CHAPTER 13

SURFACE AREAS AND VOLUMES

(A) Main Concepts and Results

- **Cuboid whose length = l, breadth = b and height = h**
  - (a) Volume of cuboid = lbh
  - (b) Total surface area of cuboid = $2\,(lb + bh + hl)$
  - (c) Lateral surface area of cuboid = $2h\,(l + b)$
  - (d) Diagonal of cuboid = $\sqrt{l^2 + b^2 + h^2}$
- **Cube whose edge = a**
  - (a) Volume of cube = a^3
  - (b) Lateral Surface area = $4a^2$
  - (c) Total surface area of cube = $6a^2$
  - (d) Diagonal of cube = $a\sqrt{3}$
- **Cylinder whose radius = r, height = h**
  - (a) Volume of cylinder = $\pi r^2 h$
  - (b) Curved surface area of cylinder = $2\pi rh$
  - (c) Total surface area of cylinder = $2\pi r\,(r + h)$
- **Cone having height = h, radius = r and slant height = l**
  - (a) Volume of cone = $\frac{1}{3}\pi r^2 h$
  - (b) Curved surface area of cone = πrl

(c) Total surface area of cone = $\pi r\,(l + r)$

(d) Slant height of cone (l) = $\sqrt{h^2 + r^2}$

- **Sphere whose radius = r**

(a) Volume of sphere = $\frac{4}{3}\pi r^3$

(b) Surface area of sphere = $4\pi r^2$

- **Hemisphere whose radius = r**

(a) Volume of hemisphere = $\frac{2}{3}\pi r^3$

(b) Curved surface area of hemisphere = $2\pi r^2$

(c) Total surface area of hemisphere = $3\pi r^2$

(B) Multiple Choice Questions

Write the correct answer

Sample Question 1 : In a cylinder, if radius is halved and height is doubled, the volume will be

(A) same (B) doubled (C) halved (D) four times

Solution: Answer (C)

EXERCISE 13.1

Write the correct answer in each of the following :

1. The radius of a sphere is $2r$, then its volume will be

(A) $\frac{4}{3}\pi r^3$ (B) $4\pi r^3$ (C) $\frac{8\pi r^3}{3}$ (D) $\frac{32}{3}\pi r^3$

2. The total surface area of a cube is 96 cm^2. The volume of the cube is:

(A) 8 cm^3 (B) 512 cm^3 (C) 64 cm^3 (D) 27 cm^3

3. A cone is 8.4 cm high and the radius of its base is 2.1 cm. It is melted and recast into a sphere. The radius of the sphere is :

(A) 4.2 cm (B) 2.1 cm (C) 2.4 cm (D) 1.6 cm

4. In a cylinder, radius is doubled and height is halved, curved surface area will be

(A) halved (B) doubled (C) same (D) four times

5. The total surface area of a cone whose radius is $\frac{r}{2}$ and slant height $2l$ is

(A) $2\pi r\,(l + r)$ (B) $\pi r\,(l + \frac{r}{4})$ (C) $\pi r\,(l + r)$ (D) $2\pi rl$

6. The radii of two cylinders are in the ratio of 2:3 and their heights are in the ratio of 5:3. The ratio of their volumes is:

(A) 10 : 17 (B) 20 : 27 (C) 17 : 27 (D) 20 : 37

7. The lateral surface area of a cube is 256 m^2. The volume of the cube is

(A) 512 m^3 (B) 64 m^3 (C) 216 m^3 (D) 256 m^3

8. The number of planks of dimensions (4 m × 50 cm × 20 cm) that can be stored in a pit which is 16 m long, 12m wide and 4 m deep is

(A) 1900 (B) 1920 (C) 1800 (D) 1840

9. The length of the longest pole that can be put in a room of dimensions (10 m × 10 m × 5m) is

(A) 15 m (B) 16 m (C) 10 m (D) 12 m

10. The radius of a hemispherical balloon increases from 6 cm to 12 cm as air is being pumped into it. The ratios of the surface areas of the balloon in the two cases is

(A) 1 : 4 (B) 1 : 3 (C) 2 : 3 (D) 2 : 1

(C) Short Answer Questions with Reasoning

Write **True** or **False** and justify your answer.

Sample Question 1 : A right circular cylinder just encloses a sphere of radius r as shown in Fig 13.1. The surface area of the sphere is equal to the curved surface area of the cylinder.

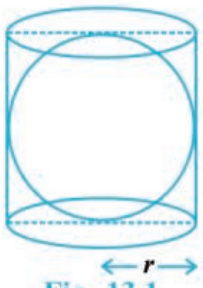


Fig. 13.1

Solution : True.

Here, radius of the sphere = radius of the cylinder = r

Diameter of the sphere = height of the cylinder = $2r$

Surface area of the sphere = $4\pi r^2$

Curved surface area of the cylinder = $2\pi r\,(2r) = 4\pi r^2$

Sample Question 2 : An edge of a cube measures r cm. If the largest possible right circular cone is cut out of this cube, then the volume of the cone (in cm^3) is $\frac{1}{6}\pi r^3$.

Solution : False.

Height of the cone = r cm.

Diameter of the base = r cm.

Therefore, volume of the cone $= \frac{1}{3}\pi\left(\frac{r}{2}\right)^2 . r$

$$= \frac{1}{12}\pi r^3$$

EXERCISE 13.2

Write **True** or **False** and justify your answer in each of the following :

1. The volume of a sphere is equal to two-third of the volume of a cylinder whose height and diameter are equal to the diameter of the sphere.

2. If the radius of a right circular cone is halved and height is doubled, the volume will remain unchanged.

3. In a right circular cone, height, radius and slant height do not always be sides of a right triangle.

4. If the radius of a cylinder is doubled and its curved surface area is not changed, the height must be halved.

5. The volume of the largest right circular cone that can be fitted in a cube whose edge is $2r$ equals to the volume of a hemisphere of radius r.

6. A cylinder and a right circular cone are having the same base and same height. The volume of the cylinder is three times the volume of the cone.

7. A cone, a hemisphere and a cylinder stand on equal bases and have the same height. The ratio of their volumes is 1 : 2 : 3.

8. If the length of the diagonal of a cube is $6\sqrt{3}$ cm, then the length of the edge of the cube is 3 cm.

9. If a sphere is inscribed in a cube, then the ratio of the volume of the cube to the volume of the sphere will be 6 : π.

10. If the radius of a cylinder is doubled and height is halved, the volume will be doubled.

(D) Short Answer Questions

Sample Question 1: The surface area of a sphere of radius 5 cm is five times the area of the curved surface of a cone of radius 4 cm. Find the height and the volume of the cone (taking $\pi = \frac{22}{7}$).

Solution: Surface area of the sphere = $4\pi \times 5 \times 5$ cm^2.

Curved surface area of the cone = $\pi \times 4 \times l$ cm^2,

where l is the slant height of the cone.

According to the statement

$$4\pi \times 5 \times 5 = 5 \times \pi \times 4 \times l$$

or $$l = 5 \text{ cm}.$$

Now, $$l^2 = h^2 + r^2$$

Therefore, $$(5)^2 = h^2 + (4)^2$$

where h is the height of the cone

or $$(5)^2 - (4)^2 = h^2$$

or $$(5 + 4)(5 - 4) = h^2$$

or $$9 = h^2$$

or $$h = 3 \text{ cm}$$

$$\text{Volume of Cone} = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3} \times \frac{22}{7} \times 4 \times 4 \times 3 \text{ cm}^3$$

$$= \frac{22 \times 16}{7} \text{ cm}^3$$

$$= \frac{352}{7} \text{ cm}^3 = 50.29 \text{ cm}^3 \text{ (approximately)}$$

Sample Question 2: The radius of a sphere is increased by 10%. Prove that the volume will be increased by 33.1% approximately.

Solution: The volume of a sphere = $\frac{4}{3}\pi r^3$

10% increase in radius = 10% r

Increased radius = $r + \frac{1}{10}r = \frac{11}{10}r$

The volume of the sphere now becomes

$$\frac{4}{3}\pi\left(\frac{11}{10}r\right)^3 = \frac{4}{3}\pi\times\frac{1331}{1000}r^3$$

$$= \frac{4}{3}\pi\times 1.331r^3$$

Increase in volume = $\frac{4}{3}\pi\times 1.331r^3 - \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi r^3\,(1.331-1) = \frac{4}{3}\pi r^3\times .331$

Percentage increase in volume= $\left[\frac{\frac{4}{3}\pi r^3\times .331}{\frac{4}{3}\pi r^3}\times 100\right] = 33.1$

EXERCISE 13.3

1. Metal spheres, each of radius 2 cm, are packed into a rectangular box of internal dimensions 16 cm × 8 cm × 8 cm. When 16 spheres are packed the box is filled with preservative liquid. Find the volume of this liquid. Give your answer to the nearest integer. [Use $\pi = 3.14$]

2. A storage tank is in the form of a cube. When it is full of water, the volume of water is 15.625 m^3. If the present depth of water is 1.3 m, find the volume of water already used from the tank.

3. Find the amount of water displaced by a solid spherical ball of diameter 4.2 cm, when it is completely immersed in water.

4. How many square metres of canvas is required for a conical tent whose height is 3.5 m and the radius of the base is 12 m?

5. Two solid spheres made of the same metal have weights 5920 g and 740 g, respectively. Determine the radius of the larger sphere, if the diameter of the smaller one is 5 cm.

6. A school provides milk to the students daily in a cylindrical glasses of diameter 7 cm. If the glass is filled with milk upto an height of 12 cm, find how many litres of milk is needed to serve 1600 students.

7. A cylindrical roller 2.5 m in length, 1.75 m in radius when rolled on a road was found to cover the area of 5500 m^2. How many revolutions did it make?

8. A small village, having a population of 5000, requires 75 litres of water per head per day. The village has got an overhead tank of measurement 40 m × 25 m × 15 m. For how many days will the water of this tank last?

9. A shopkeeper has one spherical laddoo of radius 5cm. With the same amount of material, how many laddoos of radius 2.5 cm can be made?

10. A right triangle with sides 6 cm, 8 cm and 10 cm is revolved about the side 8 cm. Find the volume and the curved surface of the solid so formed.

(E) Long Answer Questions

Sample Question 1: Rain water which falls on a flat rectangular surface of length 6 m and breadth 4 m is transferred into a cylindrical vessel of internal radius 20 cm. What will be the height of water in the cylindrical vessel if the rain fall is 1 cm. Give your answer to the nearest integer. (Take $\pi = 3.14$)

Solution : Let the height of the water level in the cylindrical vessel be h cm

Volume of the rain water = $600 \times 400 \times 1$ cm^3

Volume of water in the cylindrical vessel = $\pi (20)^2 \times h$ cm^3

According to statement

$$600 \times 400 \times 1 = \pi (20)^2 \times h$$

or
$$h = \frac{600}{3.14} \text{ cm} = 191 \text{ cm}$$

EXERCISE 13.4

1. A cylindrical tube opened at both the ends is made of iron sheet which is 2 cm thick. If the outer diameter is 16 cm and its length is 100 cm, find how many cubic centimeters of iron has been used in making the tube ?

2. A semi-circular sheet of metal of diameter 28cm is bent to form an open conical cup. Find the capacity of the cup.

3. A cloth having an area of 165 m^2 is shaped into the form of a conical tent of radius 5 m

(i) How many students can sit in the tent if a student, on an average, occupies $\frac{5}{7}m^2$ on the ground?

(ii) Find the volume of the cone.

4. The water for a factory is stored in a hemispherical tank whose internal diameter is 14 m. The tank contains 50 kilolitres of water. Water is pumped into the tank to fill to its capacity. Calculate the volume of water pumped into the tank.

5. The volumes of the two spheres are in the ratio 64 : 27. Find the ratio of their surface areas.

6. A cube of side 4 cm contains a sphere touching its sides. Find the volume of the gap in between.

7. A sphere and a right circular cylinder of the same radius have equal volumes. By what percentage does the diameter of the cylinder exceed its height ?

8. 30 circular plates, each of radius 14 cm and thickness 3cm are placed one above the another to form a cylindrical solid. Find :

(i) the total surface area

(ii) volume of the cylinder so formed.