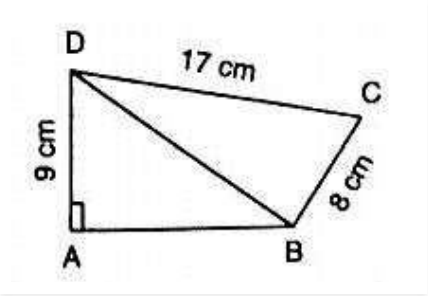
Q 1. In figure, compute the area of quadrilateral ABCD.



Solution:

Given:

DC = 17 cm, AD = 9 cm and BC = 8 cm

In ΔBCD we have

$$CD^2 = BD^2 + BC^2$$

$$\Rightarrow 17^2 = BD^2 + 8^2$$

$$\Rightarrow BD^2 = 289 - 64$$

$$\Rightarrow BD = 15$$

In ΔABD we have

$$AB^2 + AD^2 = BD^2$$

$$\Rightarrow 15^2 = AB^2 + 9^2$$

$$\Rightarrow AB^2 = 225 - 81 = 144$$

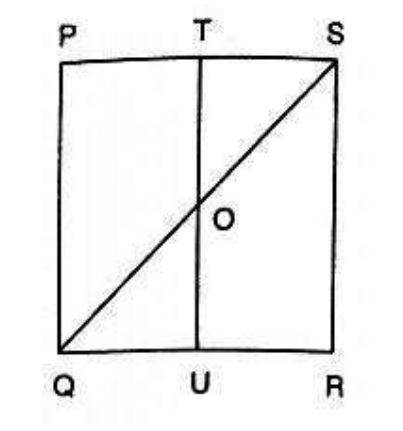
$$\Rightarrow AB = 12$$

$$\text{ar}(\text{quad } ABCD) = \text{ar}(\Delta ABD) + \text{ar}(\Delta BCD)$$

$$\text{ar}(\text{quad } ABCD) = \frac{1}{2}(12 \times 9) + \frac{1}{2}(8 \times 17) = 54 + 68 = 122 \text{ cm}^2$$

$$\text{ar}(\text{quad } ABCD) = \frac{1}{2}(12 \times 9) + \frac{1}{2}(8 \times 15) = 54 + 60 = 114 \text{ cm}^2$$

Q2. In figure, PQRS is a square and T and U are, respectively, the midpoints of PS and QR . Find the area of ΔOTS ***if PQ = 8 cm.***



Solution:

From the figure,

T and U are mid points of PS and QR respectively

$\therefore TU||PQ$

$\Rightarrow TO||PQ$

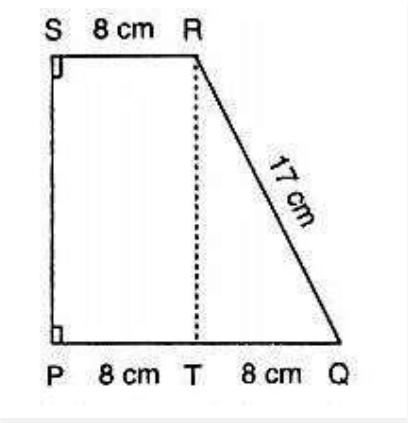
Thus , in ΔPQS , T is the mid point of PS and TO||PQ

$\therefore TO = \frac{1}{2}PQ = 4 \text{ cm}$

Also, TS = $\frac{1}{2}PS = 4 \text{ cm}$

$\therefore \text{ar}(\Delta OTS) = \frac{1}{2}(TO \times TS) = \frac{1}{2}(4 \times 4) \text{ cm}^2 = 8 \text{ cm}^2$

Q3. Compute the area of trapezium PQRS in figure



Solution:

We have,

$\text{ar}(\text{trap. } PQRS) = \text{ar}(\text{rect. } PSRT) + \text{ar}(\Delta QRT)$

$\Rightarrow \text{ar}(\text{trap. } PQRS) = PT \times RT + \frac{1}{2}(QT \times RT)$

$= = 8 \times RT + \frac{1}{2}(8 \times RT) = 12 \times RT$

In ΔQRT , we have

$QR^2 = QT^2 + RT^2$

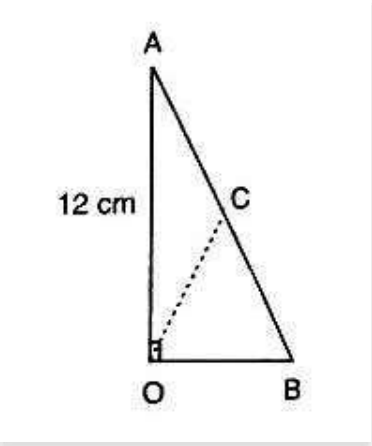
$\Rightarrow RT^2 = QR^2 - QT^2$

$\Rightarrow RT^2 = 17^2 - 8^2 = 225$

$\Rightarrow RT = 15$

Hence , area of trapezium = $12 \times 15 \text{ cm}^2 = 180 \text{ cm}^2$

Q4. In figure, $\angle AOB = 90^\circ$ ***, AC = BC , OA = 12 cm and OC = 6.5 cm .Find the area of*** ΔAOB.



Solution:

Since, the midpoint of the hypotenuse of a right triangle is equidistant from the vertices

$\therefore CA = CB = OC$

$\Rightarrow CA = CB = 6.5 \text{ cm}$

$\Rightarrow AB = 13 \text{ cm}$

In right angled triangle OAB , we have

$AB^2 = OB^2 + OA^2$

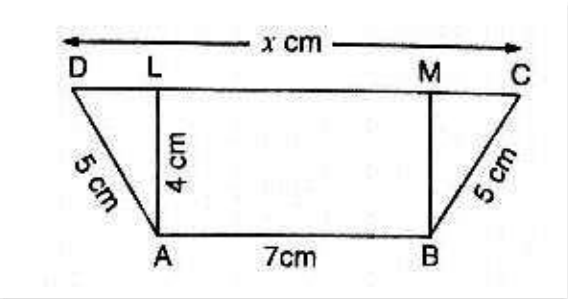
$\Rightarrow 13^2 = OB^2 + 12^2$

$\Rightarrow OB^2 = 13^2 - 12^2 = 169 - 144 = 25$

$\Rightarrow OB = 5$

$\therefore \text{ar}(\Delta AOB) = \frac{1}{2}(12 \times 5) = 30 \text{ cm}^2$

Q5. In figure, ABCD is a trapezium in which AB = 7 cm, AD = BC = 5 cm, DC = x cm , and distance between AB and DC is 4 cm . Find the value of x and area of trapezium ABCD.



Solution:

Draw AL ⊥ DC, BM ⊥ DC then ,

AL = BM = 4 cm and LM = 7 cm.

In Δ ADL , we have

$AD^2 = AL^2 + DL^2$

$\Rightarrow 25 = 16 + DL^2$

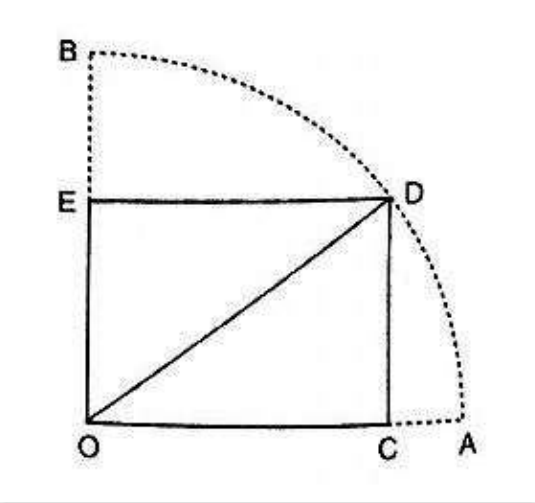
$\Rightarrow DL = 3 \text{ cm}$

Similarly, MC = $\sqrt{BC^2 - BM^2}$ = $\sqrt{25 - 16} = 3$ cm

$\therefore x = CD = CM + ML + LD = (3 + 7 + 3)$ cm = 13 cm

$\text{ar}(\text{trap. ABCD}) = \frac{1}{2}(AB + CD) \times AL = \frac{1}{2}(7 + 13) \times 4 \text{ cm}^2 = 40 \text{ cm}^2$

Q 6. In figure, OCDE is a rectangle inscribed in a quadrant of a circle of radius 10 cm. If OE = $2\sqrt{5}$ ***cm , find the area of the rectangle .***



Solution:

Given OD = 10 cm and OE = $2\sqrt{5}$ cm

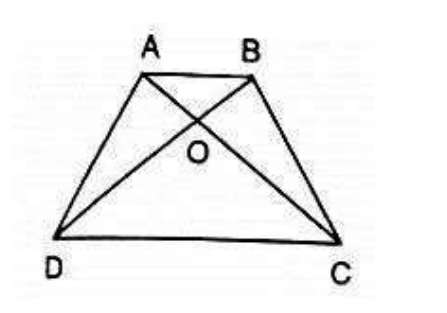
By using Pythagoras theorem

$\therefore OD^2 = OE^2 + DE^2$

$\Rightarrow DE = \sqrt{OD^2 - OE^2} = \sqrt{10^2 - (2\sqrt{5})^2} = 4\sqrt{5}$ cm

$\therefore$ Area of rectangle OCDE = $OE \times DE = 2\sqrt{5} \times 4\sqrt{5} \text{ cm}^2 = 40 \text{ cm}^2$

Q 7. In figure, ABCD is a trapezium in which AB || DC. Prove that $\text{ar}(\Delta AOD) = \text{ar}(\Delta BOC)$



Solution:

Given: ABCD is a trapezium in which AB || DC

To prove: $\text{ar}(\Delta AOD) = \text{ar}(\Delta BOC)$

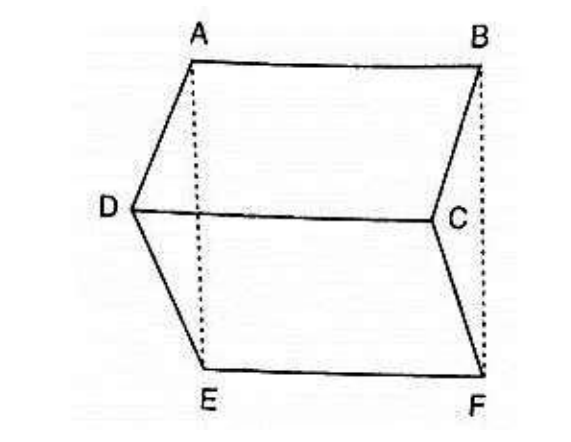
Proof:- Since , ΔADC and ΔBDC are on the same base DC and between same parallels AB and DC

Then, ar(ΔADC) = ar(ΔBDC)

$\Rightarrow \text{ar}(\Delta AOD) + \text{ar}(\Delta DOC) = \text{ar}(\Delta BOC) + \text{ar}(\Delta DOC)$

$\Rightarrow \text{ar}(\Delta AOD) = \text{ar}(\Delta BOC)$

Q 8. In figure, ABCD, ABFE and CDEF are parallelograms. Prove that $\text{ar}(\Delta ADE) = \text{ar}(\Delta BCF)$.



Solution:

Given that

ABCD is parallelogram $\Rightarrow AD = BC$

CDEF is parallelogram $\Rightarrow DE = CF$

ABFE is parallelogram $\Rightarrow AE = BF$

Thus, in Δs ADF and BCF , we have

AD = BC, DE = CF and AE = BF

So, by SSS criterion of congruence, we have

$\Delta ADE \cong \Delta BCF$

$\text{ar}(\Delta ADE) = \text{ar}(\Delta BCF)$

Q 9. Diagonals AC and BD of a quadrilateral ABCD intersect each other at P. Show that :
$\text{ar}(\Delta APB) \times \text{ar}(\Delta CPD) = \text{ar}(\Delta APD) \times \text{ar}(\Delta BPC)$.

Solution:

Construction: – Draw BQ $\perp$AC and DR $\perp$ AC

Proof:-

L.H.S

$= \text{ar}(\Delta APB) \times \text{ar}(\Delta CDP)$

$= \frac{1}{2}[(AP \times BQ)] \times (\frac{1}{2} \times PC \times DR)$

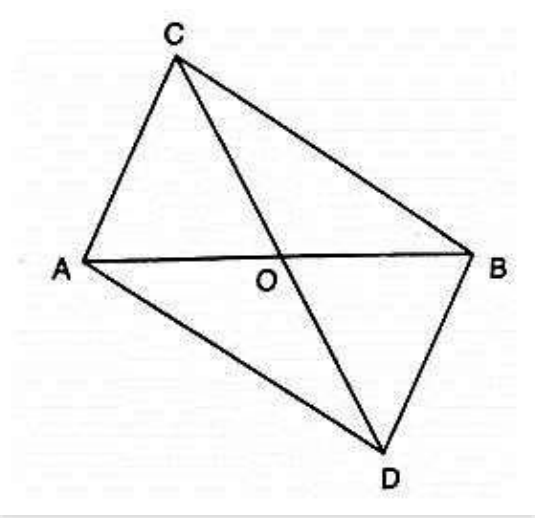
$= (\frac{1}{2} \times PC \times BQ) \times (\frac{1}{2} \times AP \times DR)$

$= \text{ar}(\Delta APD) \times \text{ar}(\Delta BPC)$.

= R.H.S

Hence proved.

Q 10. In figure, ABC and ABD are two triangles on the base AB. If line segment CD is bisected by AB at O, show that $\text{ar}(\Delta ABC) = \text{ar}(\Delta ABD)$.



Solution:

Given that CD is bisected by AB at O

To prove: $\text{ar}(\Delta ABC) = \text{ar}(\Delta ABD)$.

Construction: Draw CP $\perp$ AB and DQ $\perp$ AB .

Proof:

$$\text{ar}(\Delta ABC) = \frac{1}{2} \times AB \times CP \cdots\cdots (1)$$

$$\text{ar}(\Delta ABD) = \frac{1}{2} \times AB \times DQ \cdots\cdots (2)$$

In ΔCPO and ΔDQO

$\angle CPO = \angle DQO$ [each 90°]

Given that, CO = OD

$\angle COP = \angle DOQ$ [Vertically opposite angles are equal]

Then , $\Delta CP0 \cong \Delta DQO$ [By AAS condition]

$\therefore CP = DQ$ (3) [c. p. c. t]

Compare equation (1), (2) and (3)

$\therefore \text{ar}(\Delta ABC) = \text{ar}(\Delta ABD)$.

Q 11. If P is any point in the interior of a parallelogram ABCD , then prove that area of the triangle APB is less than half the area of parallelogram.

Solution:

Draw DN $\perp$ AB and PM $\perp$ AB

Now ,

$$\text{ar}(||^{gm} ABCD) = AB \times DN, \text{ar}(\Delta APB) = \frac{1}{2}(AB \times PM)$$

Now , PM < DN

$\Rightarrow AB \times PM < AB \times DN$

$\Rightarrow \frac{1}{2}(AB \times PM) < \frac{1}{2}(AB \times DN)$

$\Rightarrow \text{ar}(\Delta APB) < \frac{1}{2}\text{ar}(||^{gm} ABCD)$

Q 12. If AD is a median of a triangle ABC, then prove that triangles ADB and ADC are equal in area. If G is the mid-point of the median AD, prove that $\text{ar}(\Delta BGC) = 2\text{ar}(\Delta AGC)$.

Solution:

Draw AM ⊥ BC

Since, AD is the median of ΔABC

∴ BD = DC

$\Rightarrow BD = AM = DC \times AM$

$\Rightarrow \frac{1}{2}(BD \times AM) = \frac{1}{2}(DC \times AM)$

$\Rightarrow \text{ar}(\Delta ABD) = \text{ar}(\Delta ACD) \cdots\cdots\cdots (1)$

In ΔBGC , GD is the median

$\therefore \text{ar}(\Delta BGD) = \text{ar}(\Delta CGD) \cdots\cdots\cdots (2)$

In ΔACD , CG is the median

$\therefore \text{ar}(\Delta AGC) = \text{ar}(\Delta CGD) \cdots\cdots\cdots (3)$

From (2) and (3) we have,

$\text{ar}(\Delta BGD) = \text{ar}(\Delta AGC)$

But, $\text{ar}(\Delta BGC) = 2\text{ar}(\Delta BGD)$

$\therefore \text{ar}(\Delta BGC) = 2\text{ar}(\Delta AGC)$

Q 13. A point D is taken on the side BC of a ΔABC ***, such that BD = 2DC . Prove that*** $\text{ar}(\Delta ABD) = 2\text{ar}(\Delta ADC)$.

Solution:

Given that,

In ΔABC, BD = 2DC

To prove: $\text{ar}(\Delta ABD) = 2\text{ar}(\Delta ADC)$.

Construction:

Take a point E on BD such that BE = ED

Proof: Since, BE = ED and BD = 2 DC

Then, BE = ED = DC

We know that median of triangle divides it into two equal triangles.

$\therefore$ In ΔABD , AE is the median .

Then , $\text{ar}(\Delta ABD) = 2\text{ar}(\Delta AED) \cdots\cdots (1)$

In ΔAEC , AD is the median .

Then, $\text{ar}(\Delta AED) = 2\text{ar}(\Delta ADC) \cdots\cdots (2)$

Compare equation 1 and 2

$\text{ar}(\Delta ABD) = 2\text{ar}(\Delta ADC).$

Q 14. ABCD is a parallelogram whose diagonals intersect at O .If P is any point on BO, prove that :

(i) . $\text{ar}(\Delta ADO) = \text{ar}(\Delta CDO).$

(ii) . $\text{ar}(\Delta ABP) = 2\text{ar}(\Delta CBP).$

Solution:

Given that ABCD is the parallelogram

To Prove: (i) $\text{ar}(\Delta ADO) = \text{ar}(\Delta CDO).$

(ii) $\text{ar}(\Delta ABP) = 2\text{ar}(\Delta CBP).$

Proof: we know that diagonals of parallelogram bisect each other

$\therefore$ AO = OC and BO = OD

(i) . In Δ DAC , since DO is a median .

Then $\text{ar}(\Delta ADO) = \text{ar}(\Delta CDO).$

(ii) . In Δ BAC , since BO is a median .

Then $\text{ar}(\Delta BAO) = \text{ar}(\Delta BCO) \cdots\cdots (1)$

In Δ PAC , since PO is a median .

Then $\text{ar}(\Delta PAO) = \text{ar}(\Delta PCO) \cdots\cdots (2)$

Subtract equation 2 from 1.

$\Rightarrow \text{ar}(\Delta BAO) - \text{ar}(\Delta PAO) = \text{ar}(\Delta BCO) - \text{ar}(\Delta PCO)$

$\Rightarrow \text{ar}(\Delta ABP) = 2\text{ar}(\Delta CBP).$

Q 15. ABCD is a parallelogram in which BC is produced to E such that CE = BC. AE intersects CD at F.

(i) . Prove that $\text{ar}(\Delta ADF) = \text{ar}(\Delta ECF).$

(ii) . If the area of $\Delta DFB = 3\ \text{cm}^2$ ***, find the area of*** $||^{gm}$ ***ABCD .***

Solution:

In triangles ADF and ECF, we have

$\angle ADF = \angle ECF$ [Alternate interior angles, Since $AD||BE$]

AD = EC [since $AD = BC = CE$]

And $\angle DFA = \angle CFA$ [Vertically opposite angles]

So, by AAS congruence criterion, we have

$\Delta ADF \cong \Delta ECF$

$\Rightarrow \text{ar}(\Delta ADF) = \text{ar}(\Delta ECF)$ and DF = CF .

Now, DF = CF

$\Rightarrow$ BF is a median in Δ BCD.

$\Rightarrow \text{ar}(\Delta BCD) = 2\text{ar}(\Delta BDF)$

$\Rightarrow \text{ar}(\Delta BCD) = 2 \times 3 \text{ cm}^2 = 6 \text{ cm}^2$

Hence, area of a parallelogram = $2\text{ar}(\Delta BCD) = 2 \times 6 \text{ cm}^2 = 12 \text{ cm}^2$

Q 16. ABCD is a parallelogram whose diagonals AC and BD intersect at O . A line through O intersects AB at P and DC at Q. Prove that $\text{ar}(\Delta POA) = \text{ar}(\Delta QOC)$.

Solution:

In triangles POA and QOC, we have

$\angle AOP = \angle COQ$

AO = OC

$\angle PAC = \angle QCA$

So, by ASA congruence criterion , we have

$\Delta POA \cong \Delta QOC$

$\Rightarrow \text{ar}(\Delta POA) = \text{ar}(\Delta QOC).$

Q 17. ABCD is a parallelogram. E is a point on BA such that BE = 2EA and F is point on DC such that DF = 2FC. Prove that AECF is a parallelogram whose area is one third of the area of parallelogram ABCD.

Solution:

Draw FG $\perp$ AB

We have,

BE = 2 EA and DF = 2FC

$\Rightarrow$ AB – AE = 2 AE and DC – FC = 2 FC

$\Rightarrow$ AB = 3 AE and DC = 3 FC

$\Rightarrow AE = \frac{1}{3}AB$ and $FC = \frac{1}{3}DC \cdots\cdots\cdots(1)$

But AB = DC

Then, AE = FC $\quad [\text{opposite sides of } \|^{gm}]$

Thus, AE = FC and AE || FC

Then, AECF is a parallelogram

Now, area of parallelogram AECF = $AE \times FG$

$\Rightarrow \text{ar}(\|^{gm} AECF) = \frac{1}{3}AB \times FG$ from(1)

$\Rightarrow 3\ ar(||^{gm}\ AECF) = AB \times FG \cdots\cdots (2)$

$And\ ar(||^{gm}\ ABCD) = AB \times FG \cdots\cdots (3)$

Compare equation 2 and 3

$\Rightarrow 3ar(||^{gm}\ AECF) = ar(||^{gm}\ ABCD)$

$\Rightarrow ar(||^{gm}\ AECF) = \frac{1}{3}ar(||^{gm}\ ABCD)$

Q 18. In a triangle ABC, P and Q are respectively the mid points of AB and BC and R is the mid point of AP. Prove that :

(i) . $ar(\Delta PBQ) = ar(\Delta ARC)$.

(ii) . $ar(\Delta PRQ) = \frac{1}{2}ar(\Delta ARC)$.

(iii) . $ar(\Delta RQC) = \frac{3}{8}ar(\Delta ABC)$.

Solution:

We know that each median of a triangle divides it into two triangles of equal area.

(i) . Since CR is the median of Δ CAP

$\therefore ar(\Delta CRA) = \frac{1}{2}ar(\Delta CAP) \cdots\cdots (1)$

Also , CP is the median of a Δ CAB

$\therefore ar(\Delta CAP) = ar(\Delta CPB) \cdots\cdots (2)$

From 1 and 2 , we get

$\therefore ar(\Delta ARC) = \frac{1}{2}ar(\Delta CPB) \cdots\cdots (3)$

PQ is the median of a Δ PBC

$\therefore ar(\Delta CPB) = 2ar(\Delta PBQ) \cdots\cdots (4)$

From 3 and 4, we get

$\therefore ar(\Delta ARC) = ar(\Delta PBQ) \cdots\cdots (5)$

(ii) . Since QP and QR medians of triangles QAB and QAP respectively

$\therefore ar(\Delta QAP) = ar(\Delta QBP) \cdots\cdots (6)$

$And\ ar(\Delta QAP) = 2ar(\Delta QRP) \cdots\cdots (7)$

From 6 and 7, we get

$ar(\Delta PRQ) = \frac{1}{2}ar(\Delta PBQ) \cdots\cdots (8)$

From 5 and 8, we get

$ar(\Delta PRQ) = \frac{1}{2}ar(\Delta ARC)$

(iii) . Since, LR is a median of Δ CAP

$\therefore \text{ar}(\Delta ARC) = \frac{1}{2}\text{ar}(\Delta CAD)$

$= \frac{1}{2} \times \frac{1}{2}\text{ar}(\Delta ABC)$

$= \frac{1}{4}\text{ar}(\Delta ABC)$

Since RQ is the median of Δ RBC.

$\therefore \text{ar}(\Delta RQC) = \frac{1}{2}\text{ar}(\Delta RBC)$

$= \frac{1}{2}\{\text{ar}(\Delta ABC) - \text{ar}(\Delta ARC)\}$

$= \frac{1}{2}\{\text{ar}(\Delta ABC) - \frac{1}{4}\text{ar}(\Delta ABC)\}$

$= \frac{3}{8}\text{ar}(\Delta ABC)$

Q 19. ABCD is a parallelogram. G is a point on AB such that AG = 2GB and E is point on DC such that CE = 2DE and F is the point of BC such that BF = 2FC. Prove that:

(i) $\text{ar}(ADEG) = \text{ar}(GBCE)$.

(ii) $\text{ar}(\Delta EGB) = \frac{1}{6}\text{ar}(ABCD)$.

(iii) $\text{ar}(\Delta EFC) = \frac{1}{2}\text{ar}(\Delta EBF)$.

(iv) $\text{ar}(\Delta EGB) = \frac{3}{2} \times \text{ar}(\Delta EFC)$

(v) . Find what portion of the area of parallelogram is the area of Δ ***EFG.***

Solution:

Given: ABCD is a parallelogram

AG = 2 GB, CE = 2 DE and BF = 2 FC

To prove:

(i) $\text{ar}(ADEG) = \text{ar}(GBCE)$.

(ii) $\text{ar}(\Delta EGB) = \frac{1}{6}\text{ar}(ABCD)$.

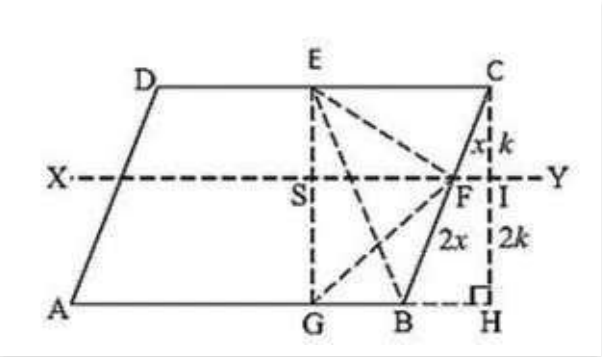
(iii) $\text{ar}(\Delta EFC) = \frac{1}{2}\text{ar}(\Delta EBF)$.

(iv) $\text{ar}(\Delta EGB) = \frac{3}{2} \times \text{ar}(\Delta EFC)$

(v) . Find what portion of the area of parallelogram is the area of Δ EFG.

Construction: Draw a parallel line to AB through point F and a perpendicular line to AB from C

Proof:



(i) .Since ABCD is a parallelogram

So, AB = CD and AD = BC

Consider the two trapezium s ADEG and GBCE

Since AB = DC, EC = 2DE, AG = 2GB

$\Rightarrow \mathrm{ED} = \frac{1}{3}\mathrm{CD} = \frac{1}{3}\mathrm{AB}$ and $\mathrm{EC} = \frac{2}{3}\mathrm{CD} = \frac{2}{3}\mathrm{AB}$

$\Rightarrow \mathrm{AG} = \frac{2}{3}\mathrm{AB}$ and $\mathrm{BG} = \frac{1}{3}\mathrm{AB}$

So , $\mathrm{DE} + \mathrm{AG} = \frac{1}{3}\mathrm{AB} + \frac{2}{3}\mathrm{AB} = \mathrm{AB}$ and $\mathrm{EC} + \mathrm{BG} = \frac{2}{3}\mathrm{AB} + \frac{1}{3}\mathrm{AB} = \mathrm{AB}$

Since the two trapezium ADEG and GBCE have same height and their sum of two parallel sides are equal

Since Area of trapezium = $\frac{\text{sum of parallel sides}}{2} \times \text{height}$

So, $\mathrm{ar}(\mathrm{ADEG}) = \mathrm{ar}(\mathrm{GBCE}).$

(ii) . Since we know from above that

$\mathrm{BG} = \frac{1}{2}\mathrm{AB}$. So

$\mathrm{ar}(\Delta\mathrm{EGB}) = \frac{1}{2} \times \mathrm{GB} \times \mathrm{Height}$

$\mathrm{ar}(\Delta\mathrm{EGB}) = \frac{1}{2} \times \frac{1}{3} \times \mathrm{AB} \times \mathrm{Height}$

$\mathrm{ar}(\Delta\mathrm{EGB}) = \frac{1}{6} \times \mathrm{AB} \times \mathrm{Height}$

$\mathrm{ar}(\Delta\mathrm{EGB}) = \frac{1}{6}\mathrm{ar}(\mathrm{ABCD}).$

(iii) . Since height if triangle EFC and EBF are equal.So

$\mathrm{ar}(\Delta\mathrm{EFC}) = \frac{1}{2} \times \mathrm{FC} \times \mathrm{Height}$

$\mathrm{ar}(\Delta\mathrm{EFC}) = \frac{1}{2} \times \frac{1}{2} \times \mathrm{FB} \times \mathrm{Height}$

$\mathrm{ar}(\Delta\mathrm{EFC}) = \frac{1}{2}\mathrm{ar}(\mathrm{EBF})$

Hence , $\mathrm{ar}(\Delta\mathrm{EFC}) = \frac{1}{2}\mathrm{ar}(\Delta\mathrm{EBF}).$

(iv) . Consider the trapezium in which

$\mathrm{ar}(\mathrm{EGBC}) = \mathrm{ar}(\Delta\mathrm{EGB}) + \mathrm{ar}(\Delta\mathrm{EBF}) + \mathrm{ar}(\Delta\mathrm{EFC})$

$\Rightarrow \frac{1}{2}\text{ar}(\text{ABCD}) = \frac{1}{6}\text{ar}(\text{ABCD}) + 2\text{ar}(\Delta\text{EFC}) + \text{ar}(\Delta\text{EFC})$

$\Rightarrow \frac{1}{3}\text{ar}(\text{ABCD}) = 3\text{ar}(\Delta\text{EFC})$

$\Rightarrow \text{ar}(\Delta\text{EFC}) = \frac{1}{9}\text{ar}(\text{ABCD})$

Now from (ii)part we have

$\text{ar}(\Delta\text{EGB}) = \frac{1}{6}\text{ar}(\Delta\text{EFC})$

$\text{ar}(\Delta\text{EGB}) = \frac{3}{2} \times \frac{1}{9}\text{ar}(\text{ABCD})$

$\text{ar}(\Delta\text{EGB}) = \frac{3}{2}\text{ar}(\Delta\text{EFC})$

$\therefore \text{ar}(\Delta\text{EGB}) = \frac{3}{2}\text{ar}(\Delta\text{EFC})$

(v) . In the figure it is given that FB = 2CF .Let CF = x and FB = 2x.

Now consider the two triangles CFI and CBH which are similar triangle.

So by the property of similar triangle CI = k and IH = 2k

Now consider the triangle EGF in which

$\text{ar}(\Delta\text{EFG}) = \text{ar}(\Delta\text{ESF}) + \text{ar}(\Delta\text{SGF})$

$\text{ar}(\Delta\text{EFG}) = \frac{1}{2}\text{SF} \times \text{k} + \frac{1}{2}\text{SF} \times 2\text{k}$

$\text{ar}(\Delta\text{EFG}) = \frac{3}{2}\text{SF} \times \text{k} \cdots\cdots (\text{i})$

Now ,

$\text{ar}(\Delta\text{EGBC}) = \text{ar}(\text{SGBF}) + \text{ar}(\text{ESFC})$

$\text{ar}(\Delta\text{EGBC}) = \frac{1}{2}(\text{SF} + \text{GB}) \times 2\text{k} + \frac{1}{2}(\text{SF} + \text{EC}) \times \text{k}$

$\text{ar}(\Delta\text{EGBC}) = \frac{3}{2}\text{k} \times \text{SF} + (\text{GB} + \frac{1}{2}\text{EC}) \times \text{k}$

$\text{ar}(\Delta\text{EGBC}) = \frac{3}{2}\text{k} \times \text{SF} + (\frac{1}{3}\text{AB} + \frac{1}{2} \times \frac{2}{3}\text{AB}) \times \text{k}$

$\frac{1}{2}\text{ar}(\Delta\text{ABCD}) = \frac{3}{2}\text{k} \times \text{SF} + \frac{2}{3}\text{AB} \times \text{k}$

$\Rightarrow \text{ar}(\Delta\text{ABCD}) = 3\text{k} \times \text{SF} + \frac{4}{3}\text{AB} \times \text{k}$ [Multiply both sides by 2]

$\Rightarrow \text{ar}(\Delta\text{ABCD}) = 3\text{k} \times \text{SF} + \frac{4}{9}\text{ar}(\text{ABCD})$

$\Rightarrow \text{k} \times \text{SF} = \frac{5}{27}\text{ar}(\text{ABCD}) \cdots\cdots (2)$

From 1 and 2 we have ,

$\text{ar}(\Delta\text{EFG}) = \frac{3}{2} \times \frac{5}{27}\text{ar}(\text{ABCD})$

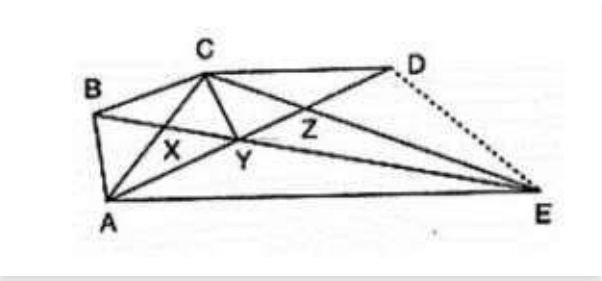
$\text{ar}(\Delta\text{EFG}) = \frac{5}{18}\text{ar}(\text{ABCD})$

Q 20. In figure, CD || AE and CY || BA.

(i) . Name a triangle equal in area of ΔCBX

(ii) . Prove that $\text{ar}(\Delta ZDE) = \text{ar}(\Delta CZA)$.

(iii) . Prove that $\text{ar}(BCZY) = \text{ar}(\Delta EDZ)$.



Solution:

Since, triangle BCA and triangle BYA are on the same base BA and between same parallel s BA and CY.

Then $\text{ar}(\Delta BCA) = \text{ar}(\Delta BYA)$

$\Rightarrow \text{ar}(\Delta CBX) + \text{ar}(\Delta BXA) = \text{ar}(\Delta BXA) + \text{ar}(\Delta AXY)$

$\Rightarrow \text{ar}(\Delta CBX) = \text{ar}(\Delta AXY) \cdots\cdots (1)$

Since, triangles ACE and ADE are on the same base AE and between same parallels CD and AE

Then, $\text{ar}(\Delta ACE) = \text{ar}(\Delta ADE)$

$\text{ar}(\Delta CZA) + \text{ar}(\Delta AZE) = \text{ar}(\Delta AZE) + \text{ar}(\Delta DZE)$

$\text{ar}(\Delta CZA) = \text{ar}(\Delta DZE) \cdots\cdots (2)$

Adding $\text{ar}(\Delta CYG)$ on both sides , we get

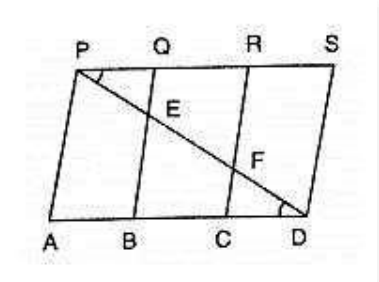
$\Rightarrow \text{ar}(\Delta CBX) + \text{ar}(\Delta CYZ) = \text{ar}(\Delta CAY) + \text{ar}(\Delta CYZ)$

$\Rightarrow \text{ar}(BCZY) = \text{ar}(\Delta CZA) \cdots\cdots (3)$

Compare equation 2 and 3

$\Rightarrow \text{ar}(BCZY) = \text{ar}(\Delta DZE)$

Q 21. In figure, PSDA is a parallelogram in which PQ = QR =RS and AP || BQ ||CR. Prove that $\text{ar}(\Delta PQE) = \text{ar}(\Delta CFD)$.



Solution:

Given that PSDA is a parallelogram

Since, AP || BQ || CR ||DS and AD ||PS

Therefore, PQ = CD (equ. 1)

In triangle BED, C is the midpoint of BD and CF || BE

Therefore, F is the midpoint of ED

$\Rightarrow$ EF = PE

Smiliarly,

EF = PE

Therefore, PE = FD (equ . 2)

In triangles PQE and CFD, we have

PE = FD

Therefore, $\angle$ EPQ = $\angle$FDC [Alternate angles]

So, by SAS criterion , we have

$\Delta PQE \cong \Delta DCF$

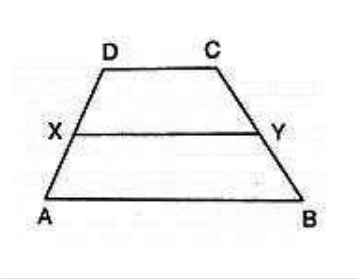
$\Rightarrow \text{ar}(\Delta PQE) = \text{ar}(\Delta DCF)$

Q 22. In figure, ABCD is a trapezium in which AB || DC and DC = 40 cm and AB = 60 cm .If X and Y are , respectively , the mid points of AD and BC , prove that :

(i) . XY = 50 cm

(ii) . DCYX is a trapezium

(iii) . $\text{ar}(\text{trap. DCYX}) = \frac{9}{11}\text{ar}(\text{XYBA}).$



Solution:

(i) Join DY and produce it to meet AB produced at P.

In triangles BYP and CYD we have,

$\angle$BYP = $\angle$ CYD [Vertically opposite angles]

$\angle$DCY = $\angle$ PBY [Since , DC ||AP]

And BY = CY

So, by ASA congruence criterion, we have

$(\Delta BYP) \cong (\Delta CYD)$

$\Rightarrow$ DY = Yp and DC = BP

$\Rightarrow$ Y is the midpoint of DP

Also , x is the mid point of AD

Therefore, XY ||AP and XY $||\frac{1}{2}AP$

$\Rightarrow XY = \frac{1}{2}(AB + BP)$

$\Rightarrow XY = \frac{1}{2}(AB + DC)$

$\Rightarrow XY = \frac{1}{2}(60 + 40)$

= 50 cm

(ii) We have, XY ||AP

⇒ XY || AB and AB ||DC

⇒ XY || DC

⇒ DCYX is a trapezium

(iii) Since x and y are the mid points of Ad and BC respectively.

Therefore, trapezium DCYX and ABYX are of the same height say h cm

Now,

$\text{ar}(\text{trap. DCXY}) = \frac{1}{2}(DC + XY) \times h$

$\Rightarrow \text{ar}(\text{trap. DCXY}) = \frac{1}{2}(50 + 40) \times h \text{ cm}^2 = 45h \text{ cm}^2$

$\Rightarrow \text{ar}(\text{trap. ABY X}) = \frac{1}{2}(AB + XY) \times h$

$\Rightarrow \text{ar}(\text{trap. ABY X}) = \frac{1}{2}(60 + 50) \times h \text{ cm}^2 = 55h \text{ cm}^2$

$\frac{\text{ar(trap.DCY X)}}{\text{ar(trap.ABY X)}} = \frac{45h}{55h} = \frac{9}{11}$

$\Rightarrow \text{ar}(\text{trap. DCY X}) = \frac{9}{11}\text{ar}(\text{trap. ABY X})$

Q 23. In figure ABC and BDE are two equilateral triangles such that D is the midpoint of BC. AE intersects BC in F. Prove that:

(i) . $\text{ar}(\Delta BDE) = \frac{1}{4}\text{ar}(\Delta ABC)$.

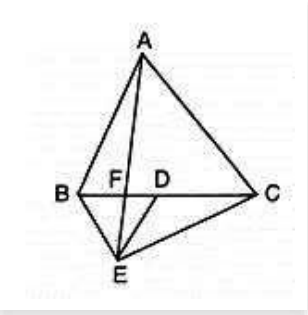
(ii) . $\text{ar}(\Delta BDE) = \frac{1}{2}\text{ar}(\Delta BAE)$.

(iii) . $\text{ar}(\Delta BFE) = \text{ar}(\Delta AFD)$.

(iv) . $\text{ar}(\Delta ABC) = 2\,\text{ar}(\Delta BEC)$.

(v) . $\text{ar}(\Delta FED) = \frac{1}{8}\text{ar}(\Delta AFC)$.

(vi) . $\text{ar}(\Delta BFE) = 2\,\text{ar}(\Delta EFD)$.



Solution:

Given that ABC and BDE are two equilateral triangles.

Let AB = BC = CA = x. Then, BD = $\frac{x}{2}$ = DE = BE

(i) We have,

$$\text{ar}(\Delta ABC) = \frac{\sqrt{3}}{4}x^2 \text{ and } \text{ar}(\Delta BDE) = \frac{\sqrt{3}}{4}(\frac{x}{2})^2 = \frac{1}{4} \times \frac{\sqrt{3}}{4}x^2$$

Therefore , $\text{ar}(\Delta BDE) = \frac{1}{4}\text{ar}(\Delta ABC).$

(ii) . It is given that triangles, ABC and BED are equilateral triangles

$\angle$ ACB = $\angle$DBE = 60°

$\Rightarrow$ BE || AC (Since , alternative angles are equal)

Trinagles BAF and BEC are on the same base BE and between same parallels BF and AC .

Therefore, $\text{ar}(\Delta BAE) = \text{ar}(\Delta BEC)$

$\Rightarrow \text{ar}(\Delta BAE) = 2\text{ar}(\Delta BDE)$ [Since , ED is a median of triangle EBC ; $\text{ar}(\Delta BEC) = 2\text{ar}(\Delta BDE)$]

$\therefore \text{ar}(\Delta BDE) = \frac{1}{2}\text{ar}(\Delta BAE)$

(iii) Since, triangles ABC and BDE are equilateral triangles

$\therefore \angle ABC = 60^\circ \text{ and } \angle BDE = 60^\circ$

$\angle ABC = \angle BDE$

$\Rightarrow$ AB ||DE (since, alternate angles are equal)

Triangles BED and AED are on the same base ED and between same parallels AB and DE.

Therefore, $\text{ar}(\Delta BED) = \text{ar}(\Delta AED)$

$\Rightarrow \text{ar}(\Delta BED) - \text{ar}(\Delta EFD) = \text{ar}(\Delta AED) - \text{ar}(\Delta EFD)$

$\Rightarrow \text{ar}(\Delta BEF) = \text{ar}(\Delta AFD)$

(iv) Since ED is the median of triangle BEC

Therefore, $\text{ar}(\Delta BEC) = 2\text{ar}(\Delta BDE)$

$\Rightarrow \text{ar}(\Delta BEC) = 2 \times \frac{1}{4}\text{ar}(\Delta ABC)$ [From 1 , $\text{ar}(\Delta BDE) = \frac{1}{4}\text{ar}(\Delta ABC)$]

$\Rightarrow \text{ar}(\Delta BEC) = \frac{1}{2}\text{ar}(\Delta ABC)$

$\Rightarrow \text{ar}(\Delta ABC) = 2\text{ar}(\Delta BEC)$

(v) $\text{ar}(\Delta AFC) = \text{ar}(\Delta AFD) + \text{ar}(\Delta ADC)$

$\Rightarrow \text{ar}(\Delta BFE) + \frac{1}{2}\text{ar}(\Delta ABC)$ [using part (iii) , and AD is the median of triangle ABC]

$= \text{ar}(\Delta BFE) + \frac{1}{2} \times 4\text{ar}(\Delta BDE)$ (using part (i))

$= \text{ar}(\Delta BFE) = 2\text{ar}(\Delta FED) \cdots\cdots (3)$

$\text{ar}(\Delta BDE) = \text{ar}(\Delta BFE) + \text{ar}(\Delta FED)$

$\Rightarrow 2\text{ar}(\Delta FED) + \text{ar}(\Delta FED)$

$\Rightarrow 3\text{ar}(\Delta FED) \cdots\cdots (4)$

From 2, 3 and 4 we get ,

$\text{ar}(\Delta AFC) = 2\text{ar}(\Delta FED) + 2 \times 3\text{ar}(\Delta FED) = 8\text{ar}(\Delta FED)$

$\text{ar}(\Delta FED) = \frac{1}{8}\text{ar}(\Delta AFC)$

(vi) Let h be the height of vertex E, corresponding to the side BD in triangle BDE.

Let H be the height of vertex A, corresponding to the side BC in triangle ABC

From part (i)

$\text{ar}(\Delta BDE) = \frac{1}{4}\text{ar}(\Delta ABC)$

$\Rightarrow \frac{1}{2} \times BD \times h = \frac{1}{4}(\frac{1}{2} \times BC \times h)$

$\Rightarrow BD \times h = \frac{1}{4}(2BD \times H)$

$\Rightarrow h = \frac{1}{2}H \cdots\cdots (1)$

From part (iii)

$\text{ar}(\Delta BFE) = \text{ar}(\Delta AFD)$

$\text{ar}(\Delta BFE) = \frac{1}{2} \times FD \times H$

$\text{ar}(\Delta BFE) = \frac{1}{2} \times FD \times 2h$

$\text{ar}(\Delta BFE) = 2(\frac{1}{2} \times FD \times h)$

$\text{ar}(\Delta BFE) = 2\text{ar}(\Delta EFD)$

Q 24. D is the midpoint of side BC of ΔABC ***and E is the midpoint of BD. If O is the midpoint of AE, Prove that*** $\text{ar}(\Delta BOE) = \frac{1}{8}\text{ar}(\Delta ABC)$.

Solution:

Given that

D is the midpoint of sides BC of triangle ABC

E is the midpoint of BD and O is the midpoint of AE

Since AD and AE are the medians of triangles, ABC and ABD respectively

$\therefore \text{ar}(\Delta ABD) = \frac{1}{2}\text{ar}(\Delta ABC) \cdots\cdots\cdots (1)$

$\therefore \text{ar}(\Delta ABE) = \frac{1}{2}\text{ar}(\Delta ABD) \cdots\cdots\cdots (2)$

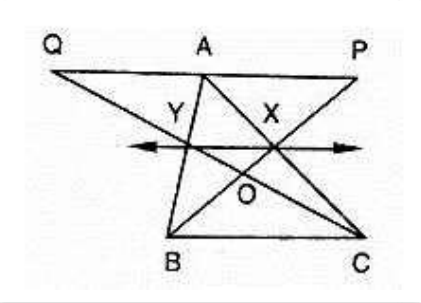
OB is the median of triangle ABE

Therefore, $\therefore \text{ar}(\Delta BOE) = \frac{1}{2}\text{ar}(\Delta ABE)$

From 1, 2 and 3 , we have

$\therefore \text{ar}(\Delta BOE) = \frac{1}{8}\text{ar}(\Delta ABC)$

Q 25. In figure, X and Y are the mid points of AC and AB respectively, QP ||BC and CYQ and BXP are straight lines . Prove that $\text{ar}(\Delta ABP) = \text{ar}(\Delta ACQ)$.



Solution:

Since X and Y are the mid points of AC and AB respectively.

Therefore, XY || BC

Clearly, triangles BYC and BXC are on the same base BC and between the same parallels Xy and BC

$\therefore \text{ar}(\Delta BYC) = \text{ar}(\Delta BXC)$

$\Rightarrow \text{ar}(\Delta BYC) - \text{ar}(\Delta BOC) = \text{ar}(\Delta BXC) - \text{ar}(\Delta BOC)$

$\Rightarrow \text{ar}(\Delta BOY) = \text{ar}(\Delta COX)$

$\Rightarrow \text{ar}(\Delta BOY) + \text{ar}(\Delta XOY) = \text{ar}(\Delta COX) + \text{ar}(\Delta XOY)$

$\Rightarrow \text{ar}(\Delta BXY) = \text{ar}(\Delta CXY)$ (2)

We observed that the quadrilaterals XYAP and XYAQ are on the same base XY and between same parallels XY and PQ.

$\therefore \text{ar}(\text{quad. } XYAP) = \text{ar}(\text{quad}XYQA) \cdots\cdots (2)$

Adding 1 and 2 , we get

$\therefore \text{ar}(\Delta BXY) + \text{ar}(\text{quad. } XYAP) = \text{ar}(\Delta CXY) + \text{ar}(\text{quad}XYQA)$

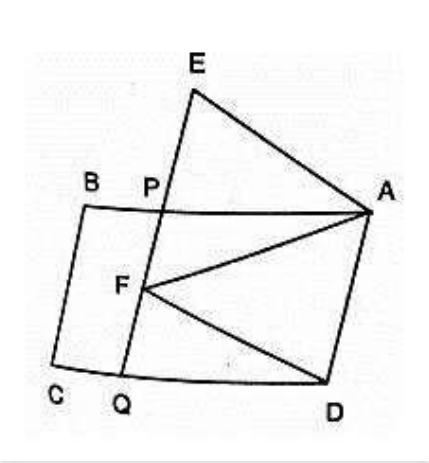
$\Rightarrow \text{ar}(\Delta ABP) = \text{ar}(\Delta ACQ)$

Q 26. In figure, ABCD and AEFD are two parallelograms. Prove that

(i) . PE = FQ

(ii) . $\text{ar}(\Delta APE) : \text{ar}(\Delta PFA) = \text{ar}(\Delta QFD) : \text{ar}(\Delta PFD)$

(iii) . $\text{ar}(\Delta \text{PEA}) = \text{ar}(\Delta \text{QFD})$.



Solution:

Given that, ABCD and AEFD are two parallelograms

(i) . In triangles, EPA and FQD

$\angle \text{PEA} = \angle \text{QFD}$ [corresponding angles]

$\angle \text{EPA} = \angle \text{FQD}$ [corresponding angles]

PA = QD [opposite sides of parallelogram]

Then, $\Delta \text{EPA} \cong \Delta \text{FQD}$ [By AAS condition]

Therefore, EP = FQ [C.P.C.T]

(ii) . Since triangles, PEA and QFD stand on equal bases PE and FQ lies between the same parallels EQ and AD

Therefore, $\text{ar}(\Delta \text{PEA}) = \text{ar}(\Delta \text{QFD})$ (1)

Since, triangles PEA and PFD stand on the same base PF and between same parallels PF and AD

Therefore, $\text{ar}(\Delta \text{PFA}) = \text{ar}(\Delta \text{PFD})$ (2)

Divide the equation 1 by equation 2

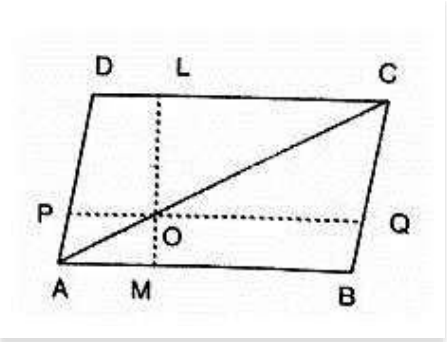
$$\frac{\text{ar}(\Delta \text{PEA})}{\text{ar}(\Delta \text{PFA})} = \frac{\text{ar}(\Delta \text{QFD})}{\text{ar}(\Delta \text{PFD})}$$

(iii) . From part (i), $\Delta \text{EPA} \cong \Delta \text{FQD}$

Then, $\text{ar}(\Delta \text{PEA}) = \text{ar}(\Delta \text{QFD})$.

Q 27. In figure, ABCD is a parallelogram . O is any point on AC. PQ ||AB and LM || AD. Prove that :

$\text{ar}(||^{gm}\text{DLOP}) = \text{ar}(||^{gm}\text{BMOQ})$.



Solution:

Since a diagonal of a parallelogram divides it into two triangles of equal area

Therefore, $\text{ar}(\Delta ADC) = \text{ar}(\Delta ABC)$

$\Rightarrow \text{ar}(\Delta APO) + \text{ar}(||^{gm} DLOP) + \text{ar}(\Delta OLC)$

$\Rightarrow \text{ar}(\Delta AOM) + \text{ar}(||^{gm} BMOQ) + \text{ar}(\Delta OQC)$ (1)

Since AO and Oc are diagonals of parallelograms AMOP and OQCL respectively.

$\therefore \text{ar}(\Delta APO) = \text{ar}(\Delta AMO)$ (2)

And $\text{ar}(\Delta OLC) = \text{ar}(\Delta OQC)$ (3)

Subtracting 2 and 3 from 1, we get

$\text{ar}(||^{gm} DLOP) = \text{ar}(||^{gm} BMOQ).$

Q 28. In a triangle ABC, if L and M are points on AB and AC respectively such that LM || BC. Prove that:

(i) . $\text{ar}(\Delta LCM) = \text{ar}(\Delta LBM).$

(ii) . $\text{ar}(\Delta LBC) = \text{ar}(\Delta MBC).$

(iii) . $\text{ar}(\Delta ABM) = \text{ar}(\Delta ACL).$

(iv) . $\text{ar}(\Delta LOB) = \text{ar}(\Delta MOC).$

Solution:

(i) . Clearly triangles LMB and LMC are on the same base LM and between the same parallels LM and BC.

$\therefore \text{ar}(\Delta LMB) = \text{ar}(\Delta LMC)$ (1)

(ii) . We observe that triangles LBC and MBC are on the same base BC and between same parallels LM and BC.

$\therefore \text{ar}(\Delta LBC) = \text{ar}(\Delta MBC)$ (2)

(iii) . We have,

$\text{ar}(\Delta LMB) = \text{ar}(\Delta LMC)$ [From 1]

$\Rightarrow \text{ar}(\Delta ALM) + \text{ar}(\Delta LMB) = \text{ar}(\Delta ALM) + \text{ar}(\Delta LMC)$

$\Rightarrow \text{ar}(\Delta ABM) = \text{ar}(\Delta ACL)$

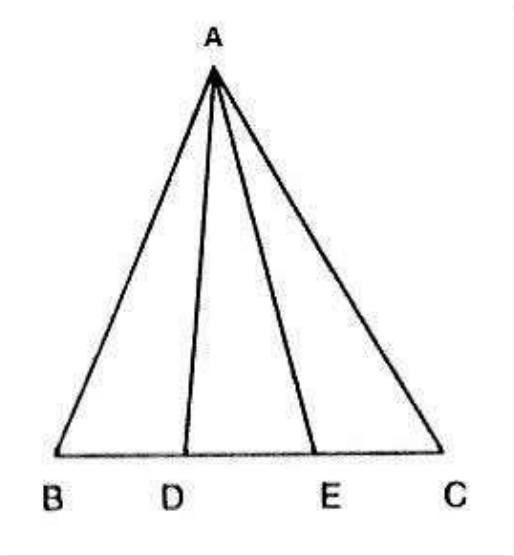
(iv) . We have,

$\text{ar}(\Delta LBC) = \text{ar}(\Delta MBC)$ [From 1]

$\Rightarrow \text{ar}(\Delta LBC) - \text{ar}(\Delta BOC) = \text{ar}(\Delta MBC) - \text{ar}(\Delta BOC)$

$\Rightarrow \text{ar}(\Delta LOB) = \text{ar}(\Delta MOC).$

Q 29. In figure, D and E are two points on BC such that BD = DE = EC. Show that
$\text{ar}(\Delta ABD) = \text{ar}(\Delta ADE) = \text{ar}(\Delta AEC).$



Solution:

Draw a line l through A parallel to BC.

Given that, BD = DE = EC

We observed that the triangles ABD and AEC are on the equal bases and between the same parallels l and BC. Therefore, their areas are equal.

Hence, $\text{ar}(\Delta ABD) = \text{ar}(\Delta ADE) = \text{ar}(\Delta AEC).$

Q 30. In figure, ABC is a right angled triangle at A, BCED, ACFG and ABMN are squares on the sides BC, CA and AB respectively. Line segment AX $\perp$ DE meets BC at Y. Show that

(i) . $\Delta MBC \cong \Delta ABD$

(ii) . $\text{ar}(BY\,XD) = 2\text{ar}(\Delta MBC)$

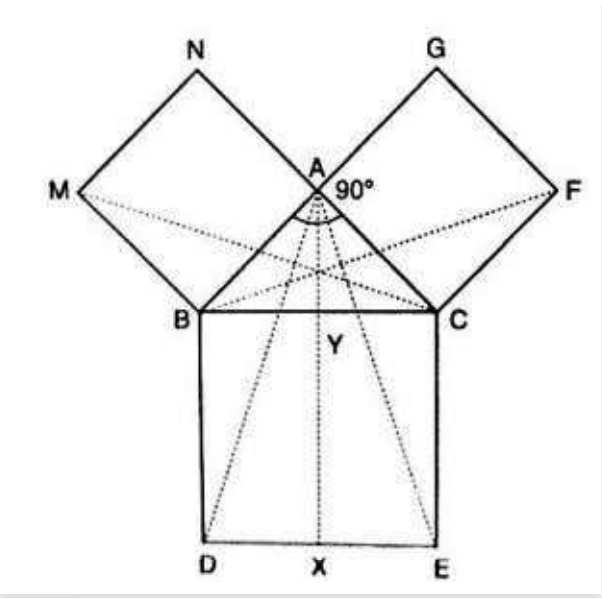
(iii) . $\text{ar}(BY\,XD) = \text{ar}(ABMN)$

(iv) . $\Delta FCB \cong \Delta ACE$

(v) . $\text{ar}(CY\,XE) = 2\text{ar}(\Delta FCB)$

(vi) . $\text{ar}(CY\,XE) = \text{ar}(ACFG)$

(vii) . $\text{ar}(BCED) = \text{ar}(ABMN) + \text{ar}(ACFG)$



Solution:

(i) . In ΔMBC and ΔABD , we have

MB = AB

BC = BD

And $\angle MBC = \angle ABD$ [since ,$\angle MBC$ and $\angle ABC$ are obtained by adding $\angle ABC$ to a right angle.]

So, by SAS congruence criterion, we have

$$\Delta MBC \cong \Delta ABD$$

$$\Rightarrow \text{ar}(\Delta MBC) = \text{ar}(\Delta ABD) \cdots\cdots (1)$$

(ii) . Clearly, triangle ABC and rectangle BYXD are on the same base BD and between the same parallels AX and BD .

$$\therefore \text{ar}(\Delta ABD) = \frac{1}{2}\text{ar}(\text{rect } BYXD)$$

$$\Rightarrow \text{ar}(\text{rect } BYXD) = 2\text{ar}(\Delta ABD)$$

$$\Rightarrow \text{ar}(\text{rect } BYXD) = 2\text{ar}(\Delta MBC) \cdots\cdots\cdots (2)$$ [From equ .1]

(iii) . Since triangles MBC and square MBAN are on the same base Mb and between the same parallels MB and NC.

$$\therefore 2\text{ar}(\Delta MBC) = \text{ar}(MBAN) \cdots\cdots\cdots (3)$$

From equ. 2 and 3, we have

$$\text{ar}(\text{sq. } MBAN) = \text{ar}(\text{rect} BYXD)$$

(iv) . In triangles FCB and ACE, we have

FC = AC

CB = CE

And , $\angle FCB = \angle ACE$ [since ,$\angle FCB$ and $\angle ACE$ are obtained by adding $\angle ACB$ to a right angle.]

So, by SAS congruence criterion, we have

$$\Delta FCB \cong \Delta ACE$$

(v) . We have,

$\Delta FCB \cong \Delta ACE$

$\Rightarrow \text{ar}(\Delta FCB) = \text{ar}(\Delta ACE)$

Clearly, triangle ACE and rectangle CYXE are on the same base CE and between same parallels CE and AX.

$\therefore 2\text{ar}(\Delta ACE) = \text{ar}(CYXE)$

$\Rightarrow 2\text{ar}(\Delta FCB) = \text{ar}(\Delta CYXE) \cdots\cdots\cdots (4)$

(vi) . Clearly , triangle FCb and rectangle FCAG are on the same base FC and between the same parallels FC and BG.

$\therefore 2\text{ar}(\Delta FCB) = \text{ar}(FCAG) \cdots\cdots (5)$

From 4 and 5, we get

$\text{ar}(CYXE) = \text{ar}(ACFG)$

(vii) . Applying Pythagoras theorem in triangle ACB, we have

$BC^2 = AB^2 + AC^2$

$\Rightarrow BC \times BD = AB \times MB + AC \times FC$

$\Rightarrow \text{ar}(BCED) = \text{ar}(ABMN) + \text{ar}(ACFG)$