

Chapter - 2

Powers

Exercise 2.1

Q.1 Express each of the following as a rational number of the form $\frac{p}{q}$, where p and q are integers.

(i) 2^{-3} (ii) $(-4)^{-2}$ (iii) $\frac{1}{3^{-2}}$ (iv) $(\frac{1}{2})^{-5}$

(v) $(\frac{2}{3})^{-2}$

Solution: (i) $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

(ii) $(-4)^{-2} = \frac{1}{(-4)^2} = \frac{1}{16}$

(iii) $\frac{1}{3^{-2}} = 3^2 = 9$

(iv) $(\frac{1}{2})^{-5} = (\frac{2}{1})^5 = (2)^5 = 32$

(v) $(\frac{2}{3})^{-2} = (\frac{3}{2})^{+2} = \frac{3 \times 3}{2 \times 2} = \frac{9}{4}$

Q.2 Find the values of each of the following

(i) $3^{-1} + 4^{-1}$

(ii) $(3^0 + 4^1) \times 2^2$

(iii) $(3^{-1} + 4^{-1} + 5^{-1})^0$

(iv) $\{(\frac{1}{3})^{-1} - (\frac{1}{4})^{-1}\}^{-1}$

Solution: (i) $3^{-1} + 4^{-1} = \frac{1}{3} + \frac{1}{4} = \frac{4+3}{12} = \frac{7}{12}$

(ii) $(3^0 + 4^0) \times 2^2$
 $= (1 + \frac{1}{4}) \times 4$
 $= \frac{5}{4} \times 4 = 5$

(iii) $(3^{-1} + 4^{-1} + 5^{-1})^0 = 1 \quad \{a^0 = 1\}$

(iv) $\left\{ \left(\frac{1}{3}\right)^{-1} - \left(\frac{1}{4}\right)^{-1} \right\}^{-1}$
 $= (3^1 - 4^1)^{-1}$
 $= (-1)^{-1} = \frac{1}{(-1)} = -1$

Q3 Find the value of each of the following.

(i) $\left(\frac{1}{2}\right)^{-1} + \left(\frac{1}{3}\right)^{-1} + \left(\frac{1}{4}\right)^{-1}$ (ii) $\left(\frac{1}{2}\right)^{-2} + \left(\frac{1}{3}\right)^{-2} + \left(\frac{1}{4}\right)^{-2}$

(iii) $(2^{-1} \times 4^{-1}) \div 2^{-2}$ (iv) $(5^{-1} \times 2^{-1}) \div 6^{-1}$

Solution: (i) $\left(\frac{1}{2}\right)^{-1} + \left(\frac{1}{3}\right)^{-1} + \left(\frac{1}{4}\right)^{-1}$
 $= (2)^1 + (3)^1 + (4)^1$
 $= 2 + 3 + 4 = 9$

$$\begin{aligned}
 \text{(ii)} \quad & \left(\frac{1}{2}\right)^{-2} + \left(\frac{1}{3}\right)^{-2} + \left(\frac{1}{4}\right)^{-2} \\
 &= (2)^2 + (3)^2 + (4)^2 \\
 &= 4 + 9 + 16 \\
 &= 29
 \end{aligned}$$

$$\begin{aligned}
 \text{(iii)} \quad & (2^{-1} \times 4^{-1}) \div 2^{-2} \\
 &= \left(\frac{1}{2} \times \frac{1}{4}\right) \div \frac{1}{2^2} \\
 &= \frac{1}{8} \times 2^2 = \frac{1}{8} \times 4 = \frac{1}{2}
 \end{aligned}$$

$$\begin{aligned}
 \text{(iv)} \quad & (5^{-1} \times 2^{-1}) \div 6^{-1} \\
 &= \left(\frac{1}{5} \times \frac{1}{2}\right) \div \frac{1}{6} \\
 &= \frac{1}{10} \times 6 = \frac{3}{5}
 \end{aligned}$$

Q.4 Simplify

$$\text{(i)} \quad (4^{-1} \times 3^{-1})^2$$

$$\text{(ii)} \quad (5^{-1} \div 6^{-1})^3$$

$$\text{(iii)} \quad (2^{-1} + 3^{-1})^{-1}$$

$$\text{(iv)} \quad (3^{-1} \times 4^{-1})^{-1} \times 5^{-1}$$

Solution (i) $(4^{-1} \times 3^{-1})^2 = \left(\frac{1}{4} \times \frac{1}{3}\right)^2 = \left(\frac{1}{12}\right)^2 = \frac{1}{144}$

$$\begin{aligned}
 \text{(ii)} \quad & (5^{-1} \div 6^{-1})^3 = \left(\frac{1}{5} \div \frac{1}{6}\right)^3 = \left(\frac{1}{5} \times 6\right)^3 = \left(\frac{6}{5}\right)^3 \\
 &= \frac{6^3}{5^3} = \frac{216}{125}
 \end{aligned}$$

$$(iii) (2^{-1} + 3^{-1})^{-1} = \left(\frac{1}{2} + \frac{1}{3}\right)^{-1} \\ = \left(\frac{3+2}{6}\right)^{-1} = \left(\frac{5}{6}\right)^{-1} = \frac{6}{5}$$

$$(iv) (3^{-1} \times 4^{-1})^{-1} \times 5^{-1} = \left(\frac{1}{3} \times \frac{1}{4}\right)^{-1} \times 5^{-1} \\ = \left(\frac{1}{12}\right)^{-1} \times 5^{-1} = 12 \times \frac{1}{5} = \frac{12}{5}$$

Q.5 Simplify:

$$(i) (3^2 + 2^2) \times \left(\frac{1}{2}\right)^3 \quad (ii) (3^2 - 2^2) \times \left(\frac{2}{3}\right)^{-3}$$

$$(iii) \left[\left(\frac{1}{3}\right)^{-3} - \left(\frac{1}{2}\right)^{-3}\right] \div \left(\frac{1}{4}\right)^{-3}$$

$$(iv) (2^2 + 3^2 - 4^2) \div \left(\frac{3}{2}\right)^2$$

Solution: (i) $(3^2 + 2^2) \times \left(\frac{1}{2}\right)^3$

$$= (9 + 4) \times \frac{1}{8} = \frac{13 \times 1}{8} = \frac{13}{8}$$

$$(ii) (3^2 - 2^2) \times \left(\frac{2}{3}\right)^{-3} = (9 - 4) \times \left(\frac{3}{2}\right)^3$$

$$= 5 \times \frac{3^3}{2^3} = \frac{5 \times 27}{8} = \frac{135}{8}$$

$$(iii) \left[\left(\frac{1}{3}\right)^{-3} - \left(\frac{1}{2}\right)^{-3} \right] \div \left(\frac{1}{4}\right)^{-3}$$

$$= (3^3 - 2^3) \div (4)^3$$

$$\Rightarrow (27 - 8) \div 64$$

$$= \frac{19}{64}$$

$$(iv) (2^2 + 3^2 - 4^2) \div \left(\frac{3}{2}\right)^2$$

$$= (4 + 9 - 16) \div \frac{3^2}{2^2}$$

$$= -3 \div \frac{9}{4} = -3 \times \frac{4}{9} = -\frac{4}{3}$$

Q.6 By what number should 5^{-1} be multiplied so that the product may be equal to $(-7)^{-1}$?

Solution: let required no. be x

$$5^{-1} \times x = (-7)^{-1}$$

$$\Rightarrow \frac{1}{5} \times x = \frac{1}{-7}$$

$$\Rightarrow \frac{x}{5} = \frac{1}{-7} \Rightarrow x = \frac{-5}{7}$$

Q.7 By what number should $(\frac{1}{2})^{-1}$ be multiplied so that the product may be equal to $(-\frac{4}{7})^{-1}$?

Solution: let required no. be x .

$$(\frac{1}{2})^{-1} \times x = (-\frac{4}{7})^{-1}$$

$$\Rightarrow 2 \times x = (\frac{7}{-4})$$

$$\Rightarrow x = \frac{7}{-4 \times 2} = -\frac{7}{8}$$

$$\therefore x = -\frac{7}{8}$$

Q.8 By what number should $(-15)^{-1}$ be divided so that the quotient may be equal to $(-5)^{-1}$?

Sol: let required no. be x

$$(-15)^{-1} \div x = (-5)^{-1}$$

$$\Rightarrow \frac{1}{-15} \times \frac{1}{x} = \frac{1}{-5}$$

(By cross multiplication)

$$\Rightarrow -15x = -5$$

$$x = \frac{-5}{-15} = \frac{1}{3}$$