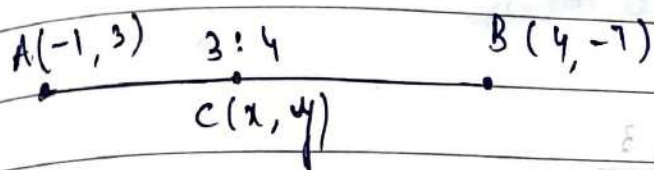


Date _____
Page _____

ch-6
exercise 6.3



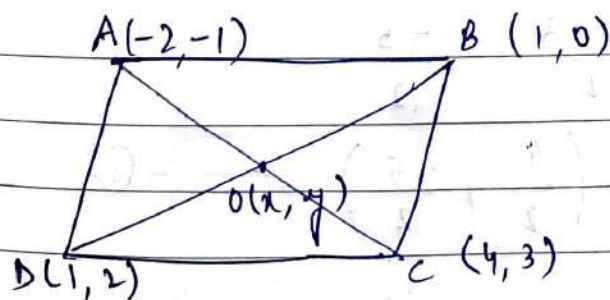
Let $C(x, y)$ be the required point.

$$x = \frac{mx_2 + nx_1}{m+n} = \frac{3(4) + 4(-1)}{3+4}$$
$$= \frac{12-4}{7} = \boxed{\frac{8}{7}}$$

$$y = \frac{my_2 + ny_1}{m+n} = \frac{3(-7) + 4(3)}{3+4}$$
$$= \frac{-21+12}{7} = \boxed{\frac{-9}{7}}$$

✓

∴ The coordinates of C are $\left(\frac{8}{7}, -\frac{9}{7}\right)$.



ABCD is a || gram.

Let $O(x, y)$ be the required point.

As diagonals of a || gram bisect each other.
∴ Ratio = 1:1

$$x = \frac{x_1 + x_2}{2} = \frac{-2 + 4}{2}$$

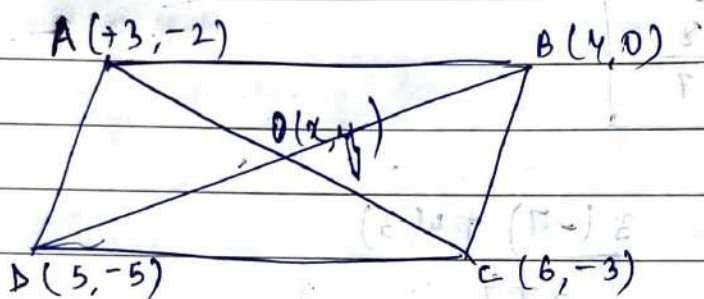
$$= \frac{+2}{2} = \boxed{1}$$

$$y = \frac{y_1 + y_2}{2} = \frac{-1 + 3}{2}$$

$$= \frac{2}{2} = \boxed{1}$$

∴ The coordinates of O are (1, 1).

Q.4



Let O(x, y) be the point of intersection of diagonals AC & BD of ABCD.

In AC,

$$x = \frac{x_1 + x_2}{2} = \frac{3 + 6}{2} = \frac{9}{2}$$

$$y = \frac{y_1 + y_2}{2} = \frac{-2 - 3}{2} = \frac{-5}{2}$$

∴ Midpoint of AC = $\left(\frac{9}{2}, \frac{-5}{2}\right)$ ——— ①

In BD,

$$x = \frac{x_1 + x_2}{2} = \frac{5 + 4}{2} = \frac{9}{2}$$

$$y = \frac{y_1 + y_2}{2} = \frac{-5 + 0}{2} = \frac{-5}{2}$$

∴ Midpoint of BD = $\left(\frac{9}{2}, \frac{-5}{2}\right)$ ——— ②

From ① & ②

Midpoint of AC = Midpoint of BD
 $\therefore$ Diagonals AC & BD bisect each other.
 $\therefore$ ABCD is a ||gram.

Q.5 P(9a-2, -b) divides line segment ^{joining} A(3a+1, -3) and B(8a, 5) in the ratio 3:1

$$\therefore \text{Coordinates of P} = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n} \right)$$

$$(9a-2, -b) = \left[\frac{3(8a) + 1(3a+1)}{3+1}, \frac{3(5) + 1(-3)}{3+1} \right]$$

$$(9a-2, -b) = \left[\frac{24a + 3a + 1}{4}, \frac{15 - 3}{4} \right]$$

$$(9a-2, -b) = \left[\frac{27a + 1}{4}, 3 \right]$$

$$9a - 2 = \frac{27a + 1}{4}$$

$$-b = 3$$

$$b = -3$$

$$36a - 8 = 27a + 1$$

$$36a - 27a = 1 + 8$$

$$9a = 9$$

$$a = 1$$

Q.6 (a, b) is the midpoint of line segment A(10, -6) & B(k, 4)
 $\therefore$ Ratio = 1:1

$$a = \frac{x_1 + x_2}{2}$$

$$a = \frac{10 + k}{2}$$

$$2a = 10 + k$$

$$k = 2a - 10 \quad \text{--- ①}$$

$$b = \frac{-6+4}{2}$$

$$b = \frac{-2}{2}$$

$$\boxed{b = -1} \quad \text{--- (2)}$$

also, $a - 2b = 18$ (given)

$$a - 2(-1) = 18$$

$$a + 2 = 18$$

$$\boxed{a = 16} \quad \text{--- (3)}$$

Substitute (3) in (1)

$$k = 2a - 10$$

$$k = 2(16) - 10$$

$$k = 32 - 10$$

$$\boxed{k = 22}$$

$$\begin{aligned} AB &= \sqrt{(22-10)^2 + (4+6)^2} \\ &= \sqrt{(12)^2 + (10)^2} = \sqrt{144+100} \\ &= \sqrt{244} = \boxed{2\sqrt{61} \text{ units}} \end{aligned}$$

Q.7 Let point P(2, 4) divide the line segment joining points A(-2, 2) & B(3, 7) in the ratio of $k:1$.

$$\begin{aligned} \text{Coordinates of P} &= \left(\frac{kx_2 + x_1}{k+1}, \frac{ky_2 + y_1}{k+1} \right) \\ &= \left(\frac{3k-2}{k+1}, \frac{7k+2}{k+1} \right) \end{aligned}$$

Coordinates of P given = (2, 4)

$$\therefore \frac{3k-2}{k+1} = 2$$

$$3k-2 = 2k+2$$

$$3k-2k = 2+2$$

$$\boxed{k = 4}$$

$$\frac{7k+2}{k+1} = y$$

$$\frac{7(4)+2}{4+1} = y \quad (\text{Putting value of } k)$$

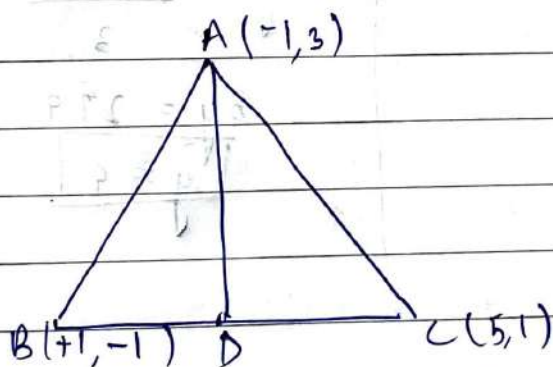
$$\frac{28+2}{5} = y$$

$$6 = y$$

$$\boxed{y = 6}$$

$\therefore$ The ratio is 4:1 and $y = 6$.

Q.8 $A(-1, 3)$, $B(1, -1)$ and $C(5, 1)$



As AD is the median,

$\therefore$ D is the midpoint of BC.

$$\begin{aligned} \text{Coordinates of } D &= \left(\frac{1+5}{2}, \frac{-1+1}{2} \right) = \left(\frac{6}{2}, 0 \right) \\ &= \boxed{(3, 0)} \end{aligned}$$

$$\begin{aligned}\text{Length of AD} &= \sqrt{(9+1)^2 + (10-3)^2} \\ &= \sqrt{(4)^2 + (-3)^2} = \sqrt{16+9} \\ &= \sqrt{25} = \boxed{5 \text{ units}}\end{aligned}$$

Q.9

$$A(2, p) \quad P \quad Q(x, 7) \quad R \quad S(6, 4) \quad B(7, 10)$$

In QB,
S divides QB in the ratio 2:1

$$\text{Coordinates of S} = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n} \right)$$

$$(6, 4) = \left[\frac{2(7) + 1(x)}{2+1}, \frac{2(10) + 1(7)}{2+1} \right]$$

$$(6, 4) = \left[\frac{14+x}{3}, \frac{20+7}{3} \right]$$

$$6 = \frac{14+x}{3}$$

$$18 = 14+x$$

$$x = 18 - 14$$

$$\boxed{x = 4}$$

$$4 = \frac{20+7}{3}$$

$$12 = 27$$

$$\boxed{y = 9}$$

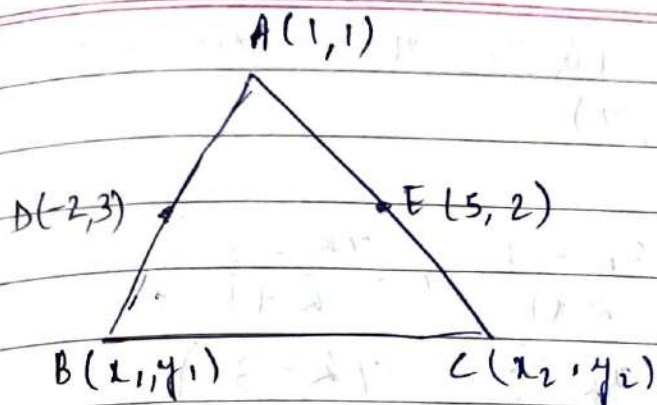
In AS,

Q divides AS in the ratio 1:1.

$$y \text{ coordinate of Q} = \frac{y_1 + y_2}{2}$$

$$7 = \frac{p+9}{2}$$

$$14 = p+9 \Rightarrow \boxed{p = 5}$$



$A = (1, 1)$ (given)

$D(-2, 3)$ & $E(5, 2)$ are the midpoints of AB & AC respectively.

$$\text{Coordinates of } D = \left(\frac{x_1 + 1}{2}, \frac{y_1 + 1}{2} \right)$$

$$(-2, 3) = \left(\frac{x_1 + 1}{2}, \frac{y_1 + 1}{2} \right)$$

$$-2 = \frac{x_1 + 1}{2}$$

$$-4 = x_1 + 1$$

$$\boxed{x_1 = -5}$$

$$3 = \frac{y_1 + 1}{2}$$

$$6 = y_1 + 1$$

$$\boxed{y_1 = 5}$$

$$\text{Coordinates of } E = \left(\frac{x_2 + 1}{2}, \frac{y_2 + 1}{2} \right)$$

$$(5, 2) = \left(\frac{x_2 + 1}{2}, \frac{y_2 + 1}{2} \right)$$

$$5 = \frac{x_2 + 1}{2}$$

$$10 = x_2 + 1$$

$$\boxed{x_2 = 9}$$

$$2 = \frac{y_2 + 1}{2}$$

$$4 = y_2 + 1$$

$$\boxed{y_2 = 3}$$

$\therefore$ The other vertices are $B(-5, 5)$ and $C(9, 3)$.

Q.11 (i) Let the y-axis divide PQ in the ratio $k:1$ at R.
P $(-2, -3)$ and Q $(3, 7)$

$$\text{Coordinates of R} = \left(\frac{3k-2}{k+1}, \frac{7k-3}{k+1} \right)$$

$$(0, y) = \left(\frac{3k-2}{k+1}, \frac{7k-3}{k+1} \right)$$

$$0 = \frac{3k-2}{k+1}$$

$$3k = 2$$

$$\boxed{k = \frac{2}{3}}$$

$$\frac{7k-3}{k+1} = y$$

$$\frac{7\left(\frac{2}{3}\right)-3}{\frac{2}{3}+1} = y$$

$$\frac{14-9}{3+3} = y$$

$$\frac{2-3}{3+3} = y$$

$$\frac{-1}{6} = y$$

$\therefore$ The ratio is $\frac{2}{3} : 1 \Rightarrow 2:3$.

Putting value of $k:1$ in y coordinate.

$$\frac{7k-3(1)}{k+1} = y$$

$$\frac{7(2)-9}{2+1} = y$$

$$\frac{14-9}{3} = y$$

$$\frac{5}{3} = y \Rightarrow \boxed{y = 1}$$

$\therefore$ The coordinate is $(0, 1)$.

(ii) $A(-3, -1)$ and $B(-8, -9)$.
Coordinate of $C = \left(-5, -\frac{21}{5}\right)$

Let C divide line segment AB in the ratio $k:1$.

$$\therefore \text{Coordinates of } C = \left[\frac{-8k-3}{k+1}, \frac{-9k-1}{k+1} \right]$$

$$\therefore \frac{-8k-3}{k+1} = -5$$

$$-8k-3 = -5k-5$$

$$-8k+5k = -5+3$$

$$-3k = -2$$

$$\boxed{k = \frac{2}{3}}$$

$$\therefore \text{Ratio} = \frac{2}{3} : 1 = \boxed{2:3}$$

(12) $A(3, 4)$ and $B(k, 7)$

Let $C(x, y)$ be the midpoint.

$$x = \frac{3+k}{2}, \quad y = \frac{4+7}{2} = \frac{11}{2}$$

~~also~~ also, $2x+2y+1=0$ (given)

$$2\left(\frac{3+k}{2}\right) + 2\left(\frac{11}{2}\right) + 1 = 0$$

[Putting the values of x and y .]

$$3+k+11+1=0$$

$$\boxed{k = -15}$$

Q.13 $A\left(\frac{1}{2}, \frac{3}{2}\right)$ and $B(2, -5)$

Point $P = \left(\frac{3}{4}, \frac{5}{12}\right)$

$$x \text{ coordinate of } P = \frac{mx_2 + nx_1}{m+n}$$

$$\frac{3}{4} = \frac{2m + \frac{1}{2}n}{m+n}$$

$$\frac{3m + 3n}{4} = \frac{4m + n}{2}$$

$$2\left(\frac{3m + 3n}{4}\right) = 4m + n$$

$$\frac{3m + 3n}{2} = 4m + n$$

$$3m + 3n = 8m + 2n$$

$$3n - 2n = 8m - 3m$$

$$n = 5m$$

$$\boxed{\frac{m}{n} = \frac{1}{5}}$$

$$\therefore \text{Ratio} = \boxed{1:5}$$

Q.14 $A(-2, -3)$ and $B(5, 6)$

(i) Let x -axis divide AB at P in the ratio $k:1$
 Coordinates of $P = (x, 0)$ [given]
 Coordinates of $P = \left[\frac{5k-2}{k+1}, \frac{6k-3}{k+1}\right]$

$$\therefore \frac{6k-3}{k+1} = 0$$

$$k+1$$

$$6k = 3$$

$$\boxed{k = \frac{1}{2}}$$

$$\therefore \text{Ratio} = \frac{1}{2} : 1 = \boxed{1:2}$$

$$x = \frac{mx_2 + nx_1}{m+n}$$

$$x = \frac{5 + 2(-2)}{1+2}$$

$$x = \frac{5 - 4}{3}$$

$$\boxed{x = \frac{1}{3}}$$

$\therefore$ The coordinates of P are $(\frac{1}{3}, 0)$ ✓

(ii) Let y axis divide AB at P in the ratio k:1.

Coordinates of P = (0, y) [given]

$$\text{Coordinates of P} = \left[\frac{5k-2}{k+1}, \frac{6k-3}{k+1} \right]$$

$$\therefore \frac{5k-2}{k+1} = 0$$

$$5k = 2$$

$$\boxed{k = \frac{2}{5}}$$

$$\therefore \text{Ratio} = \frac{2}{5} = \boxed{2:5}$$

$$y = \frac{my_2 + ny_1}{m+n}$$

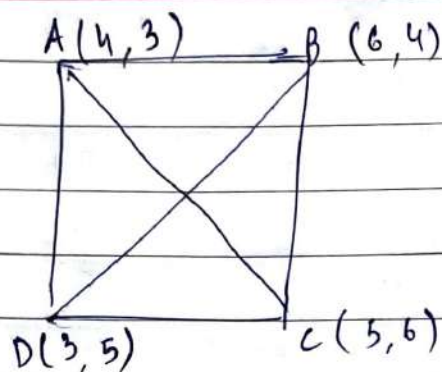
$$y = \frac{2(6) + 5(-3)}{2+5}$$

$$y = \frac{12 - 15}{7}$$

$$\boxed{y = \frac{-3}{7}}$$

$\therefore$ The coordinates of P are $(0, \frac{-3}{7})$ ✓

Q. 16



$$\begin{aligned} \text{Coordinates of midpoint of AC} &= \left(\frac{4+5}{2}, \frac{3+6}{2} \right) \\ &= \left(\frac{9}{2}, \frac{9}{2} \right) \quad \text{--- ①} \end{aligned}$$

$$\begin{aligned} \text{Coordinates of midpoint of BD} &= \left(\frac{6+3}{2}, \frac{4+5}{2} \right) \\ &= \left(\frac{9}{2}, \frac{9}{2} \right) \quad \text{--- ②} \end{aligned}$$

From ① & ②,
 $\therefore$ AC & BD have the same midpoint
 $\therefore$ ABCD is a ||gram.

$$\begin{aligned} AB &= \sqrt{(6-4)^2 + (4-3)^2} \\ &= \sqrt{(2)^2 + (1)^2} = \sqrt{4+1} \\ &= \sqrt{5} \quad \text{--- ③} \end{aligned}$$

$$\begin{aligned} BC &= \sqrt{(6-5)^2 + (4-6)^2} \\ &= \sqrt{(1)^2 + (-2)^2} = \sqrt{1+4} \\ &= \sqrt{5} \quad \text{--- ④} \end{aligned}$$

From ③ & ④,
 $AB = BC$
 $\therefore$ A ||gram with adjacent sides equal is a rhombus.

$$AC = \sqrt{(5-4)^2 + (6-3)^2}$$

$$= \sqrt{(1)^2 + (3)^2} = \sqrt{1+9}$$

$$= \sqrt{10} \quad \text{--- (5)}$$

$$BD = \sqrt{(6-3)^2 + (4-5)^2} = \sqrt{9+1}$$

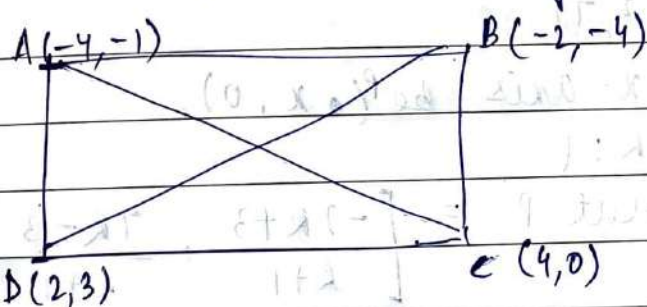
$$= \sqrt{(3)^2 + (-1)^2} = \sqrt{9+1}$$

$$= \sqrt{10} \quad \text{--- (6)}$$

From (5) & (6),
 $AC = BD$

As, a rhombus with equal diagonals is a square
 $\therefore ABCD$ is a square &

A, B, C, D are angular points of a square.



Coordinates of midpoint of AC = $\left(\frac{-4+4}{2}, \frac{-1+0}{2} \right)$

$$= \left(0, -\frac{1}{2} \right) \quad \text{--- (1)}$$

Coordinates of midpoint of BD = $\left(\frac{-2+2}{2}, \frac{-4+3}{2} \right)$

$$= \left(0, -\frac{1}{2} \right) \quad \text{--- (2)}$$

From (1) & (2),

AC & BD have the same midpoint.
 $\therefore ABCD$ is a parallelogram.

$$\begin{aligned} AC &= \sqrt{(4+4)^2 + (0+1)^2} \\ &= \sqrt{(8)^2 + (1)^2} = \sqrt{64+1} \\ &= \sqrt{65} \longrightarrow \textcircled{3} \end{aligned}$$

$$\begin{aligned} BD &= \sqrt{(-2-2)^2 + (-4-3)^2} \\ &= \sqrt{(-4)^2 + (-7)^2} = \sqrt{16+49} \\ &= \sqrt{65} \longrightarrow \textcircled{4} \end{aligned}$$

from $\textcircled{3}$ & $\textcircled{4}$,

$$AC = BD$$

is a ~~parallelogram~~ with equal diagonals is a rectangle.
 $\therefore ABCD$ is a rectangle.

Q.19

$A(3, -3)$ and $B(-2, 7)$

Let the point on x-axis be $P(x, 0)$.

Let the ratio be $k:1$

$$\text{Coordinates of point P} = \left[\frac{-2k+3}{k+1}, \frac{7k-3}{k+1} \right]$$

$$\therefore \frac{7k-3}{k+1} = 0$$

$$\therefore 7k = 3$$

$$\boxed{k = \frac{3}{7}}$$

$$\therefore \text{Ratio} = \frac{3}{7} : 1 = \boxed{3 : 7}$$

Q.20

$A(12, 5)$ and $B(4, -3)$

Let $P(x, 2)$ divide AB in the ratio $k:1$

$$\text{Coordinates of P} = \left[\frac{4k+12}{k+1}, \frac{-3k+5}{k+1} \right]$$

$$\therefore \frac{-3k+5}{k+1} = 2$$

$$-3k+5=2k+2$$

$$5-2=2k+3k$$

$$5k=3$$

$$\boxed{k = \frac{3}{5}}$$

$$\therefore \text{Ratio} = \frac{3}{5} : 1 = \boxed{3:5}$$

$$x \text{ coordinate of } P = \frac{mx_1 + nx_2}{m+n}$$

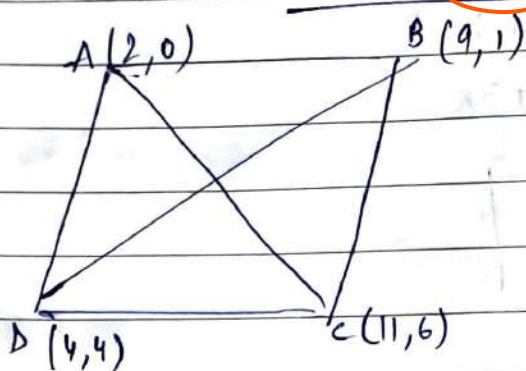
$$= \frac{3(4) + 5(12)}{3+5} = \frac{12+60}{8}$$

$$= \frac{72}{8} = \boxed{9}$$



Ch-6 Exercise 6.3

Q.24



$$\begin{aligned}
 AB &= \sqrt{(9-2)^2 + (1-0)^2} \\
 &= \sqrt{(7)^2 + (1)^2} = \sqrt{49+1} \\
 &= \sqrt{50} \quad \text{--- (1)}
 \end{aligned}$$

$$\begin{aligned}
 BC &= \sqrt{(11-9)^2 + (6-1)^2} \\
 &= \sqrt{(2)^2 + (5)^2} = \sqrt{4+25} \\
 &= \sqrt{29} \quad \text{--- (2)}
 \end{aligned}$$

From (1) & (2),

$$AB \neq BC$$

$\therefore$ Adjacent sides are not equal.
 But in a rhombus, all sides are equal.

$\therefore$ ABCD is ^{not} a rhombus.

Q.26

P(5, -6) and Q(-1, -4)

Let the y-axis divide PQ at R in the ratio $k:1$.
 Coordinates of R given = (0, y)

$$\text{Coordinates of R} = \left[\frac{-k+5}{k+1}, \frac{-4k-6}{k+1} \right]$$

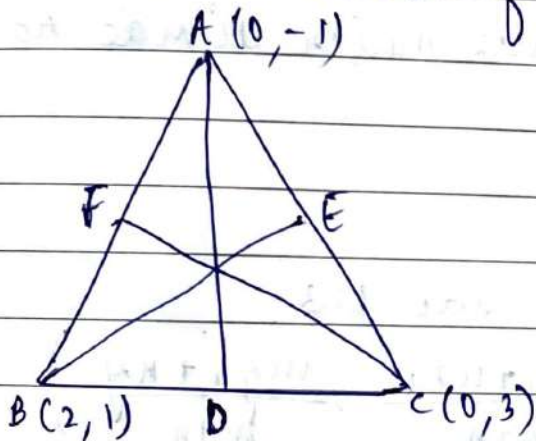
$$\therefore \frac{-k+5}{k+1} = 0$$

$$\boxed{k = 5}$$

$$\therefore \text{Ratio} = \boxed{5:1}$$

$$\begin{aligned} \text{y coordinate of R} &= \frac{my_2 + ny_1}{m+n} \\ &= \frac{5(-4) + (-6)}{5+1} \\ &= \frac{-20 - 6}{6} = \frac{-26}{6} \\ &= \frac{-13}{3} \end{aligned}$$

$\therefore$ The coordinates of R are $\left(0, \frac{-13}{3}\right)$.



A(0, -1), B(2, 1) and C(0, 3)

~~Coordinate~~ Let AD, BE & CF be the medians.

$$\text{Coordinates of D} = \left(\frac{2+0}{2}, \frac{1+3}{2}\right) = (1, 2)$$

$$\text{Coordinates of E} = \left(\frac{0}{2}, \frac{3-1}{2}\right) = (0, 1)$$

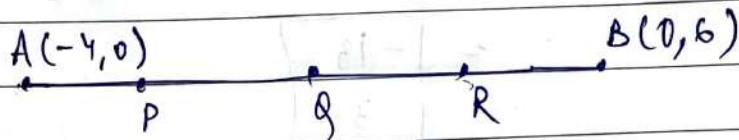
$$\text{Coordinates of F} = \left(\frac{2+0}{2}, \frac{1-1}{2}\right) = (1, 0)$$

$$\begin{aligned} \text{Length of median AD} &= \sqrt{(1-0)^2 + (2+1)^2} \\ &= \sqrt{(1)^2 + (3)^2} = \sqrt{1+9} \\ &= \boxed{\sqrt{10} \text{ units}} \end{aligned}$$

$$\begin{aligned}\text{Length of median BE} &= \sqrt{(2-0)^2 + (1-1)^2} \\ &= \sqrt{(2)^2 + 0} = \sqrt{4} \\ &= \boxed{2 \text{ units}}\end{aligned}$$

$$\begin{aligned}\text{Length of median CF} &= \sqrt{(1-0)^2 + (0-3)^2} \\ &= \sqrt{(1)^2 + (-3)^2} = \sqrt{1+9} \\ &= \boxed{\sqrt{10} \text{ units}}\end{aligned}$$

Q.30



$A(-4, 0)$ and $B(0, 6)$
Let P, Q & R be the points which divide AB into four equal parts.

In AB,

P divides AB in the ratio 1:3.

$$\begin{aligned}\text{Coordinates of P} &= \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n} \right) \\ &= \left(\frac{1(0) + 3(-4)}{1+3}, \frac{1(6) + 3(0)}{1+3} \right) \\ &= \left(\frac{-12}{4}, \frac{6}{4} \right) = \boxed{\left(-3, \frac{3}{2} \right)}\end{aligned}$$

In AB,

Q is the midpoint. $\therefore$ Ratio = 1:1

$$\begin{aligned}\text{Coordinates of Q} &= \left(\frac{-4+0}{2}, \frac{0+6}{2} \right) \\ &= \left(\frac{-4}{2}, \frac{6}{2} \right) = \boxed{(-2, 3)}\end{aligned}$$

In AB,

R divides AB in the ratio 3:1.

$$\begin{aligned}\text{Coordinates of R} &= \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n} \right) \\ &= \left[\frac{3(0) + 1(-4)}{3+1}, \frac{3(6) + 1(0)}{3+1} \right] \\ &= \left[\frac{-4}{4}, \frac{18}{4} \right] = \left[\left(-1, \frac{9}{2} \right) \right]\end{aligned}$$

Q.31 A(5,7), B(3,9), C(8,6) & D(0,10)

$$\begin{aligned}\text{Coordinates of the midpoint of AB} &= \left(\frac{5+3}{2}, \frac{7+9}{2} \right) \\ &= \left(\frac{8}{2}, \frac{16}{2} \right) = (4, 8)\end{aligned}$$

$$\begin{aligned}\text{Coordinates of the midpoint of CD} &= \left(\frac{8+0}{2}, \frac{6+10}{2} \right) \\ &= \left(\frac{8}{2}, \frac{16}{2} \right) = (4, 8)\end{aligned}$$

$\therefore$ Midpoint of AB = Midpoint of CD

Q.32 A(1,2), B(6,8) & C(2,4)

Let D(x,y) be the midpoint of BC.

$$\begin{aligned}\text{Coordinates of D} &= \left(\frac{6+2}{2}, \frac{8+4}{2} \right) = \left(\frac{8}{2}, \frac{12}{2} \right) \\ &= (4, 6)\end{aligned}$$

$$\begin{aligned}\text{Distance AD} &= \sqrt{(4-1)^2 + (6-2)^2} \\ &= \sqrt{(3)^2 + (4)^2} = \sqrt{9+16} \\ &= \sqrt{25} = 5 \text{ units}\end{aligned}$$

Q.33

A (1, 4) and B (5, 2)

$$\frac{AP}{BP} = \frac{3}{4} \quad (\text{given})$$

$$\therefore AP : BP = 3 : 4$$

$$\begin{aligned} \therefore \text{Coordinates of } P &= \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n} \right) \\ &= \left[\frac{3(5) + 4(1)}{3+4}, \frac{3(2) + 4(4)}{3+4} \right] \\ &= \left[\frac{15+4}{7}, \frac{6+16}{7} \right] \\ &= \left(\frac{19}{7}, \frac{22}{7} \right) \end{aligned}$$

Q.35

A (-4, 3) and B (2, 8)

Let C (m, 6) divide AB in the ratio k : 1.

$$\therefore \text{Coordinates of } C = \left(\frac{2k-4}{k+1}, \frac{8k+3}{k+1} \right)$$

$$\text{Also, } \frac{8k+3}{k+1} = 6$$

$$8k+3 = 6k+6$$

$$8k-6k = 6-3$$

$$2k = 3$$

$$\boxed{k = \frac{3}{2}}$$

$$\therefore \text{Ratio} = \frac{3}{2} : 1 = \boxed{3 : 2}$$

$$\frac{2k-4}{k+1} = m$$

(Putting value of k)

$$2\left(\frac{3}{2}\right) - 4 = m$$

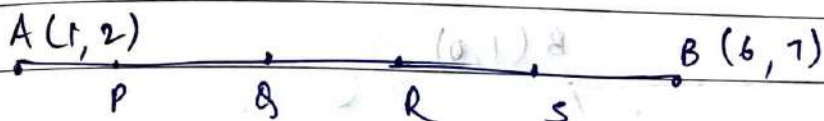
$$\frac{3}{2} - 4$$

$$\frac{3-4}{3+2} = m$$

$$2$$

$$\frac{-1(2)}{5} = m$$

$$\boxed{m = -\frac{2}{5}}$$



A(1, 2) and B(6, 7)

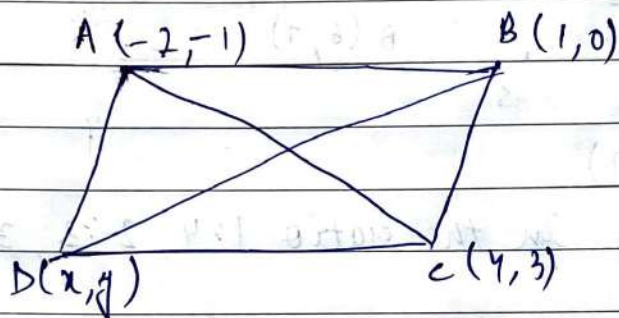
P, Q, R, S divide AB in the ratio 1:4, 2:3, 3:2, 4:1 respectively.

$$\begin{aligned}\text{Coordinates of P} &= \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n} \right) \\ &= \left[\frac{1(6) + 4(1)}{1+4}, \frac{1(7) + 4(2)}{1+4} \right] \\ &= \left[\frac{6+4}{5}, \frac{7+8}{5} \right] = \left[\frac{10}{5}, \frac{15}{5} \right] \\ &= \boxed{(2, 3)}\end{aligned}$$

$$\begin{aligned}\text{Coordinates of Q} &= \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n} \right) \\ &= \left[\frac{2(6) + 3(1)}{2+3}, \frac{2(7) + 3(2)}{2+3} \right] \\ &= \left[\frac{12+3}{5}, \frac{14+6}{5} \right] = \left[\frac{15}{5}, \frac{20}{5} \right] \\ &= \boxed{(3, 4)}\end{aligned}$$

$$\begin{aligned}
 \text{Coordinates of } R &= \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n} \right) \\
 &= \left[\frac{3(6) + 2(1)}{3+2}, \frac{3(7) + 2(2)}{3+2} \right] \\
 &= \left[\frac{18+2}{5}, \frac{21+4}{5} \right] = \left[\frac{20}{5}, \frac{25}{5} \right] \\
 &= \boxed{(4, 5)}
 \end{aligned}$$

Q. 41



A(-2, -1), B(1, 0), C(4, 3) are the three consecutive vertices of a ||gram.

~~Let D (the fourth vertex)~~

Let D (the 4th vertex) be (x, y).

Since, the diagonals of a ||gram bisect each other.
 $\therefore$ Coordinates of midpoint of AC = Coordinates of midpoint of BD

$$\left(\frac{-2+4}{2}, \frac{-1+3}{2} \right) = \left(\frac{1+x}{2}, \frac{0+y}{2} \right)$$

$$\frac{-2+4}{2} = \frac{1+x}{2}$$

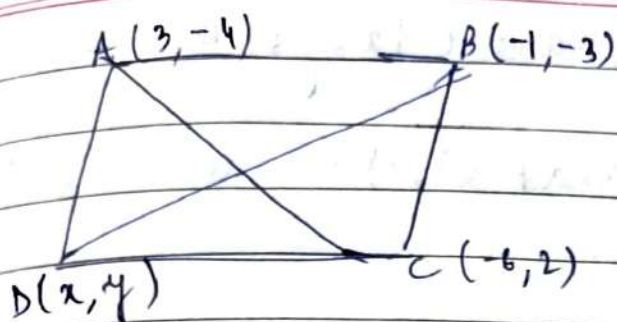
$$x = 2 - 1$$

$$\boxed{x = 1}$$

$$\frac{-1+3}{2} = \frac{y}{2}$$

$$\boxed{y = 2}$$

$\therefore$ The fourth vertex of ||gram is $\boxed{(1, 2)}$.



Let $A(3, -4)$ and $C(-6, 2)$ be the extremities of diagonal AC .

$$B = (-1, -3)$$

Let D be (x, y) .

Since, the diagonals of a ||gram bisect each other.

$\therefore$ Coordinates of midpoint of AC = Coordinates of midpoint of BD

$$\left(\frac{3-6}{2}, \frac{-4+2}{2} \right) = \left(\frac{x-1}{2}, \frac{y-3}{2} \right)$$

$$\frac{3-6}{2} = \frac{x-1}{2}$$

$$x = -3 + 1$$

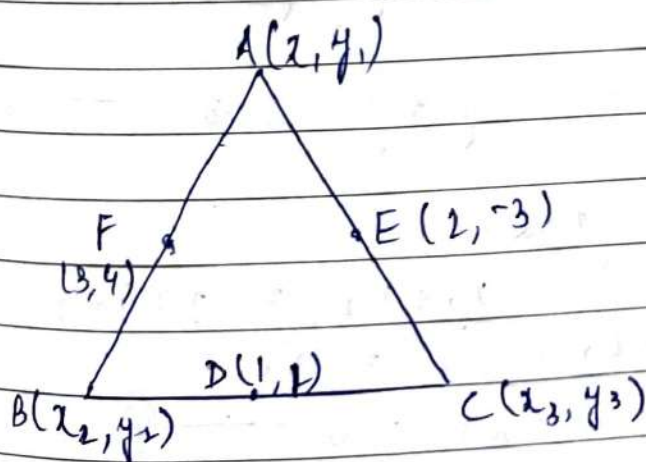
$$\boxed{x = -2}$$

$$\frac{-4+2}{2} = \frac{y-3}{2}$$

$$y = -2 + 3$$

$$\boxed{y = 1}$$

$\therefore$ The fourth vertex of ||gram is $(-2, 1)$.



Let $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ be the vertices of Δ .

$D(1, 1)$, $E(2, -3)$ and $F(3, 4)$ are the midpoints of sides BC , AC & AB .

In BC ,

$$(1, 1) = \left(\frac{x_2 + x_3}{2}, \frac{y_2 + y_3}{2} \right)$$

$$1 = \frac{x_2 + x_3}{2} \quad \text{and} \quad 1 = \frac{y_2 + y_3}{2}$$

$$x_2 + x_3 = 2 \quad \text{and} \quad y_2 + y_3 = 2 \quad \text{--- (1)}$$

In AC ,

$$(2, -3) = \left(\frac{x_1 + x_3}{2}, \frac{y_1 + y_3}{2} \right)$$

$$2 = \frac{x_1 + x_3}{2} \quad \text{and} \quad -3 = \frac{y_1 + y_3}{2}$$

$$x_1 + x_3 = 4 \quad \text{and} \quad y_1 + y_3 = -6 \quad \text{--- (2)}$$

In AB

$$(3, 4) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$3 = \frac{x_1 + x_2}{2} \quad \text{and} \quad 4 = \frac{y_1 + y_2}{2}$$

$$x_1 + x_2 = 6 \quad \text{and} \quad y_1 + y_2 = 8 \quad \text{--- (3)}$$

From (1), (2) & (3),

$$(x_2 + x_3) + (x_1 + x_3) + (x_1 + x_2) = 2 + 4 + 6$$

$$2(x_1 + x_2 + x_3) = 12$$

$$x_1 + x_2 + x_3 = 6 \quad \text{--- (4)}$$

$$(y_2 + y_3) + (y_1 + y_3) + (y_1 + y_2) = 2 - 6 + 8$$

$$2(y_1 + y_2 + y_3) = 4$$

$$y_1 + y_2 + y_3 = 2 \quad \text{--- (5)}$$

From ①, ④, ⑤

$$x_1 + x_2 + x_3 = 6$$

$$x_1 + 2 = 6$$

$$\boxed{x_1 = 4}$$

$$y_1 + y_2 + y_3 = 2$$

$$y_1 + 2 = 2$$

$$\boxed{y_1 = 0}$$

From ②, ④, ⑤

$$x_1 + x_2 + x_3 = 6$$

$$x_2 + 4 = 6$$

$$\boxed{x_2 = 2}$$

$$y_1 + y_2 + y_3 = 2$$

$$y_2 - 6 = 2$$

$$\boxed{y_2 = 8}$$

From ③, ④, ⑤

$$x_1 + x_2 + x_3 = 6$$

$$x_3 + 6 = 6$$

$$\boxed{x_3 = 0}$$

$$y_1 + y_2 + y_3 = 2$$

$$y_3 + 8 = 2$$

$$\boxed{y_3 = -6}$$

$$\therefore A = (4, 0), B = (2, 8), C = (0, -6)$$

Q44 A(3, -1) and B(8, 9)

Let the line $x - y - 2 = 0$ divide AB at P in the ratio $k : 1$.

$$\text{Coordinates of } P = \left(\frac{8k+3}{k+1}, \frac{9k-1}{k+1} \right)$$

As P lies on $x - y - 2 = 0$

$$\therefore \left(\frac{8k+3}{k+1} \right) - \left(\frac{9k-1}{k+1} \right) = 2$$

$$\frac{8k+3 - 9k+1}{k+1} = 2$$

$$\begin{array}{rcl}
 -k + 4 & = & 2k + 2 \\
 -k - k & = & 2 - 4 \\
 -3k & = & 2 \\
 k & = &
 \end{array}$$

$$-k + 4 = 2k + 2$$

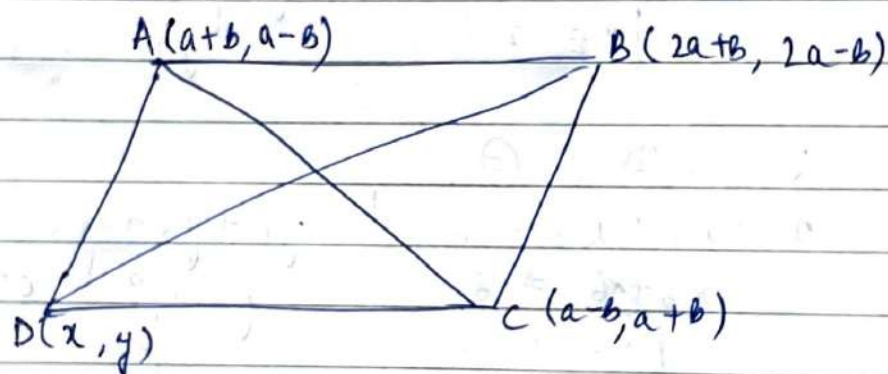
$$-k - 2k = 2 - 4$$

$$-3k = -2$$

$$k = \frac{2}{3}$$

$$\therefore \text{Ratio} = \frac{2}{3} : 1 = \boxed{2:3}$$

Q. 45



A(a+b, a-b), B(2a+b, 2a-b), C(a-b, a+b)
Let D be (x, y).

Since diagonals of a parallelogram bisect each other.

$\therefore$ Coordinates of midpoint of AC = Coordinates of midpoint of BD

$$\left(\frac{a+b+a-b}{2}, \frac{a-b+a+b}{2} \right) = \left(\frac{2a+b+x}{2}, \frac{2a-b+y}{2} \right)$$

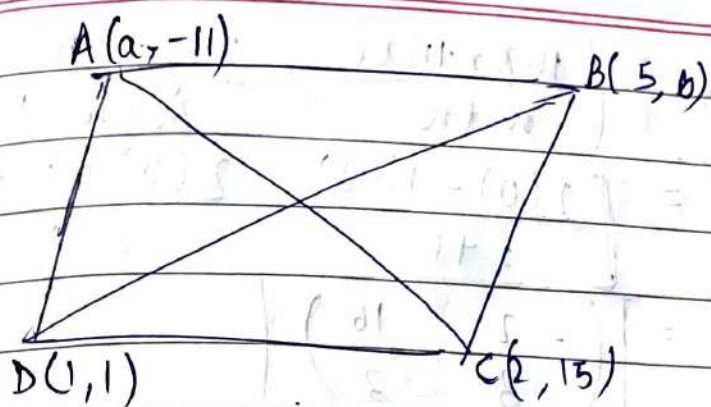
$$\frac{2a}{2} = \frac{2a+b+x}{2}$$

$$\boxed{x = -b}$$

$$\frac{2a}{2} = \frac{2a-b+y}{2}$$

$$\boxed{y = b}$$

$\therefore$ The fourth vertex D is (-b, b).



$$A(a, -11), B(5, b), C(2, 15), D(1, 1)$$

As diagonals of a parallelogram bisect each other,
 $\therefore$ coordinates of midpoint of AC = coordinates of midpoint of BD

$$\left(\frac{a+2}{2}, \frac{-11+15}{2} \right) = \left(\frac{5+1}{2}, \frac{b+1}{2} \right)$$

$$\frac{a+2}{2} = \frac{6}{2}$$

$$a = 4$$

$$\frac{4}{2} = \frac{b+1}{2}$$

$$b = 3$$

Q.52 $A(-2, 0)$ and $B(10, 8)$

P & Q trisect the line segment AB.

P is near to A.

~~Ratio 2:1~~ P & Q divide in the ratio 1:2 & 2:1 respectively.

$$\text{Coordinates of P} = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n} \right)$$

$$= \left[\frac{1(10) + 2(-2)}{1+2}, \frac{1(8) + 2(0)}{1+2} \right]$$

$$= \left[\frac{-4}{3}, \frac{8}{3} \right]$$

$$\begin{aligned}\text{Coordinates of } Q &= \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n} \right) \\ &= \left[\frac{2(0) + 1(-2)}{2+1}, \frac{2(8) + 1(0)}{2+1} \right] \\ &= \left[\left(\frac{-2}{3}, \frac{16}{3} \right) \right]\end{aligned}$$

(ii)

$$\begin{array}{ccccccc} & & P(\beta, -2) & & Q(5/3, q) & & \\ & & \bullet & & \bullet & & \\ A(3, -4) & & & & & & B(1, 2) \end{array}$$

$A(3, -4)$ and $B(1, 2)$
 $P(\beta, -2)$ and $Q(5/3, q)$ are the points of trisection.

$AP : PB = 1 : 2$

$$\begin{aligned}\text{Coordinates of } P &= \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n} \right) \\ &= \left[\frac{1(1) + 2(3)}{1+2}, \frac{1(2) + 2(-4)}{1+2} \right] \\ &= \left[\frac{1+6}{3}, \frac{2-8}{3} \right] \\ &= \left[\left(\frac{7}{3}, -2 \right) \right]\end{aligned}$$

$\therefore \boxed{\beta = \frac{7}{3}}$

$AQ : QB = 2 : 1$

$$\begin{aligned}\text{Coordinates of } Q &= \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n} \right) \\ &= \left[\frac{2(1) + 1(3)}{2+1}, \frac{2(2) + 1(-4)}{2+1} \right] \\ &= \left[\frac{2+3}{3}, \frac{4-4}{3} \right]\end{aligned}$$

$$\therefore q = \boxed{0}$$

$$= \boxed{\left(\frac{5}{3}, 0\right)}$$

Q. 53

A(2, 1) and B(5, -8)

P and Q are the points of trisection of AB.
P lies on the line $2x - y + k = 0$

$$AP : PB = 1 : 2$$

$$\begin{aligned} \text{Coordinates of P} &= \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n} \right) \\ &= \left[\frac{1(5) + 2(2)}{1+2}, \frac{1(-8) + 2(1)}{1+2} \right] \\ &= \left[\frac{5+4}{3}, \frac{-8+2}{3} \right] \\ &= \left[\frac{9}{3}, \frac{-6}{3} \right] = \boxed{(3, -2)} \end{aligned}$$

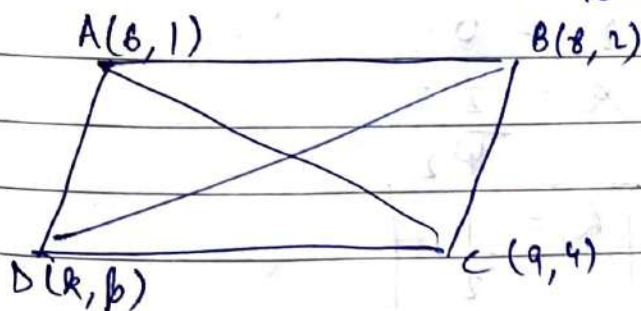
Also P lies on $2x - y + k = 0$

$$2(3) - (-2) + k = 0$$

$$6 + 2 + k = 0$$

$$\boxed{k = -8}$$

Q. 55



A(6, 1), B(8, 2), C(9, 4), D(k, p) are the vertices of a parallelogram.

As diagonals bisect each other in a parallelogram,

∴ Coordinates of midpoint of AC = Coordinates of midpoint of BD

$$\left(\frac{6+9}{2}, \frac{1+4}{2} \right) = \left(\frac{8+k}{2}, \frac{2+p}{2} \right)$$

$$\frac{6+9}{2} = \frac{8+k}{2}$$

$$k = 15 - 8$$

$$\boxed{k = 7}$$

$$\frac{1+4}{2} = \frac{2+p}{2}$$

$$p = 5 - 2$$

$$\boxed{p = 3}$$

Q. 56

A (3, -5) and B (-4, 2)

P lies on the line $x+y=0$.

$$AP : PB = k : 1$$

Coordinates of P = $\left(\frac{-4k+3}{k+1}, \frac{8k-5}{k+1} \right)$

As, P lies on $x+y=0$

$$\therefore \frac{-4k+3}{k+1} + \frac{8k-5}{k+1} = 0$$

$$k+1 = k+1$$

$$\frac{-4k+3+8k-5}{k+1} = 0$$

$$k+1$$

$$4k-2=0$$

$$k = \frac{2}{4}$$

$$\frac{2}{4}$$

$$\boxed{k = \frac{1}{2}}$$

Q. 57

A (-10, 4), B (-2, 0), C (-9, -4) and D (-4, 4)

P is the midpoint of AB.

$$\text{Coordinates of } P = \left(\frac{-10-2}{2}, \frac{4+0}{2} \right)$$

$$= \left(\frac{-12}{2}, \frac{4}{2} \right) = \boxed{(-6, 2)}$$

Let P divide C & D in ratio $k:1$

$$\text{Coordinates of } P = \left(\frac{-4k-9}{k+1}, \frac{ky-4}{k+1} \right)$$

$$\frac{-4k-9}{k+1} = x\text{-coordinate of } P$$

$$\frac{-4k-9}{k+1} = -6$$

$$-4k-9 = -6k-6$$

$$6k-4k = 9-6$$

$$2k = 3$$

$$\boxed{k = \frac{3}{2}}$$

$$\therefore \text{Ratio} = \frac{3}{2} : 1 = \boxed{3:2}$$

$$y\text{-coordinate} = \frac{my_2 + ny_1}{m+n}$$

$$2 = \frac{3(4) + 2(-4)}{3+2}$$

$$10 = 3y - 8$$

$$3y = 18$$

$$\boxed{y = 6}$$

A(2, 5) and B(2, 4)

C(-1, 2) divides AB in the ratio 3:4

$$\text{Coordinates of } C = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n} \right)$$

$$(-1, 2) = \left[\frac{3x + 4(2)}{3+4}, \frac{3y + 4(5)}{3+4} \right]$$

$$\begin{array}{c|c} -1 = \frac{3x+8}{7} & 2 = \frac{3y+20}{7} \end{array}$$

$$-7 = 3x + 8$$

$$3x = -15$$

$$\boxed{x = -5}$$

$$14 = 3y + 20$$

$$3y = -6$$

$$\boxed{y = -2}$$

$$\begin{aligned} x^2 + y^2 &= (-5)^2 + (-2)^2 \\ &= 25 + 4 = 29 \end{aligned}$$

Q.59 $A(x_1, y_1), B(x_2, y_2), C(x_3, y_3)$ are the three vertices of parallelogram ABCD. Let D be (a, b).

As, diagonals of a parallelogram bisect each other
 $\therefore$ Coordinates of midpoint of AC = coordinates of midpoint of BD

$$\left(\frac{x_1 + x_3}{2}, \frac{y_1 + y_3}{2} \right) = \left(\frac{x_2 + a}{2}, \frac{y_2 + b}{2} \right)$$

$$\frac{x_1 + x_3}{2} = \frac{x_2 + a}{2}$$

$$a = \boxed{x_1 + x_3 - x_2}$$

$$\frac{y_1 + y_3}{2} = \frac{y_2 + b}{2}$$

$$b = \boxed{y_1 + y_3 - y_2}$$

$\therefore$ The fourth vertex of parallelogram is $(x_1 + x_3 - x_2, y_1 + y_3 - y_2)$