

Exercise 2.2

Q.1 Write each of the following in exponential form

$$(i) \left(\frac{3}{2}\right)^{-1} \times \left(\frac{3}{2}\right)^{-1} \times \left(\frac{3}{2}\right)^{-1} \times \left(\frac{3}{2}\right)^{-1}$$

Sol: $\left(\frac{3}{2}\right)^{-1+(-1)+(-1)+(-1)} = \left(\frac{3}{2}\right)^{-4} \left\{ \begin{array}{l} \because x^a \times x^b \\ = x^{a+b} \end{array} \right\}$

$$(ii) \left(\frac{2}{5}\right)^{-2} \times \left(\frac{2}{5}\right)^{-2} \times \left(\frac{2}{5}\right)^{-2}$$

$$= \left(\frac{2}{5}\right)^{-2+(-2)+(-2)} \left\{ \because x^a \times x^b = x^{a+b} \right\}$$

$$= \left(\frac{2}{5}\right)^{-6}$$

Q.2 Evaluate

$$(i) 5^{-2}$$

$$(ii) (-3)^{-2}$$

$$(iii) \left(\frac{1}{3}\right)^{-4}$$

$$(iv) \left(-\frac{1}{2}\right)^{-1}$$

Solution: (i) $5^{-2} = \frac{1}{5^2} = \frac{1}{25}$

$$(ii) (-3)^{-2} = \frac{1}{(-3)^2} = \frac{1}{9}$$

$$(iii) \left(\frac{1}{3}\right)^{-4} = \left(\frac{3}{1}\right)^4 = (3)^4 = 81$$

$$(iv) \left(-\frac{1}{2}\right)^{-1} = \left(\frac{2}{-1}\right)^1 = -2$$

Q.3 Express each of the following as a rational number in the form $\frac{p}{q}$.

(i) 6^{-1} (ii) $(-7)^{-1}$ (iii) $(\frac{1}{4})^{-1}$

(iv) $(-4)^{-1} \times (\frac{3}{2})^{-1}$ (v) $(\frac{3}{5})^{-1} \times (\frac{5}{2})^{-1}$

Sol (i) $6^{-1} = \frac{1}{6}$

(ii) $(-7)^{-1} = \frac{1}{-7}$

(iii) $(\frac{1}{4})^{-1} = (\frac{4}{1})^1 = 4$

(iv) $(-4)^{-1} \times (\frac{3}{2})^{-1} = \frac{1}{-4} \times \frac{2}{3} = \frac{2}{-12} = \frac{1}{-6} = -\frac{1}{6}$

(v) $(\frac{3}{5})^{-1} \times (\frac{5}{2})^{-1} = \frac{5}{3} \times \frac{2}{5} = \frac{2}{3}$

Q.4 Simplify

(i) $\{4^{-1} \times 3^{-1}\}^2$

(ii) $\{5^{-1} \div 6^{-1}\}^3$

(iii) $(2^{-1} + 3^{-1})^{-1}$

(iv) $\{3^{-1} \times 4^{-1}\}^{-1} \times 5^{-1}$

(v) $(4^{-1} - 5^{-1}) \div 3^{-1}$

Solution: (i) $\{4^{-1} \times 3^{-1}\}^2 = \left(\frac{1}{4} \times \frac{1}{3}\right)^2$
 $= \left(\frac{1}{12}\right)^2 = \frac{1^2}{12^2} = \frac{1}{144}$

(ii) $\{5^{-1} \div 6^{-1}\}^3 = \left\{\frac{1}{5} \div \frac{1}{6}\right\}^3$
 $= \left(\frac{1}{5} \times \frac{6}{1}\right)^3 = \left(\frac{6}{5}\right)^3 = \frac{6^3}{5^3} = \frac{216}{125}$

(iii) $(2^{-1} + 3^{-1})^{-1} = \left(\frac{1}{2} + \frac{1}{3}\right)^{-1} = \left(\frac{5}{6}\right)^{-1} = \frac{6}{5}$

(iv) $\{3^{-1} \times 4^{-1}\}^{-1} \times 5^{-1} = \left(\frac{1}{3} \times \frac{1}{4}\right)^{-1} \times \frac{1}{5}$
 $= \left(\frac{1}{12}\right)^{-1} \times \frac{1}{5} = \frac{12}{1} \times \frac{1}{5} = \frac{12}{5}$

(v) $(4^{-1} - 5^{-1}) \div 3^{-1}$
 $= \left(\frac{1}{4} - \frac{1}{5}\right) \div \frac{1}{3}$
 $= \left(\frac{5-4}{20}\right) \times 3 = \frac{1}{20} \times 3 = \frac{3}{20}$

Q.5 Express each of the following rational numbers with a negative exponent:

(i) $(\frac{1}{4})^3$

(ii) 3^5

(iii) $(\frac{3}{5})^4$

(iv) $\{(\frac{3}{2})^4\}^{-3}$

(v) $\{(\frac{7}{3})^4\}^{-3}$

Solution: (i) $(\frac{1}{4})^3 = 4^{-3}$

(ii) $3^5 = \frac{1}{3^{-5}}$

(iii) $(\frac{3}{5})^4 = (\frac{5}{3})^{-4}$

(iv) $\{(\frac{3}{2})^4\}^{-3} = (\frac{3}{2})^{4 \times -3} = (\frac{3}{2})^{-12}$

(v) $\{(\frac{7}{3})^4\}^{-3} = (\frac{7}{3})^{-12}$

Q.6 Express each of the following rational numbers with positive exponent.

(i) $(\frac{3}{4})^{-2}$

(ii) $(\frac{5}{4})^{-3}$

(iii) $4^3 \times 4^{-9}$

(iv) $\{(\frac{4}{3})^{-3}\}^{-4}$

(v) $\{(\frac{3}{2})^4\}^{-2}$

Solution: (i) $\left(\frac{3}{2}\right)^{-4} = \left(\frac{2}{3}\right)^4$

(ii) $\left(\frac{5}{4}\right)^{-3} = \left(\frac{4}{5}\right)^3$

(iii) $4^3 \times 4^{-9} = 4^{3 \times -9} = 4^{-27} = \frac{1}{4^{27}}$

(iv) $\left\{\left(\frac{4}{3}\right)^{-3}\right\}^{-4} = \left(\frac{4}{3}\right)^{-3 \times -4} = \left(\frac{4}{3}\right)^{12}$

(v) $\left\{\left(\frac{3}{2}\right)^4\right\}^{-2} = \left(\frac{3}{2}\right)^{-8} = \left(\frac{2}{3}\right)^8$

Q.7 Simplify

(i) $\left\{\left(\frac{1}{3}\right)^3 - \left(\frac{1}{2}\right)^3\right\} \div \left(\frac{1}{4}\right)^3$ (ii) $(3^2 - 2^2) \times \left(\frac{2}{3}\right)^{-3}$

(iii) $\left\{\left(\frac{1}{2}\right)^1 \times (-4)^1\right\}^{-1}$ (iv) $\left[\left\{\left(-\frac{1}{4}\right)^2\right\}^{-2}\right]^{-1}$

(v) $\left\{\left(\frac{2}{3}\right)^2\right\}^3 \times \left(\frac{1}{3}\right)^{-4} \times 3^{-1} \times 6^{-1}$

Solution: (i) $\left\{\left(\frac{1}{3}\right)^3 - \left(\frac{1}{2}\right)^3\right\} \div \left(\frac{1}{4}\right)^3$

$$\{3^3 - 2^3\} \div (4)^3$$

$$= (27 - 8) \div 64 = \frac{19}{64}$$

$$\begin{aligned}
 \text{(ii)} \quad (3^2 - 2^2) \times \left(\frac{2}{3}\right)^{-3} &= (9 - 4) \times \left(\frac{2}{3}\right)^3 \\
 &= 5 \times \frac{3^3}{2^3} = \frac{5 \times 27}{8} \\
 &= \frac{135}{8}
 \end{aligned}$$

$$\begin{aligned}
 \text{(iii)} \quad \left\{ \left(\frac{1}{2}\right)^{-1} \times (-4)^{-1} \right\}^{-1} &= \left\{ 2 \times \frac{1}{-4} \right\}^{-1} = \left(\frac{-1}{2}\right)^{-1} \\
 &= \left(-\frac{2}{1}\right) = -2
 \end{aligned}$$

$$\begin{aligned}
 \text{(iv)} \quad \left[\left\{ \left(-\frac{1}{4}\right)^2 \right\}^{-2} \right]^{-1} \\
 &= \left\{ \left(-\frac{1}{4}\right)^{2 \times -2} \right\}^{-1} = \left\{ \left(-\frac{1}{4}\right)^{-4} \right\}^{-1} = \left(-\frac{1}{4}\right)^{-4 \times -1} \\
 &= \left(-\frac{1}{4}\right)^4 = \frac{(-1)^4}{4^4} \\
 &= \frac{1}{256}
 \end{aligned}$$

$$\begin{aligned}
 \text{(v)} \quad \left\{ \left(\frac{2}{3}\right)^2 \right\}^3 \times \left(\frac{1}{3}\right)^{-4} \times 3^{-1} \times 6^{-1} \\
 &= \left(\frac{2}{3}\right)^{2 \times 3} \times (3)^4 \times \frac{1}{3^1} \times \frac{1}{6^1}
 \end{aligned}$$

$$\begin{aligned}
 \Rightarrow \left(\frac{2}{3}\right)^6 \times \frac{3^4}{3^1 \times 6^1} &\Rightarrow \frac{2^6}{3^6} \times \frac{3^4}{3^1 \times 6^1} = \frac{2^6 \times 3^4}{3^7 \times 6} \\
 &= \frac{2^6}{3^3 \times 6} = \frac{64}{162} \\
 &= \frac{32}{81}
 \end{aligned}$$

Q.8 By what number should 5^{-1} be multiplied so that the product may be equal to $(-7)^{-1}$?

Solution: let required no. be x

$$5^{-1} \times x = (-7)^{-1}$$

$$\Rightarrow \frac{1}{5} \times x = \frac{1}{-7} \Rightarrow -7x = 5$$

$$\Rightarrow x = \frac{-5}{7}$$

Q.9 By what number should $(\frac{1}{2})^{-1}$ be multiplied so that the product may be equal to $(-\frac{4}{7})^{-1}$?

Solution: let required no. be x

$$(\frac{1}{2})^{-1} \times x = (-\frac{4}{7})^{-1}$$

$$\Rightarrow (\frac{2}{1})^{-1} \times x = (\frac{7}{-4})^{-1}$$

$$\Rightarrow 2x = -\frac{7}{4} \Rightarrow 8x = -7$$

$$\Rightarrow x = -\frac{7}{8}$$

Q.10 By what number should $(-15)^{-1}$ be ^{divided} so that the product quotient may be equal to $(-5)^{-1}$?

Sol: let required no. be x

$$(-15)^{-1} \div x = (-5)^{-1}$$

$$= \frac{1}{-15} \times \frac{1}{x} = \frac{1}{-5}$$

$$\Rightarrow -15x = -5$$

$$\Rightarrow x = \frac{5}{15} = \frac{1}{3}$$

Q.11 By what number should $\left(\frac{5}{3}\right)^{-2}$ be multiplied so that the product may be $\left(\frac{7}{3}\right)^{-1}$?

Solution: Let required no. be x

$$\left(\frac{5}{3}\right)^{-2} \times x = \left(\frac{7}{3}\right)^{-1}$$

$$\Rightarrow \left(\frac{3}{5}\right)^2 \times x = \left(\frac{3}{7}\right)$$

$$\Rightarrow \frac{9x}{25} = \frac{3}{7} \Rightarrow 63x = 75$$

$$\Rightarrow x = \frac{75}{63} = \frac{25}{21}$$

$$\therefore \text{Required number} = \frac{25}{21}$$

Q.12 Find x , if

$$(i) \left(\frac{1}{4}\right)^{-4} \times \left(\frac{1}{4}\right)^{-8} = \left(\frac{1}{4}\right)^{-4x}$$

$$(ii) \left(-\frac{1}{2}\right)^{-19} \div \left(-\frac{1}{2}\right)^8 = \left(-\frac{1}{2}\right)^{-2x+1}$$

$$(iii) \left(\frac{3}{2}\right)^{-3} \times \left(\frac{3}{2}\right)^5 = \left(\frac{3}{2}\right)^{2x+1}$$

$$(iv) \left(\frac{2}{5}\right)^{-3} \times \left(\frac{2}{5}\right)^{15} = \left(\frac{2}{5}\right)^{2+3x}$$

$$(v) \left(\frac{5}{4}\right)^{-x} \div \left(\frac{5}{4}\right)^{-4} = \left(\frac{5}{4}\right)^5 \quad (vi) \left(\frac{8}{3}\right)^{2x+1} \times \left(\frac{8}{3}\right)^5 = \left(\frac{8}{3}\right)^{x+4}$$

Solution (i) $\left(\frac{1}{4}\right)^{-4} \times \left(\frac{1}{4}\right)^{-8} = \left(\frac{1}{4}\right)^{-4x}$

$$\Rightarrow \left(\frac{1}{4}\right)^{-4-8} = \left(\frac{1}{4}\right)^{-4x} \quad \left\{ \because x^a \times x^b = x^{a+b} \right\}$$

$$\Rightarrow \left(\frac{1}{4}\right)^{-12} = \left(\frac{1}{4}\right)^{-4x}$$

$$\Rightarrow -12 = -4x$$

$$\Rightarrow x = \frac{12}{4} = 3$$

$$(ii) \left(-\frac{1}{2}\right)^{-19} \div \left(-\frac{1}{2}\right)^8 = \left(-\frac{1}{2}\right)^{-2x+1}$$

$$\Rightarrow \left(-\frac{1}{2}\right)^{-19-8} = \left(-\frac{1}{2}\right)^{-2x+1}$$

$$\Rightarrow \left(-\frac{1}{2}\right)^{-27} = \left(-\frac{1}{2}\right)^{-2x+1}$$

$$\Rightarrow -27 = -2x+1$$

$$\Rightarrow -27-1 = -2x$$

$$\Rightarrow -28 = -2x$$

$$x = \frac{28}{2} = 14$$

$$(iii) \left(\frac{3}{2}\right)^{-3} \times \left(\frac{3}{2}\right)^5 = \left(\frac{3}{2}\right)^{2x+1}$$

$$\Rightarrow \left(\frac{3}{2}\right)^{-3+5} = \left(\frac{3}{2}\right)^{2x+1}$$

$$\Rightarrow \left(\frac{3}{2}\right)^2 = \left(\frac{3}{2}\right)^{2x+1} \Rightarrow 2x+1 = 2$$

$$\Rightarrow 2x = 2-1$$

$$\Rightarrow 2x = 1$$

$$\Rightarrow x = \frac{1}{2}$$

$$(iv) \left(\frac{2}{5}\right)^{-3} \times \left(\frac{2}{5}\right)^{15} = \left(\frac{2}{5}\right)^{2+3x}$$

$$\Rightarrow \left(\frac{2}{5}\right)^{-3+15} = \left(\frac{2}{5}\right)^{2+3x}$$

$$\Rightarrow \left(\frac{2}{5}\right)^{12} = \left(\frac{2}{5}\right)^{2+3x}$$

$$\Rightarrow 2+3x = 12$$

$$\Rightarrow 3x = 10$$

$$\Rightarrow x = \frac{10}{3}$$

$$(v) \left(\frac{5}{4}\right)^{-x} \div \left(\frac{5}{4}\right)^{-4} = \left(\frac{5}{4}\right)^5$$

$$\Rightarrow \left(\frac{5}{4}\right)^{-x-(-4)} = \left(\frac{5}{4}\right)^5$$

$$\Rightarrow \left(\frac{5}{4}\right)^{-x+4} = \left(\frac{5}{4}\right)^5 \Rightarrow -x+4 = 5$$

$$\Rightarrow x = 4-5 = -1$$

$$(vi) \left(\frac{8}{3}\right)^{2x+1} \times \left(\frac{8}{3}\right)^5 = \left(\frac{8}{3}\right)^{x+2}$$

$$\Rightarrow \left(\frac{8}{3}\right)^{2x+1+5} = \left(\frac{8}{3}\right)^{x+2}$$

$$\Rightarrow \left(\frac{8}{3}\right)^{2x+6} = \left(\frac{8}{3}\right)^{x+2}$$

$$\Rightarrow 2x+6 = x+2$$

$$\Rightarrow 2x-x = 2-6 \Rightarrow x = -4$$

Q.13 (i) If $x = \left(\frac{3}{2}\right)^2 \times \left(\frac{2}{3}\right)^{-4}$, find the value of $\bar{x}^2$.

Solution: $x = \left(\frac{3}{2}\right)^2 \times \left(\frac{3}{2}\right)^4$

$$\Rightarrow x = \left(\frac{3}{2}\right)^{2+4} = \left(\frac{3}{2}\right)^6$$

then $\bar{x}^2 = \left\{ \left(\frac{3}{2}\right)^6 \right\}^{-2}$

$$= \left(\frac{3}{2}\right)^{6 \times -2} = \left(\frac{3}{2}\right)^{-12}$$

$$= \left(\frac{2}{3}\right)^{12}$$

(ii) If $x = \left(\frac{4}{5}\right)^{-2} \div \left(\frac{1}{4}\right)^2$, find the value of $\bar{x}^1$

Sol: $x = \left(\frac{5}{4}\right)^2 \times 4^2 = \frac{5^2}{4^2} \times 4^2 = 5^2$

$$\text{then } x' = (5^2)^{-1} = 5^{-2} \\ = \frac{1}{5^2}$$

Q.14 Find the value of x for which $5^{2x} \div 5^{-3} = 5^5$

Solution: $5^{2x} \div 5^{-3} = 5^5$

$$\Rightarrow 5^{2x - (-3)} = 5^5$$

$$\Rightarrow 5^{2x+3} = 5^5 \quad \Rightarrow 2x+3=5$$

$$\Rightarrow 2x = 5-3$$

$$\Rightarrow 2x = 2$$

$$\Rightarrow x = 1$$