M.Sc. Statistics

1. The probability density function (pdf) of a random variable X is $f(x) = (x^{-1/2})/4$, $0< x<4$ and zero otherwise. The value of $P(X>1)$ is :

A) 1 B) ½ C) ¾ D) ¼

2. Probability is .7 that an industrial worker suffers from respiratory problem if exposed to toxic gases and probability is .2 that the worker suffers from respiratory problem if not exposed to toxic gases. The probability that an industrial worker suffers from respiratory problem is:

A) .14 B) .7 C) .9 D) 1

3. Let Q_i be the ith quartile of a distribution, i= 1, 2, 3. The distribution is positively skewed if:

A) $Q_3 - Q_1 > Q_2 - Q_1$ B) $Q_3 - Q_1 < Q_2 - Q_1$

C) $Q_3 - Q_2 < Q_2 - Q_1$ D) $Q_3 - Q_2 > Q_2 - Q_1$

4. The total number of simple random samples with replacement each of size n which can be drawn from a population of size N are:

A) n^N B) N^n C) ${}^NC_n$ D) nN

5. The total number of simple random samples without replacement each of size n which can be drawn from a population of size N are:

A) ${}^NC_n$ B) N^n C) n^N D) nN

6. Let the random variable X follows binomial distribution with parameters n and p. Define random variable Y = X/n . Variance of Y is:

A) np(1-p) B) p(1-p) C) p(1-p)/n D) $p(1-p)/n^2$

7. Let the random variable W follows Poisson distribution such that 2P[W=0] = P[W=1]. Variance of W is:

A) 2 B) 1 C) ½ D) e^2

8. The mean and variance of a binomial random variable X are 4 and 3, respectively. The value of the parameters (n, p) is

A) (4, ¾) B) (12, 3/4) C) (16, 3/ 4) D) (16, ¼)

9. A person selects a lottery ticket from a bag containing 10 lottery tickets numbered from 1 to 10. The probability the number on the drawn ticket is either even or a multiple of 3 is:

A) 8/10 B) 2/10 C) 7/10 D) 5/10

10. Area under normal probability density function above third quartile is:

A) .75 B) .25 C) .5 D) .40

11. A simple random sample without replacement of size two was drawn from a lot containing 2 defective and 4 non defective items. The probability of the event that the first drawn item is defective and second is non-defective is:

A) 8/36 B) 1/8 C) 7/9 D) 4/15

12. A discrete random variable X takes values -2, -1, 0, 1, 2 with probabilities .2, .1, .4, .1, .2 , respectively. Variance of X is:

A) 1.8 B) 0 C) ½ D) .18

13. Poisson distribution is:

A) Symmetric B) Positively skewed

C) Negatively skewed D) Continuous

The three units, say I, II and III of an industrial organization manufacture identical items in the proportion 2: 3: 5, respectively. It is also known that the units I, II and III manufacture 3%, 4% and 10% defective items, respectively. From the entire daily production of all the three units a person selects one item randomly.

Answer questions 14 and 15 using the above information:

14. The probability that the selected item will be non-defective is:

A) .932 B) .068 C) .17 D) .83

15. If the drawn item was defective, then the probability that it was manufactured by unit III is:

A) .068 B) .932 C) .204 D) .735

16. Let the random variable X follows normal distribution. The distribution of –X will be:

A) Binomial B) Poisson C) Normal D) Negatively Skewed

17. An unbiased coin and an unbiased die are tossed simultaneously. Probability that the coin shows head and die shows an even number on upper faces is:

A) 1/12 B) 1 C) 3/4 D) ¼

18. Two unbiased dice are tossed simultaneously. The probability that the sum of numbers on the upper faces of both dice will be odd is:

A) ¼ B) ¾ C) ½ D) 2/3

19. The probability density function (pdf) of a random variable X is $f(x) = 1$, $0<x<1$. Variance of X will be:

A) ½ B) 1/12 C) ¾ D) ¼

20. Let b_{xy} and b_{yx} be the two regression coefficients in simple linear regression. Then the relation between Karl Pearson's correlation coefficient, denoted by r, and these regression coefficients is:

A) $r^2 = (b_{xy}/b_{yx})^2$ B) $r^2 = (b_{xy}b_{yx})^2$ C) $r = (b_{xy}b_{yx})^2$ D) $r^2 = (b_{xy}\, b_{yx})$

Let A and B be two events in the sample space S such that $P(A \cup B) = .7$, $P(A) = .4$ and $P(B) = x$. Answer questions 21 and 22 using this information.

21. The value of x for which A and B are mutually exclusive is :

A) .3 B) ½ C) .28 D) 0

22. The value of x for which A and B are independent is:

A) .3 B) ½ C) ¾ D) ¼

A lot contains 4 defective and 6 Non-defective items. A person selects a simple random sample of 4 items from the lot. Use this information to answer questions 23 and 24.

23. The probability that the sample contains two defective and two non defective if the items were drawn with replacement is:

A) .52 B) .0576 C) .3456 D) 3/7

24. The probability that the sample contains two defective and two non defective if the items were drawn without replacement is:

A) .52 B) .0576 C) .3456 D) 3/7

25. The mean, mode and standard deviation of a distribution are 15, 12 and 3 respectively. The coefficient of skewness of this distribution is:

A) 1 B) -1 C) 4 D) 5

26. Let the matrix G be such that $\begin{pmatrix} 5 & 4 \\ 1 & 1 \end{pmatrix}$ G = $\begin{pmatrix} 1 & -2 \\ 1 & 3 \end{pmatrix}$, then G is equal to:

A) $\begin{pmatrix} 3 & -14 \\ 4 & -17 \end{pmatrix}$ B) $\begin{pmatrix} 1 & -2 \\ 3 & 1 \end{pmatrix}$ C) $\begin{pmatrix} 1 & 3 \\ -2 & 1 \end{pmatrix}$ D) $\begin{pmatrix} -3 & -14 \\ 4 & 17 \end{pmatrix}$

27. The quadratic form x^tGx is said to be positive semidefinite when all the eigen values of G are:

A) Positive B) Negative C) Non-negative D) non-positive

28. The sum of the characteristic roots of the matrix $\begin{bmatrix} 3 & 10 & 15 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$ is

A) 13 B) 7 C) -9 D) 15

29. The product of the eigen values of the matrix $\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$

A) 5 B) 7 C) 12 D) 4

30. If H = $\begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}$, then H^n is:

A) $\begin{pmatrix} 1+2n & -4n \\ n & 1-2n \end{pmatrix}$ B) $\begin{pmatrix} 3^n & (-4)^n \\ 1 & (-1)^n \end{pmatrix}$ C) $\begin{pmatrix} 1+3n & 1-4n \\ 1+n & 1-n \end{pmatrix}$ D) $\begin{pmatrix} 1+2n & -4n \\ 1+n & 1-2n \end{pmatrix}$

31. The extreme value of $(x)^{1/x}$ is:

A) e B) $(1/e)^e$ C) $(e)^{1/e}$ D) 1

32. If f(x) = sinx - x/2 is an increasing function, then the range of x is

A) $0< x <\pi/3$ B) $-\pi/3 < x < 0$ C) $-\pi/6 < x <\pi/6$ D) $-\pi/3< x <\pi/3$

33. The Jacobean of the transformation $x= r\cos\theta$, $y = r\sin\theta$ is:

A) 1 B) –r C) r D) 1/r

34. The value of $\iint 2x(x-y)dx\,dy$ over the region R={(x,y): 0<x<1, |y|<x} is:

A) 2/3 B) 0 C) ½ D) 1

35. The value of $\sum_{x=0}^{3}\sum_{y=0}^{2}(x+y)$ is:

A) 12 B) 30 C) 15 D) 1/30

36. $\frac{d^2}{dxdy}(1-e^{-x}-e^{-y}+e^{-x-y})$ is equal to:

A) 0 B) $-e^{-x-y}$ C) e^{-x-y} D) e^{-xy}

37. Let f(x,y) = 1/y, then the value of $\iint f(x,y)dxdy$ over the region R = {(x,y): 0<x<y, 0<y<1} is:

A) 1 B) ½ C) 2 D) ¾

38. The value of $\iint 2dx\,dy$ over the region R={(x,y): 0<x, 0<y, x+y <1} is:

A) ½ B) 1 C) 2 D) ¾

39. $\int_0^{\pi/2}\frac{\sqrt{\sin x}}{(\sin x)^{1/2}+(\cos x)^{1/2}}dx$ is equal to:

A) 0 B) π/2 C) π/4 D) 1

40. $\int_0^{\pi}\frac{x\sin x\ dx}{1+\cos^2 x}$ is equal to :

A) $\pi^2/2$ B) $\pi^2/4$ C) ¼ D) π/4

41. $\int_0^{1}\frac{\log(1+x)\ dx}{1+x^2}$ is equal to

A) (π/8)log2 B) (π/4)log2 C) (π/2)log2 D) log2

42. $\int_0^{\pi}\sin^7 x\,dx$ is equal to

A) 0 B) 16/35 C) 32π/35 D) 32/35

43. $\int_0^{\pi/2}\sin^4 x\cos^2 x dx$ is equal to

A) 1/16 B) 1/32 C) π/32 D) π/4

44. The value of $\iint xydxdy$ over the region R = { (x,y): 0<x<1, x^2<y<2-x} is:

A) ¾ B) 3/8 C) 3/5 D) 3/7

45. The area of the segment cut off from the parabola $x^2 = 8y$ by the line x-2y+8 =0 is:

A) 48 B) 120 C) 36 D) 24

46. $\int_0^1 \int_0^{1-x} dx\,dy$ represents the area of a:

A) Rectangle B) Triangle C) Square D) Circle

47. $\int_0^1 \frac{\sin^{-1} x}{x} dx$ is equal to:

A) π/2 B) log2 C) (π/2)log2 D) -(π/2)log2

48. $\int_0^{\pi} \log \sin x dx$ is equal to:

A) –π log2 B) -(π/2)log2 C) (π/2)log2 D) log2

49. $\int_0^{\pi/6} \cos^4 3\theta \sin^3 6\theta d\theta$ is equal to:

A) 0 B) π/30 C) 8/45 D) 1/15

50. The value of $\int_0^{\infty} \frac{dx}{(1+x^2)^{7/2}}$ is :

A) 8/15 B) 8π/15 C) π/15 D) 3π/4

51. $\int_0^a \frac{dx}{x+(a^2-x^2)^{1/2}}$ is equal to:

A) π/2 B) π/4 C) $a^2/4$ D) aπ/4

52. Let $y = x^x$. Then dy/dx is:

A) $x\, x^{x-1}$ B) $x^x \log x$ C) $x^x (\log x + 1)$ D) $x^{x+1}\log x$

53. Let $y = a^x$, then dy/dx is:

A) xa^{x-1} B) $a^x \log a$ C) (aloga)/x D) $xa^{x-1}\log a$

54. Let the matrix $G = \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$ Then the value of G^2 -4G -5I is

A) O B) G^{-1} C) I D) G

55. The value of $\lim_{x\to 0}(1+x)^{1/x}$ is:

A) ∞ B) 1 C) e D) 1/e

56. The value of $\lim_{x\to \pi/2}(\sin x)^{\tan x}$ is:

A) 0 B) 1 C) e D) 1/e

57. The value of $\lim_{x\to 0}\{(xe^x - \log(1+x))/x^2\}$ is:

A) ½ B) 3 C) 1 D) 3/2

58. The value of $\lim_{x\to 0}(1/\sin x - 1/x)$ is:

A) ∞ B) 1 C) 0 D) 1/2

59. The slope of the normal at any point θ to the curve $x = a(\cos\theta + \theta\cos\theta)$, $y = a(\sin\theta - \theta\cos\theta)$ is

A) $\tan\theta$ B) $\cot\theta$ C) $-\tan\theta$ D) $-\cot\theta$

60. The value of $\lim_{x\to 0}(\log x/\cot x)$ is:

A) 1 B) 0 C) 2 D) ½

61. The series $1 - 1/\sqrt{2} + 1/\sqrt{3} - 1/\sqrt{4}$ is

A) Convergent B) Divergent C) Oscillatory D) Both B and C

62. The series 5/2 -7/4 +9/6- 11/8 + ……. is

A) Convergent B) Divergent C) Oscillatory D) Both A and C

63. The series $1 + 1/2^2 - 1/3^2 - 1/4^2 + 1/5^2 + 1/6^2 - 1/7^2 - 1/8^2$ ……. is

A) Convergent B) Divergent C) Oscillatory D) Both B and C

64. The series $1 + x + x^2/2 + x^3/3 +$…… is convergent for:

A) All values of B) $x > 0$ C) $x < 0$ D) $x < -1$

65. The series $x - x^2/2 + x^3/3 -$ ….. $+ (-1)^n x^n/n +$ …. is convergent for values of x the interval:

A) (-1, 1) B) $|x| > 1$ C) (-1, 1] D) $|x| \le 1$.

66. The value of $\int_a^{a+2\pi} \sin^2 nx\,dx$ is

A) 0 B) π C) $\pi/2$ D) n/2

67. The value of $\int_a^{a+2\pi} \sin mx \cos mx\,dx$ is.

A) 0 B) π C) $\pi/2$ D) m/2

68. $\sin(ix)$ is equal to (here $i = \sqrt{-1}$)

A) sinhx B) i sinhx C) e^{ix} D) $e^{ix + 2\pi}$

69. For complex number Z = x + i y, if $\frac{e^{Z} + e^{-z}}{2} = i$, then e^{Z} is equal to

A) 1 B) -1 C) i D) –i

70. For complex number Z , the value of $\tanh^{-1} z$ is:

A) log [(1-Z)/(1+z)] B) .5 log [(1-Z)/(1+z)]

C) log [(1+Z)/(1-z)] D) .5 [log {(1+Z)/(1-z)}]

71. For complex number Z = x +iy, the imaginary part of e^{Z^2} is:

A) e^{i2xy} B) e^{ixy} C) $e^{ix^2y^2}$ D) $e^{i(x+y)}$

72. The value of $\int_C \frac{e^{2Z}}{(Z^2 - 3Z + 2}dz$, where C: |Z| = 3 is:

A) $2\pi i$ B) $2\pi i e^4$ C) $2\pi i (e^4 - e^2)$ D) $2\pi i (e^4 + e^2)$

73. The value of $\int_0^{2+i} (\overline{Z})^2 dz$ along the real axis to 2 and then vertically to 2+i is:

A) 5(2-i)/3 B) 14/3 C) 14 + 11i D) (14 + 11i)/3

74. The solution of the differential equation dy/dx +y/x = x^2 under the condition that y=1 when x= 1, is:

A) $4xy = x^3+3$ B) $4xy = x^4 +3$ C) $4xy = y^4+3$ D) $4xy = y^3 +3$

75. The particular integral of the differential equation $(D^2 +D)y = x^2+2x+4$ is (here D =d/dx):

A) $(x^2/3) +4x$ B) $(x^3/3) +4$ C) $(x^3/3)+4x$ D) $(x^3/3)+4x^2$

x-x-x