SOLUTIONS TO CONCEPTS
CHAPTER 14

1. $F = mg$

   $\text{Stress} = \frac{F}{A}$

   $\text{Strain} = \frac{\Delta L}{L}$

   $Y = \frac{FL}{A\Delta L} \Rightarrow \frac{\Delta L}{L} = \frac{F}{YA}$

2. $\rho = \text{stress} = mg/A$

   $e = \text{strain} = \rho/Y$

   Compression $\Delta L = eL$

3. $y = \frac{F}{A}\frac{L}{\Delta L} \Rightarrow \Delta L = \frac{FL}{AY}$

4. $L_{steel} = L_{cu}$ and $A_{steel} = A_{cu}$

   a) $\frac{\text{Stress of cu}}{\text{Stress of st}} = \frac{F_{cu}}{A_{cu}}\frac{A_g}{F_g} = \frac{F_{cu}}{F_{st}} = 1$

   b) $\text{Strain} = \frac{\Delta Lst}{\Delta lcu} = \frac{F_{st}L_{st}}{A_{st}Y_{st}} \cdot \frac{A_{cu}Y_{cu}}{F_{cu}l_{cu}}$ $(\because L_{cu} = l_{st}\ ;\ A_{cu} = A_{st})$

5. $\left(\frac{\Delta L}{L}\right)_{st} = \frac{F}{AY_{st}}$

   $\left(\frac{\Delta L}{L}\right)_{cu} = \frac{F}{AY_{cu}}$

   $\frac{\text{strain steel wire}}{\text{Strain om copper wire}} = \frac{F}{AY_{st}} \times \frac{AY_{cu}}{F} (\because A_{cu} = A_{st}) = \frac{Y_{cu}}{Y_{st}}$

6. $\text{Stress in lower rod} = \frac{T_1}{A_1} \Rightarrow \frac{m_1g + \omega g}{A_1} \Rightarrow w = 14\ kg$

   $\text{Stress in upper rod} = \frac{T_2}{A_u} \Rightarrow \frac{m_2g + m_1g + wg}{A_u} \Rightarrow w = .18\ kg$

   For same stress, the max load that can be put is 14 kg. If the load is increased the lower wire will break first.

   $\frac{T_1}{A_1} = \frac{m_1g + \omega g}{A_1} = 8 \times 10^8 \Rightarrow w = 14\ kg$

   $\frac{T_2}{A_u} \Rightarrow \frac{m_2g + m_1g + \omega g}{A_u} = 8 \times 10^8 \Rightarrow \omega_0 = 2\ kg$

   The maximum load that can be put is 2 kg. Upper wire will break first if load is increased.

7. $Y = \frac{F}{A}\frac{L}{\Delta L}$

8. $Y = \frac{F}{A}\frac{L}{\Delta L} \Rightarrow F = \frac{YA\ \Delta L}{L}$

9. $m_2g - T = m_2a$ ...(1)

   and $T - F = m_1a$ ...(2)

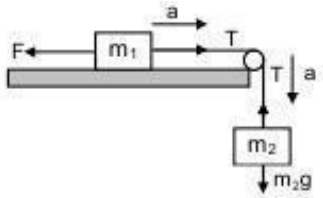
   $\Rightarrow a = \frac{m_2g - F}{m_1 + m_2}$

   From equation (1) and (2), we get $\frac{m_2g}{2(m_1 + m_2)}$

   Again, $T = F + m_1a$

   $\Rightarrow T = \frac{m_2g}{2} + m_1\frac{m_2g}{2(m_1 + m_2)} \Rightarrow \frac{m_2^2g + 2m_1m_2g}{2(m_1 + m_2)}$

$2 \qquad 2(m_1+m_2) \qquad 2(m_1+m_2)$



Now $Y = \dfrac{FL}{A\,\Delta L} \Rightarrow \dfrac{\Delta L}{L} = \dfrac{F}{AY}$

$\Rightarrow \dfrac{\Delta L}{L} = \dfrac{(m_2^2+2m_1m_2)g}{2(m_1+m_2)AY} = \dfrac{m_2g(m_2+2m_1)}{2AY(m_1+m_2)}$

10. At equilibrium $\Rightarrow T = mg$

When it moves to an angle θ, and released, the tension the T′ at lowest point is

$\Rightarrow T' = mg + \dfrac{mv^2}{r}$

The change in tension is due to centrifugal force $\Delta T = \dfrac{mv^2}{r}$ ...(1)

$\Rightarrow$ Again, by work energy principle,

$\Rightarrow \dfrac{1}{2}mv^2 - 0 = mgr(1-\cos\theta)$

$\Rightarrow v^2 = 2gr\,(1-\cos\theta)$ ...(2)

So, $\Delta T = \dfrac{m[2gr(1-\cos\theta)]}{r} = 2mg(1-\cos\theta)$

$\Rightarrow F = \Delta T$

$\Rightarrow F = \dfrac{YA\ \Delta L}{L} = 2mg - 2mg\cos\theta \Rightarrow 2mg\cos\theta = 2mg - \dfrac{YA\ \Delta L}{L}$

$= \cos\theta = 1 - \dfrac{YA\ \Delta L}{L(2mg)}$

11. From figure $\cos\theta = \dfrac{x}{\sqrt{x^2+l^2}} = \dfrac{x}{l}\left[1+\dfrac{x^2}{l^2}\right]^{-1/2}$

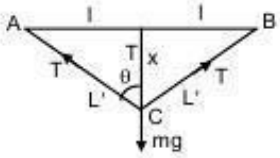
$= x/l$ ... (1)

Increase in length $\Delta L = (AC + CB) - AB$

Here, $AC = (l^2 + x^2)^{1/2}$

So, $\Delta L = 2(l^2 + x^2)^{1/2} - 100$ ...(2)

$Y = \dfrac{F}{A}\dfrac{l}{\Delta l}$ ...(3)



From equation (1), (2) and (3) and the freebody diagram,

$2l\cos\theta = mg$.

12. $Y = \dfrac{FL}{A\Delta L} \Rightarrow \dfrac{\Delta L}{L} = \dfrac{F}{Ay}$

$\sigma = \dfrac{\Delta D/D}{\Delta L/L} \Rightarrow \dfrac{\Delta D}{D} = \dfrac{\Delta L}{L}$

Again, $\dfrac{\Delta A}{A} = \dfrac{2\Delta r}{r}$

$\Rightarrow \Delta A = \dfrac{2\Delta r}{r}$

13. $B = \dfrac{Pv}{\Delta v} \Rightarrow P = B\left(\dfrac{\Delta v}{v}\right)$

14. $\rho_0 = \dfrac{m}{V_0} = \dfrac{m}{V_d}$

so, $\dfrac{\rho_d}{\rho_0} = \dfrac{V_0}{V_d}$ ...(1)

vol.strain $= \dfrac{V_0 - V_d}{V_0}$

$B = \dfrac{\rho_0 gh}{(V_0 - V_d)/V_0} \Rightarrow 1 - \dfrac{V_d}{V_0} = \dfrac{\rho_0 gh}{B}$

$$\Rightarrow \frac{vD}{v_0} = \left(1 - \frac{\rho_0 gh}{B}\right) \quad \ldots(2)$$

Putting value of (2) in equation (1), we get

$$\frac{\rho_d}{\rho_0} = \frac{1}{1 - \rho_0 gh/B} \Rightarrow \rho_d = \frac{1}{(1 - \rho_0 gh/B)} \times \rho_0$$

15. $\eta = \dfrac{F}{A\theta}$

Lateral displacement = $l\theta$.

16. $F = T\,l$

17. a) $P = \dfrac{2T_{Hg}}{r}$ b) $P = \dfrac{4T_g}{r}$ c) $P = \dfrac{2T_g}{r}$

18. a) $F = P_0 A$

b) Pressure = $P_0 + (2T/r)$

$F = P'A = (P_0 + (2T/r)A$

c) $P = 2T/r$

$F = PA = \dfrac{2T}{r}A$

19. a) $h_A = \dfrac{2T\cos\theta}{r_A - \rho g}$ b) $h_B = \dfrac{2T\cos\theta}{r_B \rho g}$ c) $h_C = \dfrac{2T\cos\theta}{r_C \rho g}$

20. $h_{Hg} = \dfrac{2T_{Hg}\cos\theta_{Hg}}{r\rho_{Hg} g}$

$h_\omega = \dfrac{2T_\omega \cos\theta_\omega}{r\rho_\omega g}$ where, the symbols have their usual meanings.

$$\frac{h_\omega}{h_{Hg}} = \frac{T_\omega}{T_{Hg}} \times \frac{\rho_{Hg}}{\rho_\omega} \times \frac{\cos\theta_\omega}{\cos\theta_{Hg}}$$

21. $h = \dfrac{2T\cos\theta}{r\rho g}$

22. $P = \dfrac{2T}{r}$

$P = F/r$

23. $A = \pi r^2$

24. $\dfrac{4}{3}\pi R^3 = \dfrac{4}{3}\pi r^3 \times 8$

$\Rightarrow r = R/2 = 2$

Increase in surface energy = $TA' - TA$

25. $h = \dfrac{2T\cos\theta}{r\rho g}$, $h' = \dfrac{2T\cos\theta}{r\rho g}$

$\Rightarrow \cos\theta = \dfrac{h' r\rho g}{2T}$

So, $\theta = \cos^{-1}(1/2) = 60°$.

26. a) $h = \dfrac{2T\cos\theta}{r\rho g}$

b) $T \times 2\pi r \cos\theta = \pi r^2 h \times \rho \times g$

$\therefore \cos\theta = \dfrac{hr\rho g}{2T}$

27. $T(2l) = [1 \times (10^{-3}) \times h]\rho g$

28. Surface area = $4\pi r^2$

29. The length of small element = $r\,d\,\theta$

$dF = T \times r\,d\,\theta$

considering symmetric elements,

$dF_y = 2T\, rd\theta \,.\, \sin\theta \quad [dF_x = 0]$

so, $F = 2Tr\int_0^{\pi/2} \sin\theta d\theta = 2Tr[\cos\theta]_0^{\pi/2} = T \times 2r$

Tension $\Rightarrow 2T_1 = T \times 2r \Rightarrow T_1 = Tr$

30. a) Viscous force = $6\pi\eta rv$

b) Hydrostatic force = $B = \left(\frac{4}{3}\right)\pi r^3\sigma g$

c) $6\pi\eta\, rv + \left(\frac{4}{3}\right)\pi r^3\sigma g = mg$

$$v = \frac{2r^2(\rho-\sigma)g}{9\eta} \Rightarrow \frac{2}{9}r^2\frac{\left(\frac{m}{(4/3)\pi r^3}-\sigma\right)g}{n}$$

31. To find the terminal velocity of rain drops, the forces acting on the drop are,

i) The weight $(4/3)\pi\, r^3\, \rho g$ downward.

ii) Force of buoyancy $(4/3)\pi\, r^3\, \sigma g$ upward.

iii) Force of viscosity $6\,\pi\,\eta\, r\, v$ upward.

Because, σ of air is very small, the force of buoyancy may be neglected.

Thus,

$$6\,\pi\,\eta\, r\, v = \left(\frac{4}{3}\right)\pi r^2\rho g \quad \text{or} \quad v = \frac{2r^2\rho g}{9\eta}$$

32. $v = \frac{R\eta}{\rho D} \Rightarrow R = \frac{v\rho D}{\eta}$