CHAPTER 5

INTRODUCTION TO EUCLID'S GEOMETRY

(A) Main Concepts and Results

Points, Line, Plane or surface, Axiom, Postulate and Theorem, The Elements, Shapes of altars or vedis in ancient India, Equivalent versions of Euclid's fifth Postulate, Consistency of a system of axioms.

Ancient India

- The geometry of the Vedic period originated with the construction of altars (or vedis) and fireplaces for performing Vedic rites. Square and circular altars were used for household rituals, while altars, whose shapes were combinations of rectangles, triangles and trapeziums, were required for public worship.

Egypt, Babylonia and Greece

- Egyptians developed a number of geometric techniques and rules for calculating simple areas and doing simple constructions. Babylonians and Egyptains used geometry mostly for practical purposes and did very little to develop it as a systematic science. The Greeks were interested in establishing the truth of the statements they discovered using deductive reasoning. A Greek mathematician, Thales is credited with giving the first known proof.

Euclid's Elements

- Euclid around 300 B.C. collected all known work in the field of mathematics and arranged it in his famous treatise called **Elements**. Euclid assumed certain properties, which were not to be proved. These assumptions are actually "obvious universal truths". He divided them into two types.

Axioms

1. The things which are equal to the same thing are equal to one another.
2. If equals be added to the equals, the wholes are equal.
3. If equals be subtracted from equals, the remainders are equals.
4. Things which coincide with one another are equal to one another.
5. The whole is greater than the part.
6. Things which are double of the same thing are equal to one another.
7. Things which are halves of the same thing are equal to one another.

Postulates

1. A straight line may be drawn from any point to any other point.
2. A terminated line (line segment) can be produced indefinitely.
3. A circle may be described with any centre and any radius.
4. All right angles are equal to one another.
5. If a straight line falling on two straight lines makes the interior angles on the same side of it, taken together less than two right angles, then the the two straight lines if produced indefinitely, meet on that side on which the sum of angles is taken together less than two right angles.

   Euclid used the term postulate for the assumptions that were specific to geometry and otherwise called axioms. A **theorem** is a mathematical statement whose truth has been logically established.

Present Day Geometry

- A mathematical system consists of axioms, definitions and undefined terms.
- Point, line and plane are taken as undefined terms.
- A system of axioms is said to be consistent if there are no contradictions in the axioms and theorems that can be derived from them.
- Given two distinct points, there is a unique line passing through them.
- Two distinct lines can not have more than one point in common.
- Playfair's Axiom (An equivalent version of Euclid's fifth postulate).

(B) Multiple Choice Questions

Write the correct answer :

Sample Question 1 : Euclid's second axiom (as per order given in the Textbook for Class IX) is

(A) The things which are equal to the same thing are equal to one another.

(B) If equals be added to equals, the wholes are equal.

(C) If equals be subtracted from equals, the remainders are equals.

(D) Things which coincide with one another are equal to one another.

Solution : Answer (B)

Sample Question 2 : Euclid's fifth postulate is

(A) The whole is greater than the part.

(B) A circle may be described with any centre and any radius.

(C) All right angles are equal to one another.

(D) If a straight line falling on two straight lines makes the interior angles on the same side of it taken together less than two right angles, then the two straight lines if produced indefinitely, meet on that side on which the sum of angles is less than two right angles.

Solution : Answer (D)

Sample Question 3 : The things which are double of the same thing are

(A) equal

(B) unequal

(C) halves of the same thing

(D) double of the same thing

Solution : Answer (A)

Sample Question 4 : Axioms are assumed

(A) universal truths in all branches of mathematics

(B) universal truths specific to geometry

(C) theorems

(D) definitions

Solution : Answer (A)

Sample Question 5 : John is of the same age as Mohan. Ram is also of the same age as Mohan. State the Euclid's axiom that illustrates the relative ages of John and Ram

(A) First Axiom (B) Second Axiom

(C) Third Axiom (D) Fourth Axiom

Solution : Answer (A)

Sample Question 6 : If a straight line falling on two straight lines makes the interior angles on the same side of it, whose sum is 120°, then the two straight lines, if produced indefinitely, meet on the side on which the sum of angles is

(A) less than 120° (B) greater than 120°

(C) is equal to 120° (D) greater than 180°

Solution : Answer (C)

EXERCISE 5.1

1. The three steps from solids to points are :

(A) Solids - surfaces - lines - points

(B) Solids - lines - surfaces - points

(C) Lines - points - surfaces - solids

(D) Lines - surfaces - points - solids

2. The number of dimensions, a solid has :

(A) 1 (B) 2 (C) 3 (D) 0

3. The number of dimensions, a surface has :

(A) 1 (B) 2 (C) 3 (D) 0

4. The number of dimension, a point has :

(A) 0 (B) 1 (C) 2 (D) 3

5. Euclid divided his famous treatise "The Elements" into :

(A) 13 chapters (B) 12 chapters (C) 11 chapters (D) 9 chapters

6. The total number of propositions in the Elements are :

(A) 465 (B) 460 (C) 13 (D) 55

7. Boundaries of solids are :

(A) surfaces (B) curves (C) lines (D) points

8. Boundaries of surfaces are :

(A) surfaces (B) curves (C) lines (D) points

9. In Indus Valley Civilisation (about 3000 B.C.), the bricks used for construction work were having dimensions in the ratio

(A) 1 : 3 : 4 (B) 4 : 2 : 1 (C) 4 : 4 : 1 (D) 4 : 3 : 2

10. A pyramid is a solid figure, the base of which is

(A) only a triangle (B) only a square

(C) only a rectangle (D) any polygon

11. The side faces of a pyramid are :

(A) Triangles (B) Squares (C) Polygons (D) Trapeziums

12. It is known that if $x + y = 10$ then $x + y + z = 10 + z$. The Euclid's axiom that illustrates this statement is :

(A) First Axiom (B) Second Axiom

(C) Third Axiom (D) Fourth Axiom

13. In ancient India, the shapes of altars used for house hold rituals were :

(A) Squares and circles (B) Triangles and rectangles

(C) Trapeziums and pyramids (D) Rectangles and squares

14. The number of interwoven isosceles triangles in Sriyantra (in the Atharvaveda) is:

(A) Seven (B) Eight (C) Nine (D) Eleven

15. Greek's emphasised on :

(A) Inductive reasoning (B) Deductive reasoning

(C) Both A and B (D) Practical use of geometry

16. In Ancient India, Altars with combination of shapes like rectangles, triangles and trapeziums were used for :

(A) Public worship (B) Household rituals

(C) Both A and B (D) None of A, B, C

17. Euclid belongs to the country :

(A) Babylonia (B) Egypt (C) Greece (D) India

18. Thales belongs to the country :

(A) Babylonia (B) Egypt (C) Greece (D) Rome

19. Pythagoras was a student of :

(A) Thales (B) Euclid (C) Both A and B (D) Archimedes

20. Which of the following needs a proof ?

(A) Theorem (B) Axiom (C) Definition (D) Postulate

21. Euclid stated that all right angles are equal to each other in the form of

(A) an axiom (B) a definition (C) a postulate (D) a proof

22. 'Lines are parallel if they do not intersect' is stated in the form of

(A) an axiom (B) a definition (C) a postulate (D) a proof

(C) Short Answer Questions with Reasoning

Sample Question 1 : Write whether the following statements are True or False? Justify your answer.

(i) Pyramid is a solid figure, the base of which is a triangle or square or some other polygon and its side faces are equilateral triangles that converges to a point at the top.

(ii) In Vedic period, squares and circular shaped altars were used for household rituals, while altars whose shapes were combination of rectangles, triangles and trapeziums were used for public worship.

(iii) In geometry, we take a point, a line and a plane as undefined terms.

(iv) If the area of a triangle equals the area of a rectangle and the area of the rectangle equals that of a square, then the area of the triangle also equals the area of the square.

(v) Euclid's fourth axiom says that everything equals itself.

(vi) The Euclidean geometry is valid only for figures in the plane.

Solution :

(i) False. The side faces of a pyramid are triangles not necessarily equilateral triangles.

(ii) True. The geometry of Vedic period originated with the construction of vedis and fireplaces for performing vedic rites. The location of the sacred fires had to be in accordance to the clearly laid down instructions about their shapes and area.

(iii) True. To define a point, a line and a plane in geometry we need to define many other things that give a long chain of definitions without an end. For such reasons, mathematicians agree to leave these geometric terms undefined.

(iv) True. Things equal to the same thing are equal.

(v) True. It is the justification of the principle of superposition.

(vi) True. It fails on the curved surfaces. For example on curved surfaces, the sum of angles of a triangle may be more than 180°.

EXERCISE 5.2

Write whether the following statements are **True** or **False**? Justify your answer :

1. Euclidean geometry is valid only for curved surfaces.

2. The boundaries of the solids are curves.

3. The edges of a surface are curves.

4. The things which are double of the same thing are equal to one another.

5. If a quantity B is a part of another quantity A, then A can be written as the sum of B and some third quantity C.

6. The statements that are proved are called axioms.

7. "For every line l and for every point P not lying on a given line l, there exists a unique line m passing through P and parallel to l" is known as Playfair's axiom.
8. Two distinct intersecting lines cannot be parallel to the same line.
9. Attempts to prove Euclid's fifth postulate using the other postulates and axioms led to the discovery of several other geometries.

(D) Short Answer Questions

Sample Question 1 : Ram and Ravi have the same weight. If they each gain weight by 2 kg, how will their new weights be compared ?

Solution : Let x kg be the weight each of Ram and Ravi. On gaining 2 kg, weight of Ram and Ravi will be $(x + 2)$ each. According to Euclid's second axiom, when equals are added to equals, the wholes are equal. So, weight of Ram and Ravi are again equal.

Sample Question 2 : Solve the equation $a - 15 = 25$ and state which axiom do you use here.

Solution : $a - 15 = 25$. On adding 15 to both sides, we have $a - 15 + 15 = 25 + 15 = 40$ (using Euclid's second axiom).

or $a = 40$

Sample Question 3 : In the Fig. 5.1, if

$\angle 1 = \angle 3$, $\angle 2 = \angle 4$ and $\angle 3 = \angle 4$,
write the relation between $\angle 1$ and $\angle 2$, using an Euclid's axiom.

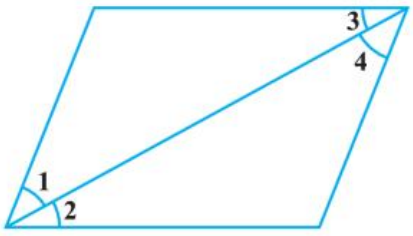


Fig. 5.1

Solution : Here, $\angle 3 = \angle 4$, $\angle 1 = \angle 3$ and $\angle 2 = \angle 4$. Euclid's first axiom says, the things which are equal to equal thing are equal to one aother.

So, $\angle 1 = \angle 2$.

Sample Question 4 : In Fig. 5.2, we have : AC = XD, C is the mid-point of AB and D is the mid-point of XY. Using an Euclid's axiom, show that AB = XY.

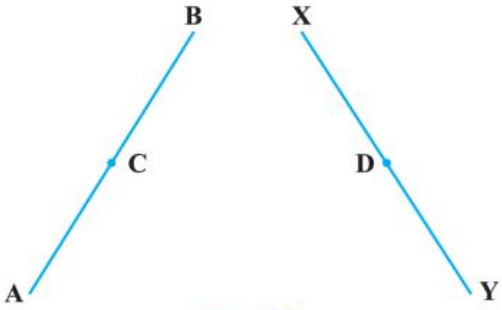


Fig. 5.2

Solution :
AB = 2AC (C is the mid-point of AB)

XY = 2XD (D is the mid-point of XY)

Also, AC = XD (Given)

Therefore, AB = XY, because things which are double of the same things are equal to one another.

EXERCISE 5.3

Solve each of the following question using appropriate Euclid's axiom :

1. Two salesmen make equal sales during the month of August. In September, each salesman doubles his sale of the month of August. Compare their sales in September.

2. It is known that $x + y = 10$ and that $x = z$. Show that $z + y = 10$?

3. Look at the Fig. 5.3. Show that length AH > sum of lengths of AB + BC + CD.



Fig. 5.3

4. In the Fig.5.4, we have

AB = BC, BX = BY. Show that AX = CY.

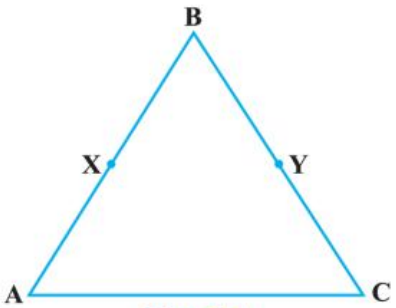


Fig. 5.4

5. In the Fig.5.5, we have

X and Y are the mid-points of AC and BC and AX = CY. Show that AC = BC.

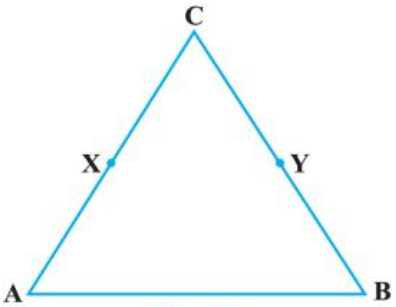


Fig. 5.5

6. In the Fig.5.6, we have

$BX = \frac{1}{2}$ AB

$BY = \frac{1}{2}$ BC and AB = BC. Show that BX = BY.

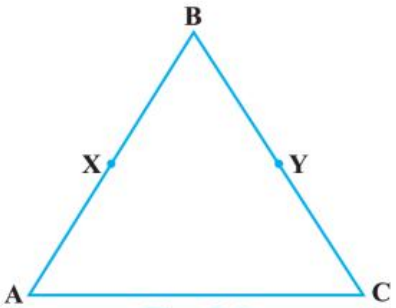


Fig. 5.6

7. In the Fig.5.7, we have
$\angle 1 = \angle 2$, $\angle 2 = \angle 3$. Show that $\angle 1 = \angle 3$.

8. In the Fig. 5.8, we have
$\angle 1 = \angle 3$ and $\angle 2 = \angle 4$. Show that $\angle A = \angle C$.

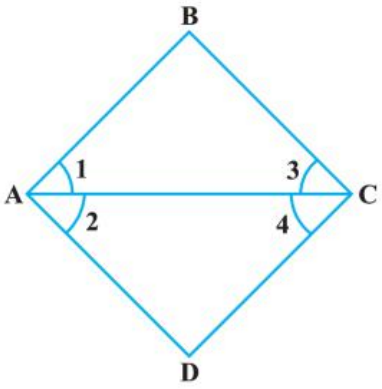


Fig. 5.8

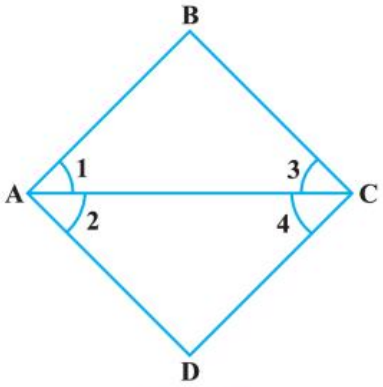


Fig. 5.7

9. In the Fig. 5.9, we have
$\angle ABC = \angle ACB$, $\angle 3 = \angle 4$. Show that $\angle 1 = \angle 2$.

10. In the Fig. 5.10, we have
AC = DC, CB = CE. Show that AB = DE.

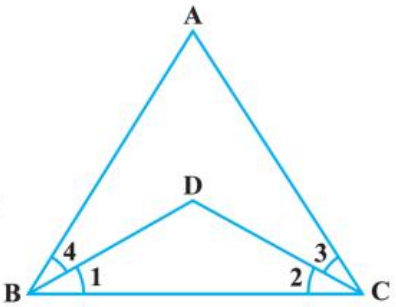


Fig. 5.9

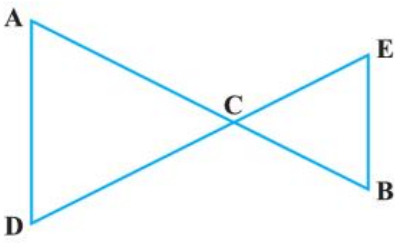


Fig. 5.10

11. In the Fig. 5.11, if $OX = \frac{1}{2}XY$, $PX = \frac{1}{2}XZ$ and OX = PX, show that XY = XZ.

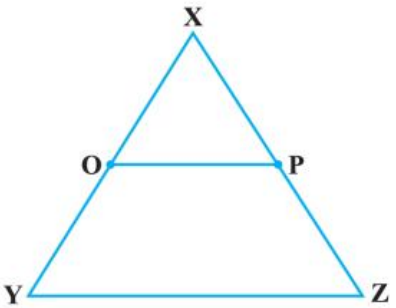


Fig. 5.11

12. In the Fig.5.12 :

(i) AB = BC, M is the mid-point of AB and N is the mid- point of BC. Show that AM = NC.

(ii) BM = BN, M is the mid-point of AB and N is the mid-point of BC. Show that AB = BC.

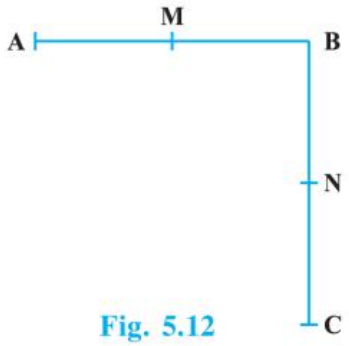


Fig. 5.12

(E) Long Answer Questions

Sample Question 1 : Read the following statement:

"A square is a polygon made up of four line segments, out of which, length of three line segments are equal to the length of fourth one and all its angles are right angles".

Define the terms used in this definition which you feel necessary. Are there any undefined terms in this? Can you justify that all angles and sides of a square are equal?

Solution : The terms need to be defined are :

Polygon : A simple closed figure made up of three or more line segments.
Line segment : Part of a line with two end points.
Line : Undefined term
Point : Undefined term
Angle : A figure formed by two rays with a common initial point.
Ray : Part of a line with one end point.
Right angle : Angle whose measure is 90°.
Undefined terms used are : line, point.

Euclid's fourth postulate says that "all right angles are equal to one another."

In a square, all angles are right angles, therefore, all angles are equal (From Euclid's fourth postulate).

Three line segments are equal to fourth line segment (Given).

Therefore, all the four sides of a square are equal. (by Euclid's first axiom "things which are equal to the same thing are equal to one another.")

EXERCISE 5.4

1. Read the following statement :

An equilateral triangle is a polygon made up of three line segments out of which two line segments are equal to the third one and all its angles are 60° each.

Define the terms used in this definition which you feel necessary. Are there any undefined terms in this? Can you justify that all sides and all angles are equal in a equilateral triangle.

2. Study the following statement:

"Two intersecting lines cannot be perpendicular to the same line".

Check whether it is an equivalent version to the Euclid's fifth postulate. [**Hint :** Identify the two intersecting lines l and m and the line n in the above statement.]

3. Read the following statements which are taken as axioms :

(i) If a transversal intersects two parallel lines, then corresponding angles are not necessarily equal.

(ii) If a transversal intersect two parallel lines, then alternate interior angles are equal.

Is this system of axioms consistent? Justify your answer.

4. Read the following two statements which are taken as axioms :

(i) If two lines intersect each other, then the vertically opposite angles are not equal.

(ii) If a ray stands on a line, then the sum of two adjacent angles so formed is equal to 180°.

Is this system of axioms consistent? Justify your answer.

5. Read the following axioms:

(i) Things which are equal to the same thing are equal to one another.

(ii) If equals are added to equals, the wholes are equal.

(iii) Things which are double of the same thing are equal to one another.

Check whether the given system of axioms is consistent or inconsistent.