

Exercise 6.2

$$(i) A(a+b, b+c); B(a-b, c-b)$$

$$x_1 = a+b, y_1 = b+c$$

$$x_2 = a-b, y_2 = c-b$$

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$AB = \sqrt{(a-b-a-b)^2 + (c-b-b-c)^2}$$

$$AB = \sqrt{(-2b)^2 + (-2b)^2}$$

$$AB = \sqrt{4b^2 + 4b^2} = \sqrt{8b^2}$$

$$|AB| = 2\sqrt{2}b$$

$$(ii) A(a \sin \alpha, -b \cos \alpha); B(-a \cos \alpha, b \sin \alpha)$$

$$x_1 = a \sin \alpha, y_1 = -b \cos \alpha$$

$$x_2 = -a \cos \alpha, y_2 = b \sin \alpha$$

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$AB = \sqrt{(-a \cos \alpha - a \sin \alpha)^2 + (b \sin \alpha + b \cos \alpha)^2}$$

$$AB = \sqrt{(-a \cos \alpha)^2 + (-a \sin \alpha)^2 + [2(-a \cos \alpha)(-a \sin \alpha)] + (b \sin \alpha)^2 + (b \cos \alpha)^2 + [2(b \sin \alpha)(b \cos \alpha)]}$$

$$AB = \sqrt{a^2 \cos^2 \alpha + a^2 \sin^2 \alpha + 2a^2 \cos \alpha \sin \alpha + b^2 \sin^2 \alpha + b^2 \cos^2 \alpha + 2b^2 \sin \alpha \cos \alpha}$$

$$AB = \sqrt{a^2(\cos^2 \alpha + \sin^2 \alpha) + 2a^2 \cos \alpha \sin \alpha + b^2(\sin^2 \alpha + \cos^2 \alpha) + 2b^2 \sin \alpha \cos \alpha}$$

$$AB = \sqrt{a^2 + b^2 + 2a^2 \cos \alpha \sin \alpha + 2b^2 \sin \alpha \cos \alpha}$$

$$= \sqrt{(a^2 + b^2) + 2 \cos \alpha \sin \alpha (a^2 + b^2)}$$

$$|AB| = \sqrt{(a^2 + b^2)(1 + 2 \cos \alpha \sin \alpha)}$$

Q.3 $A(2,1)$ and $B(1,-2)$

$C = (x, y)$

As A & B are equidistant from C,

$\therefore AC = BC$

$$\sqrt{(x-2)^2 + (y-1)^2} = \sqrt{(x-1)^2 + (y+2)^2}$$

$$x^2 + 4 - 4x + y^2 + 1 - 2y = x^2 + 1 - 2x + y^2 + 4 + 4y$$

$$x^2 + 5 - 4x + y^2 - 2y = x^2 + 5 - 2x + y^2 + 4y$$

$$-4x - 2y = -2x + 4y$$

$$-4x - 2y + 2x - 4y = 0$$

$$-2x - 6y = 0$$

$$-2(x + 3y) = 0$$

$$\boxed{x + 3y = 0}$$



Q.4 $P(x, y)$, $Q(-3, 0)$ and $R(3, 0)$

ATQ,

$$PQ = \sqrt{(-3-x)^2 + (0-y)^2}$$

$$4 = \sqrt{9 + x^2 + 6x + 0 + y^2 + 0}$$

$$4^2 = 9 + x^2 + 6x + y^2$$

$$16 = 9 + x^2 + 6x + y^2$$

$$16 - 9 = x^2 + y^2 + 6x$$

$$x^2 + y^2 = 7 - 6x \quad \text{--- (1)}$$

$$PR = \sqrt{(3-x)^2 + (0-y)^2}$$

$$4 = \sqrt{9 + x^2 - 6x + 0 + y^2 + 0}$$

$$4^2 = 9 + x^2 - 6x + y^2$$

$$16 - 9 = x^2 + y^2 - 6x$$

$$x^2 + y^2 = 7 + 6x \quad \text{--- (2)}$$

From (1) & (2)

$$7 - 6x = 7 + 6x$$

$$-6x - 6x = 0$$

$$-12x = 0$$

$$\boxed{x = 0}$$

Putting value of x in ②

$$x^2 + y^2 = 7 + 6x$$

$$0 + y^2 = 7 + 6(0)$$

$$y^2 = 7$$

$$\boxed{y = \pm\sqrt{7}}$$

Q.5 P(2, -3) and R(10, y)

Distance PR = 10.

ATQ,

$$PR = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$10 = \sqrt{(10 - 2)^2 + (y + 3)^2}$$

$$10 = \sqrt{8^2 + y^2 + 3^2 + 6y}$$

$$10^2 = 64 + y^2 + 9 + 6y$$

$$100 - 73 = y^2 + 6y$$

$$y^2 + 6y - 27 = 0$$

$$y^2 + 9y - 3y - 27 = 0$$

$$y(y + 9) - 3(y + 9) = 0$$

$$(y + 9)(y - 3) = 0$$

$$\therefore \boxed{y = -9} \text{ or } \boxed{y = 3}$$

D(2, 3)

C(14, 0)

A(-4, -1)

B(-2, -4)

$$\left. \begin{array}{l|l} A = (-4, -1) & C = (4, 0) \\ B = (-2, -4) & D = (2, 3) \end{array} \right\} \text{given}$$

$$\begin{aligned} AB &= \sqrt{(-2+4)^2 + (-4+1)^2} \\ &= \sqrt{(2)^2 + (-3)^2} = \sqrt{4+9} \\ &= \sqrt{13} \quad \text{--- (1)} \end{aligned}$$

$$\begin{aligned} CD &= \sqrt{(4-2)^2 + (0-3)^2} \\ &= \sqrt{(2)^2 + (-3)^2} = \sqrt{4+9} \\ &= \sqrt{13} \quad \text{--- (2)} \end{aligned}$$

$$\begin{aligned} BC &= \sqrt{(4+2)^2 + (0+4)^2} \\ &= \sqrt{(6)^2 + (4)^2} = \sqrt{36+16} \\ &= \sqrt{52} \quad \text{--- (3)} \end{aligned}$$

$$\begin{aligned} AD &= \sqrt{(-4-2)^2 + (-1-3)^2} \\ &= \sqrt{(-6)^2 + (-4)^2} = \sqrt{36+16} \\ &= \sqrt{52} \quad \text{--- (4)} \end{aligned}$$

From (1), (2), (3) & (4)

$$AB = CD \text{ \& } BC = AD$$

$\therefore$ ABCD is a ~~paralle~~ parallelogram. ✓

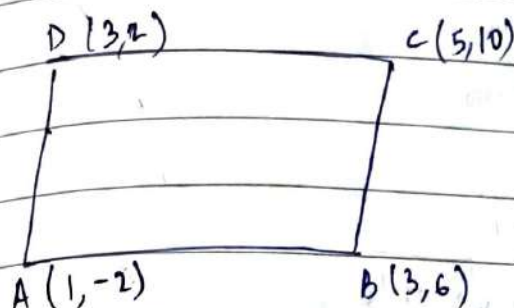
$$\begin{aligned} AC &= \sqrt{(4+4)^2 + (0+1)^2} \\ &= \sqrt{(8)^2 + (1)^2} = \sqrt{64+1} \\ &= \sqrt{65} \quad \text{--- (5)} \end{aligned}$$

$$\begin{aligned} BD &= \sqrt{(2+2)^2 + (3+4)^2} \\ &= \sqrt{(4)^2 + (7)^2} = \sqrt{16+49} \\ &= \sqrt{65} \quad \text{--- (6)} \end{aligned}$$

From ⑤ & ⑥,

$$AC = BD$$

$\therefore$ A ||gram with equal diagonals is a rectangle
 $\therefore$ ABCD is a rectangle. ✓



$$\left. \begin{array}{l} A = (1, -2) \\ B = (3, 6) \end{array} \right| \left. \begin{array}{l} C = \cancel{(3, 2)} (5, 10) \\ D = (3, 2) \end{array} \right\} \text{ given}$$

$$\begin{aligned} AB &= \sqrt{(3-1)^2 + (6+2)^2} \\ &= \sqrt{(2)^2 + (8)^2} = \sqrt{4+64} \\ &= \sqrt{68} \quad \text{--- ①} \end{aligned}$$

$$\begin{aligned} CD &= \sqrt{(5-3)^2 + (10-2)^2} \\ &= \sqrt{(2)^2 + (8)^2} \\ &= \sqrt{4+64} = \sqrt{68} \quad \text{--- ②} \end{aligned}$$

$$\begin{aligned} AD &= \sqrt{(3-1)^2 + (2+2)^2} \\ &= \sqrt{(2)^2 + (4)^2} = \sqrt{4+16} \\ &= \sqrt{20} \quad \text{--- ③} \end{aligned}$$

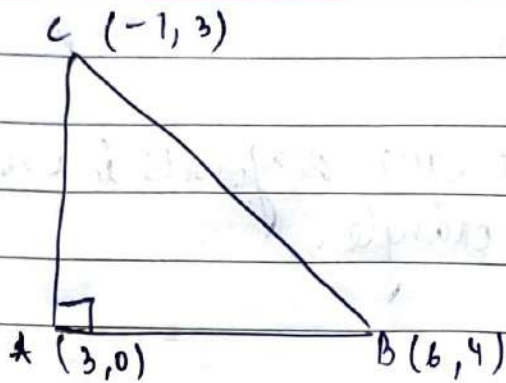
$$\begin{aligned} BC &= \sqrt{(5-3)^2 + (10-6)^2} \\ &= \sqrt{(2)^2 + (4)^2} = \sqrt{4+16} \\ &= \sqrt{20} \quad \text{--- ④} \end{aligned}$$

From ①, ②, ③ & ④,

$$AB = CD \text{ \& } AD = BC$$

$\therefore$ ABCD is a ||gram. ✓

Q.9



$$\begin{aligned}
 AB &= \sqrt{(6-3)^2 + (4-0)^2} \\
 &= \sqrt{(3)^2 + (4)^2} = \sqrt{9+16} \\
 &= \sqrt{25} = 5
 \end{aligned}$$

$$\begin{aligned}
 BC &= \sqrt{(-1-6)^2 + (3-4)^2} \\
 &= \sqrt{(-7)^2 + (-1)^2} = \sqrt{49+1} \\
 &= \sqrt{50}
 \end{aligned}$$

$$\begin{aligned}
 AC &= \sqrt{(-1-3)^2 + (3-0)^2} \\
 &= \sqrt{(-4)^2 + (3)^2} = \sqrt{16+9} \\
 &= \sqrt{25} = 5
 \end{aligned}$$

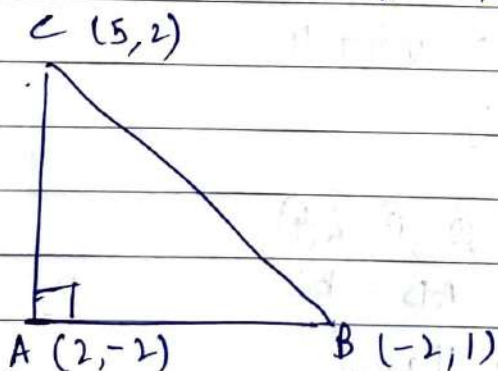
$$\begin{array}{c|c|c}
 AB^2 = (5)^2 & BC^2 = (\sqrt{50})^2 & AC^2 = (5)^2 \\
 = 25 & = 50 & = 25
 \end{array}$$

$$\text{Since, } AB^2 + AC^2 = BC^2$$

$\therefore$ ABC is a ~~right~~ right angled
~~isosceles~~ $\triangle$

$$\& AB = AC$$

Q.10



$$\begin{aligned}
 AB &= \sqrt{(-2-2)^2 + (1+2)^2} \\
 &= \sqrt{(-4)^2 + (3)^2} = \sqrt{16+9} \\
 &= \sqrt{25} = 5 \quad \text{--- (1)}
 \end{aligned}$$

$$\begin{aligned}
 BC &= \sqrt{(5+2)^2 + (2-1)^2} \\
 &= \sqrt{(7)^2 + (1)^2} = \sqrt{49+1} \\
 &= \sqrt{50} \quad \text{--- (2)}
 \end{aligned}$$

$$\begin{aligned}
 AC &= \sqrt{(5-2)^2 + (2+2)^2} \\
 &= \sqrt{(3)^2 + (4)^2} = \sqrt{9+16} \\
 &= \sqrt{25} = 5 \quad \text{--- (3)}
 \end{aligned}$$

$$\begin{aligned}
 AB^2 &= (5)^2 \\
 &= 25
 \end{aligned}$$

$$\begin{aligned}
 BC^2 &= (\sqrt{50})^2 \\
 &= 50
 \end{aligned}$$

$$\begin{aligned}
 AC^2 &= (5)^2 \\
 &= 25
 \end{aligned}$$

Since, $AB^2 + AC^2 = BC^2$

$\therefore$ ABC is a right angled Δ . ✓

Length of hypotenuse $BC = \sqrt{50} = \boxed{5\sqrt{2} \text{ units}}$

$$\text{Area of } \Delta ABC = \frac{1}{2} \times AB \times AC = \frac{1}{2} \times \sqrt{25} \times \sqrt{25}$$

$$= \frac{25}{2} = 12.5 = \boxed{12.5 \text{ sq. units}} \quad \checkmark$$

A $(2a, 4a)$, B $(2a, 6a)$, C $(2a + \sqrt{3}a, 5a)$

$$\begin{aligned}
 AB &= \sqrt{(2a-2a)^2 + (6a-4a)^2} \\
 &= \sqrt{(0)^2 + (2a)^2} \\
 &= \sqrt{4a^2} = 2a \quad \text{--- (1)}
 \end{aligned}$$

$$\begin{aligned} BC &= \sqrt{(2a + \sqrt{3}a - 2a)^2 + (5a - 6a)^2} \\ &= \sqrt{(\sqrt{3}a)^2 + (-a)^2} = \sqrt{3a^2 + a^2} \\ &= \sqrt{4a^2} = 2a \quad \text{--- (2)} \end{aligned}$$

$$\begin{aligned} AC &= \sqrt{(2a + \sqrt{3}a - 2a)^2 + (5a - 4a)^2} \\ &= \sqrt{(\sqrt{3}a)^2 + (a)^2} = \sqrt{3a^2 + a^2} \\ &= \sqrt{4a^2} = 2a \quad \text{--- (3)} \end{aligned}$$

From ①, ② & ③, ✓
 $\therefore$ ABC is an equilateral Δ .

Q.12 A(2, 3), B(-4, -6) and C($1, \frac{3}{2}$)

$$\begin{aligned} AB &= \sqrt{(-4-2)^2 + (-6-3)^2} \\ &= \sqrt{(-6)^2 + (-9)^2} = \sqrt{36+81} \\ &= \sqrt{117} = 10.81 \end{aligned}$$

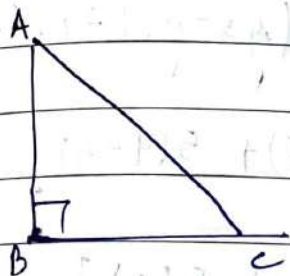
$$\begin{aligned} BC &= \sqrt{(1+4)^2 + \left(\frac{3}{2}+6\right)^2} \\ &= \sqrt{(5)^2 + \left(\frac{15}{2}\right)^2} = \sqrt{25 + \frac{225}{4}} \\ &= \sqrt{\frac{325}{4}} = \sqrt{81.25} = 9.01 \end{aligned}$$

$$\begin{aligned} AC &= \sqrt{(2-1)^2 + \left(3-\frac{3}{2}\right)^2} \\ &= \sqrt{(1)^2 + \left(\frac{3}{2}\right)^2} = \sqrt{1 + \frac{9}{4}} \\ &= \sqrt{\frac{13}{4}} = \sqrt{3.25} = 1.80 \end{aligned}$$

$$\begin{aligned}
 BC + AC &= 9.01 + 1.80 \\
 &= 10.81 \\
 &= AB
 \end{aligned}$$

$\therefore$ Since, $AC + BC$ is not greater than AB ,
 $\therefore ABC$ is not a Δ . ✓

Q.13 $A(2, 9)$, $B(a, 5)$ and $C(5, 5)$.



ABC is a Δ right angled at B (given)

$$AC^2 = AB^2 + BC^2$$

$$[\sqrt{(5-2)^2 + (5-9)^2}]^2 = [\sqrt{(a-2)^2 + (5-9)^2}]^2 + [\sqrt{(5-a)^2 + (5-5)^2}]^2$$

$$(5-2)^2 + (5-9)^2 = [(a-2)^2 + (5-9)^2] + [(5-a)^2 + (5-5)^2]$$

$$(3)^2 + (-4)^2 = [a^2 + 4 - 4a + (-4)^2] + [25 + a^2 - 10a + 0]$$

$$9 + 16 = a^2 + 4 - 4a + 16 + 25 + a^2 - 10a$$

$$25 = 2a^2 - 14a + 25 + 20$$

$$a^2 - 7a + 10 = 0$$

$$a^2 - 5a - 2a + 10 = 0$$

$$a(a-5) - 2(a-5) = 0$$

$$(a-2)(a-5) = 0$$

$$a = 2 \text{ or } a = 5 \quad \checkmark$$

Q. If $a = 5$

$$\begin{aligned}
 \therefore \text{Area of } \Delta &= \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] \\
 &= \frac{1}{2} [2(5-5) + 5(5-9) + 5(9-5)]
 \end{aligned}$$

$$= \frac{1}{2} [2(0) + 5(-4) + 5(4)]$$

$$= \frac{1}{2} [0 - 20 + 20]$$

$$= 0 \quad (\text{not possible})$$

$$\therefore \boxed{a = 20}$$

$$\text{Area of } \Delta = \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$= \frac{1}{2} [2(5 - 9) + 2(9 - 5) + 5(5 - 9)]$$

$$= \frac{1}{2} [2(0) + 2(-4) + 5(4)]$$

$$= \frac{1}{2} [0 - 8 + 20] = \frac{1}{2} \times 12$$

$$= \boxed{6 \text{ sq. units}}$$

Q.14

A (2, -1), B (3, 4), C (2, 3) & D (-3, -2)

$$AB = \sqrt{(3-2)^2 + (4+1)^2}$$

$$= \sqrt{(1)^2 + (5)^2} = \sqrt{1+25}$$

$$= \sqrt{26}$$

①

$$BC = \sqrt{(-2-3)^2 + (3-4)^2}$$

$$= \sqrt{(-5)^2 + (-1)^2} = \sqrt{25+1}$$

$$= \sqrt{26}$$

②

$$CD = \sqrt{(-3+2)^2 + (-2-3)^2}$$

$$= \sqrt{(-1)^2 + (-5)^2} = \sqrt{1+25}$$

$$= \sqrt{26}$$

③

$$\begin{aligned}
 AD &= \sqrt{(-3-2)^2 + (-2+1)^2} \\
 &= \sqrt{(-5)^2 + (-1)^2} = \sqrt{25+1} \\
 &= \sqrt{26} \quad \text{--- (4)}
 \end{aligned}$$

From ①, ②, ③ & ④,

$$AB = BC = CD = AD$$

$\therefore$ ABCD is a rhombus ✓

Q.15 A(2,0) and B(2,5)

Let C be (x,y)

$$\begin{aligned}
 AB &= \sqrt{(2-2)^2 + (5-0)^2} \\
 &= \sqrt{(0)^2 + (5)^2} = \sqrt{25} \\
 &= 5
 \end{aligned}$$

$$\begin{aligned}
 BC &= \sqrt{(x-2)^2 + (y-5)^2} \\
 &= \sqrt{x^2 + 4 - 4x + y^2 + 25 - 10y} \\
 &= \sqrt{x^2 - 4x + y^2 - 10y + 29}
 \end{aligned}$$

$$\begin{aligned}
 AC &= \sqrt{(x-2)^2 + (y-0)^2} \\
 &= \sqrt{x^2 + 4 - 4x + y^2}
 \end{aligned}$$

ATQ,

$$AC = BC = 3$$

$$AC = BC$$

$$\begin{aligned}
 \sqrt{x^2 + 4 - 4x + y^2} &= \sqrt{x^2 - 4x + y^2 - 10y + 29} \\
 x^2 + 4 - 4x + y^2 &= x^2 - 4x + y^2 - 10y + 29 \\
 10y &= 29 - 4 \\
 y &= \frac{25}{10} = 2.5
 \end{aligned}$$

$$\boxed{y = 2.5} \quad \checkmark$$

$$AC = 3$$

$$\sqrt{x^2 + 4 - 4x + y^2} = 3$$

$$x^2 + 4 - 4x + y^2 = 3^2$$

$$x^2 + 4 - 4x + (2.5)^2 = 9$$

$$x^2 - 4x + 4 + 6.25 = 9$$

$$x^2 - 4x + 10.25 - 9 = 0$$

$$x^2 - 4x + 1.25 = 0$$

$$x^2 - 4x + \frac{5}{4} = 0$$

$$4x^2 - 16x + 5 = 0 \times 4$$

$$4x^2 - 16x + 5 = 0$$

$$D = b^2 - 4ac = (-16)^2 - [4 \times 4 \times 5]$$

$$= 256 - 80 = 176$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{16 + \sqrt{176}}{2 \times 4}$$

$$x = \frac{16 + 13.26}{8}$$

$$x = \frac{29.26}{8}$$

$$x = 3.65$$

$$x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{16 - \sqrt{176}}{2 \times 4}$$

$$x = \frac{16 - 13.26}{8}$$

$$x = \frac{2.74}{8}$$

$$x = 0.35$$

$\therefore$ The third vertex is $(3.65, 2.5)$ or $(0.35, 2.5)$.

Q.17 A(-2, 5), B(0, 1) and C(2, -3)

$$\begin{aligned} AB &= \sqrt{(0+2)^2 + (1-5)^2} \\ &= \sqrt{4 + (-4)^2} = \sqrt{4+16} \\ &= \sqrt{20} \end{aligned}$$

$$\begin{aligned} BC &= \sqrt{(2-0)^2 + (-3-1)^2} \\ &= \sqrt{(2)^2 + (-4)^2} = \sqrt{4+16} \\ &= \sqrt{20} \end{aligned}$$

$$\begin{aligned} AC &= \sqrt{(2+2)^2 + (-3-5)^2} \\ &= \sqrt{(4)^2 + (-8)^2} = \sqrt{16+64} \\ &= \sqrt{80} = 2\sqrt{20} \end{aligned}$$

Since, $AB + BC = AC$,
 $\therefore A, B \& C$ are collinear ✓

Q.18

P (-3, 2)

Let Q be (x, y)

Since Q lies on the line joining P & O (origin) and

$$OP = OQ,$$

$$\therefore \frac{x_1 + x_2}{2} = 0$$

$$\frac{-3 + x}{2} = 0$$

$$\boxed{x = 3}$$

$$\frac{y_1 + y_2}{2} = 0$$

$$\frac{2 + y}{2} = 0$$

$$\boxed{y = -2}$$

$\therefore Q$ is (3, -2) ✓

Ch-6

Exercise 8.2

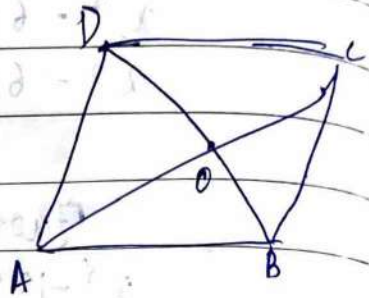
Q. 20

$A(3, 4)$; $B(3, 8)$; $C(9, 8)$

Let D be (x, y)

$ABCD$ is a ||gram (given)

Since, diagonals of a ||gram bisect each other



$\therefore$ Coordinates of midpoint of AC = Coordinates of midpoint of BD

$$\frac{3+9}{2}, \frac{4+8}{2} = \frac{3+x}{2}, \frac{8+y}{2}$$

$$\frac{12}{2}, \frac{12}{2} = \frac{3+x}{2}, \frac{8+y}{2}$$

$$\frac{3+x}{2} = 6$$

$$3+x = 12$$

$$\boxed{x = 9}$$

$$\frac{8+y}{2} = 6$$

$$8+y = 12$$

$$\boxed{y = 4}$$

Q. 21

$A(-5, 4)$ and $B(-1, 6)$

Let $C(x, y)$ be equidistant from A & B

One point

$$(x, y) = \left(\frac{-5-1}{2}, \frac{4+6}{2} \right)$$

$$(x, y) = \left(\frac{-6}{2}, \frac{10}{2} \right)$$

$$(x, y) = \boxed{(-3, 5)}$$

Genl^y: $(x+5)^2 + (y-4)^2$

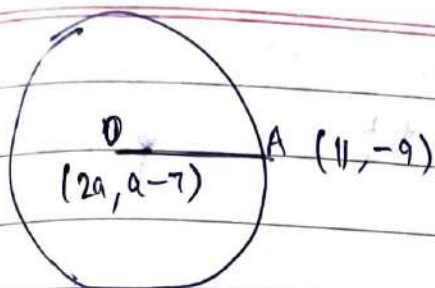
$$= (x+1)^2 + (y-6)^2$$

$$\Rightarrow 10x + 25 - 8y + 16$$

$$= 2x + 1 - 12y + 36$$

$$\Rightarrow 8x + 4y = -4$$

$\Rightarrow$ There will be infinite points



$$r \text{ of circle} = \frac{10\sqrt{2}}{2} = 5\sqrt{2} \text{ units}$$

ATQ,

$$OA = 5\sqrt{2}$$

$$\sqrt{(11-2a)^2 + (-9-a+7)^2} = 5\sqrt{2}$$

$$121 + 4a^2 - 44a + (-2-a)^2 = (5\sqrt{2})^2$$

$$121 + 4a^2 - 44a + 4 + a^2 + 4a = 50$$

$$5a^2 - 40a + 125 - 50 = 0$$

$$5a^2 - 40a + 75 = 0$$

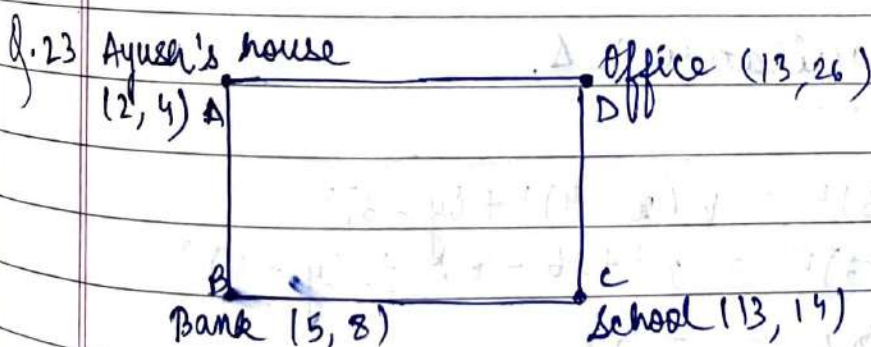
$$a^2 - 8a + 15 = 0$$

$$a^2 - 5a - 3a + 15 = 0$$

$$a(a-5) - 3(a-5) = 0$$

$$(a-5)(a-3) = 0$$

$$a = 5 \text{ or } a = 3$$



$$\begin{aligned} AB &= \sqrt{(5-2)^2 + (8-4)^2} \\ &= \sqrt{(3)^2 + (4)^2} = \sqrt{9+16} \\ &= \sqrt{25} = 5 \text{ km} \end{aligned}$$

$$\begin{aligned} BC &= \sqrt{(13-5)^2 + (14-8)^2} \\ &= \sqrt{8^2 + 6^2} = \sqrt{64+36} \\ &= \sqrt{100} = 10 \text{ km} \end{aligned}$$

$$\begin{aligned} CD &= \sqrt{(13-13)^2 + (26-14)^2} \\ &= \sqrt{(0)^2 + (12)^2} = \sqrt{0+144} \\ &= 12 \text{ km} \end{aligned}$$

$$\begin{aligned} \text{Total distance covered} &= 5 + 10 + 12 \text{ km} \\ \text{by Aakash} &= \boxed{27 \text{ km}} \end{aligned}$$

$$\begin{aligned} \text{Shortest distance} &= \sqrt{(13-2)^2 + (26-4)^2} \\ \text{to office (A)} &= \sqrt{(11)^2 + (22)^2} = \sqrt{121+484} \\ &= \sqrt{605} = \boxed{24.6 \text{ km}} \end{aligned}$$

$$\begin{aligned} \text{Extra distance} &= 27 - 24.6 \\ \text{covered} &= \boxed{2.4 \text{ km}} \end{aligned}$$

Q.25 A (-4, 3) and B (4, 3)
Let C be (x, y)

As the Δ is an equilateral Δ ,
 $\therefore AC = BC$

$$\begin{aligned} \sqrt{(x+4)^2 + (y-3)^2} &= \sqrt{(x-4)^2 + (y-3)^2} \\ x^2 + 16 + 8x + (y-3)^2 &= x^2 + 16 - 8x + (y-3)^2 \\ 8x &= -8x \end{aligned}$$

$$8x + 8x = 0$$

$$x = 0$$

$$16$$

$$\boxed{x = 0}$$

Hence, C = (0, y)

$$\begin{aligned} AB &= \sqrt{(4+4)^2 + (3-3)^2} \\ &= \sqrt{(8)^2 + (0)^2} = \sqrt{64+0} \\ &= 8 \end{aligned}$$

As, $AB = BC = AC$

$\therefore AC = 8$

$$\sqrt{(0+4)^2 + (y-3)^2} = 8$$

$$\sqrt{(4)^2 + y^2 + 9 - 6y} = 8$$

$$16 + y^2 + 9 - 6y = 8^2$$

$$y^2 - 6y + 25 - 64 = 0$$

$$y^2 - 6y - 39 = 0$$

$$D = b^2 - 4ac$$

$$= (-6)^2 - [4 \times (1) \times (-39)]$$

$$= 36 + 156 = 192$$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$y = \frac{(6) \pm \sqrt{192}}{2 \times 1}$$

$$y = \frac{6 + 8\sqrt{3}}{2}$$

$$y = 3 + 4\sqrt{3}$$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$y = \frac{6 - \sqrt{192}}{2 \times 1}$$

$$y = \frac{6 - 8\sqrt{3}}{2}$$

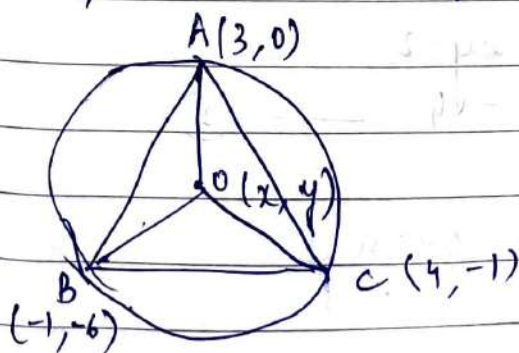
$$y = 3 - 4\sqrt{3}$$

(i) Interior

$$\rightarrow C(0, 3 - 4\sqrt{3})$$

(ii) exterior

$$\rightarrow C(0, 3 + 4\sqrt{3})$$



$A(3,0)$, $B(-1,-6)$ and $C(4,-1)$

Let $O(x,y)$ be the circumcentre of the Δ .

$$OA = OB = OC = \text{radius of circle}$$

$$\therefore OA = OB$$

$$\sqrt{(x-3)^2 + (y-0)^2} = \sqrt{(x+1)^2 + (y+6)^2}$$

$$x^2 + 9 - 6x + y^2 = x^2 + 1 + 2x + y^2 + 36 + 12y$$

$$9 - 6x = 37 + 12y + 2x$$

$$9 - 37 = 2x + 6x + 12y$$

$$8x + 12y = -28$$

$$2x + 3y = -7 \quad \text{--- ①}$$

$$OB = OC$$

$$\sqrt{(x+1)^2 + (y+6)^2} = \sqrt{(x-4)^2 + (y+1)^2}$$

$$x^2 + 1 + 2x + y^2 + 36 + 12y = x^2 + 16 - 8x + y^2 + 1 + 2y$$

$$2x + 12y + 37 = -8x + 2y + 17$$

$$2x + 8x + 12y - 2y + 37 - 17 = 0$$

$$10x + 10y + 20 = 0$$

$$x + y = -2 \quad \text{--- ②}$$

~~Subtract ② from ①~~

$$\begin{array}{r} 2x + 3y = -7 \\ x + y = -2 \\ \hline (-) \quad (-) \quad (+) \\ \hline x \end{array}$$

Multiple ② by 2

$$2x + 2y = -4 \quad \text{--- ③}$$

Subtract ③ from ①

$$\begin{array}{r} 2x + 3y = -7 \\ 2x + 2y = -4 \\ (-) \quad (-) \quad (+) \\ \hline y = -3 \end{array}$$

Putting value of y in (2)

$$x + y = -2$$

$$x - 3 = -2$$

$$x = -2 + 3$$

$$x = 1$$

$\therefore$ circumcentre of the Δ is $(1, -3)$.

$$\begin{aligned} \text{Circumradius} &= \sqrt{(1+1)^2 + (-3+6)^2} \\ &= \sqrt{(2)^2 + (3)^2} = \sqrt{4+9} \\ &= \sqrt{13} \text{ units} \end{aligned}$$

Q.28 A $(7, 6)$ and B $(-3, 4)$

Let $P(x, 0)$ be a point on x -axis.

As P is equidistant from A & B,

$$\therefore PA = PB$$

$$\begin{aligned} PA &= PB \\ \sqrt{(x-7)^2 + (0-6)^2} &= \sqrt{(x+3)^2 + (0-4)^2} \\ x^2 + 49 - 14x + 36 &= x^2 + 9 + 6x + 16 \end{aligned}$$

$$-14x - 6x = 25 - 85$$

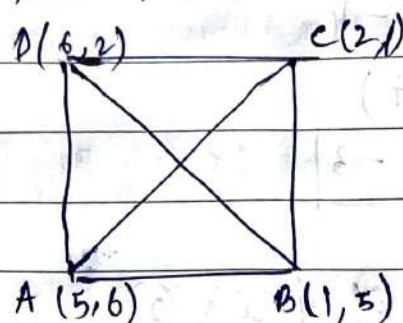
$$-20x = -60$$

$$x = \frac{-60}{-20}$$

$$x = 3$$

$\therefore$ The point on x -axis is $(3, 0)$

Q. 29(i) A (5, 6), B (1, 5), C (2, 1), D (6, 2)



$$\begin{aligned} AB &= \sqrt{(5-1)^2 + (6-5)^2} \\ &= \sqrt{(4)^2 + (1)^2} = \sqrt{16+1} \\ &= \sqrt{17} \quad \text{--- (1)} \end{aligned}$$

$$\begin{aligned} AC &= \sqrt{(5-2)^2 + (6-1)^2} \\ &= \sqrt{(3)^2 + (5)^2} \\ &= \sqrt{9+25} \\ &= \sqrt{34} \quad \text{--- (2)} \end{aligned}$$

$$\begin{aligned} BC &= \sqrt{(1-2)^2 + (5-1)^2} \\ &= \sqrt{(-1)^2 + (4)^2} = \sqrt{1+16} \\ &= \sqrt{17} \quad \text{--- (3)} \end{aligned}$$

$$\begin{aligned} BD &= \sqrt{(6-1)^2 + (2-5)^2} \\ &= \sqrt{(5)^2 + (-3)^2} \\ &= \sqrt{25+9} \\ &= \sqrt{34} \quad \text{--- (4)} \end{aligned}$$

$$\begin{aligned} CD &= \sqrt{(6-2)^2 + (2-1)^2} \\ &= \sqrt{(4)^2 + (1)^2} = \sqrt{16+1} \\ &= \sqrt{17} \quad \text{--- (5)} \end{aligned}$$

$$\begin{aligned} AD &= \sqrt{(6-5)^2 + (2-6)^2} \\ &= \sqrt{(1)^2 + (-4)^2} = \sqrt{1+16} \\ &= \sqrt{17} \quad \text{--- (6)} \end{aligned}$$

From (1), (2), (3), (4), (5) & (6)

$AB = BC = CD = AD$ and

diagonals $AC = BD$

$\therefore$ ABCD is a square. ✓

(iii) P ($\sqrt{2}$, $\sqrt{2}$), ~~Q ($\sqrt{2}$, $\sqrt{2}$)~~ Q ($-\sqrt{2}$, $-\sqrt{2}$), R ($-\sqrt{6}$, $\sqrt{6}$)

$$\begin{aligned} PQ &= \sqrt{(-\sqrt{2}-\sqrt{2})^2 + (\sqrt{2}-\sqrt{2})^2} \\ &= \sqrt{(-2\sqrt{2})^2 + (-2\sqrt{2})^2} = \sqrt{8+8} \\ &= \sqrt{16} = 4 \end{aligned}$$

$$\begin{aligned} QR &= \sqrt{(-\sqrt{6}+\sqrt{2})^2 + (\sqrt{6}+\sqrt{2})^2} \\ &= \sqrt{6+2-2\sqrt{2}+6+2+2\sqrt{2}} \\ &= \sqrt{16} = 4 \end{aligned}$$

$$\begin{aligned} PR &= \sqrt{(-\sqrt{6}-\sqrt{2})^2 + (\sqrt{6}-\sqrt{2})^2} \\ &= \sqrt{6+2+2\sqrt{2}+6+2-2\sqrt{2}} \\ &= \sqrt{16} = 4 \end{aligned}$$

Since, $PQ = QR = PR$
 $\therefore PQR$ is an equilateral Δ .

Ch-6

Exercise 6.2Q.31 $P(6, -1)$, $Q(1, 3)$ and $R(x, 8)$

$$PQ = QR \text{ (given)}$$

$$\sqrt{(6-1)^2 + (-1-3)^2} = \sqrt{(x-1)^2 + (8-3)^2}$$

$$\cancel{(5)^2} + (-4)^2 = x^2 + 1 - 2x + \cancel{(5)^2}$$

$$16 = x^2 + 1 - 2x$$

$$x^2 - 2x + 1 - 16 = 0$$

$$x^2 - 2x - 15 = 0$$

$$x^2 - 5x + 3x - 15 = 0$$

$$x(x-5) + 3(x-5) = 0$$

$$(x-5)(x+3) = 0$$

$$\therefore \boxed{x = +5} \text{ or } \boxed{x = -3}$$

Q.33 $A(5, 1)$, $B(1, 5)$, $P(x, y)$ P is equidistant from A & B .

$$\therefore AP = BP$$

$$\sqrt{(x-5)^2 + (y-1)^2} = \sqrt{(x-1)^2 + (y-5)^2}$$

$$\sqrt{x^2 + 25 - 10x + y^2 + 1 - 2y} = \sqrt{x^2 + 1 - 2x + y^2 + 25 - 10y}$$

$$\cancel{x^2} + 25 - 10x + \cancel{y^2} + 1 - 2y = \cancel{x^2} + 1 - 2x + \cancel{y^2} + 25 - 10y$$

$$-10x + 2x = -10y + 2y$$

$$-8x = -8y$$

$$\boxed{x = y}$$

Hence, proved

Q.36 $P(k-1, 2)$ is equidistant from $A(3, k)$ and $B(k, 5)$.

$$PA = PB$$

$$\therefore \sqrt{(k-1-3)^2 + (2-k)^2} = \sqrt{(k-1-k)^2 + (2-5)^2}$$

$$(k-4)^2 + (2-k)^2 = \cancel{(-1)^2} + (-3)^2$$

$$k^2 + 16 - 8k + 4 + k^2 - 4k = 1 + 9$$

$$2k^2 - 12k + 20 - 10 = 0$$

$$2k^2 - 12k + 10 = 0$$

$$k^2 - 6k + 5 = 0$$

$$k^2 - 5k - k + 5 = 0$$

$$k(k-5) - 1(k-5) = 0$$

$$(k-5)(k-1) = 0$$

$$\therefore \boxed{k=5} \text{ or } \boxed{k=1}$$

Q.37 $A(0, 2)$ is equidistant from $B(3, p)$ and $C(p, 5)$.

$$\therefore AB = AC$$

$$\sqrt{(0-3)^2 + (2-p)^2} = \sqrt{(0-p)^2 + (2-5)^2}$$

$$(-3)^2 + (2-p)^2 = (-p)^2 + (-3)^2$$

$$9 + 4 + p^2 - 4p = p^2 + 9$$

$$13 - 9 = 4p$$

$$4p = 4$$

$$\boxed{p=1}$$

$$AB = \sqrt{(0-3)^2 + (2-1)^2}$$

$$= \sqrt{9+1} = \boxed{\sqrt{10}}$$

Q. 38

(i) $A(-1, -2)$, $B(1, 0)$, $C(-1, 2)$, $D(-3, 0)$

$$\begin{aligned} AB &= \sqrt{(-1-1)^2 + (-2-0)^2} \\ &= \sqrt{(-2)^2 + (-2)^2} = \sqrt{4+4} \\ &= \sqrt{8} \quad \text{--- (1)} \end{aligned}$$

$$\begin{aligned} BC &= \sqrt{(1+1)^2 + (0-2)^2} \\ &= \sqrt{(2)^2 + (-2)^2} = \sqrt{4+4} \\ &= \sqrt{8} \quad \text{--- (2)} \end{aligned}$$

$$\begin{aligned} CD &= \sqrt{(-1+3)^2 + (2-0)^2} \\ &= \sqrt{(2)^2 + (2)^2} = \sqrt{4+4} \\ &= \sqrt{8} \quad \text{--- (3)} \end{aligned}$$

$$\begin{aligned} AD &= \sqrt{(-1+3)^2 + (-2-0)^2} \\ &= \sqrt{(2)^2 + (-2)^2} = \sqrt{4+4} \\ &= \sqrt{8} \quad \text{--- (4)} \end{aligned}$$

$$\begin{aligned} AC &= \sqrt{(-1+1)^2 + (-2-2)^2} \\ &= \sqrt{(0)^2 + (-4)^2} = \sqrt{0+16} \\ &= 4 \quad \text{--- (5)} \end{aligned}$$

$$\begin{aligned} BD &= \sqrt{(1+3)^2 + (0-0)^2} \\ &= \sqrt{(4)^2 + (0)^2} = \sqrt{16+0} \\ &= 4 \quad \text{--- (6)} \end{aligned}$$

From (1), (2), (3), (4), (5) & (6),
 $AB = BC = CD = AD$ & $AC = BD$
 $\therefore ABCD$ is a square.



(ii) $A(-3, 5)$, $B(3, 1)$, $C(0, 3)$, $D(-1, -4)$

$$\begin{aligned} AB &= \sqrt{(-3-3)^2 + (5-1)^2} \\ &= \sqrt{(-6)^2 + (4)^2} = \sqrt{36+16} \\ &= \sqrt{52} \quad \text{--- (1)} \end{aligned}$$

$$\begin{aligned} BC &= \sqrt{(3-0)^2 + (1-3)^2} \\ &= \sqrt{(3)^2 + (-2)^2} = \sqrt{9+4} \\ &= \sqrt{13} \quad \text{--- (2)} \end{aligned}$$

$$\begin{aligned} CD &= \sqrt{(0+1)^2 + (3+4)^2} \\ &= \sqrt{(1)^2 + (7)^2} = \sqrt{1+49} \\ &= \sqrt{50} \quad \text{--- (3)} \end{aligned}$$

$$\begin{aligned} AD &= \sqrt{(-3+1)^2 + (5+4)^2} \\ &= \sqrt{(-2)^2 + (9)^2} = \sqrt{4+81} \\ &= \sqrt{85} \quad \text{--- (4)} \end{aligned}$$

From (1), (2), (3), (4)

$$AB \neq BC \neq CD \neq AD$$

$\therefore ABCD$ is a general quadrilateral. ✓

Ch-6
Exercise 6.2

Q.41 A(3,1), B(6,4) and C(8,6)

$$\begin{aligned} AB &= \sqrt{(6-3)^2 + (4-1)^2} \\ &= \sqrt{(3)^2 + (3)^2} = \sqrt{9+9} \\ &= \sqrt{18} = 3\sqrt{2} \quad \text{--- (1)} \end{aligned}$$

$$\begin{aligned} BC &= \sqrt{(8-6)^2 + (6-4)^2} \\ &= \sqrt{(2)^2 + (2)^2} = \sqrt{4+4} \\ &= \sqrt{8} = 2\sqrt{2} \quad \text{--- (2)} \end{aligned}$$

$$\begin{aligned} AC &= \sqrt{(8-3)^2 + (6-1)^2} \\ &= \sqrt{(5)^2 + (5)^2} = \sqrt{25+25} \\ &= \sqrt{50} = 5\sqrt{2} \quad \text{--- (3)} \end{aligned}$$

From (1), (2) & (3),

$$3\sqrt{2} + 2\sqrt{2} = 5\sqrt{2}$$

$$\therefore AB + BC = AC$$

$\therefore$ Points A, B & C are collinear
and hence, Rehini, Sandhya and
Bina are seated in a line.

Q.42 A(5, -2) and B(-3, 2)

Let C(0, y) be the point on y-axis

$$\begin{aligned} \text{As, } CA &= CB \quad (\text{given}) \\ \sqrt{(0-5)^2 + (y+2)^2} &= \sqrt{(0+3)^2 + (y-2)^2} \\ (-5)^2 + (y+2)^2 &= (3)^2 + (y-2)^2 \\ 25 + y^2 + 4y + 4 &= 9 + y^2 - 4y + 4 \\ 25 + y^2 + 4y + 4 &= 9 + y^2 - 4y + 4 \end{aligned}$$

$$4y + 4y = 9 - 25$$

$$8y = -16$$

$$\boxed{y = -2}$$

$\therefore$ The point on y-axis is $(0, -2)$.

Q.46 A(7, -1) and B(6, 8)

P(x, 3) is equidistant from points A & B.

$$\therefore PA = PB$$

$$\sqrt{(x-7)^2 + (3+1)^2} = \sqrt{(x-6)^2 + (3-8)^2}$$

$$(x-7)^2 + (4)^2 = (x-6)^2 + (-5)^2$$

$$x^2 + 49 - 14x + 16 = x^2 + 36 - 12x + 25$$

$$65 - 14x = 61 - 12x$$

$$65 - 61 = -14x - (-12x)$$

$$4 = -2x$$

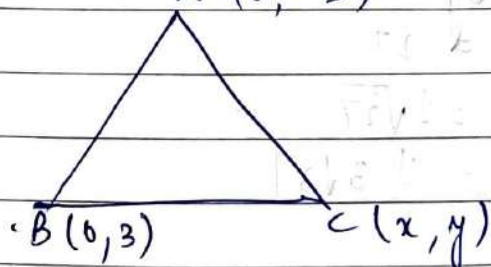
$$\boxed{x = -2}$$

$$\therefore PA = \sqrt{(2-7)^2 + (3+1)^2}$$

$$= \sqrt{(-5)^2 + (4)^2} = \sqrt{25 + 16}$$

$$= \boxed{\sqrt{41}}$$

A(0, -3)



Q.48 A(0, -3) & B(0, 3) are vertices of an equilateral Δ .
Let the coordinates of C be (x, y)

Ans, $AC = AB$

$$\therefore \sqrt{(x-0)^2 + (y+3)^2} = \sqrt{(0-0)^2 + (3+3)^2}$$

$$\cdot (x^2)^2 + (y+3)^2 = 0 + (6)^2$$

$$x^2 + y^2 + 9 + 6y = 0 + 36$$

$$x^2 + y^2 + 6y = 36 - 9$$

$$x^2 + y^2 + 6y = 27 \quad \text{--- (1)}$$

$BC = AB$

$$\sqrt{(x-0)^2 + (y-3)^2} = \sqrt{(0-0)^2 + (3+3)^2}$$

$$(x)^2 + (y-3)^2 = 0 + (6)^2$$

$$x^2 + y^2 + 9 - 6y = 0 + 36$$

$$x^2 + y^2 - 6y = 36 - 9$$

$$x^2 + y^2 - 6y = 27 \quad \text{--- (2)}$$

From (1) & (2).

$$x^2 + y^2 + 6y = x^2 + y^2 - 6y$$

$$6y + 6y = 0$$

$$12y = 0$$

$$\boxed{y = 0}$$

Putting value of y in (1).

$$x^2 + y^2 + 6y = 27$$

$$x^2 + 0 + 0 = 27$$

$$x = \pm \sqrt{27}$$

$$\boxed{x = \pm 3\sqrt{3}}$$

$\therefore$ The coordinates of C are $(3\sqrt{3}, 0)$ or $(-3\sqrt{3}, 0)$.

Q. 50 In $\triangle ABC$,
 $A(-2, 0)$, $B(2, 0)$, $C(0, 2)$

In $\triangle PQR$,
 $P(-4, 0)$, $Q(4, 0)$, $R(0, 4)$

$$AB = \sqrt{(2+2)^2 + (0-0)^2} = \sqrt{(4)^2 + (0)^2} \\ = \sqrt{16} = 4 \quad \text{--- ①}$$

$$AC = \sqrt{(-2+0)^2 + (0-2)^2} = \sqrt{(-2)^2 + (-2)^2} \\ = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2} \quad \text{--- ②}$$

$$BC = \sqrt{(2-0)^2 + (0-2)^2} = \sqrt{(2)^2 + (-2)^2} \\ = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2} \quad \text{--- ③}$$

$$PQ = \sqrt{(4+4)^2 + (0-0)^2} = \sqrt{(8)^2 + 0} \\ = \sqrt{64} = 8 \quad \text{--- ④}$$

$$PR = \sqrt{(-4-0)^2 + (0-4)^2} = \sqrt{(-4)^2 + (-4)^2} \\ = \sqrt{16+16} = \sqrt{32} = 4\sqrt{2} \quad \text{--- ⑤}$$

$$QR = \sqrt{(4+0)^2 + (0-4)^2} = \sqrt{(4)^2 + (-4)^2} \\ = \sqrt{16+16} = \sqrt{32} = 4\sqrt{2} \quad \text{--- ⑥}$$

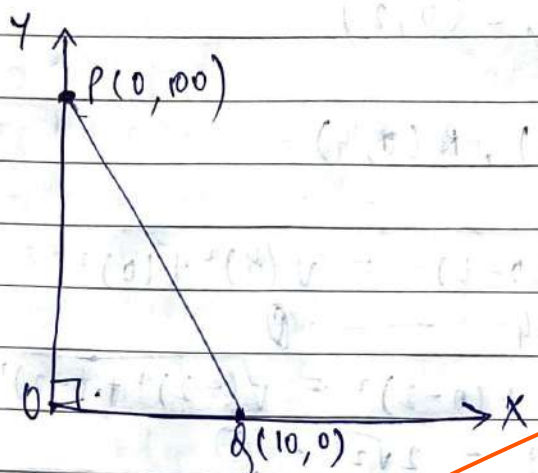
From ①, ②, ③, ⑤, ⑥,

$\frac{AB}{PQ} = \frac{4}{8} = \frac{1}{2}$	$\frac{BC}{QR} = \frac{2\sqrt{2}}{4\sqrt{2}} = \frac{1}{2}$
$\frac{AC}{PR} = \frac{2\sqrt{2}}{4\sqrt{2}} = \frac{1}{2}$	

$$\therefore \frac{AB}{PQ} = \frac{AC}{PR} = \frac{BC}{QR}$$

$\therefore \triangle ABC \sim \triangle PQR$ (SSS similarity)

Q. 53 P (0, 100) & Q (10, 0)



$\therefore$ The angle subtended by the line segment PQ at origin O is 90° .

Q. 54 A (5, -8), B (2, -9) and C (2, 1)

Let the center O be (x, y).

$\therefore OA = OB = OC$ (equal radii)

$$OA = OB$$

$$\begin{aligned} \sqrt{(x-5)^2 + (y+8)^2} &= \sqrt{(x-2)^2 + (y+9)^2} \\ x^2 + 25 - 10x + y^2 + 64 + 16y &= x^2 + 4 - 4x + y^2 + 81 + 18y \\ 89 - 10x + 16y &= 85 - 4x + 18y \\ -10x + 4x + 16y - 18y &= 85 - 89 \\ -6x - 2y &= -4 \\ 3x + y &= 2 \\ y &= 2 - 3x \quad \text{--- (1)} \end{aligned}$$

$$OB = OC$$

$$\begin{aligned} \sqrt{(x-2)^2 + (y+9)^2} &= \sqrt{(x-2)^2 + (y-1)^2} \\ x^2 + 4 - 4x + y^2 + 81 + 18y &= x^2 + 4 - 4x + y^2 + 1 - 2y \\ 18y + 2y &= 1 - 81 \\ 20y &= -80 \\ y &= -4 \end{aligned}$$

Putting value of y in ①

$$y = 2 - 3x$$

$$-y = 2 - 3x$$

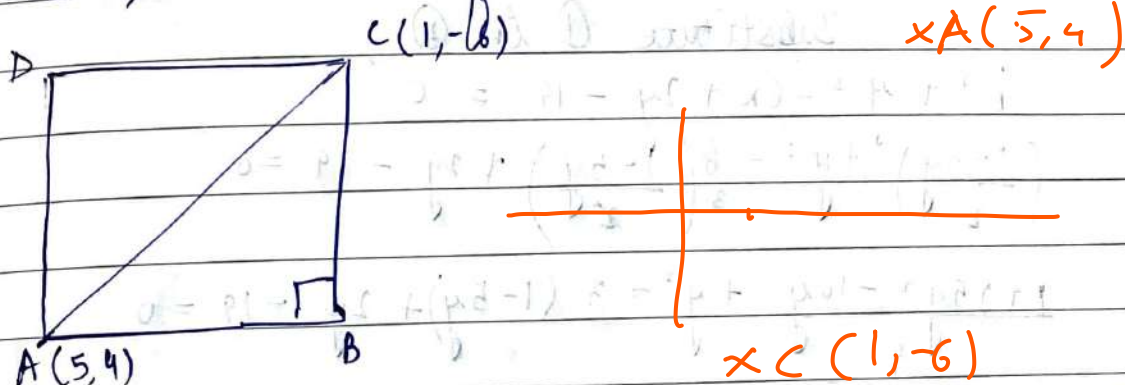
$$3x = 4 + 2$$

$$x = \frac{6}{3}$$

$$x$$

$$\boxed{x = 2} \quad \checkmark$$

Hence, the centre of circle is $(2, -4)$



ABCD is a square. A(5, 4) and C(1, -6) are the given points.

Let (x_1, y_1) & (x_2, y_2) be the coordinates of B & D.

$AB = BC$ (Sides of square)

$$\sqrt{(x-5)^2 + (y-4)^2} = \sqrt{(x-1)^2 + (y+6)^2}$$

$$x^2 + 25 - 10x + y^2 + 16 - 8y = x^2 + 1 - 2x + y^2 + 36 + 12y$$

$$-10x - 8y + 41 = -2x + 12y + 37$$

$$-10x + 2x - 8y - 12y = 37 - 41$$

$$-8x - 20y = -4$$

$$8x + 20y = 4$$

$$2x + 5y = 1$$

$$x = \frac{1 - 5y}{2} \quad \text{--- ①}$$

$\checkmark$

In right $\triangle ABC$,

$$AB^2 + BC^2 = AC^2 \quad (\text{Pythagoras theorem})$$

$$(x-5)^2 + (y-4)^2 + (x-1)^2 + (y+6)^2 = (5-1)^2 + (4+6)^2$$
$$x^2 + 25 - 10x + y^2 + 16 - 8y + x^2 + 1 - 2x + y^2 + 36 + 12y = 16 + 100$$
$$2x^2 - 12x + y^2 + 4y + 78 = 116$$

$$2x^2 - 12x + y^2 + 4y = 116 - 78$$

$$x^2 + y^2 - 6x + 2y = 19 \quad \text{--- ②}$$

Substitute ① in ②,

$$x^2 + y^2 - 6x + 2y - 19 = 0$$

$$\left(\frac{1-5y}{2}\right)^2 + y^2 - 3\left(\frac{1-5y}{x}\right) + 2y - 19 = 0$$

$$\frac{1+25y^2-10y}{4} + y^2 - 3(1-5y) + 2y - 19 = 0$$

$$\frac{1+25y^2-10y}{4} + y^2 - 3 + 15y + 2y - 19 = 0$$

$$\frac{1+25y^2-10y}{4} + y^2 - 12 + 60y + 8y - 76 = 0$$

$$29y^2 + 58y - 87 = 0 \times 4$$

$$y^2 + 2y - 3 = 0$$

$$y^2 + 3y - y - 3 = 0$$

$$y(y+3) - 1(y+3) = 0$$

$$(y+3)(y-1) = 0$$

$$\therefore \boxed{y = -3}, \boxed{y = 1}$$

$$\text{If } y = -3,$$
$$x = \frac{1-5(-3)}{2} = \frac{1+15}{2}$$

$$\boxed{x = 8}$$

$$\text{If } y = 1$$
$$x = \frac{1-5(1)}{2}$$

$$\boxed{x = -2}$$

$\therefore$ The required vertices are $(8, -3)$ and $(-2, 1)$