x_1, y_1)$   
 $D$  &  $E$  are the midpoints of  $AB$  &  $AC$ .

Coordinates of  $D = \left( \frac{x+0}{2}, \frac{y+0}{2} \right) = \left( \frac{x}{2}, \frac{y}{2} \right)$

Coordinates of  $E = \left( \frac{x+x_1}{2}, \frac{y+y_1}{2} \right)$

$$\text{Length of } BC = \sqrt{x_1^2 + y_1^2} \quad \text{--- ①}$$

$$\begin{aligned} \text{Length of } DE &= \sqrt{\left(\frac{x+x_1}{2} - \frac{x}{2}\right)^2 + \left(\frac{y+y_1}{2} - \frac{y}{2}\right)^2} \\ &= \sqrt{\left(\frac{x_1}{2}\right)^2 + \left(\frac{y_1}{2}\right)^2} \\ &= \sqrt{\frac{x_1^2 + y_1^2}{4}} = \sqrt{\frac{1}{4}(x_1^2 + y_1^2)} \\ &= \frac{1}{2} \sqrt{x_1^2 + y_1^2} \\ &= \frac{1}{2} BC \quad (\text{Putting ①}) \end{aligned}$$

Hence, proved ✓

Q. 8 To prove  $PA^2 + PB^2 + PC^2 = GA^2 + GB^2 + GC^2 + 3GP^2$

Proof - Let  $\triangle ABC$  be a  $\triangle$  with coordinates  $A(x_1, y_1)$ ,  $B(x_2, y_2)$  and  $C(x_3, y_3)$ .  
 $G$  is the centroid of the  $\triangle$ .  
 Let  $P$  be the origin.

$$\text{Coordinates of } G = \left( \frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right)$$

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$PA = \sqrt{(x_1 - 0)^2 + (y_1 - 0)^2} = \sqrt{x_1^2 + y_1^2}$$

$$\therefore PA^2 = x_1^2 + y_1^2$$

$$\text{Similarly, } PB^2 = x_2^2 + y_2^2$$

$$PC^2 = x_3^2 + y_3^2$$

$$\therefore \text{LHS} = PA^2 + PB^2 + PC^2 = x_1^2 + y_1^2 + x_2^2 + y_2^2 + x_3^2 + y_3^2 \quad \text{--- ①}$$

$$GP = \sqrt{\left(\frac{x_1 + x_2 + x_3 - 0}{3}\right)^2 + \left(\frac{y_1 + y_2 + y_3 - 0}{3}\right)^2}$$

$$\therefore GP^2 = \left(\frac{x_1 + x_2 + x_3}{3}\right)^2 + \left(\frac{y_1 + y_2 + y_3}{3}\right)^2$$

$$\text{Similarly, } GA^2 = \left(x_1 - \frac{x_1 + x_2 + x_3}{3}\right)^2 + \left(y_1 - \frac{y_1 + y_2 + y_3}{3}\right)^2$$

$$= \left(\frac{2x_1 - x_2 - x_3}{3}\right)^2 + \left(\frac{2y_1 - y_2 - y_3}{3}\right)^2$$

$$GB^2 = \left(\frac{2x_2 - x_1 - x_3}{3}\right)^2 + \left(\frac{2y_2 - y_1 - y_3}{3}\right)^2$$

$$GC^2 = \left(\frac{2x_3 - x_1 - x_2}{3}\right)^2 + \left(\frac{2y_3 - y_1 - y_2}{3}\right)^2$$

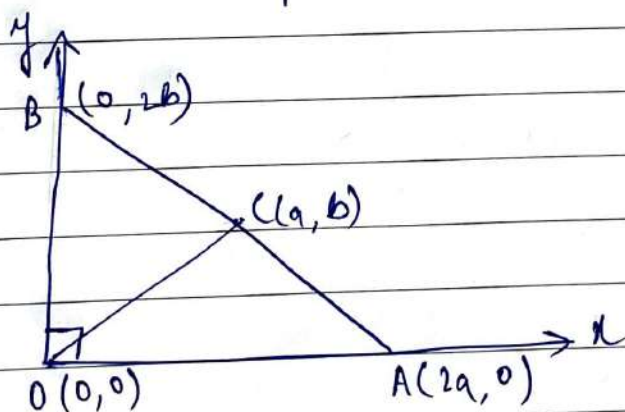
$$\begin{aligned} \text{RHS} &= GA^2 + GB^2 + GC^2 + 3GP^2 \\ &= \frac{(2x_1 - x_2 - x_3)^2}{9} + \frac{(2y_1 - y_2 - y_3)^2}{9} + \frac{(2x_2 - x_1 - x_3)^2}{9} \\ &\quad + \frac{(2y_2 - y_1 - y_3)^2}{9} + \frac{(2x_3 - x_1 - x_2)^2}{9} + \frac{(2y_3 - y_1 - y_2)^2}{9} \\ &\quad + 3 \left[ \frac{(x_1 + x_2 + x_3)^2}{9} + \frac{(y_1 + y_2 + y_3)^2}{9} \right] \\ &= \left[ \frac{1}{9} (4x_1^2 + x_2^2 + x_3^2 + 4x_2^2 + x_1^2 + x_3^2 + 4x_3^2 + x_1^2 + x_2^2) \right. \\ &\quad \left. + \frac{1}{9} (x_1^2 + x_2^2 + x_3^2) \right] + \\ &\quad \left[ \frac{1}{9} (4y_1^2 + y_2^2 + y_3^2 + 4y_2^2 + y_1^2 + y_3^2 + 4y_3^2 + y_1^2 + y_2^2) \right. \\ &\quad \left. + \frac{1}{9} (y_1^2 + y_2^2 + y_3^2) \right] \end{aligned}$$

$$\begin{aligned}
&= \left[ \frac{1}{9} (6x_1^2 + 6x_2^2 + 6x_3^2) + \frac{1}{3} (x_1^2 + x_2^2 + x_3^2) \right] \\
&\quad + \left[ \frac{1}{9} (6y_1^2 + 6y_2^2 + 6y_3^2) + \frac{1}{3} (y_1^2 + y_2^2 + y_3^2) \right] \\
&= \left[ \frac{6x_1^2 + 6x_2^2 + 6x_3^2}{9} + \frac{1}{3} (x_1^2 + x_2^2 + x_3^2) \right] + \\
&\quad \left[ \frac{6y_1^2 + 6y_2^2 + 6y_3^2}{9} + \frac{1}{3} (y_1^2 + y_2^2 + y_3^2) \right] \\
&= \frac{2}{3} (x_1^2 + x_2^2 + x_3^2) + \frac{2}{3} (y_1^2 + y_2^2 + y_3^2) \\
&= x_1^2 + x_2^2 + x_3^2 + y_1^2 + y_2^2 + y_3^2 \quad \text{--- (2)}
\end{aligned}$$

From (1) & (2),

LHS = RHS

Hence, proved



BOA is a right  $\Delta$  with vertices B(0, 2b), O(0,0) and A(2a, 0).

C is the midpoint of AB.

$$\begin{aligned}
\therefore \text{coordinates of C} &= \left( \frac{2a+0}{2}, \frac{0+2b}{2} \right) = \left( \frac{2a}{2}, \frac{2b}{2} \right) \\
&= (a, b)
\end{aligned}$$

$$\begin{aligned}
\therefore CO &= \sqrt{(a-0)^2 + (b-0)^2} \\
&= \sqrt{a^2 + b^2} \quad \text{--- (1)}
\end{aligned}$$

$$CA = \sqrt{(2a-a)^2 + (0-b)^2}$$

$$= \sqrt{a^2 + b^2} \quad \text{--- (2)}$$

$$CB = \sqrt{(a-0)^2 + (b-2b)^2}$$

$$= \sqrt{a^2 + b^2} \quad \text{--- (3)}$$

From (1), (2) and (3),  
 $CO = CA = CB$ .

$\therefore$  C is equidistant from O, A and B.

x

x

x