

NCERT SOLUTIONS

CLASS - 10th



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Exercise 12.1

Q1 The radii of two circles are 19 cm and 9 cm respectively. Find the radius of the circle which has circumference equal to the sum of the circumferences of the two circles.

Answer. Radius (r_1) of the 1st circle = 19cm

Radius (r_2) of the 1st circle = 9 cm

Let the radius of 3rd circle be r.

Circumference of 1st circle = $2\pi r_1 = 2\pi(19) = 38\pi$

Circumference of 2nd = $2\pi r_2 = 2\pi(9) = 18\pi$

Circumference of 3rd circle = $2r\pi$

That given ,

Circumference of 3rd circle = Circumference of 2nd circle = Circumference of 1st circle

$$2\pi r = 38\pi + 18\pi = 56\pi$$

$$r = \frac{56\pi}{2\pi} = 28$$

Therefore, the radius of the circle which has circumference equal to the sum of the circumference of the given two circles is 28 cm.

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Q2 The radii of two circles are 8 cm and 6 cm respectively. Find the radius of the circle having area equal to the sum of the areas of the two circles.

Answer. Radius (r_1) of the 1st circle = 8 cm

Radius (r_2) of the 1st circle = 6 cm

Let the radius of 3rd circle be r.

Area of 1st circle = $\pi r_1^2 = \pi(8)^2 = 64\pi$

Area of 2nd circle = $\pi r_2^2 = \pi(6)^2 = 36\pi$

That given,

Area of 3rd circle = Area of 2nd circle = Area of 1st circle

$$\pi r^2 = \pi r_1^2 + \pi r_2^2$$

$$\pi r^2 = 64\pi + 36\pi$$

$$\pi r^2 = 100\pi$$

$$r = \pm 10$$

However, the radius cannot be negative. Therefore, the radius of the circle having area equal to the sum of the areas of the two circles is 10 cm.

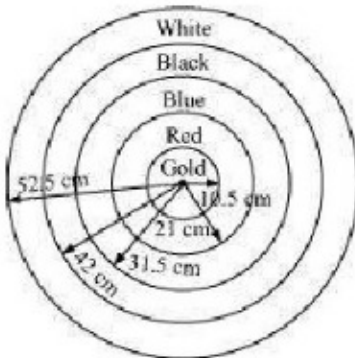
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Q3 Fig. depicts an archery target marked with its five scoring regions from the centre outwards as Gold, Red, Blue, Black and White. The diameter of the region representing Gold score is 21 cm and each of the other bands is 10.5 cm wide. Find the area of each of the five scoring regions.



Fig. 12.3

Answer.



Radius (r_1) of gold region (i.e. 1st circle) = $\frac{21}{2} = 10.5\text{cm}$

Given that each circle is 10.5 cm wider than the previous circle.

Therefore . radius r_2 of 2nd circle = $10.5 + 10.5 = 21\text{ cm}$

radius r_3 of 2nd circle = $21 + 10.5 = 31.5\text{ cm}$

radius r_4 of 2nd circle = $31.5 + 10.5 = 42\text{ cm}$

radius r_5 of 2nd circle = $42 + 10.5 = 52.5\text{ cm}$

Area of gold region = Area of 1st circle = $\pi r_1^2 = \pi(10.5)^2 = 346.5\text{cm}^2$

Area of red region = Area of 2nd circle - Area of 1st circle

$$= \pi r_2^2 - \pi r_1^2$$

$$= \pi(21)^2 - \pi(10.5)^2$$

$$= 441\pi - 110.25\pi = 330.75\pi$$

$$= 1039.5\text{cm}^2$$

Area of blue region = Area of 3rd circle - Area of 2nd circle

$$= \pi r_3^2 - \pi r_2^2$$

$$= \pi(31.5)^2 - \pi(21)^2$$

$$= 992.25\pi - 441\pi = 551.25\pi$$

$$= 1732.5\text{cm}^2$$

Area of black region = Area of 4th circle - Area of 3rd circle

$$= \pi r_3^2 - \pi r_2^2$$

$$= \pi(42)^2 - \pi(31.5)^2$$

$$= 1764\pi - 992.25\pi = 771.75\pi$$

$$= 2425.5\text{cm}^2$$

Area of white region = Area of 5th circle - Area of 4th circle

$$= \pi r_3^2 - \pi r_2^2$$

$$= \pi(52.5)^2 - \pi(42)^2$$

$$= 2756.25\pi - 1764\pi = 992.25\pi$$

$$= 3118.5\text{cm}^2$$

Therefore, areas of gold, red, blue, black, and white regions are 346.5cm^2 , 1039.5cm^2 , 1732.5cm^2 , 2425.5cm^2 , 3118.5cm^2 respectively.

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Q4 The wheels of a car are of diameter 80 cm each. How many complete revolutions does each wheel make in 10 minutes when the car is travelling at a speed of 66 km per hour?

Answer. Diameter of the wheel of the car = 80 cm

Radius (r) of the wheel of the car = 40 cm

Circumference of wheel = $2\pi r$

$$= 2\pi(40) = 80\pi\text{cm}$$

Speed of car = 66 km/hr

$$= \frac{66 \times 100000}{60} \text{cm/min}$$

$$= \frac{35000}{8} = 4375$$

Therefore, each wheel of the car will make 4375 revolutions.

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Q5 Tick the correct answer in the following and justify your choice : If the perimeter and the area of a circle are numerically equal, then the radius of the circle is

(A) 2 units

(B) π units

(C) 4 units

(D) 7 units

Answer. Let the radius of the circle be r.

Circumference of circle = $2\pi r$

Area of circle = πr^2

Given that, the circumference of the circle and the area of the circle equal.

This implies $2\pi r = \pi r^2$

$$r=2$$

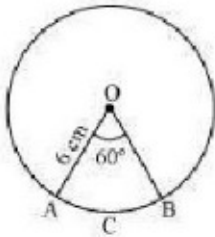
Therefore, the radius of the circle is 2 units.

Hence, the correct answer is A.

Exercise 12.2

Q1 Find the area of a sector of a circle with radius 6 cm if angle of the sector is 60° .

Answer.



$$\text{Area of sector of angle } \theta = \frac{\theta}{360^\circ} \times \pi r^2$$

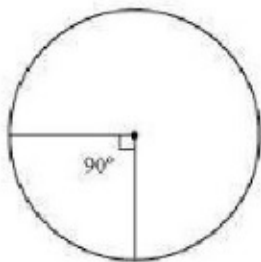
$$\begin{aligned} \text{Area of sector OACB} &= \frac{60^\circ}{360^\circ} \times \frac{22}{7} \times (6)^2 \\ &= \frac{1}{6} \times \frac{22}{7} \times 6 \times 6 = \frac{132}{7} \text{ cm}^2 \end{aligned}$$

Therefore , the area of sector of the circle making 60° at the centre of the circle is $\frac{132}{7} \text{ cm}^2$

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Q2 Find the area of a quadrant of a circle whose circumference is 22 cm.

Answer.



Let the radius of the circle be r .

Circumference = 22 cm

$$2\pi r = 22$$

$$r = \frac{22}{2\pi} = \frac{11}{\pi}$$

Quadrant of circle will subtend 90° at the centre of the circle .

$$\text{Area of such quadrant of the circle} = \frac{90^\circ}{360^\circ} \times \pi \times r^2$$

$$= \frac{1}{4} \times \pi \times \left(\frac{11}{\pi}\right)^2$$

$$= \frac{121}{4\pi} = \frac{121 \times 7}{4 \times 22}$$

$$= \frac{77}{8} \text{ cm}^2$$

Q3 The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 minutes.

Answer.



We know that in 1 hour (i. e. , 60 minutes), the minute hand rotates 360° .

In 5 minutes, minute hand will rotate $= \frac{360^\circ}{60} \times 5 = 30^\circ$

Therefore, the area swept by the minute hand in 5 minutes will be the area of a sector of 30° in a circle of 14 cm radius.

Area of sector of angle $\theta = \frac{\theta}{360^\circ} \times \pi r^2$

Area of sector of $30^\circ = \frac{30^\circ}{360^\circ} \times \frac{22}{7} \times 14 \times 14$

$$= \frac{22}{12} \times 2 \times 14$$

$$= \frac{11 \times 14}{3}$$

$$= \frac{154}{3} \text{ cm}^2$$

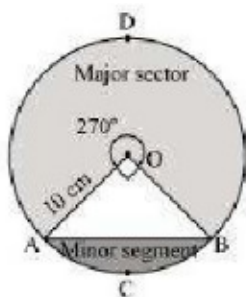
Therefore , the area swept by the minute hand in 5 minutes $= \frac{154}{3} \text{ cm}^2$

Q4 A chord of a circle of radius 10 cm subtends a right angle at the centre. Find the area of the corresponding :

(i) minor segment

(ii) major sector. (Use $\pi = 3.14$)

Answer.



Let AB be the chord of the subtending 90° angle at centre O of the circle.

Area of major sector OADB $= \left(\frac{360^\circ - 90^\circ}{360^\circ} \right) \times \pi r^2$

$$= \left(\frac{270^\circ}{360^\circ} \right) \pi r^2$$

$$= \frac{3}{4} \times 3.14 \times 10 \times 10$$

$$= 78.5 \text{ cm}^2$$

$$\text{Area of triangle OAB} = \frac{1}{2} \times \text{OA} \times \text{OB} = \frac{1}{2} \times 10 \times 10 \\ = 50 \text{ cm}^2$$

Area of minor segment ACB = Area of minor sector OACB

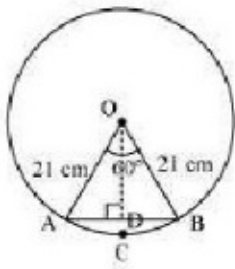
$$\text{Area of triangle OAB} = 78.5 - 50 = 28.5 \text{ cm}^2$$

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Q5 In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. Find:

- (i) the length of the arc
- (ii) area of the sector formed by the arc
- (iii) area of the segment formed by the corresponding chord .

Answer.



$$\text{Length of arc ACB} = \frac{60^\circ}{360^\circ} \times 2 \times \frac{22}{7} \times 21 \\ = \frac{1}{6} \times 2 \times 22 \times 3 \\ = 22 \text{ cm}$$

$$\text{Area of sector OACB} = \frac{60^\circ}{360^\circ} \times \pi r^2 \\ = \frac{1}{6} \times \frac{22}{7} \times 21 \times 21 \\ = 231 \text{ cm}^2$$

In $\triangle OAB$

$$\angle OAB = \angle OBA (\text{As } OA = OB)$$

$$\angle OAB + \angle AOB + \angle OBA = 180^\circ$$

$$2\angle OAB + 60^\circ = 180^\circ$$

$$\angle OAB = 60^\circ$$

Therefore , triangle OAB is an equilateral triangle.

$$\text{Area of triangle OAB} = \frac{\sqrt{3}}{4} \times (\text{Side})^2 \\ = \frac{\sqrt{3}}{4} \times (21)^2 = \frac{441\sqrt{3}}{4} \text{ cm}^2$$

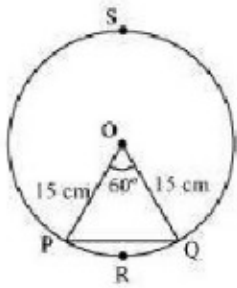
Area of segment ACB = Area of sector OACB - Area of triangle OAB

$$= \left(231 - \frac{441\sqrt{3}}{4} \right) \text{ cm}^2$$

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Q6 A chord of a circle of radius 15 cm subtends an angle of 60° at the centre. Find the areas of the corresponding minor and major segments of the circle. (Use $\pi = 3.14$ and $\sqrt{3} = 1.73$)

Answer.



Radius (r) of circle = 15 cm

$$\text{Area of sector OPRQ} = \frac{60^\circ}{360^\circ} \times \pi r^2$$

$$= \frac{1}{6} \times 3.14 \times (15)^2$$

$$= 117.75 \text{ cm}^2$$

In triangle OPQ ,

$$\angle OPQ = \angle OQP (\text{As } OP = OQ)$$

$$\angle OPQ + \angle OQP + \angle POQ = 180^\circ$$

$$2\angle OPQ = 120^\circ$$

$$\angle OPQ = 60^\circ$$

$\triangle OPQ$ is an equilateral triangle .

$$\text{Area of } \triangle OPQ = \frac{\sqrt{3}}{4} \times (\text{side})^2$$

$$= \frac{\sqrt{3}}{4} \times (15)^2 = \frac{225\sqrt{3}}{4} \text{ cm}^2$$

$$56.25\sqrt{3}$$

$$97.3125 \text{ cm}^2$$

Area of major segment PSQ = Area of circle — Area of segment PRQ

$$= \pi(15)^2 - 20.4375$$

$$= 3.14 \times 225 - 20.4375$$

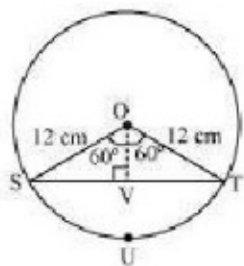
$$= 706.5 - 20.4375$$

$$= 686.0625 \text{ cm}^2$$

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Q7 A chord of a circle of radius 12 cm subtends an angle of 120° at the centre. Find the area of the corresponding segment of the circle. (Use $\pi = 3.14$ and $\sqrt{3} = 1.73$)

Answer.



Let us draw a perpendicular OV on chord ST. It will bisect the chord ST.

$$SV = VT$$

In triangle OVS ,

$$\frac{OV}{OS} = \cos 60^\circ$$

$$\frac{OV}{12} = \frac{1}{2}$$

$$OV = 6 \text{ cm}$$

$$\frac{SV}{SO} = \sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\frac{SV}{12} = \frac{\sqrt{3}}{2}$$

$$SV = 6\sqrt{3} \text{ cm}$$

$$ST = 2SV = 2 \times 6\sqrt{3} = 12\sqrt{3} \text{ cm}$$

$$\text{Area of angle OST} = \frac{1}{2} \times ST \times OV$$

$$= \frac{1}{2} \times 12\sqrt{3} \times 6$$

$$= 36\sqrt{3} = 36 \times 1.73 = 62.28 \text{ cm}^2$$

$$\text{Area of sector OSUT} = \frac{120^\circ}{360^\circ} \times \pi (12)^2$$

$$= \frac{1}{3} \times 3.14 \times 144 = 150.72 \text{ cm}^2$$

$$\text{Area of segment SUT} = \text{Area of sector OSUT} - \text{Area of triangle OST}$$

$$= 150.72 - 62.28$$

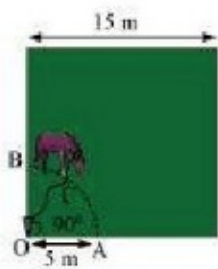
$$= 88.44 \text{ cm}^2$$

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Q8 A horse is tied to a peg at one corner of a square shaped grass field of side 15 m by means of a 5 m long rope (see Fig. 12.11). Find (i) the area of that part of the field in which the horse can graze. (ii) the increase in the grazing area if the rope were 10 m long instead of 5 m. (Use $\pi = 3.14$)



Answer.



From the figure, it be Observed that the horse graze a sector Of 90° in a circle of 5 m radius.

Area that can be grazed by horse = Area of sector OACB

$$= \frac{90^\circ}{360^\circ} \pi r^2$$

$$= \frac{1}{4} \times 3.14 \times (5)^2$$

$$= 19.625 \text{ m}^2$$

Area that can be grazed by the horse when length of rope is 10 m long

$$= \frac{90^\circ}{360^\circ} \times \pi \times (10)^2$$

$$= \frac{1}{4} \times 3.14 \times 100$$

$$= 78.5 \text{ m}^2$$

$$\text{Increase in grazing area} = (78.5 - 19.625) \text{ m}^2$$

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Q9 A brooch is made with silver wire in the form of a circle with diameter 35 mm. The wire is also used in making 5 diameters which divide the circle into 10 equal sectors as shown in Fig. 12.12.

Find :

- (i) the total length of the silver wire required.
- (ii) the area of each sector of the brooch.



Fig. 12.12

Answer. Total length of wire required will be the length of 5 diameters and the circumference of the brooch.

$$\text{Radius of circle} = \frac{35}{2} \text{ mm}$$

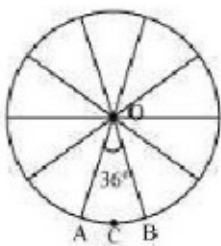
$$\text{Circumference of brooch} = 2\pi r$$

$$= 2 \times \frac{22}{7} \times \left(\frac{35}{2} \right)$$

$$= 110 \text{ mm}$$

$$\text{Length of wire required} = 110 + 5 \times 35 = 285 \text{ mm}$$

It can be observed from the figure that each of 10 sectors of the circle is subtending 36° at the centre of the circle.



$$\text{Therefore, area of each sector} = \frac{36^\circ}{360^\circ} \times \pi r^2$$

$$= \frac{1}{10} \times \frac{22}{7} \times \left(\frac{35}{2} \right)^2 = \frac{1}{10} \times \frac{22}{7} \times \frac{1225}{4}$$

$$= \frac{385}{4} \text{ mm}^2$$

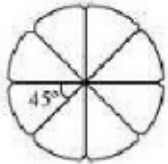
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Q10 An umbrella has 8 ribs which are equally spaced (see Fig. 12.13). Assuming umbrella to be a flat circle of radius 45 cm, find the area between the two consecutive ribs of the umbrella.



Fig. 12.13

Answer. There 8 ribs in umbrella. The between two consecutive ribs is subtending $\frac{360^\circ}{8} = 45^\circ$ at the centre of the assumed flat circle.

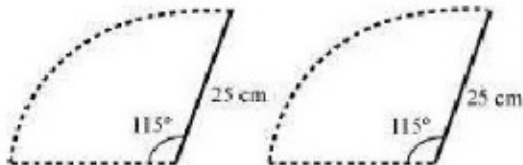


$$\begin{aligned} \text{Area between two consecutive ribs of circle} &= \frac{45^\circ}{360^\circ} \times \pi r^2 \\ &= \frac{1}{8} \times \frac{22}{7} \times (45)^2 \\ &= \frac{11}{28} \times 2025 = \frac{22275}{28} \text{ cm}^2 \end{aligned}$$

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Q11 A car has two wipers which do not overlap. Each wiper has a blade of length 25 cm sweeping through an angle of 115° . Find the total area cleaned at each sweep of the blades.

Answer.



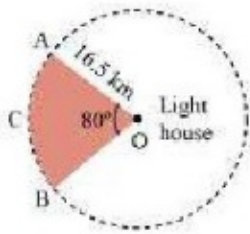
It can be observed from the figure that each blade of wiper will sweep an area of a sector of 115° in a circle of 25 cm radius.

$$\begin{aligned} \text{Area of such sector} &= \frac{115^\circ}{360^\circ} \times \pi \times (25)^2 \\ &= \frac{23}{72} \times \frac{22}{7} \times 25 \times 25 \\ &= \frac{158125}{252} \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \text{Area swept by 2 blades} &= 2 \times \frac{158125}{252} \\ &= \frac{158125}{126} \text{ cm}^2 \end{aligned}$$

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Q12 To warn ships for underwater rocks, a lighthouse spreads a red coloured light over a sector of angle 80° to a distance of 16.5 km. Find the area of the sea over which the ships are warned. (Use $\pi = 3.14$)



It can be observed from the figure that the lighthouse spreads light across a sector of 80° in a circle of 16.5 km radius.

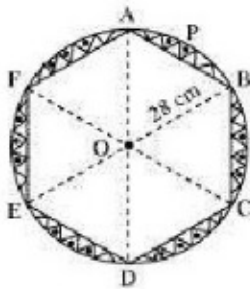
$$\begin{aligned}\text{Area of sector OACB} &= \frac{80^\circ}{360^\circ} \times \pi r^2 \\ &= \frac{2}{9} \times 3.14 \times 16.5 \times 16.5 \\ &= 189.97 \text{ km}^2\end{aligned}$$

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Q13 A round table cover has six equal designs as shown in Fig. 12.14. If the radius of the cover is 28 cm, find the cost of making the designs at the rate of ₹ 0.35 per cm^2 . (Use $\pi = 1.7$)



Answer.



It can be observed that these designs are segments of the circle.

Consider segment APE, Chord AE is a side of the hexagon. Each chord will subtend $\frac{360^\circ}{6} = 60^\circ$ at the centre of circle.

In triangle OAB,

$$\angle OAB = \angle OBA \text{ (As } OA = OB\text{)}$$

$$\angle AOB = 60^\circ$$

$$\angle OAB + \angle OBA + \angle AOB = 180^\circ$$

$$2\angle OAB = 180^\circ - 60^\circ = 120^\circ$$

$$\angle OAB = 60^\circ$$

Therefore, triangle OAB is an equilateral triangle.

$$\text{Area of triangle OAB} = \frac{\sqrt{3}}{4} \times (\text{side})^2$$

$$= \frac{\sqrt{3}}{4} \times (28)^2 = 196\sqrt{3} = 196 \times 1.7 = 333.2 \text{ cm}^2$$

$$\text{Area of sector OAPB} = \frac{60^\circ}{360^\circ} \times \pi r^2$$

$$\frac{1}{6} \times \frac{22}{7} \times 28 \times 28$$

$$\frac{1232}{3} \text{ cm}^2$$

Area of segment APB = Area of sector OAPB - Area of triangle OAB

$$= \left(\frac{1232}{3} - 333.2 \right) \text{ cm}^2$$

$$\text{Therefore, area of designs} = 6 \times \left(\frac{1232}{3} - 333.2 \right) \text{ cm}^2$$

$$= (2464 - 1999.2) \text{ cm}^2$$

$$= 464.8 \text{ cm}^2$$

Cost of making 1 cm² designs = Rs 0.35

Cost of making 464.76 cm² designs = Rs 162.68

Therefore, the cost of making such designs is Rs 162.68,

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Q14 Tick the correct answer in the following : Area of a sector of angle p (in degrees) of a circle with radius R is

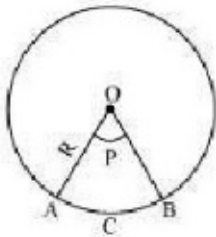
$$\frac{p}{180} \times 2\pi R$$

$$\frac{p}{180} \times \pi R^2$$

$$\frac{p}{360} \times 2\pi R$$

$$\frac{p}{720} \times 2\pi R^2$$

Answer.



We know that area of sector of angle $\theta = \frac{\theta}{360^\circ} \times \pi R^2$

$$\text{Area of sector of angle P} = \frac{p}{360^\circ} \left(\pi R^2 \right)$$

$$= \left(\frac{p}{720^\circ} \right) \left(2\pi R^2 \right)$$

Hence, (D) is the correct answer.

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Exercise 12.3

Q1 Find the area of the shaded region in Fig., if $PQ = 24$ cm, $PR = 7$ cm and O is the centre of the circle.

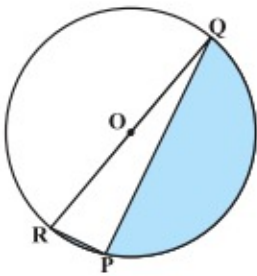


Fig. 12.19

Answer. It can be observed that RQ is the diameter of the circle. Therefore, $\angle RPQ$ will be 90°

By applying Pythagoras theorem in triangle PQR ,

$$RP^2 + PQ^2 = RQ^2$$

$$(7)^2 + (24)^2 = RQ^2$$

$$RQ = \sqrt{625} = 25$$

$$\text{Radius of circle, } OR = \frac{RQ}{2} = \frac{25}{2}$$

Since RQ is the diameter of the circle, it divides the circle in two equal parts.

$$\text{Area of the semi-circle } RPQOR = \frac{1}{2}\pi r^2$$

$$= \frac{1}{2}\pi \left(\frac{25}{2}\right)^2$$

$$= \frac{1}{2} \times \frac{22}{7} \times \frac{625}{4}$$

$$= \frac{6875}{28} \text{ cm}^2$$

$$\text{Area of triangle } PQR = \frac{1}{2} \times PQ \times PR$$

$$= \frac{1}{2} \times 24 \times 7$$

$$= 84 \text{ cm}^2$$

$$\text{Area of shaded region} = \text{Area of semi-circle } RPQOR - \text{Area of } \triangle PQR$$

$$= \frac{6875}{28} - 84$$

$$= \frac{6875 - 2352}{28}$$

$$= \frac{4523}{28} \text{ cm}^2$$

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Q2 Find the area of the shaded region in Fig., if radii of the two concentric circles with centre O are 7 cm and 14 cm respectively and $\angle AOC = 40^\circ$

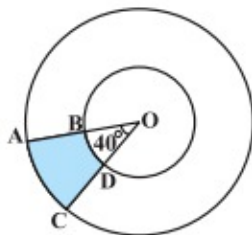
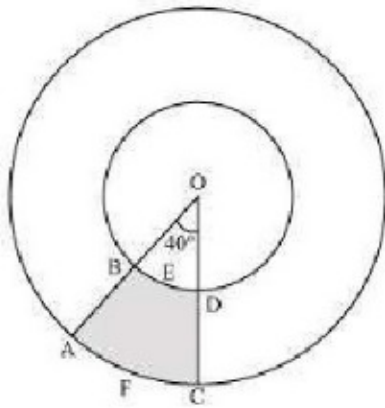


Fig. 12.20

Answer.



Radius of inner circle = 7 cm

Radius of outer circle = 14 cm

Area of shaded region = Area of sector OAF - Area of sector OBE

$$\begin{aligned}
 &= \frac{40^\circ}{360^\circ} \times \pi(14)^2 - \frac{40^\circ}{360^\circ} \times \pi(7)^2 \\
 &= \frac{1}{9} \times \frac{22}{7} \times 14 \times 14 - \frac{1}{9} \times \frac{22}{7} \times 7 \times 7 \\
 &= \frac{616}{9} - \frac{154}{9} = \frac{462}{9} \\
 &= \frac{154}{3} \text{ cm}^2
 \end{aligned}$$

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Q3 Find the area of the shaded region in Fig. , if ABCD is a square of side 14 cm and APD and BPC are semicircles.

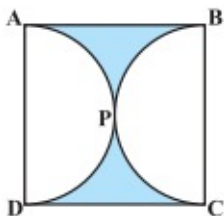
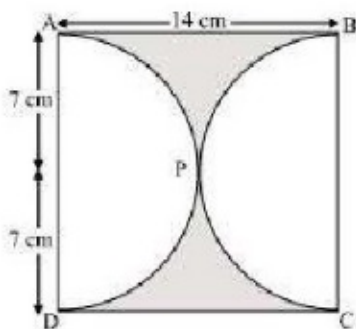


Fig. 12.21

Answer. It can be observed from the figure that the radius of each semi-circle is 7 cm.



$$\begin{aligned}
 \text{Area of each semi-circle} &= \frac{1}{2} \pi r^2 \\
 &= \frac{1}{2} \times \frac{22}{7} \times (7)^2 \\
 &= 77 \text{ cm}^2
 \end{aligned}$$

$$\text{Area of square ABCD} + (\text{Side})^2 = (14)^2$$

$$= 196 \text{ cm}^2$$

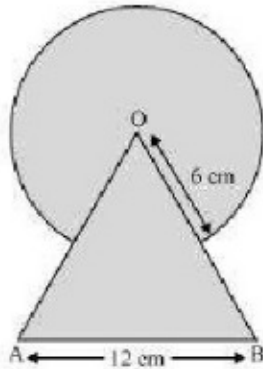
Area of the shaded region

$$= \text{Area of square ABCD} - \text{Area of semi-circle APD} - \text{Area of semi-circle BPC}$$

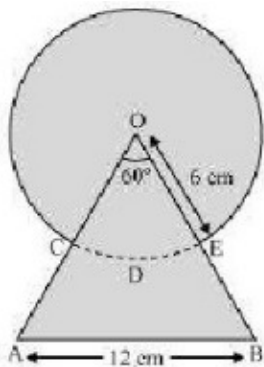
$$= 196 - 77 - 77 = 196 - 154 = 42 \text{ cm}^2$$

Page : 235 , Block Name : Exercise 12.3

Q4 Find the area of the shaded region in Fig. , where a circular arc of radius 6 cm has been drawn with vertex O of an equilateral triangle OAB of side 12 cm as centre.



Answer.



$$\text{Area of sector OCDE} = \frac{60^\circ}{360^\circ} \pi r^2$$

$$= \frac{1}{6} \times \frac{22}{7} \times 6 \times 6$$

$$= \frac{132}{7} \text{ cm}^2$$

$$\text{Area of triangle OAB} = \frac{\sqrt{3}}{4} (12)^2 = \frac{\sqrt{3} \times 12 \times 12}{4} = 36\sqrt{3} \text{ cm}^2$$

$$\text{Area of circle} = \pi r^2 = \frac{22}{7} \times 6 \times 6 = \frac{792}{7} \text{ cm}^2$$

$$\text{Area of shaded region} = \text{Area of triangle OAB} + \text{Area of circle} - \text{Area of sector OCDE}$$

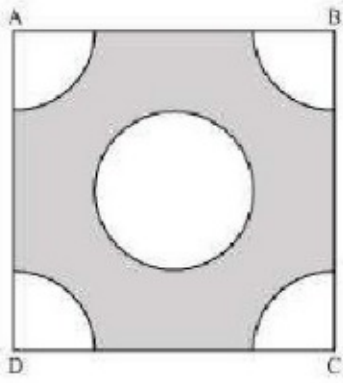
$$= 36\sqrt{3} + \frac{792}{7} - \frac{132}{7}$$

$$\left(36\sqrt{3} + \frac{660}{7} \right) \text{ cm}^2$$

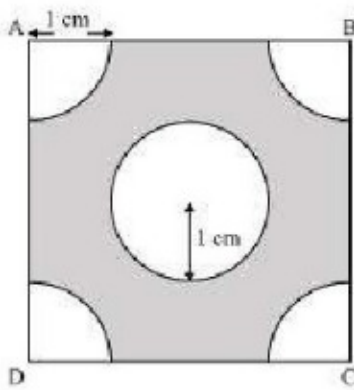
Page : 235 , Block Name : Exercise 12.3

Q5 From each corner of a square of side 4 cm a quadrant of a circle of radius 1 cm is cut and also a

circle of diameter 2 cm is cut as shown in Fig. . Find the area of the remaining portion of the square.



Answer.



Each quadrant is a sector of 90° in a circle of 1 cm radius.

$$\text{Area of each quadrant} = \frac{90^\circ}{360^\circ} \pi r^2$$

$$= \frac{1}{4} \times \frac{22}{7} \times (1)^2 = \frac{22}{28} \text{ cm}^2$$

$$\text{Area of square} = (\text{Side})^2 = (4)^2 = 16 \text{ cm}^2$$

$$= \frac{22}{7} \text{ cm}^2$$

Area of the shaded region = Area of square - Area of circle - 4 x Area of quadrant

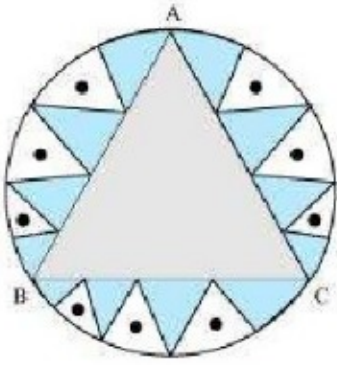
$$= 16 - \frac{22}{7} - 4 \times \frac{22}{28}$$

$$= 16 - \frac{22}{7} - \frac{22}{7} = 16 - \frac{44}{7}$$

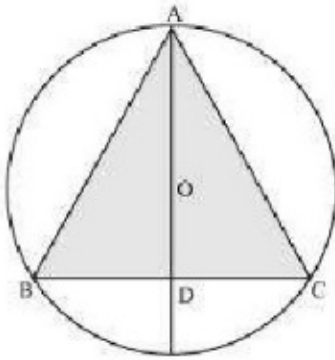
$$= \frac{112-44}{7} = \frac{68}{7} \text{ cm}^2$$

Page : 235 , Block Name : Exercise 12.3

Q6 In a circular table cover of radius 32 cm, a design is formed leaving an equilateral triangle ABC in the middle as shown in Fig. Find the area of the design.



Answer.



Answer. Radius (r) of circle 32 cm

AD is the median of triangle ABC.

$$AO = \frac{2}{3} AD = 32$$

$$AD = 48 \text{ cm}$$

In triangle ABD,

$$AB^2 = AD^2 + BD^2$$

$$AB^2 = (48)^2 + \left(\frac{AB}{2}\right)^2$$

$$\frac{3AB^2}{4} = (48)^2$$

$$AB = \frac{48 \times 2}{\sqrt{3}} = \frac{96}{\sqrt{3}}$$

$$= 32\sqrt{3} \text{ cm}$$

Area of equilateral a triangle ,

$$\frac{\sqrt{3}}{4} \times 32 \times 32 \times 3 = 96 \times 8 \times \sqrt{3}$$

$$768\sqrt{3} \text{ cm}^2$$

Area of circle = πr^2

$$= \frac{22}{7} \times (32)^2$$

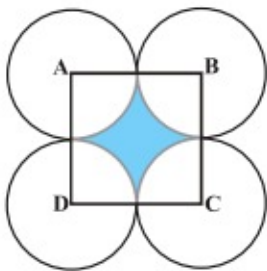
$$= \frac{22}{7} \times 1024$$

$$= \frac{22528}{7} \text{ cm}^2$$

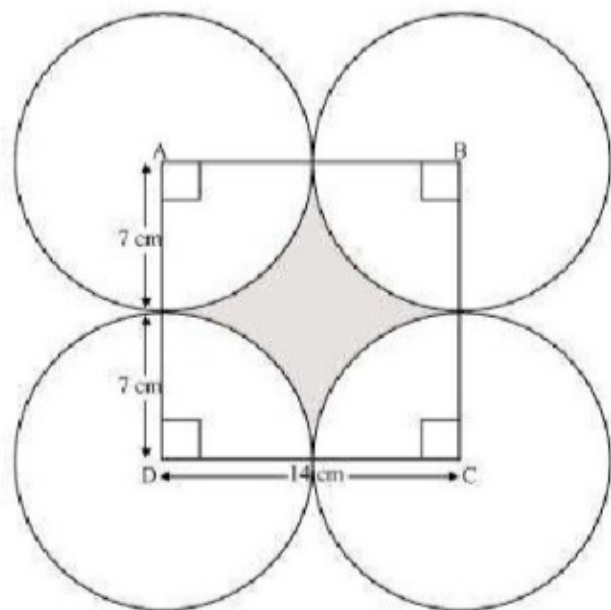
Area of design = area of circle - Area of triangle ABC

$$= \left(\frac{22528}{7} - 768\sqrt{3} \right) \text{ cm}^2$$

Q7 In Fig., ABCD is a square of side 14 cm. With centres A, B, C and D, four circles are drawn such that each circle touch externally two of the remaining three circles. Find the area of the shaded region.



Answer.



Area of each of the 4 sectors is equal to each and is a sector of 90° in a circle of 7 cm radius.

$$\text{Area of each sector} = \frac{90^\circ}{360^\circ} \times \pi(7)^2$$

$$= \frac{1}{4} \times \frac{22}{7} \times 7 \times 7$$

$$= \frac{77}{2} \text{ cm}^2$$

$$\text{Area of Square of ABCD} = (\text{Side})^2 = (14)^2 = 196 \text{ cm}^2$$

$$\text{Area Of shaded portion} = \text{Area Of square ABCD} - 4 \times \text{Area Of each sector}$$

$$= 196 - 4 \times \frac{77}{2} = 196 - 154$$

$$= 42 \text{ cm}^2$$

Therefore, the area of shaded portion is 42 cm^2 .

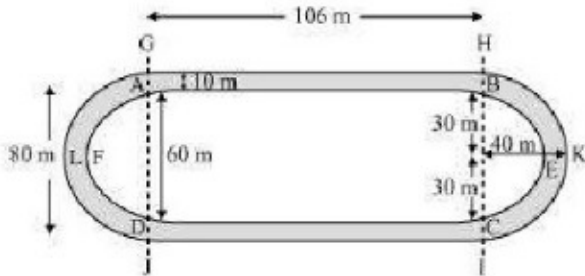
Page : 236, Block Name : Exercise 12.3

Q8 Fig. depicts a racing track whose left and right ends are semicircular. The distance between the two inner parallel line segments is 60 m and they are each 106 m long. If the track is 10 m wide, find :

- (i) the distance around the track along its inner edge
- (ii) the area of the track.



Answer.



Distance around the track along its inner edge = AB + arc BEC + CD + arc DFA

$$\begin{aligned}
 &= 106 + \frac{1}{2} \times 2\pi r + 106 + \frac{1}{2} \times 2\pi r \\
 &= 212 + \frac{1}{2} \times 2 \times \frac{22}{7} \times 30 + \frac{1}{2} \times 2 \times \frac{22}{7} \times 30 \\
 &= 212 + 2 \times \frac{22}{7} \times 30 \\
 &= 212 + \frac{1320}{7} \\
 &= \frac{1484 + 1320}{7} = \frac{2804}{7} \text{ m}
 \end{aligned}$$

Area of the track = (Area of GHIJ - Area of ABCD) + (Area of semi-circle HKI - Area of semi-circle BEC) + (Area of semi-circle GLJ - Area of semi-circle AFD)

$$\begin{aligned}
 &= 106 \times 80 - 106 \times 60 + \frac{1}{2} \times \frac{22}{7} \times (40)^2 - \frac{1}{2} \times \frac{22}{7} \times (30)^2 + \frac{1}{2} \times \frac{22}{7} \times (40)^2 - \frac{1}{2} \times \frac{22}{7} \times (30)^2 \\
 &= 106(80 - 60) + \frac{22}{7} \times (40)^2 - \frac{22}{7} \times (30)^2 \\
 &= 106(20) + \frac{22}{7} [(40)^2 - (30)^2] \\
 &= 2120 + \frac{22}{7} (40 - 30)(40 + 30) \\
 &= 2120 + \left(\frac{22}{7}\right) (10)(70) \\
 &= 2120 + 2200 \\
 &= 4320 \text{ m}^2
 \end{aligned}$$

Therefore, the area of shaded region is 4320 m^2

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Q9 In Fig., AB and CD are two diameters of a circle (with centre O) perpendicular to each other and OD is the diameter of the smaller circle. If OA = 7 cm, find the area of the shaded region.

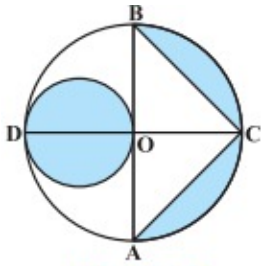
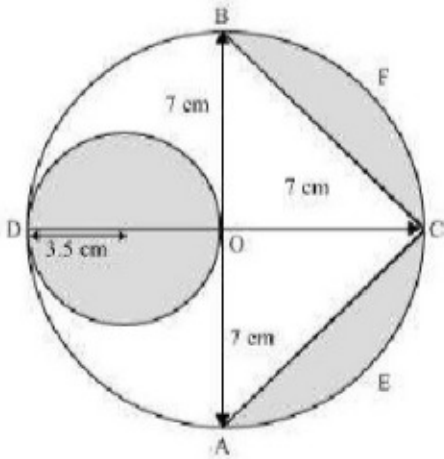


Fig. 12.27

Answer.



Radius r_1 of larger circle = 7cm

Radius r_2 of smaller circle = $\frac{7}{2}$ cm

Area of smaller circle = πr_2^2

$$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$$

$$= \frac{77}{2} \text{ cm}^2$$

Area of semi-circle AECFB of larger circle = $\frac{1}{2} \pi r_1^2$

$$= \frac{1}{2} \times \frac{22}{7} \times (7)^2$$

$$= 77 \text{ cm}^2$$

Area of triangle ABC = $\frac{1}{2} \times AB \times OC$

$$= \frac{1}{2} \times 14 \times 7 = 49 \text{ cm}^2$$

Area of the shaded region = Area of smaller circle + Area of semi-circle AECFB

- Area of triangle ABC

$$= \frac{77}{2} + 77 - 49$$

$$= 28 + \frac{77}{2} = 28 + 38.5 = 66.5 \text{ cm}^2$$

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Q10 The area of an equilateral triangle ABC is 17320.5 cm^2 . With each vertex of the triangle as centre, a circle is drawn with radius equal to half the length of the side of the triangle (see Fig.). Find the area of the shaded region.

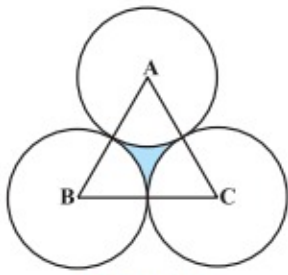


Fig. 12.28

Answer. Let the side of the equilateral triangle be a ,

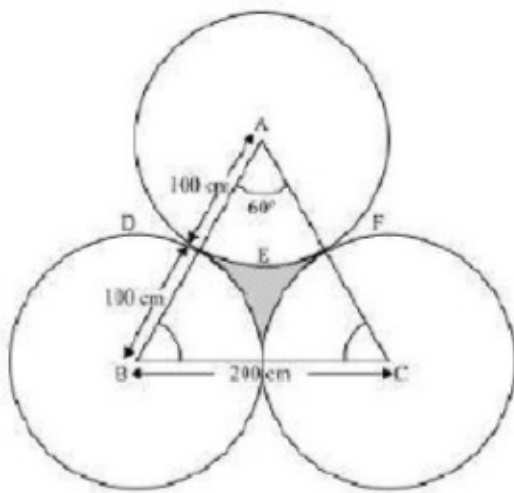
Area of equilateral triangle = 17320.5 cm^2

$$\frac{\sqrt{3}}{4}(a)^2 = 17320.5$$

$$\frac{1.73205}{4}a^2 = 17320.5$$

$$a^2 = 4 \times 10000$$

$$a = 200 \text{ cm}$$



Each sector of measure 60°

$$\text{Area of sector ADEF} = \frac{60^\circ}{360^\circ} \times \pi \times r^2$$

$$= \frac{1}{6} \times \pi \times (100)^2$$

$$= \frac{3.14 \times 10000}{6}$$

$$= \frac{15700}{3} \text{ cm}^2$$

Area Of shaded region Area Of equilateral triangle - 3 x Area Of each sector

$$17320.5 - 3 \times \frac{15700}{3}$$

$$17320.5 - 15700 = 1620.5 \text{ cm}^2$$

Page : 236 , Block Name : Exercise 12.3

Q11 On a square handkerchief, nine circular designs each of radius 7 cm are made (see Fig). Find the area of the remaining portion of the handskter chief.

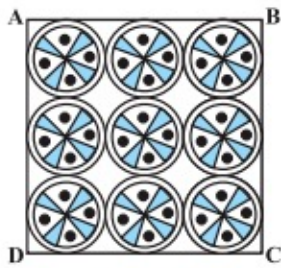
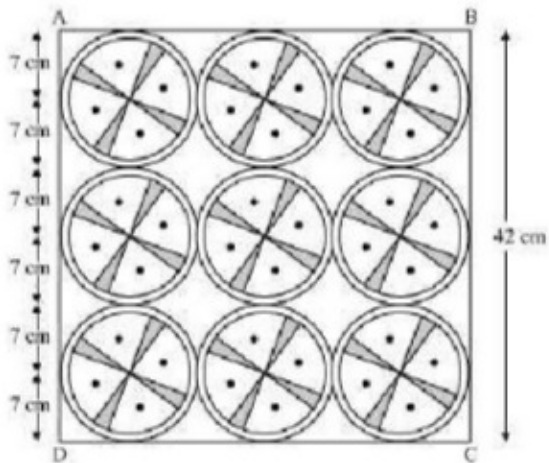


Fig. 12.29

Answer.



From the figure, it can be observed that the side of the square is 42 cm.

$$\text{Area of square} = (\text{Side})^2 = (42)^2 = 1764 \text{ cm}^2$$

$$\text{Area of each circle} = 9 \times 154 = 1386 \text{ cm}^2$$

$$\text{Area of the remaining portion of the} = 1764 - 1386 = 378 \text{ cm}^2$$

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Q12 In Fig. OACB is a quadrant of a circle with centre O and radius 3.5 cm. If OD = 2 cm, find the area of the

- (i) quadrant OACB,
- (ii) shaded region.

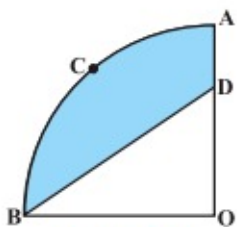
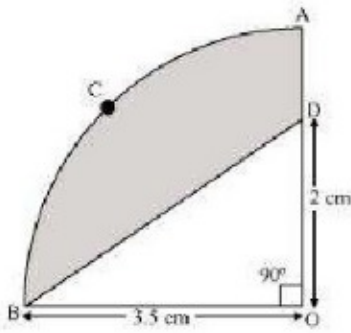


Fig. 12.30

Answer.



(i) Since OACB is a quadrant, it will subtend 90° angle at O.

$$\begin{aligned}\text{Area of quadrant OACB} &= \frac{90^\circ}{360^\circ} \times \pi r^2 \\ &= \frac{1}{4} \times \frac{22}{7} \times (3.5)^2 = \frac{1}{4} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \\ &= \frac{11 \times 7 \times 7}{2 \times 7 \times 2 \times 2} = \frac{77}{8} \text{ cm}^2\end{aligned}$$

(ii) area of triangle = $\frac{1}{2} \times OB \times OD$

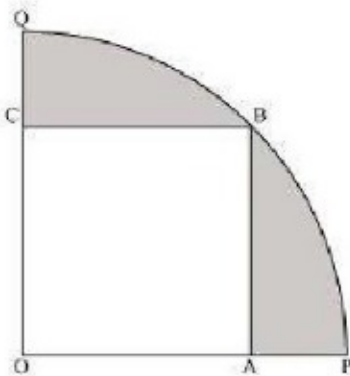
$$\begin{aligned}&\frac{1}{2} \times 3.5 \times 2 \\ &= \frac{1}{2} \times \frac{7}{2} \times 2 \\ &= \frac{7}{2} \text{ cm}^2\end{aligned}$$

Area of the shaded region = Area of OACE - Area of triangle OBO

$$\begin{aligned}&= \frac{77}{8} - \frac{7}{2} \\ &= \frac{77-28}{8} \\ &= \frac{49}{8} \text{ cm}^2\end{aligned}$$

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Q13 In Fig. a square OABC is inscribed in a quadrant OPBQ. If OA = 20 cm, find the area of the shaded region.



Answer. In $\triangle OAB$,

$$OB^2 = OA^2 + AB^2$$

$$= (20)^2 + (20)^2$$

$$OB = 20\sqrt{2}$$

Radius of circle = $20\sqrt{2}$ cm

$$\text{Area of quadrant OPBQ} = \frac{90^\circ}{360^\circ} \times 3.14 \times (20\sqrt{2})^2$$

$$\frac{1}{4} \times 3.14 \times 800$$

$$628\text{cm}^2$$

$$\text{Area of OABC} = (\text{Side})^2 = (20)^2 = 400\text{cm}^2$$

$$\text{Area of shaded region} = \text{Area of quadrant OPBQ} - \text{Area of OABC}$$

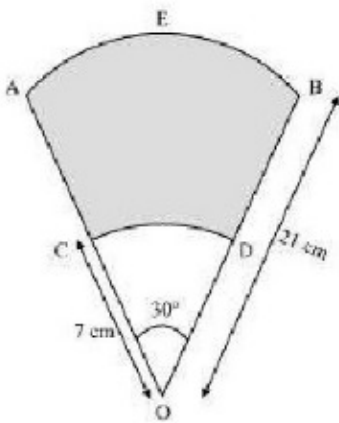
$$= (628 - 400)\text{cm}^2$$

$$= 228\text{cm}^2$$

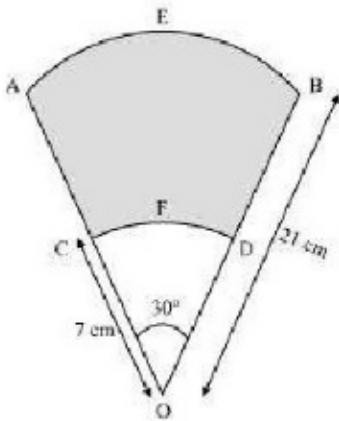
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Q14 AB and CD are respectively arcs of two concentric circles of radii 21 cm and 7 cm and centre O (see Fig.). If angle AOB = 30°

find the area of the shaded region.



Answer.



$$\text{Area of the shaded region} = \text{Area of sector OAEB} - \text{Area of sector OCFD}$$

$$= \frac{30^\circ}{360^\circ} \times \pi \times (21)^2 - \frac{30^\circ}{360^\circ} \times \pi \times (7)^2$$

$$= \frac{1}{12} \times \pi [(21)^2 - (7)^2]$$

$$= \frac{1}{12} \times \frac{22}{7} \times [(21 - 7)(21 + 7)]$$

$$= \frac{22 \times 14 \times 28}{12 \times 7}$$

$$= \frac{308}{3} \text{cm}^2$$

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Q15 In Fig. ABC is a quadrant of a circle of radius 14 cm and a semicircle is drawn with BC as diameter. Find the area of the shaded region.

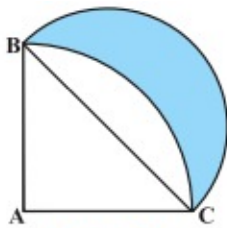
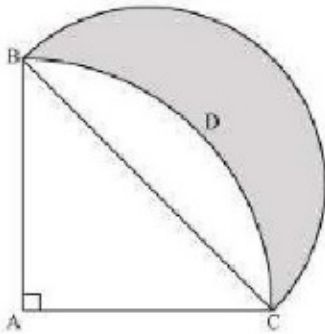


Fig. 12.33

Answer.



As ABC is a quadrant of the circle, angle BAC will be of measure 90° .
In triangle ABC,

$$\begin{aligned} BC^2 &= AC^2 + AB^2 \\ &= (14)^2 + (14)^2 \\ &= BC = 14\sqrt{2} \end{aligned}$$

$$\text{Radius } r_1 \text{ Of semi-circle drawn on } BC = \frac{14\sqrt{2}}{2} = 7\sqrt{2}\text{cm}$$

$$\begin{aligned} \text{Area of triangle ABC} &= \frac{1}{2} \times AB \times AC \\ &= \frac{1}{2} \times 14 \times 14 \\ &= 98\text{cm}^2 \end{aligned}$$

$$\begin{aligned} \text{Area of Sector ABCD} &= \frac{90^\circ}{360^\circ} \times \pi^2 \\ &= \frac{1}{4} \times \frac{22}{7} \times 14 \times 14 \\ &= 154\text{cm}^2 \end{aligned}$$

$$\begin{aligned} \text{Area of semi-circle drawn on BC} &= \frac{1}{2} \times \pi \times r_1^2 = \frac{1}{2} \times \frac{22}{7} \times (7\sqrt{2})^2 \\ \frac{1}{2} \times \frac{22}{7} \times 98 &= 154\text{cm}^2 \end{aligned}$$

$$\begin{aligned} \text{Area of shaded region} &= \text{Area of semi-circle} - (\text{Area of sector ABCD} - \text{Area of triangle ABC}) = 154 - \\ &= 98\text{cm}^2 \end{aligned}$$

Q16 Calculate the area of the designed region in Fig. 12.34 common between the two quadrants of circles of radius 8 cm each.

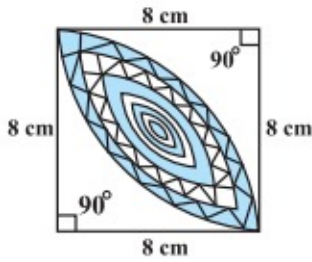
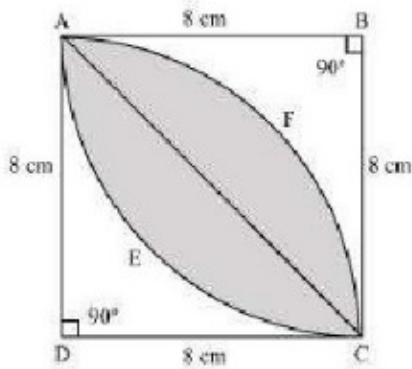


Fig. 12.34

Answer.



The designed is the common region between two sectors BAEC and DAFC.

$$\begin{aligned}
 \text{Area of sector BAEC} &= \frac{90^\circ}{360^\circ} \times \frac{22}{7} \times (8)^2 \\
 &= \frac{1}{4} \times \frac{22}{7} \times 64 \\
 &= \frac{22 \times 16}{7} \\
 &= \frac{352}{7} \text{ cm}^2
 \end{aligned}$$

$$\begin{aligned}
 \text{Area of triangle BAC} &= \frac{1}{2} \times BA \times BC \\
 &= \frac{1}{2} \times 8 \times 8 = 32 \text{ cm}^2
 \end{aligned}$$

Area of the designed portion = 2 x (Area of segment AEC)

= 2 x (Area of sector BAEC - Area of triangle BAC)

$$= 2 \times \left(\frac{352}{7} - 32 \right)$$

$$= 2 \left(\frac{352 - 224}{7} \right)$$

$$= \frac{2 \times 128}{7}$$

$$= \frac{256}{7} \text{ cm}^2$$