MSc(HS)(Mathematics)

1. If x is real, then the largest interval in which the expression $\left(\frac{1-x+x^2}{1+x+x^2}\right)$ assumes values, is

A) [-1, 1] B) $[\frac{1}{2}, 2]$ C) $[\frac{1}{3}, 3]$ D) $[\frac{1}{4}, 4]$

2. The equation $e^{\sin x} - e^{-\sin x} - 4 = 0$ has

A) No real root B) One real root
C) Two real roots D) Infinitely many roots

3. If $x = 2 + 2^{\frac{2}{3}} + 2^{\frac{1}{3}}$, then the value of $x^3 - 6x^2 + 6x$, is

A) 1 B) 2 C) 3 D) 4

4. The product $2^{\frac{1}{4}}.4^{\frac{1}{8}}.8^{\frac{1}{16}}.16^{\frac{1}{32}}$ ..... is equal to

A) 1 B) 2 C) 3/2 D) 5/2

5. The solution of $\frac{dy}{dx} = e^{-y}\cos x$ with $y(0) = 0$ is

A) $y = \sin x + 1$ B) $e^y = \sin x + 1$ C) $y = e^{\sin x} + 2$ D) $y = \sin x$

6. Bisection method is

A) Always convergent B) Always divergent
C) Not always convergent D) None of these

7. Let $f = \begin{cases} 1\ if\ \ x \text{ is rational} \\ 0\ if\ x \text{ is irrational} \end{cases}$, then

A) f is continuous everywhere on real line
B) f is discontinuous everywhere on real line
C) f is continuous on (0, 1) otherwise discontinuous
D) None of the above

8. Relation between Cartesian and polar spherical coordinates are

A) $x = r\sin\theta\ \cos\phi, y = r\sin\phi\cos\phi, z = r\cos\theta$
B) $x = r\sin\theta\sin\phi, y = r\cos\theta\sin\phi, z = r\cos\theta$
C) $x = r\sin\theta\ \cos\phi, y = r\sin\theta\sin\phi, z = r\cos\phi$
D) $x = r\sin\theta\ \cos\phi, y = r\sin\theta\sin\phi, z = r\cos\theta$

9. Equation of right circular cone in standard form is

A) $x^2 + y^2 + z^2 = \tan^2\alpha$ B) $x^2 + y^2 = z^2\tan\alpha$
C) $x^2 + y^2 = z^2\tan^2\alpha$ D) $x^2 + y^2 - z^2 = \tan^2\alpha$

10. The probability that the 13^{th} day of a randomly chosen month is a second Saturday, is

A) 1/7 B) 1/12 C) 1/84 D) 19/84

11. The limit point of $(a, b)\ a, b \in \mathbb{R}$

A) is a B) is b C) Both a and b D) Every element of [a,b]

12. $\lim_{n\to\infty} \frac{(n!)^{\frac{1}{n}}}{n}$

A) E B) 1/e C) 1 D)e^e

13. If $u = \frac{x^2+y^2}{\sqrt{x+y}}$, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = K\,u$, where K is

A) 2 B) ½ C) 3/2 D) 2/3

14. The curvature of a circle at any point on it is

A) Constant and equal to radius of the circle
B) Variable
C) Constant and equal to reciprocal of the radius of the circle
D) Proportional to the radius of the circle

15. Weight is a

A) Scalar quantity B) Vector quantity
C) Scalar and vector both D) Mass contained in the body

16. The vectors $\alpha = (a_1, a_2)$ and $\beta = (b_1, b_2)$ in $V_2(F)$ are linearly dependent if

A) $a_1b_1 - a_2b_2 = 0$ B) $a_1b_2 - a_2b_1 = 0$
C) $a_1a_2 - b_1b_2 = 0$ D) None of these

17. The vector space $\{0\}$ is defined to have dimensions

A) Zero B) One C) ∞ D) Not defined

18. A mapping f: □ ⟶ □ which is defined as $f(x) = \sin x\,; x \in$ □ is

A) One-one only B) Onto only
C) One-one onto D) Neither one-one nor onto

19. Let $R = \{x \text{ I } x \in N, x \text{ is multiple of } 3 \text{ and } x \leq 100\}$ and $S = \{x \text{ I } x \in N, x \text{ is multiple of } 5 \text{ and } x \leq 100\}$ What is the number of elements in $(R \times S) \cap (S \times R)$

A) 36 B) 25 C) 49 D) 16

20. Consider the statements

(A) $\lim_{n\to\infty} \frac{(3n)!}{(n!)^3} = 27$

(B) $\lim_{n\to\infty} \frac{n^n}{(n+1)(n+2)\ldots(n+n)} = \frac{e}{4}$

Then

A) (A) is true (B) is false B) (A) is false (B) is true
C) (A) and (B) both are false D) (A) and (B) both are true

21. The value of 'C' of Lagrange's mean value theorem, if $f(x) = x(x-1)(x-2)$; $a = 0, b = \frac{1}{2}$ is

A) 1/4 B) 1/3 C) $\frac{6-\sqrt{(21)}}{6}$ D) $\frac{6+\sqrt{(21)}}{6}$

22. At *t=0*, the function $f(t) = \frac{\sin t}{t}$ has

A) A minimum B) A discontinuity
C) A point of inflection D) A maximum

23. If $f(x,y) = \begin{cases} xy \sin\frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ then

A) $f_x(0,0) = 1 = f_y(0,0)$ B) $f_x(0,0) = 0 = f_y(0,0)$
C) $f_x(0,0)$ and $f_y(0,0)$ does not exist D) None of these

24. Given the function $f(x,y) = x^2 - 2xy + y^2 + x^3 - y^3 + x^5$, then

A) Has maximum value at origin
B) Has minimum value at origin
C) Has maximum value but no minimum value at origin
D) Has neither maximum nor minimum value at origin

25. If $A = \begin{bmatrix} -5 & -8 & 0 \\ 3 & 5 & 0 \\ 1 & 2 & -1 \end{bmatrix}$, then A^2 is

A) Nilpotent B) Periodic C) Idempotent D) Involutory

26. The square matrix A is defined as $A = \begin{bmatrix} 1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$, the diagonal matrix D of A is

A) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ B) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ C) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ D) $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -3 \end{bmatrix}$

27. Dim V, where $V = \{a_1, a_2, \ldots a_{100} : a_1 + a_2 = 0, a_3 + a_4 = 0\}$ is

A) 100 B) 102 C) 2 D) 98

28. If C is a non-singular matrix and $B = C \begin{bmatrix} 0 & x & y \\ 0 & 0 & z \\ 0 & 0 & 0 \end{bmatrix} C^{-1}$, then

A) $B^2 = 1$ B) $B^3 = 1$ C) $B^3 = 0$ D) $B^2 = 0$

29. The dimension of the subspace of $\square^3$ spanned by $(-3,0,1), (1,2,1) and\ (3,0,-1)$ is

A) 0 B) 1 C) 2 D) 3

30. The system of equations

$4x_1 + x_2 - 3x_3 - x_4 = 0$
$2x_1 + 3x_2 + x_3 - 5x_4 = 0$ has
$x_1 - 2x_2 - 2x_3 + 3x_4 = 0$

A) No solution

B) Only one solution (0,0,0,0)
C) Infinite no of solutions
D) Only two solutions (0,0,0,0) and (3/5,1,4/5,1)

31. Let A be $n \times n$ matrix which is both Hermitian and unitary. Then
A) $A^2 = I$
B) A is real
C) Eigen values of A are 0,1,-1
D) Characteristics and minimal polynomials are the same

32. Which of the following is correct?
A) R is a vector space over N
B) R is vector space over Z
C) R is vector space over Q
D) R is vector space over C

33. A homogeneous system of 5 linear equations in 6-variables admits
A) No solution in $\square^6$
B) A unique solution in $\square^6$
C) Infinite no of solutions in $\mathbb{R}^6$
D) Finite, but more than two solutions in $\mathbb{R}^6$

34. Minimal polynomial $m(x)$ of $A_{n\times n}$, each of whose element is 1, is
A) $x - 1$
B) $x^2 + x$
C) $x^2 + nx$
D) $x^2 - nx$

35. Let $\begin{bmatrix} 2 & 3 & 4 \\ 0 & 5 & 6 \\ 0 & 0 & 7 \end{bmatrix}$, then
A) A is diagonalizable but not A^2
B) A^2 is diagonalizable but not A
C) Both A and A^2 are diagonalizable
D) Neither A nor A^2 is diagonalizable

36. Let T: $\square^2 \longrightarrow R^3$be a linear transformation given by $T(x_1, x_2) = (x_1 + x_2, x_1 - x_2, x_2)$ then Rank T is
A) 0
B) 1
C) 2
D) 3

37. Let $M = \begin{bmatrix} 1 & a & c \\ 0 & 2 & b \\ 0 & 0 & 1 \end{bmatrix}, a, b, c \in \square$, then M is diagonalizable, if and only if
A) $a = bc$
B) $b = ac$
C) $c = ab$
D) $a = b = c$

38. The $\lim_{z\to 0} \frac{\bar{z}}{z}$ is
A) 0
B) 1
C) ½
D) Does not exists

39. The real part of the complex number $(1 + i)^n$ is
A) $2^{\frac{n}{2}} \cos\frac{n\pi}{4}$
B) $2^n \cos\frac{n\pi}{2}$
C) $2^{-\frac{n}{2}} \cos n\pi$
D) $2^{-n} \cos\frac{n\pi}{2}$

40. The value of m so that $2x - x^2 + my^2$ may be harmonic is
A) 0
B) 1
C) 2
D) 3

41. Let G be a group of order 77. Then, the centre of G is isomorphic to

A) $\mathbb{Z}_1$ B) $\mathbb{Z}_7$ C) $\mathbb{Z}_{77}$ D) $\mathbb{Z}_{11}$

42. The number of zeros at the end of 100! is
A) 22 B) 24 C) 26 D) 20

43. The unit digit of 2^{100} is
A) 2 B) 4 C) 6 D)8

44. The largest integer n such that 33! is divisible by 2^n is
A) 29 B) 28 C) 33 D) 31

45. The number of generators in cyclic group of order 10 are
A) 1 B) 2 C) 3 D) 4

46. For the differential equation $xy' - y = 0$, which of the following is not an integrating factor
A) $\frac{1}{x^2}$ B) $\frac{1}{y^2}$ C) $\frac{1}{xy}$ D) $\frac{1}{x+y}$

47. The solution of $xp + yq = z$ is
A) $f(x^2, y^2) = 0$ B) $f(xy, yz) = 0$ C) $f(x, y) = 0$ D) $f\left(\frac{x}{y}, \frac{y}{z}\right) = 0$

48. The equation $u_t = c^2 u_{xx}$ is classified as
A) Elliptic B) Hyperbolic C) Parabolic D) None of these

49. Simpson's rule for integration gives exact result, when f(x) is a polynomial of degree
A) 1 B) 2 C) 3 D) All of these

50. The Newton-Raphson method converges fast, if $f'(\alpha)$ is (α is the exact value of the root)
A) Small B) Large C) Zero D) None of these

51. If 3 is taken as an approximate root of the equation $x^3 - 30 = 0$, then a better approximation by Newton-Raphson method is
A) 10/3 B) 28/9 C) 8/3 D) 26/9

52. If h is the interval of differencing, then $(\Delta - \nabla)x^2$ equals to
A) $2h$ B) $2h^2$ C) $2h^3$ D) $4h^3$

53. The first term of the series whose second and subsequent terms are $8, 3, 0, -1, 0$ is
A) 5 B) 10 C) 15 D) 20

54. In Simpson's one-third rule the curve $y = f(x)$ is assumed to be a
A) Ellipse B) Hyperbola C) Circle D) Parabola

55. The solution of the equation $\phi(x) = x + \int_0^x (\xi - x)\phi(\xi)d\xi$ is
A) $\sin x$ B) $\cos x$ C) $\tan x$ D) None of the these

56. A perfectly flexible rope of uniform density per unit length is suspended with its end points fixed. It assumes the shape of

A) Cycloid B) Straight line C) Parabola D) Catenary

57. In case of a rigid body, having N particles, the number of degrees of freedom is

A) ∞ B) 3 C) 3N D) N

58. Let X be a random variable whose density function f forms an isosceles triangle above the unit interval $I = [0,1]$ and 0 elsewhere. Then the formula for pdf is

A) $f(x) = \begin{cases} x, \ if\ 0 \le x \le \frac{1}{2} \\ -x + 1, \ if \frac{1}{2} \le x \le 1 \\ 0, elsew\square ere \end{cases}$ B) $f(x) = \begin{cases} 2x, \ if\ 0 \le x \le \frac{1}{2} \\ -2x + 2, \ if \frac{1}{2} \le x \le 1 \\ 0, elsew\square ere \end{cases}$

C) $f(x) = \begin{cases} 3x, \ if\ 0 \le x \le \frac{1}{2} \\ -3x + 3, \ if \frac{1}{2} \le x \le 1 \\ 0, elsewhere \end{cases}$ D) None of these

59. If X is normal with mean 2 and standard deviation 3, then the distribution of $Y = \frac{1}{2}X - 1$ is

A) N(0,9/4) B) N(1,9/4) C) N(0,3/4) D) N(1,3/4)

60. If X and Y be two independent random variables, each representing the number of failures preceding the first success in a sequence of Bernoulli trials with p as probability of success in a single trial and q as probality of failure, then $p(X = Y)$ is

A) $\frac{p}{1+q}$ B) $\frac{q}{1+p}$ C) $\frac{p}{1+p}$ D) $\frac{q}{1+q}$

61. Let $X_1, X_2, \ldots X_n$ be n independent and identically distributed random variables each with mean μ and variance σ^2, and let $\overline{X_n}$ be the sample mean, then variance of $\overline{X_n}$ is

A) 0 B) σ C) $n\sigma^2$ D) $\frac{\sigma^2}{n}$

62. The value of mode for the frequency curve $y = \left(\frac{1}{2}\right)\sin x\,, \ 0 \le x \le \pi \ is$

A) π B) $\frac{\pi}{3}$ C) $\frac{\pi}{4}$ D) $\frac{\pi}{2}$

63. For a given probability distribution $f(x) = \frac{1}{8}\binom{3}{x}$, $x = 0, 1, 2, 3$ for a random variable X, the moment generating function of this is

A) $\frac{1}{8}(1 + e^t)^3$ B) $\frac{1}{4}e^t$ C) $(1 + e^t)^2$ D) e^t

64. A bag X contains 2 white and 3 black balls and another bag Y contains 4 white and 2 black balls. One bag is selected at random and a ball is drawn from it. Then, the probability for the ball chosen be white is to

A) $\frac{2}{15}$ B) $\frac{7}{15}$ C) $\frac{8}{15}$ D) $\frac{14}{15}$

65. Consider the group S_9 of all the permutations on a set with 9 elements. What is the largest order of a permutation in S_9?

A) 20 B) 21 C) 14 D) 30

66. Suppose V is a real vector space of dimension 3. Then the number of pairs of linearly independent vectors in V is

A) ∞ B) 1 C) 2 D) 3

67. Let X be a connected subset of real numbers. If every element of X is irrational, then the cardinality of X is

A) Infinite B) Countably infinite C) 2 D) 1

68. Suppose the matrix $A = \begin{bmatrix} 40 & -29 & -11 \\ -18 & 30 & -12 \\ 26 & 24 & -50 \end{bmatrix}$ has a certain complex number $\lambda \neq 0$ as an eigen value. Which of the following numbers must also be an eigen value of A?

A) $\lambda + 20$ B) $\lambda - 20$ C) $20 - \lambda$ D) $-20 - \lambda$

69. Let A be a 3×3 matrix with real entries such that $det(A) = 6$ and trace of A is 0. If $\det(A + I) = 0$, where I denotes 3×3 identity matrix, then eigen values of A are

A) -1,2,3 B) $-1, -2, 3$ C) $-1, 2, -3$ D) $-1, -2, -3$

70. Let $a_n = sin\frac{\pi}{n}$. For the sequence $a_1, a_2, \ldots$ the supremum is

A) 0 and it is attained B) 0 and it is not attained

C) 1 and it is attained D) 1 and it is not attained

71. The power series $\sum_{x=0}^{\infty} 3^{-n}(z-1)^{2n}$ converges, if

A) $|z| \leq \sqrt{3}$ B) $|z| < \sqrt{3}$ C) $|z-1| < \sqrt{3}$ D) $|z-1| \leq \sqrt{3}$

72. The distance between the origin and the point nearest to it on the surface $z^2 = 1 + xy$ is

A) 1 B) $\frac{\sqrt{3}}{2}$ C) $\sqrt{3}$ D) 2

73. If $f(x) = \begin{cases} \frac{\sin[x]}{[x]}, & [x] \neq 0 \\ 0, & [x] = 0 \end{cases}$ where $[x]$ denotes the greatest integer less than or equal to x, then $\lim_{x \to 0} f(x)$ is

A) 1 B) 0 C) -1 D) None of these

74. If $f(x) = (x^2 - 1)|x^2 - 3x + 2| + \cos(|x|)$, then set of point of non-differentiability is

A) {0,1,2} B) {1,2} C) {0,2} D) {2}

75. The function y= $|\log x|$ is

A) Discontinuous at x=1
B) Differentiable at x=1
C) Not differentiable at x=1
D) None of these

x-x-x