CHAPTER 8

QUADRILATERALS

(A) Main Concepts and Results

Sides, Angles and diagonals of a quadrilateral; Different types of quadrilaterals: Trapezium, parallelogram, rectangle, rhombus and square.

- Sum of the angles of a quadrilateral is 360°,
- A diagonal of a parallelogram divides it into two congruent triangles,
- In a parallelogram
  - (i) opposite angles are equal
  - (ii) opposite sides are equal
  - (iii) diagonals bisect each other.
- A quadrilateral is a parallelogram, if
  - (i) its opposite angles are equal
  - (ii) its opposite sides are equal
  - (iii) its diagonals bisect each other
  - (iv) a pair of opposite sides is equal and parallel.
- Diagonals of a rectangle bisect each other and are equal and vice-versa
- Diagonals of a rhombus bisect each other at right angles and vice-versa
- Diagonals of a square bisect each other at right angles and are equal and vice-versa
- The line-segment joining the mid-points of any two sides of a triangle is parallel to the third side and is half of it.

- A line drawn through the mid-point of a side of a triangle parallel to another side bisects the third side,
- The quadrilateral formed by joining the mid-points of the sides of a quadrilateral, taken in order, is a parallelogram.

(B) Multiple Choice Questions

Write the correct answer :

Sample Question 1 : Diagonals of a parallelogram ABCD intersect at O. If $\angle BOC = 90°$ and $\angle BDC = 50°$, then $\angle OAB$ is

(A) 90° (B) 50° (C) 40° (D) 10°

Solution : Answer (C)

EXERCISE 8.1

Write the correct answer in each of the following:

1. Three angles of a quadrilateral are 75°, 90° and 75°. The fourth angle is

(A) 90° (B) 95° (C) 105° (D) 120°

2. A diagonal of a rectangle is inclined to one side of the rectangle at 25°. The acute angle between the diagonals is

(A) 55° (B) 50° (C) 40° (D) 25°

3. ABCD is a rhombus such that $\angle ACB = 40°$. Then $\angle ADB$ is

(A) 40° (B) 45° (C) 50° (D) 60°

4. The quadrilateral formed by joining the mid-points of the sides of a quadrilateral PQRS, taken in order, is a rectangle, if

(A) PQRS is a rectangle
(B) PQRS is a parallelogram
(C) diagonals of PQRS are perpendicular
(D) diagonals of PQRS are equal.

5. The quadrilateral formed by joining the mid-points of the sides of a quadrilateral PQRS, taken in order, is a rhombus, if

(A) PQRS is a rhombus
(B) PQRS is a parallelogram
(C) diagonals of PQRS are perpendicular
(D) diagonals of PQRS are equal.

6. If angles A, B, C and D of the quadrilateral ABCD, taken in order, are in the ratio 3:7:6:4, then ABCD is a

(A) rhombus (B) parallelogram

(C) trapezium (D) kite

7. If bisectors of $\angle A$ and $\angle B$ of a quadrilateral ABCD intersect each other at P, of $\angle B$ and $\angle C$ at Q, of $\angle C$ and $\angle D$ at R and of $\angle D$ and $\angle A$ at S, then PQRS is a

(A) rectangle (B) rhombus (C) parallelogram

(D) quadrilateral whose opposite angles are supplementary

8. If APB and CQD are two parallel lines, then the bisectors of the angles APQ, BPQ, CQP and PQD form

(A) a square (B) a rhombus

(C) a rectangle (D) any other parallelogram

9. The figure obtained by joining the mid-points of the sides of a rhombus, taken in order, is

(A) a rhombus (B) a rectangle

(C) a square (D) any parallelogram

10. D and E are the mid-points of the sides AB and AC of ΔABC and O is any point on side BC. O is joined to A. If P and Q are the mid-points of OB and OC respectively, then DEQP is

(A) a square (B) a rectangle

(C) a rhombus (D) a parallelogram

11. The figure formed by joining the mid-points of the sides of a quadrilateral ABCD, taken in order, is a square only if,

(A) ABCD is a rhombus

(B) diagonals of ABCD are equal

(C) diagonals of ABCD are equal and perpendicular

(D) diagonals of ABCD are perpendicular.

12. The diagonals AC and BD of a parallelogram ABCD intersect each other at the point O. If $\angle DAC = 32^\circ$ and $\angle AOB = 70^\circ$, then $\angle DBC$ is equal to

(A) 24° (B) 86° (C) 38° (D) 32°

13. Which of the following is not true for a parallelogram?

(A) opposite sides are equal

(B) opposite angles are equal

(C) opposite angles are bisected by the diagonals

(D) diagonals bisect each other.

14. D and E are the mid-points of the sides AB and AC respectively of ΔABC. DE is produced to F. To prove that CF is equal and parallel to DA, we need an additional information which is

(A) ∠DAE = ∠EFC

(B) AE = EF

(C) DE = EF

(D) ∠ADE = ∠ECF.

(C) Short Answer Questions with Reasoning

Sample Question 1 : ABCD is a parallelogram. If its diagonals are equal, then find the value of ∠ABC.

Solution : As diagonals of the parallelogram ABCD are equal, it is a rectangle.

Therefore, ∠ABC = 90°

Sample Question 2 : Diagonals of a rhombus are equal and perpendicular to each other. Is this statement true? Give reason for your answer.

Solution : This statement is false, because diagonals of a rhombus are perpendicular but not equal to each other.

Sample Question 3 : Three angles of a quadrilateral ABCD are equal. Is it a parallelogram? Why or why not?

Solution: It need not be a parallelogram, because we may have ∠A = ∠B = ∠C = 80° and ∠D = 120°. Here, ∠B ≠ ∠D.

Sample Question 4 : Diagonals AC and BD of a quadrilateral ABCD intersect each other at O such that OA : OC = 3: 2. Is ABCD a parallelogram? Why or why not?

Solution : ABCD is not a parallelogram, because diagonals of a parallelogram bisect each other. Here OA ≠ OC.

EXERCISE 8.2

1. Diagonals AC and BD of a parallelogram ABCD intersect each other at O. If OA = 3 cm and OD = 2 cm, determine the lengths of AC and BD.

2. Diagonals of a parallelogram are perpendicular to each other. Is this statement true? Give reason for your answer.

3. Can the angles 110°, 80°, 70° and 95° be the angles of a quadrilateral? Why or why not?

4. In quadrilateral ABCD, $\angle A + \angle D = 180°$. What special name can be given to this quadrilateral?

5. All the angles of a quadrilateral are equal. What special name is given to this quadrilateral?

6. Diagonals of a rectangle are equal and perpendicular. Is this statement true? Give reason for your answer.

7. Can all the four angles of a quadrilateral be obtuse angles? Give reason for your answer.

8. In ΔABC, AB = 5 cm, BC = 8 cm and CA = 7 cm. If D and E are respectively the mid-points of AB and BC, determine the length of DE.

9. In Fig.8.1, it is given that BDEF and FDCE are parallelograms. Can you say that BD = CD? Why or why not?

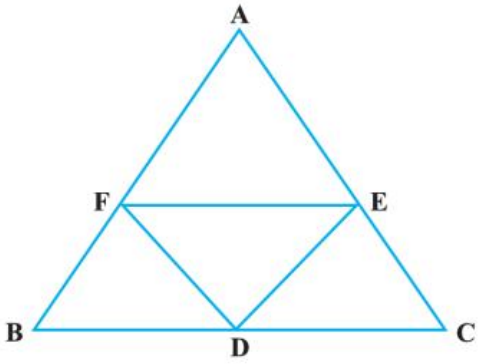


Fig. 8.1

10. In Fig.8.2, ABCD and AEFG are two parallelograms. If $\angle C = 55°$, determine $\angle F$.

11. Can all the angles of a quadrilateral be acute angles? Give reason for your answer.

12. Can all the angles of a quadrilateral be right angles? Give reason for your answer.

13. Diagonals of a quadrilateral ABCD bisect each other. If $\angle A = 35°$, determine $\angle B$.

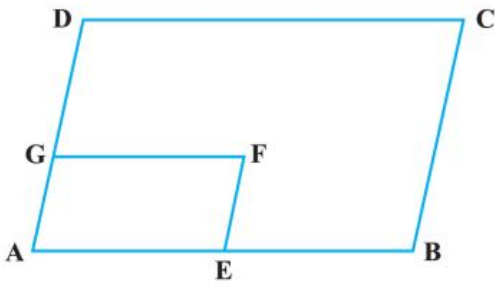


Fig. 8.2

14. Opposite angles of a quadrilateral ABCD are equal. If AB = 4 cm, determine CD.

(D) Short Answer Questions

Sample Question 1 : Angles of a quadrilateral are in the ratio 3 : 4 : 4 : 7. Find all the angles of the quadrilateral.

Solution : Let the angles of the quadrilateral be $3x$, $4x$, $4x$ and $7x$.

So, $3x + 4x + 4x + 7x = 360^\circ$

or $18x = 360^\circ$, i.e., $x = 20^\circ$

Thus, required angles are 60°, 80°, 80° and 140°.

Sample Question 2 : In Fig.8.3, X and Y are respectively the mid-points of the opposite sides AD and BC of a parallelogram ABCD. Also, BX and DY intersect AC at P and Q, respectively. Show that AP = PQ = QC.

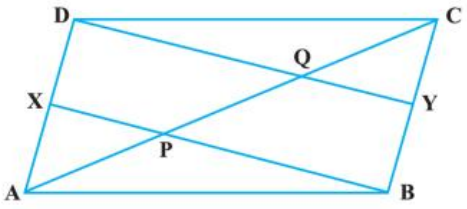


Fig. 8.3

Solution : AD = BC

(Opposite sides of a parallelogram)

Therefore, $DX = BY \left(\frac{1}{2} AD = \frac{1}{2} BC\right)$

Also, DX ∥ BY (As AD ∥ BC)

So, XBYD is a parallelogram (A pair of opposite sides equal and parallel)

i.e., PX ∥ QD

Therefore, AP = PQ (From ΔAQD where X is mid-point of AD)

Similarly, from ΔCPB, CQ = PQ (1)

Thus, AP = PQ = CQ [From (1) and (2)] (2)

Sample Question 3 : In Fig.8.4, AX and CY are respectively the bisectors of the opposite angles A and C of a parallelogram ABCD.

Show that AX ∥ CY.

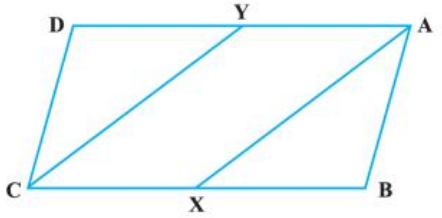


Fig. 8.4

Solution : $\angle A = \angle C$

(Opposite angles of parallelogram ABCD)

Therefore, $\frac{1}{2}\angle A = \frac{1}{2}\angle C$

i.e., $\angle YAX = \angle YCX$ (1)

Also, $\angle AYC + \angle YCX = 180^\circ$ (Because YA || CX) (2)

Therefore, $\angle AYC + \angle YAX = 180^\circ$ [From (1) and (2)]

So, AX || CY (As interior angles on the same side of the transversal are supplementary)

EXERCISE 8.3

1. One angle of a quadrilateral is of 108° and the remaining three angles are equal. Find each of the three equal angles.

2. ABCD is a trapezium in which AB || DC and $\angle A = \angle B = 45^\circ$. Find angles C and D of the trapezium.

3. The angle between two altitudes of a parallelogram through the vertex of an obtuse angle of the parallelogram is 60°. Find the angles of the parallelogram.

4. ABCD is a rhombus in which altitude from D to side AB bisects AB. Find the angles of the rhombus.

5. E and F are points on diagonal AC of a parallelogram ABCD such that AE = CF. Show that BFDE is a parallelogram.

6. E is the mid-point of the side AD of the trapezium ABCD with AB || DC. A line through E drawn parallel to AB intersect BC at F. Show that F is the mid-point of BC. [**Hint:** Join AC]

7. Through A, B and C, lines RQ, PR and QP have been drawn, respectively parallel to sides BC, CA and AB of a Δ ABC as shown in Fig.8.5. Show that $BC = \frac{1}{2}QR$.

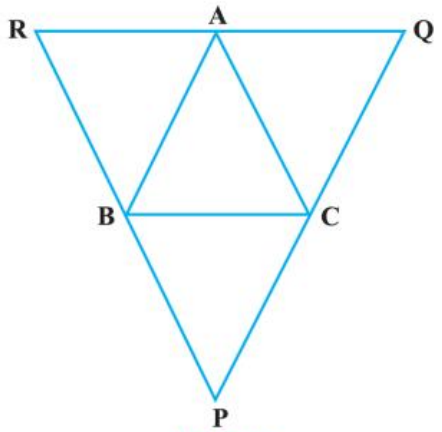


Fig. 8.5

8. D, E and F are the mid-points of the sides BC, CA and AB, respectively of an

equilateral triangle ABC. Show that Δ DEF is also an equilateral triangle.

9. Points P and Q have been taken on opposite sides AB and CD, respectively of a parallelogram ABCD such that AP = CQ (Fig. 8.6). Show that AC and PQ bisect each other.

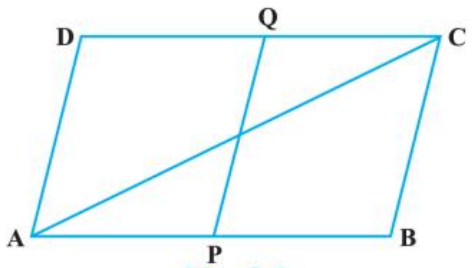


Fig. 8.6

10. In Fig. 8.7, P is the mid-point of side BC of a parallelogram ABCD such that ∠BAP = ∠DAP. Prove that AD = 2CD.

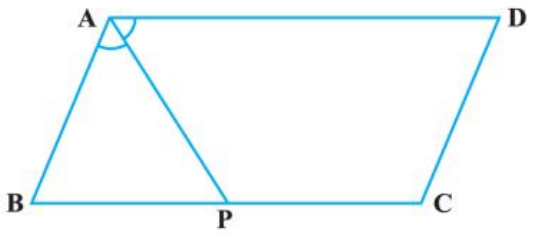


Fig. 8.7

(E) Long Answer Questions

Sample Question 1 : PQ and RS are two equal and parallel line-segments. Any point M not lying on PQ or RS is joined to Q and S and lines through P parallel to QM and through R parallel to SM meet at N. Prove that line segments MN and PQ are equal and parallel to each other.

Solution : We draw the figure as per the given conditions (Fig.8.8).

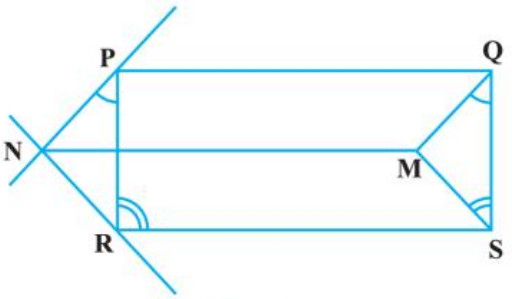


Fig. 8.8

It is given that PQ = RS and PQ ∥ RS. Therefore, PQSR is a parallelogram.

So, PR = QS and PR ∥ QS (1)

Now, PR ∥ QS

Therefore, $\angle RPQ + \angle PQS = 180^\circ$

(Interior angles on the same side of the transversal)

i.e., $\angle RPQ + \angle PQM + \angle MQS = 180^\circ$ (2)

Also, PN ∥ QM (By construction)

Therefore, $\angle NPQ + \angle PQM = 180^\circ$

i.e., $\angle NPR + \angle RPQ + \angle PQM = 180^\circ$ (3)

So, $\angle NPR = \angle MQS$ [From (2) and (3)] (4)

Similarly, $\angle NRP = \angle MSQ$ (5)

Therefore, $\Delta PNR \cong \Delta QMS$ [ASA, using (1), (4) and (5)]

So, PN = QM and NR = MS (CPCT)

As, PN = QM and

PN ∥ QM, we have PQMN is a parallelogram

So, MN = PQ and NM ∥ PQ.

Sample Question 2 : Prove that a diagonal of a parallelogram divides it into two congruent triangles.

Solution : See proof of Theorem 8.1 in the textbook.

Sample Question 3 : Show that the quadrilateral formed by joining the mid-points the sides of a rhombus, taken in order, form a rectangle.

Solution : Let ABCD be a rhombus and P, Q, R and S be the mid-points of sides AB, BC, CD and DA, respectively (Fig. 8.9). Join AC and BD.

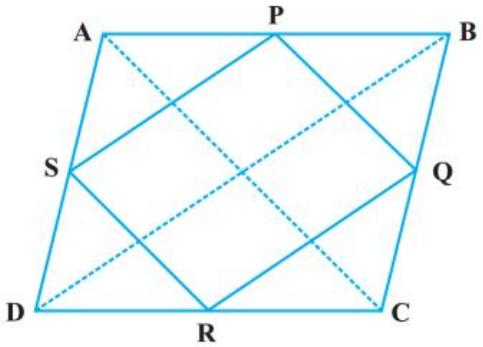


Fig. 8.9

From triangle ABD, we have

$$SP = \frac{1}{2} BD \text{ and}$$

$SP \parallel BD$ (Because S and P are mid-points)

Similarly, $RQ = \frac{1}{2} BD$ and $RQ \parallel BD$

Therefore, $SP = RQ$ and $SP \parallel RQ$

So, PQRS is a parallelogram. (1)

Also, $AC \perp BD$ (Diagonals of a rhombus are perpendicular)

Further $PQ \parallel AC$ (From ΔBAC)

As $SP \parallel BD$, $PQ \parallel AC$ and $AC \perp BD$,

therefore, we have $SP \perp PQ$, i.e. $\angle SPQ = 90^\circ$. (2)

Therefore, PQRS is a rectangle[From (1) and (2)]

Sample Question 4 : A diagonal of a parallelogram bisects one of its angle. Prove that it will bisect its opposite angle also.

Solution : Let us draw the figure as per given condition (Fig.8.10). In it, AC is a diagonal which bisects $\angle BAD$ of the parallelogram ABCD, i.e., it is given that $\angle BAC = \angle DAC$. We need to prove that $\angle BCA = \angle DCA$.

AB $\parallel$ CD and AC is a transversal.

Therefore, $\angle BAC = \angle DCA$ (Alternate angles) (1)

Similarly, $\angle DAC = \angle BCA$ (From AD $\parallel$ BC) (2)

But it is given that $\angle BAC = \angle DAC$ (3)

Therefore, from (1), (2) and (3), we have

$$\angle BCA = \angle DCA$$

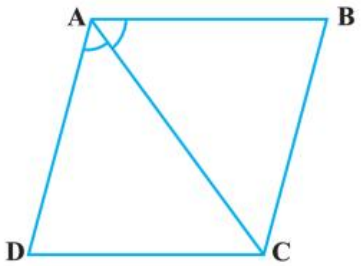


Fig. 8.10

EXERCISE 8.4

1. A square is inscribed in an isosceles right triangle so that the square and the triangle have one angle common. Show that the vertex of the square opposite the vertex of the common angle bisects the hypotenuse.

2. In a parallelogram ABCD, AB = 10 cm and AD = 6 cm. The bisector of $\angle A$ meets DC in E. AE and BC produced meet at F. Find the length of CF.

3. P, Q, R and S are respectively the mid-points of the sides AB, BC, CD and DA of a quadrilateral ABCD in which AC = BD. Prove that PQRS is a rhombus.

4. P, Q, R and S are respectively the mid-points of the sides AB, BC, CD and DA of a quadrilateral ABCD such that AC $\perp$ BD. Prove that PQRS is a rectangle.

5. P, Q, R and S are respectively the mid-points of sides AB, BC, CD and DA of quadrilateral ABCD in which AC = BD and AC $\perp$ BD. Prove that PQRS is a square.

6. A diagonal of a parallelogram bisects one of its angles. Show that it is a rhombus.

7. P and Q are the mid-points of the opposite sides AB and CD of a parallelogram ABCD. AQ intersects DP at S and BQ intersects CP at R. Show that PRQS is a parallelogram.

8. ABCD is a quadrilateral in which AB || DC and AD = BC. Prove that $\angle A = \angle B$ and $\angle C = \angle D$.

9. In Fig. 8.11, AB || DE, AB = DE, AC || DF and AC = DF. Prove that BC || EF and BC = EF.

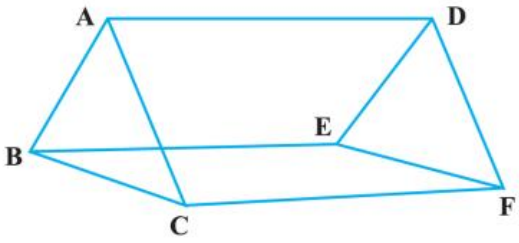


Fig. 8.11

10. E is the mid-point of a median AD of ΔABC and BE is produced to meet AC at F. Show that $AF = \frac{1}{3} AC$.

11. Show that the quadrilateral formed by joining the mid-points of the consecutive sides of a square is also a square.

12. E and F are respectively the mid-points of the non-parallel sides AD and BC of a trapezium ABCD. Prove that EF ∥ AB and EF = $\frac{1}{2}$ (AB + CD).

[**Hint:** Join BE and produce it to meet CD produced at G.]

13. Prove that the quadrilateral formed by the bisectors of the angles of a parallelogram is a rectangle.

14. P and Q are points on opposite sides AD and BC of a parallelogram ABCD such that PQ passes through the point of intersection O of its diagonals AC and BD. Show that PQ is bisected at O.

15. ABCD is a rectangle in which diagonal BD bisects ∠B. Show that ABCD is a square.

16. D, E and F are respectively the mid-points of the sides AB, BC and CA of a triangle ABC. Prove that by joining these mid-points D, E and F, the triangles ABC is divided into four congruent triangles.

17. Prove that the line joining the mid-points of the diagonals of a trapezium is parallel to the parallel sides of the trapezium.

18. P is the mid-point of the side CD of a parallelogram ABCD. A line through C parallel to PA intersects AB at Q and DA produced at R. Prove that DA = AR and CQ = QR.