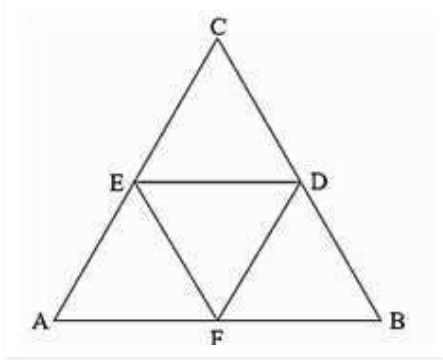


Q1) In $\triangle ABC$, D, E and F are, respectively the mid points of BC, CA and AB. If the lengths of sides AB, BC and CA are 7cm, 8cm and 9cm, respectively, find the perimeter of $\triangle DEF$.

Solution:



Given that,

AB = 7cm, BC = 8cm, AC = 9cm

In $\triangle ABC$,

F and E are the mid points of AB and AC.

$$\therefore EF = \frac{1}{2}BC$$

Similarly

$$DF = \frac{1}{2}AC \text{ and } DE = \frac{1}{2}AB$$

Perimeter of $\triangle DEF$ = DE + EF + DF

$$= \frac{1}{2}AB + \frac{1}{2}BC + \frac{1}{2}AC$$

$$= \frac{1}{2} * 7 + \frac{1}{2} * 8 + \frac{1}{2} * 9$$

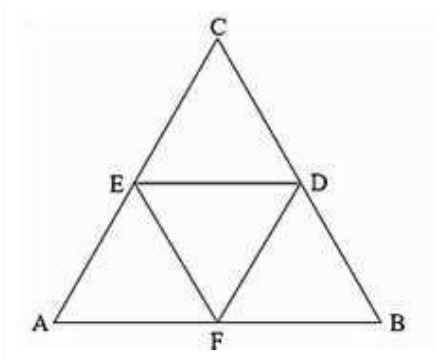
$$= 3.5 + 4 + 4.5$$

$$= 12\text{cm}$$

$$\therefore \text{Perimeter of } \triangle DEF = 12\text{cm}$$

Q2) In a $\triangle ABC$, $\angle A = 50^\circ$, $\angle B = 60^\circ$ and $\angle C = 70^\circ$. Find the measures of the angles of the triangle formed by joining the mid-points of the sides of this triangle.

Solution:



In $\triangle ABC$,

D and E are mid points of AB and BC.

By Mid point theorem,

$$DE \parallel AC, DE = \frac{1}{2}AC$$

F is the midpoint of AC

$$\text{Then, } DE = \frac{1}{2}AC = CF$$

In a Quadrilateral DECF

$$DE \parallel AC, DE = CF$$

Hence DECF is a parallelogram.

$$\therefore \angle C = \angle D = 70^0 \quad [\text{Opposite sides of a parallelogram}]$$

Similarly

$$BEFD \text{ is a parallelogram, } \angle B = \angle F = 60^0$$

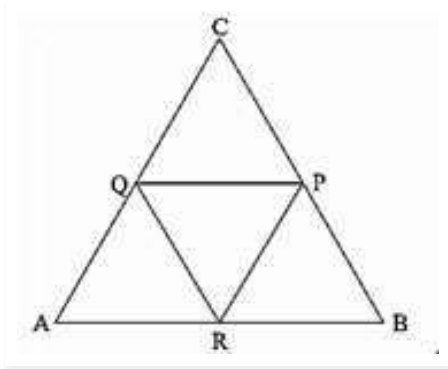
$$ADEF \text{ is a parallelogram, } \angle A = \angle E = 50^0$$

$\therefore$ Angles of $\triangle DEF$ are

$$\angle D = 70^0, \angle E = 50^0, \angle F = 60^0$$

Q3) In a triangle, P, Q and R are the mid points of sides BC, CA and AB respectively. If AC = 21cm, BC = 29cm and AB = 30cm, find the perimeter of the quadrilateral ARPQ.

Solution:



In $\triangle ABC$,

R and P are mid points of AB and BC

$$RP \parallel AC, RP = \frac{1}{2}AC \quad [\text{By Midpoint Theorem}]$$

In a quadrilateral,

[A pair of side is parallel and equal]

$$RP \parallel AQ, RP = AQ$$

Therefore, RPQA is a parallelogram

$$\Rightarrow AR = \frac{1}{2}AB = \frac{1}{2} * 30 = 15\text{cm}$$

$$AR = QP = 15\text{cm} \quad [\text{Opposite sides are equal}]$$

$$\Rightarrow RP = \frac{1}{2}AC = \frac{1}{2} * 21 = 10.5\text{cm}$$

$$RP = AQ = 10.5\text{cm} \quad [\text{Opposite sides are equal}]$$

Now,

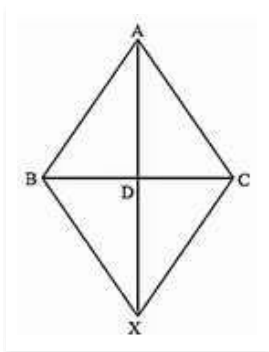
$$\text{Perimeter of ARPQ} = AR + QP + RP + AQ$$

$$= 15 + 15 + 10.5 + 10.5$$

$$= 51\text{cm}$$

Q4) In a $\triangle ABC$ median AD is produced to X such that $AD = DX$. Prove that $ABXC$ is a parallelogram.

Solution:



In a quadrilateral ABXC, we have

$$AD = DX \text{ [Given]}$$

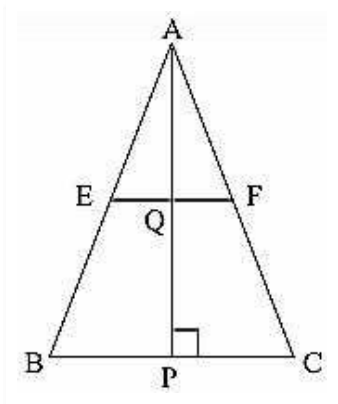
$$BD = DC \text{ [Given]}$$

So, diagonals AX and BC bisect each other.

Therefore, ABXC is a parallelogram.

Q5) In a $\triangle ABC$, E and F are the mid-points of AC and AB respectively. The altitude AP to BC intersects EF at Q . Prove that $AQ = QP$.

Solution:



In a $\triangle ABC$

E and F are mid points of AB and AC

$\therefore EF \parallel FE, \frac{1}{2}BC = FE$ [By mid point theorem]

In $\triangle ABP$

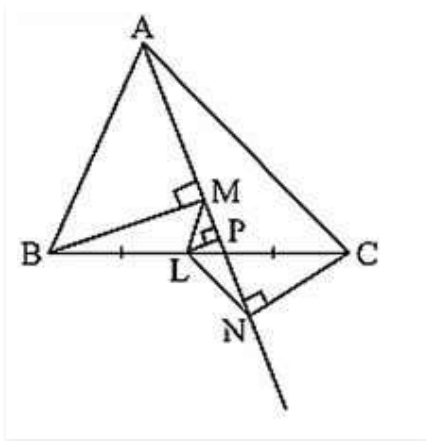
F is the mid-point of AB and $\therefore FQ \parallel BP$ [$\because EF \parallel BP$]

Therefore, Q is the mid-point of AP [By mid-point theorem]

Hence, $AQ = QP$.

Q6) In a $\triangle ABC$, BM and CN are perpendiculars from B and C respectively on any line passing through A . If L is the mid-point of BC , prove that $ML = NL$.

Solution:



Given that,

In $\triangle BLM$ and $\triangle CLN$

$$\angle BML = \angle CNL = 90^\circ$$

$$BL = CL \quad [L \text{ is the mid-point of } BC]$$

$$\angle MLB = \angle NLC \quad [\text{Vertically opposite angle}]$$

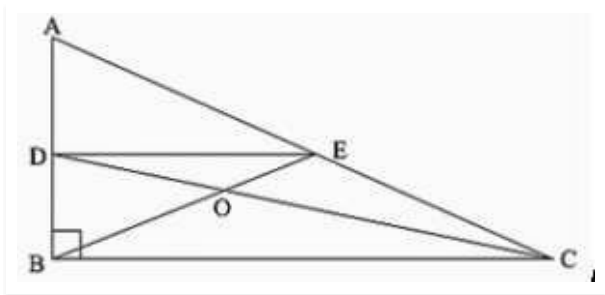
$$\therefore \triangle BLM = \triangle CLN$$

$$\therefore LM = LN \quad [\text{corresponding parts of congruent triangles}]$$

Q7) In figure 14.95, Triangle ABC is a right angled triangle at B . Given that $AB = 9\text{cm}$, $AC = 15\text{cm}$ and D, E are the mid-points of the sides AB and AC respectively, calculate

(i) The length of BC

(ii) The area of $\triangle ADE$.



Solution:

In $\triangle ABC$, $\angle B = 90^\circ$,

By using Pythagoras theorem

$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow 15^2 = 9^2 + BC^2$$

$$\Rightarrow BC = \sqrt{15^2 - 9^2}$$

$$\Rightarrow BC = \sqrt{225 - 81}$$

$$\Rightarrow BC = \sqrt{144} = 12\text{cm}$$

In $\triangle ABC$,

D and E are mid-points of AB and AC

$$\therefore DE \parallel BC, DE = \frac{1}{2}BC \quad [\text{By mid - point theorem}]$$

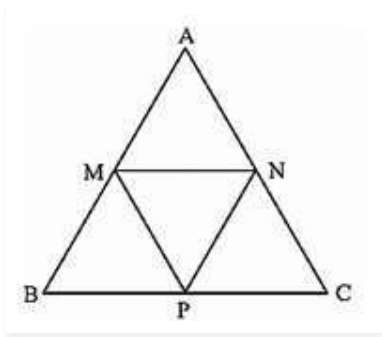
$$AD = DB = \frac{AB}{2} = \frac{9}{2} = 4.5\text{cm} \quad [\because D \text{ is the mid - point of AB}]$$

$$\text{Area of } \triangle ADE = \frac{1}{2} * AD * DE$$

$$= \frac{1}{2} * 4.5 * 6$$

$$= 13.5\text{cm}^2$$

Q8) In figure 14.96, M, N and P are mid-points of AB, AC and BC respectively. If $MN = 3\text{cm}$, $NP = 3.5\text{cm}$ and $MP = 2.5\text{cm}$, calculate BC, AB and AC.



Solution:

Given $MN = 3\text{cm}$, $NP = 3.5\text{cm}$ and $MP = 2.5\text{cm}$.

To find BC, AB and AC

In $\triangle ABC$

M and N are mid-points of AB and AC

$$\therefore MN = \frac{1}{2}BC, MN \parallel BC \quad [\text{By mid - point theorem}]$$

$$\Rightarrow 3 = \frac{1}{2}BC$$

$$\Rightarrow 3 * 2 = BC$$

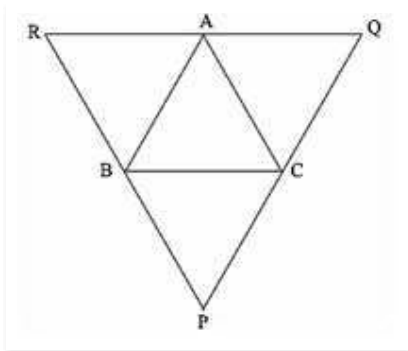
$$\Rightarrow BC = 6\text{cm}$$

Similarly

$$AC = 2MP = 2(2.5) = 5\text{cm}$$

$$AB = 2 NP = 2 (3.5) = 7\text{cm}$$

Q9) ABC is a triangle and through A, B, C lines are drawn parallel to BC, CA and AB respectively intersecting at P, Q and R. Prove that the perimeter of ΔPQR is double the perimeter of ΔABC .



Solution:

Clearly ABCQ and ARBC are parallelograms.

Therefore, $BC = AQ$ and $BC = AR$

$$\Rightarrow AQ = AR$$

$\Rightarrow A$ is the mid-point of QR

Similarly B and C are the mid points of PR and PQ respectively.

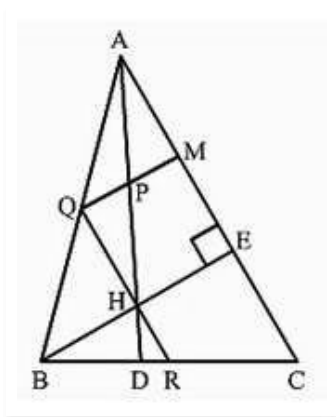
$$\therefore AB = \frac{1}{2}PQ, BC = \frac{1}{2}QR, CA = \frac{1}{2}PR$$

$$\Rightarrow PQ = 2AB, QR = 2BC \text{ and } PR = 2CA$$

$$\Rightarrow PQ + QR + RP = 2 (AB + BC + CA)$$

$$\Rightarrow \text{Perimeter of } \Delta PQR = 2 (\text{perimeter of } \Delta ABC)$$

Q10) In figure 14.97, $BE \perp AC$, AD is any line from A to BC intersecting BE in H. P, Q and R are respectively the mid-points of AH, AB and BC. Prove that $\angle PQR = 90^\circ$



Solution:

Given,

$BE \perp AC$ and P, Q and R are respectively mid-point of AH, AB and BC.

To prove: $\angle PQR = 90^\circ$

Proof: In ΔABC , Q and R are mid-points of AB and BC respectively.

$$\therefore QR \parallel AC \dots\dots (i)$$

In $\triangle ABH$, Q and P are the mid-points of AB and AH respectively

$$\therefore QP \parallel BH \dots\dots (ii)$$

But, $BE \perp AC$

Therefore, from equation (i) and equation (ii) we have,

$$QP \perp QR$$

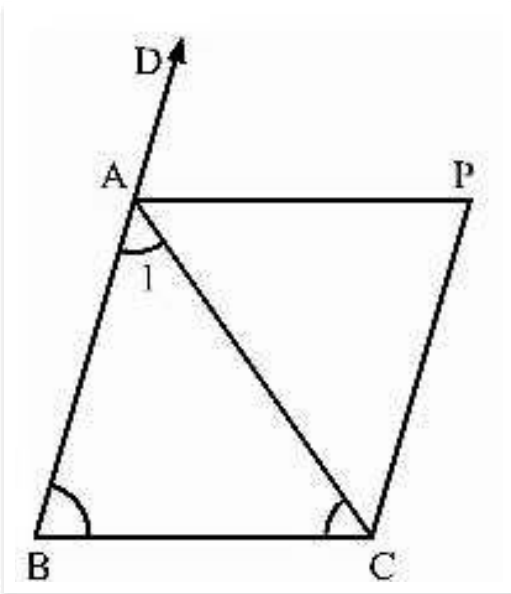
$$\Rightarrow \angle PQR = 90^\circ$$

Hence Proved.

Q11) In figure 14.98, $AB=AC$ and $CP \parallel BA$ and AP is the bisector of exterior $\angle CAD$ of $\triangle ABC$. Prove that

(i) $\angle PAC = \angle BCA$.

(ii) $ABCP$ is a parallelogram.



Solution:

Given,

$AB = AC$ and $CD \parallel BA$ and AP is the bisector of exterior $\angle CAD$ of $\triangle ABC$

To prove:

(i) $\angle PAC = \angle BCA$

(ii) $ABCP$ is a parallelogram.

Proof:

(i) We have,

$$AB=AC$$

$$\Rightarrow \angle ACB = \angle ABC$$

[Opposite angles of equal sides

of triangle are equal]

Now, $\angle CAD = \angle ABC + \angle ACB$

$\Rightarrow \angle PAC + \angle PAD = 2\angle ACB$ $[\because \angle PAC = \angle PAD]$

$\Rightarrow 2\angle PAC = 2\angle ACB$

$\Rightarrow \angle PAC = \angle ACB$

(ii) Now,

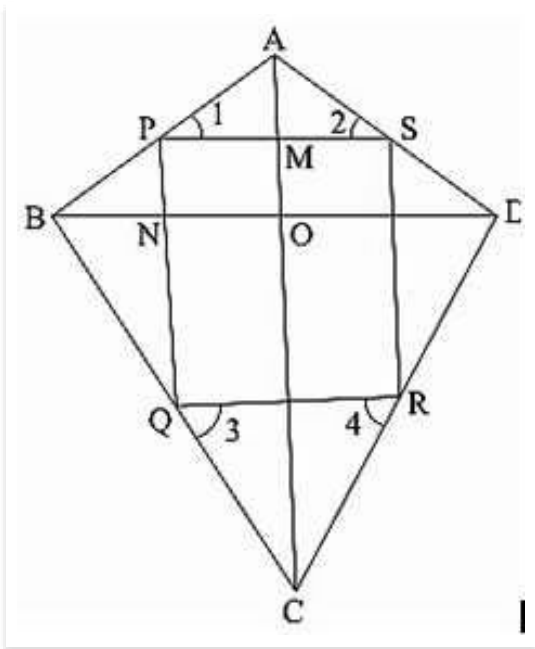
$\angle PAC = \angle BCA$

$\Rightarrow AP \parallel BC$ and $CP \parallel BA$ [Given]

Therefore, ABCP is a parallelogram.

Q12) ABCD is a kite having $AB=AD$ and $BC=CD$. Prove that the figure found by joining the mid points of the sides, in order, is a rectangle.

Solution:



Given,

A kite ABCD having $AB=AD$ and $BC=CD$. P, Q, R, S are the mid-points of sides AB, BC, CD and DA respectively. PQ, QR, RS and SP are joined.

To prove:

PQRS is a rectangle.

Proof:

In $\triangle ABC$, P and Q are the mid-points of AB and BC respectively.

$\therefore PQ \parallel AC$ and $PQ = \frac{1}{2}AC$... (i)

In $\triangle ADC$, R and S are the mid-points of CD and AD respectively.

$\therefore RS \parallel AC$ and $RS = \frac{1}{2}AC$... (ii)

From (i) and (ii) we have

$$PQ \parallel RS \text{ and } PQ = RS$$

Thus, in quadrilateral PQRS, a pair of opposite sides is equal and parallel. So, PQRS is a parallelogram. Now, we shall prove that one angle of parallelogram PQRS is a right angle.

Since $AB=AD$

$$\Rightarrow \frac{1}{2}AB = \frac{1}{2}AD$$

$$\Rightarrow AP = AS \dots (iii) \quad [\because P \text{ and } S \text{ are mid points of } AB \text{ and } AD]$$

$$\Rightarrow \angle 1 = \angle 2 \dots (iv)$$

Now, in $\triangle PBQ$ and $\triangle SDR$, we have

$$PB = SD \quad [\because AD = AB \Rightarrow \frac{1}{2}AD = \frac{1}{2}AB]$$

$$BQ = DR \quad [\text{Since } PB = SD]$$

$$\text{And } PQ = SR \quad [\text{Since, PQRS is a parallelogram}]$$

So, by SSS criterion of congruence, we have

$$\triangle PBQ \cong \triangle SDR$$

$$\Rightarrow \angle 3 = \angle 4 \quad [\text{CPCT}]$$

$$\text{Now, } \Rightarrow \angle 3 + \angle SPQ + \angle 2 = 180^\circ$$

$$\text{And } \angle 1 + \angle PSR + \angle 4 = 180^\circ$$

$$\therefore \angle 3 + \angle SPQ + \angle 2 = \angle 1 + \angle PSR + \angle 4$$

$$\Rightarrow \angle SPQ = \angle PSR \quad [\angle 1 = \angle 2 \text{ and } \angle 3 = \angle 4]$$

Now, transversal PS cuts parallel lines SR and PQ at S and P respectively.

$$\therefore \angle SPQ + \angle PSR = 180^\circ$$

$$\Rightarrow 2\angle SPQ = 180^\circ$$

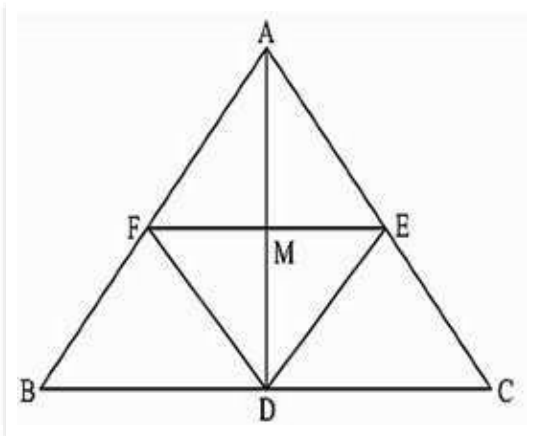
$$\Rightarrow \angle SPQ = 90^\circ \quad [\because \angle PSR = \angle SPQ]$$

Thus, PQRS is a parallelogram such that $\angle SPQ = 90^\circ$.

Hence, PQRS is a parallelogram.

Q13) Let ABC be an isosceles triangle in which $AB=AC$. If D, E, F be the mid points of the, sides BC,CA and AB respectively, show that the segment AD and EF bisect each other at right angles.

Solution:



Since D, E and F are mid-points of sides BC, CA and AB respectively.

$\therefore AB \parallel DE$ and $AC \parallel DF$

$\therefore AF \parallel DE$ and $AE \parallel DF$

ABDE is a parallelogram.

$AF = DE$ and $AE = DF$

$\frac{1}{2}AB = DE$ and $\frac{1}{2}AC = DF$

$DE = DF$ [Since, $AB = AC$]

$AE = AF = DE = DF$

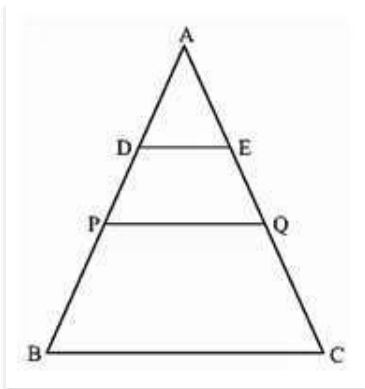
ABDF is a rhombus.

$\Rightarrow AD$ and FE bisect each other at right angle.

Q14) ABC is a triangle. D is a point on AB such that $AD = \frac{1}{4}AB$ and E is a point on AC such that $AE = \frac{1}{4}AC$.

Prove that $DE = \frac{1}{4}BC$.

Solution:



Let P and Q be the mid-points of AB and AC respectively.

Then $PQ \parallel BC$

$PQ = \frac{1}{2}BC$(i)

In $\triangle APQ$, D and E are the mid-points of AP and AQ respectively.

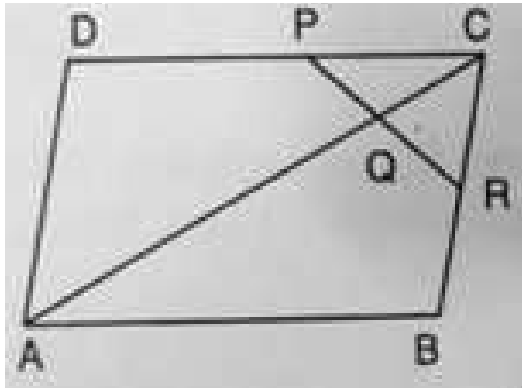
$$\therefore DE \parallel PQ, \text{ and } DE = \frac{1}{2}PQ \dots (ii)$$

$$\text{From (i) and (ii): } DE = \frac{1}{2} PQ = \frac{1}{2} \left(\frac{1}{2} BC \right)$$

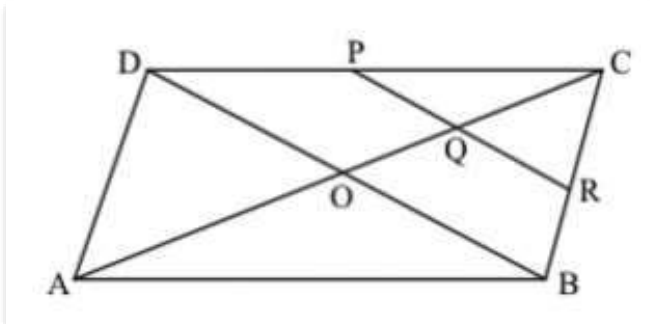
$$\therefore DE = \frac{1}{4} BC$$

Hence proved.

Q15) In Figure 14.99, ABCD is a parallelogram in which P is the mid-point of DC and Q is a point on AC such that $CQ = \frac{1}{4}AC$. If PQ produced meets BC at R, prove that R is a mid-point of BC.



Solution:



Join B and D.

Suppose AC and BD intersect at O.

$$\text{Then } OC = \frac{1}{2}AC$$

Now,

$$CQ = \frac{1}{4}AC$$

$$\Rightarrow CQ = \frac{1}{2} \left(\frac{1}{2}AC \right)$$

$$= \frac{1}{2}OC$$

In $\triangle DCO$, P and Q are mid points of DC and OC respectively.

$$\therefore PQ \parallel DO$$

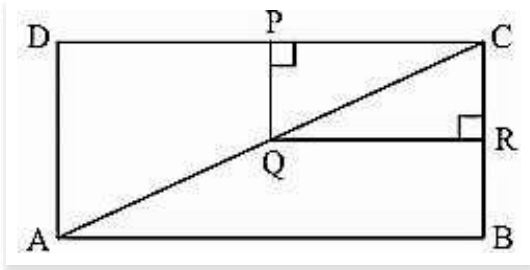
Also in $\triangle COB$, Q is the mid-point of OC and $QR \parallel OB$

Therefore, R is the mid-point of BC.

Q16) In figure 14.100, ABCD and PQRC are rectangles and Q is the mid-point of AC. Prove that

(i) $DP = PC$

(ii) $PR = \frac{1}{2}AC$



Solution:

(i) In $\triangle ADC$, Q is the mid-point of AC such that $PQ \parallel AD$

Therefore, P is the mid-point of DC.

$\Rightarrow DP = PC$ [Using mid-point theorem]

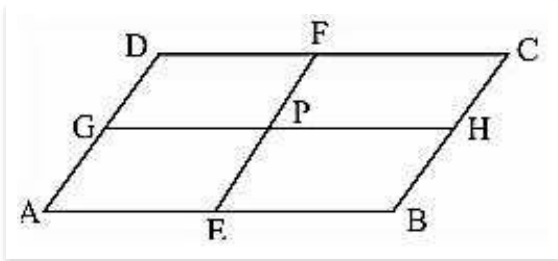
(ii) Similarly, R is the mid-point of BC

$\therefore PR = \frac{1}{2}BD$

$PR = \frac{1}{2}AC$ [Diagonal of rectangle are equal, $BD = AC$]

Q17) ABCD is a parallelogram; E and F are the mid-points of AB and CD respectively. GH is any line intersecting AD, EF and BC at G, P and H respectively. Prove that $GP = PH$.

Solution:



Since E and F are mid-points of AB and CD respectively

$AE = BE = \frac{1}{2}AB$

And $CF = DF = \frac{1}{2}CD$

But, $AB = CD$

$\frac{1}{2}AB = \frac{1}{2}CD$

$\Rightarrow BE = CF$

Also, $BE \parallel CF$ [$\because AB \parallel CD$]

Therefore, BEFC is a parallelogram

$BC \parallel EF$ and $BE = PH$ (i)

Now, $BC \parallel EF$

$\Rightarrow AD \parallel EF$ [$\because BC \parallel AD$ as ABCD is a parallelogram]

Therefore, AEFD is a parallelogram.

$\Rightarrow AE = GP$

But E is the mid-point of AB.

So, $AE = BF$

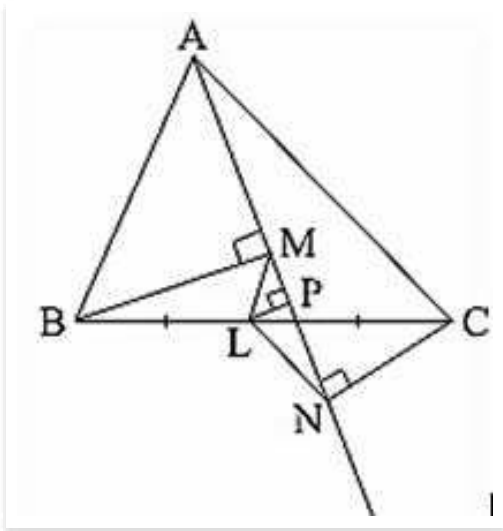
Therefore, $GP = PH$.

Q18) BM and CN are perpendiculars to a line passing through the vertex A of triangle ABC. If L is the mid-point of BC, prove that $LM = LN$.

Solution:

To prove $LM = LN$

Draw LS as perpendicular to line MN.



Therefore, the lines BM, LS and CN being the same perpendiculars on line MN are parallel to each other.

According to intercept theorem,

If there are three or more parallel lines and the intercepts made by them on a transversal are equal, then the corresponding intercepts on any other transversal are also equal.

In the figure, MB, LS and NC are three parallel lines and the two transversal lines are MN and BC.

We have, $BL = LC$ [As L is the given mid-point of BC]

Using the intercept theorem, we get

$MS = SN$ (i)

Now in $\triangle MLS$ and $\triangle LSN$

$MS = SN$ using equation (i).

$\angle LSM = \angle LSN = 90^\circ$ [$LS \perp MN$]

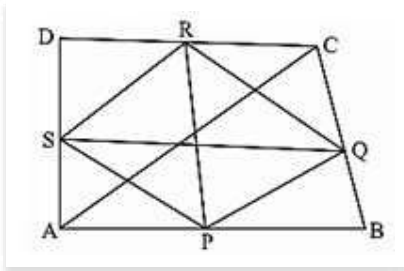
And $SL = LS$ is common.

$\therefore \triangle MLS \cong \triangle LSN$ [SAS Congruency Theorem]

$$\therefore LM = LN \quad [\text{CPCT}]$$

Q19) Show that, the line segments joining the mid-points of opposite sides of a quadrilateral bisect each other.

Solution:



Let ABCD is a quadrilateral in which P, Q, R and S are mid-points of sides AB, BC, CD and DA respectively.

So, by using mid-point theorem we can say that

$$SP \parallel BD \text{ and } SP = \frac{1}{2}BD \dots\dots (i)$$

Similarly in $\triangle BCD$

$$QR \parallel BD \text{ and } QR = \frac{1}{2}BD \dots\dots (ii)$$

From equations (i) and (ii), we have

$$SP \parallel QR \text{ and } SP = QR$$

As in quadrilateral SPQR, one pair of opposite sides is equal and parallel to each other.

So, SPQR is a parallelogram since the diagonals of a parallelogram bisect each other.

Hence PR and QS bisect each other.

Q20) Fill in the blanks to make the following statements correct:

(i) The triangle formed by joining the mid-points of the sides of an isosceles triangle is _____.

(ii) The triangle formed by joining the mid-points of the sides of a right triangle is _____.

(iii) The figure formed by joining the mid-points of consecutive sides of a quadrilateral is _____.

Solution:

(i) Isosceles

(ii) Right triangle

(iii) Parallelogram