NCERT SOLUTIONS

CLASS - 10th



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Class : 10th
Subject : Maths
Chapter : 6
Chapter Name : Triangles

Exercise 6.1

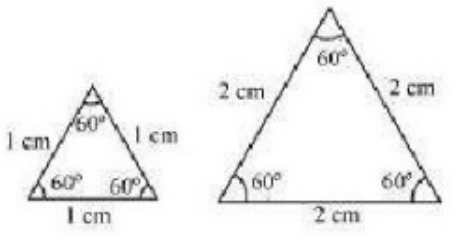
Q1 Fill in the blanks using the correct word given in brackets :
(i) All circles are _____. (congruent, similar)
(ii) All squares are_____ . (similar, congruent)
(iii) All_____ triangles are similar. (isosceles, equilateral)
(iv) Two polygons of the same number of sides are similar, if (a) their corresponding angles are _____ and (b) their corresponding sides are_____ .(equal, proportional)

Answer. (i) Similar
(ii) Similar
(iii) Equilateral
(iv) (a) Equal
(b) Proportional

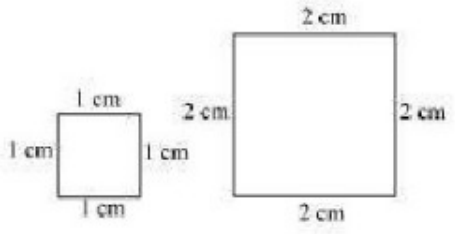
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Q2 Give two different examples of pair of
(i) similar figures.
(ii) non-similar figures.

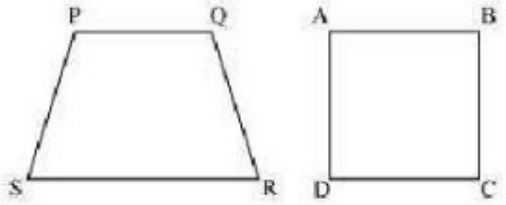
Answer. (i) Two equilateral triangles with sides 1 cm and 2 cm.



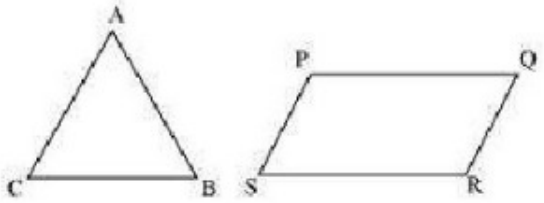
Two squares with sides 1 cm and 2 cm.



(ii) Trapezium and square

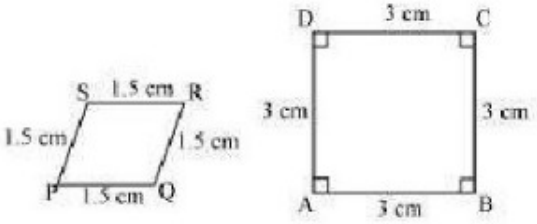


Triangle and parallelogram



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Q3 State whether the following quadrilaterals are similar or not:



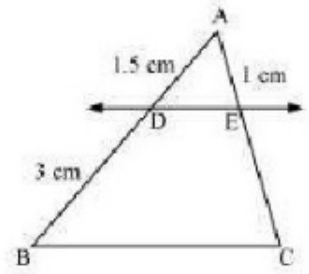
Answer. Quadrilateral PQRS and ABCD are not similar as their corresponding sides are proportional, i.e 1:2, but their corresponding angles are not equal.

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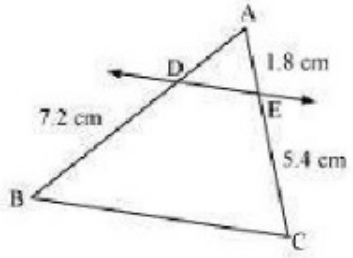
Exercise 6.2

Q1 In Figure, (i) and (ii), DE || BC. Find EC in (i) and AD in (ii).

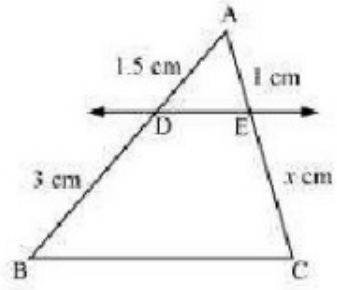
(i)



(ii)



Answer. (i)



Let EC = x cm

It is given that DE || BC.

By using basic proportionality theorem, we obtain

$\frac{\mathrm{AD}}{\mathrm{DB}} = \frac{\mathrm{AE}}{\mathrm{EC}}$

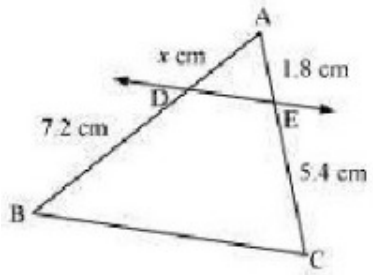
$\frac{1.5}{3} = \frac{1}{x}$

$x = \frac{3 \times 1}{1.5}$

$x = 2$

$\therefore \mathrm{EC} = 2\mathrm{cm}$

(ii)



Let AD = x cm

It is given that DE || BC

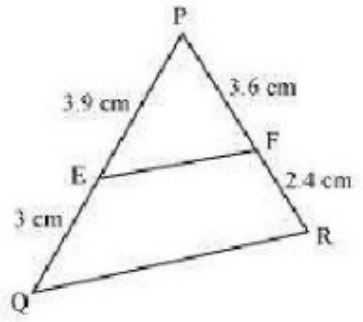
By using basic proportionality theorem, we obtain

$\frac{\mathrm{AD}}{\mathrm{DB}}=\frac{\mathrm{AE}}{\mathrm{EC}}$
$\frac{x}{7.2}=\frac{1.8}{5.4}$
$x=\frac{1.8\times 7.2}{5.4}$
$x=2.4$
$\therefore \mathrm{AD}=2.4\mathrm{cm}$

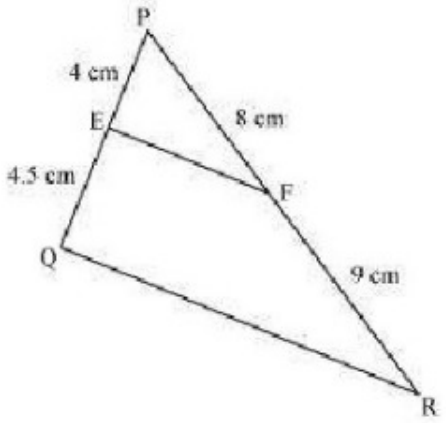
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Q2 E and F are points on the sides PQ and PR respectively of a Δ PQR. For each of the following cases, state whether EF || QR :
(i) PE = 3.9 cm, EQ = 3 cm, PF = 3.6 cm and FR = 2.4 cm
(ii) PE = 4 cm, QE = 4.5 cm, PF = 8 cm and RF = 9 cm
(iii) PQ = 1.28 cm, PR = 2.56 cm, PE = 0.18 cm and PF = 0.36 cm

Answer. (i)

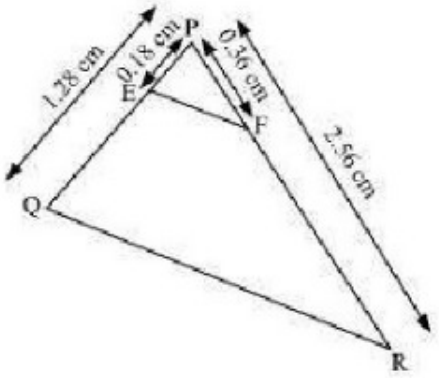


Given that, PE = 3.9 cm, EQ = 3 cm, PF = 3.6 cm, FR = 2.4 cm
$\frac{PE}{EQ}=\frac{3.9}{3}=1.3$
$\frac{\mathrm{PF}}{\mathrm{FR}}=\frac{3.6}{2.4}=1.5$
Hence, $\frac{\mathrm{PE}}{\mathrm{EQ}}\neq\frac{\mathrm{PF}}{\mathrm{FR}}$
Therefore, EF is not parallel to QR.
(ii)



PE = 4 cm, QE = 4.5 cm, PF = 8 cm, RF = 9 cm
$\frac{PE}{EQ}=\frac{4}{4.5}=\frac{8}{9}$
$\frac{PF}{FR}=\frac{8}{9}$
Hence $\frac{\mathrm{PE}}{\mathrm{EQ}}=\frac{\mathrm{PF}}{\mathrm{FR}}$

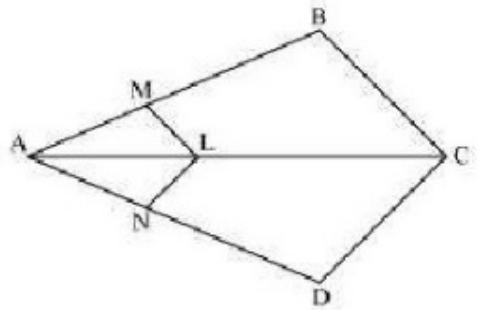
$Therefore, EF is parallel to QR.$
(iii)



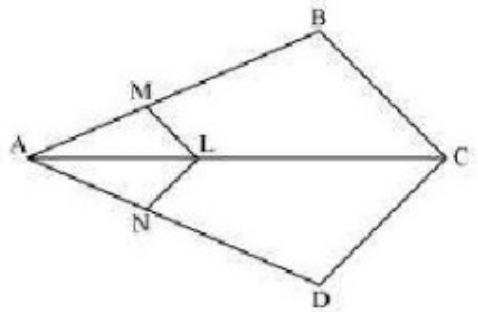
PQ = 1.28 cm, PR = 2.56 cm, PE = 0.18 cm, PF = 0.36 cm
$\frac{PE}{PQ} = \frac{0.18}{1.28} = \frac{18}{128} = \frac{9}{64}$
$\frac{\mathrm{PF}}{\mathrm{PR}} = \frac{0.36}{2.56} = \frac{9}{64}$
Hence, $\frac{\mathrm{PE}}{\mathrm{PQ}} = \frac{\mathrm{PF}}{\mathrm{PR}}$
Therefore, EF is parallel to QR.

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Q3 In Figure, if LM || CB and LN || CD, prove that
$\frac{\mathrm{AM}}{\mathrm{AB}} = \frac{\mathrm{AN}}{\mathrm{AD}}$



Answer.

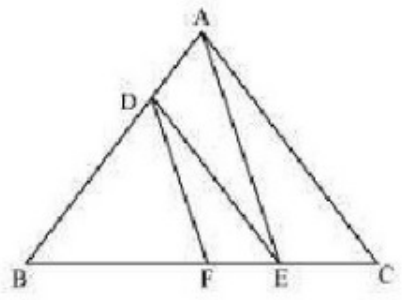


In the given figure, LM || CB
By using basic proportionality theorem, we obtain
$\frac{\mathrm{AM}}{\mathrm{AB}} = \frac{\mathrm{AL}}{\mathrm{AC}}$ ..(i)
Similarly, LN || CD
$\therefore \frac{\mathrm{AN}}{\mathrm{AD}} = \frac{\mathrm{AL}}{\mathrm{AC}}$

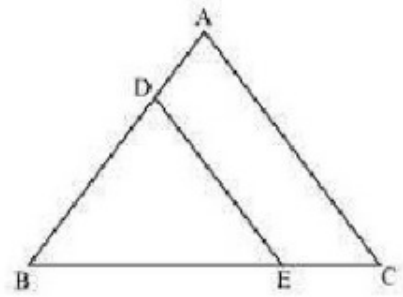
(ii)
From (i) and (ii), we obtain
$\frac{\mathrm{AM}}{\mathrm{AB}}=\frac{\mathrm{AN}}{\mathrm{AD}}$

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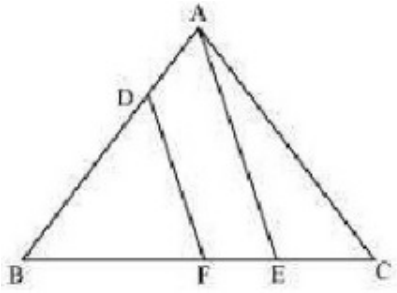
Q4 In Figure, DE || AC and DF || AE. Prove that
$\frac{\mathrm{BF}}{\mathrm{FE}}=\frac{\mathrm{BE}}{\mathrm{EC}}$



Answer.



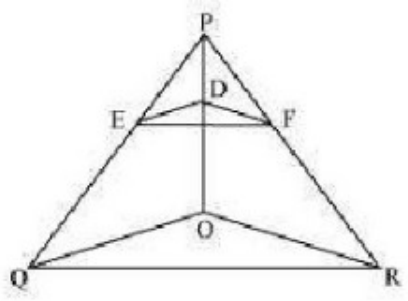
In $\triangle ABC, DE \| AC$
$\therefore \frac{\mathrm{BD}}{\mathrm{DA}}=\frac{\mathrm{BE}}{\mathrm{EC}}$ (Basic Proportionality Theorem) (i)



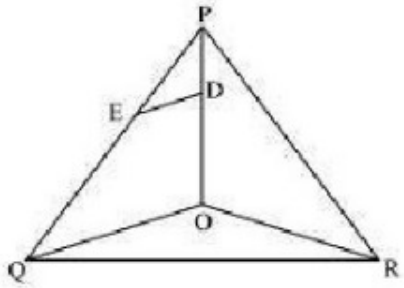
In $\triangle BAE, DF \| AE$
$\therefore \frac{\mathrm{BD}}{\mathrm{DA}}=\frac{\mathrm{BF}}{\mathrm{FE}} \quad (\text{ Basic Proportionality Theorem})$ (ii)
From (i) and (ii), we obtain
$\frac{\mathrm{BE}}{\mathrm{EC}}=\frac{\mathrm{BF}}{\mathrm{FE}}$

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Q5 In Figure, DE || OQ and DF || OR. Show that EF || QR.

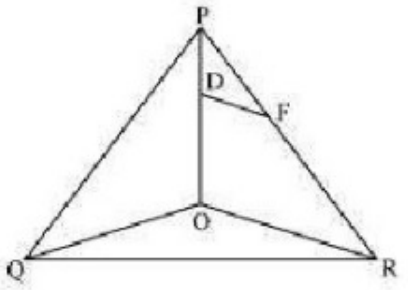


Answer.



In $\triangle \mathrm{POQ}, \mathrm{DE} \| \mathrm{OQ}$

$\therefore \frac{PE}{EQ}=\frac{PD}{DO}$ (Basic proportionality theorem) (i)



In $\triangle \mathrm{POR}, \mathrm{DF} \| \mathrm{OR}$

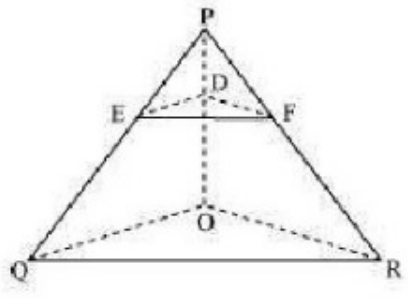
$\therefore \frac{\mathrm{PF}}{\mathrm{FR}}=\frac{\mathrm{PD}}{\mathrm{DO}}$ (Basic proportionality theorem)

(ii)

From (i) and (ii), we obtain

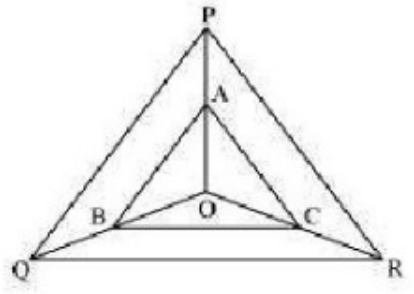
$\frac{PE}{EQ}=\frac{PF}{FR}$

$\therefore \mathrm{EF} \| \mathrm{OR}$ (Converse of basic proportionality theorem)

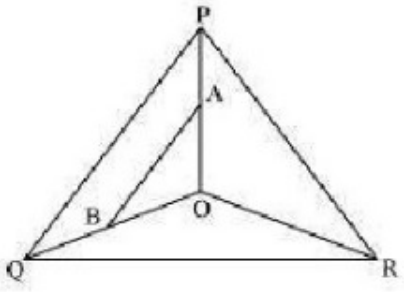


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Q6 In Figure, A, B and C are points on OP, OQ and OR respectively such that AB || PQ and AC || PR. Show that BC || QR.

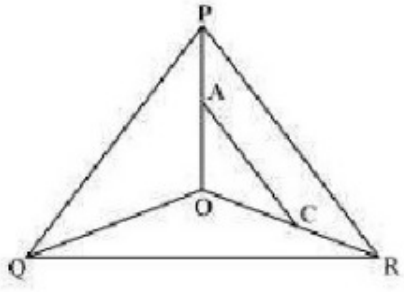


Answer.



In $\triangle \mathrm{POQ}, \mathrm{AB} \| \mathrm{PQ}$

$\therefore \frac{\mathrm{OA}}{\mathrm{AP}}=\frac{\mathrm{OB}}{\mathrm{BQ}}$ (Basic proportionality theorem) (i)



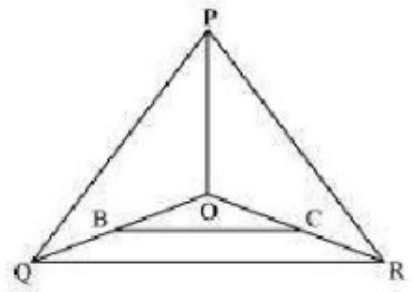
In $\triangle \mathrm{POR}, \mathrm{AC} \| \mathrm{PR}$

$\therefore \frac{\mathrm{OA}}{\mathrm{AP}}=\frac{\mathrm{OC}}{\mathrm{CR}}$ (By basic proportionality theorem) (ii)

From (i) and (ii), we obtain.

$\frac{\mathrm{OB}}{\mathrm{BQ}}=\frac{\mathrm{OC}}{\mathrm{CR}}$

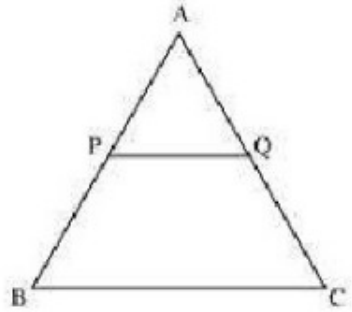
$\therefore \mathrm{BC} \| \mathrm{QR}$ (By the converse of basic proportionality theorem)



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Q7 Using Theorem 6.1, prove that a line drawn through the mid-point of one side of a triangle parallel to another side bisects the third side. (Recall that you have proved it in Class IX).

Answer.

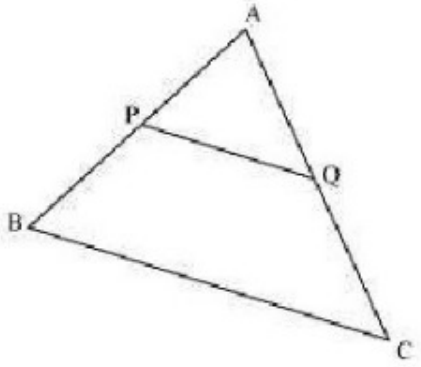


Consider the given figure in which PQ is a line segment drawn through the mid-point P of line AB, such that PQ || BC
By using basic proportionality theorem, we obtain
$\frac{\mathrm{AQ}}{\mathrm{QC}}=\frac{\mathrm{AP}}{\mathrm{PB}}$
$\frac{\mathrm{AQ}}{\mathrm{QC}}=\frac{1}{1} \quad(\mathrm{P} \text{ is the mid-point of } \mathrm{AB} . \therefore \mathrm{AP}=\mathrm{PB})$
$\Rightarrow AQ=QC$
Or, Q is the mid-point of AC.

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Q8 Using Theorem 6.2, prove that the line joining the mid-points of any two sides of a triangle is parallel to the third side. (Recall that you have done it in Class IX).

Answer.

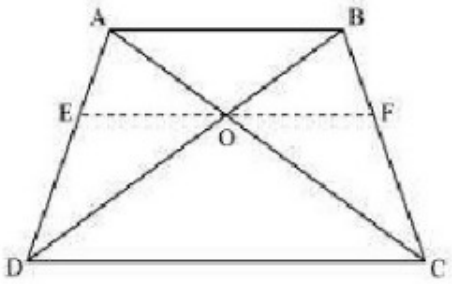


Consider the given figure in which PQ is a line segment joining the mid-points P and Q of line AB and AC respectively.
I.e., AP = PB and AQ = QC
It can be observed that
$\frac{\mathrm{AP}}{\mathrm{PB}}=\frac{1}{1}$
and $\frac{\mathrm{AQ}}{\mathrm{QC}}=\frac{1}{1}$
$\therefore \frac{\mathrm{AP}}{\mathrm{PB}}=\frac{\mathrm{AQ}}{\mathrm{QC}}$
Hence, by using basic proportionality theorem, we obtain
$\mathrm{PQ} \| \mathrm{BC}$

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Q9 ABCD is a trapezium in which AB || DC and its diagonals intersect each other at the point O. Show that
$\frac{\mathrm{AO}}{\mathrm{BO}}=\frac{\mathrm{CO}}{\mathrm{DO}}$

Answer.

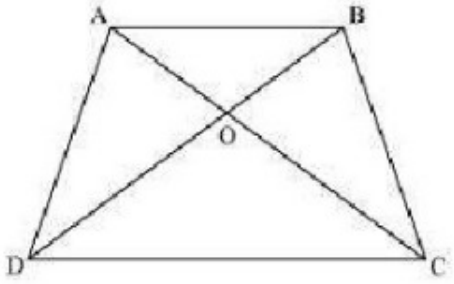


Draw a line EF through point O, such that EF || CD
In $\triangle ADC, EO \| CD$
By using basic proportionality theorem, we obtain
$\frac{\mathrm{AE}}{\mathrm{ED}}=\frac{\mathrm{AO}}{\mathrm{OC}}$ ...(1)
In $\triangle \mathrm{ABD},\ \mathrm{OE} \| \mathrm{AB}$
So, by using basic proportionality theorem, we obtain
$\frac{\mathrm{ED}}{\mathrm{AE}}=\frac{\mathrm{OD}}{\mathrm{BO}}$
$\Rightarrow \frac{\mathrm{AE}}{\mathrm{ED}}=\frac{\mathrm{BO}}{\mathrm{OD}}$... (2)
From equations (1) and (2), we obtain
$\frac{\mathrm{AO}}{\mathrm{OC}}=\frac{\mathrm{BO}}{\mathrm{OD}}$
$\Rightarrow \frac{\mathrm{AO}}{\mathrm{BO}}=\frac{\mathrm{OC}}{\mathrm{OD}}$

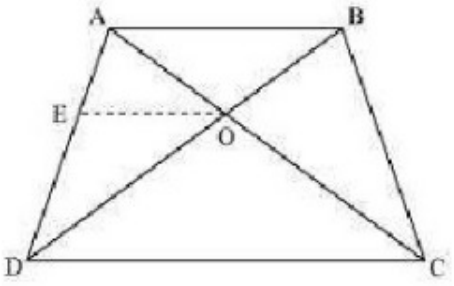
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Q10 The diagonals of a quadrilateral ABCD intersect each other at the point O such that $\frac{\mathrm{AO}}{\mathrm{BO}}=\frac{\mathrm{CO}}{\mathrm{DO}}$. Show that ABCD is a trapezium.

Answer. Let us consider the following figure for the given question.



Draw a line OE || AB



In $\triangle ABD, OE \| AB$

By using basic proportionality theorem, we obtain

$\frac{\mathrm{AE}}{\mathrm{ED}} = \frac{\mathrm{BO}}{\mathrm{OD}}$ (1)

However, it is given that

$\frac{\mathrm{AO}}{\mathrm{OC}} = \frac{\mathrm{OB}}{\mathrm{OD}}$ (2)

From equations (1) and (2), we obtain

$\frac{\mathrm{AE}}{\mathrm{ED}} = \frac{\mathrm{AO}}{\mathrm{OC}}$

$\Rightarrow$ EO $\|$ DC [By the converse of basic proportionality theorem]

$\Rightarrow AB \| OE \| DC$

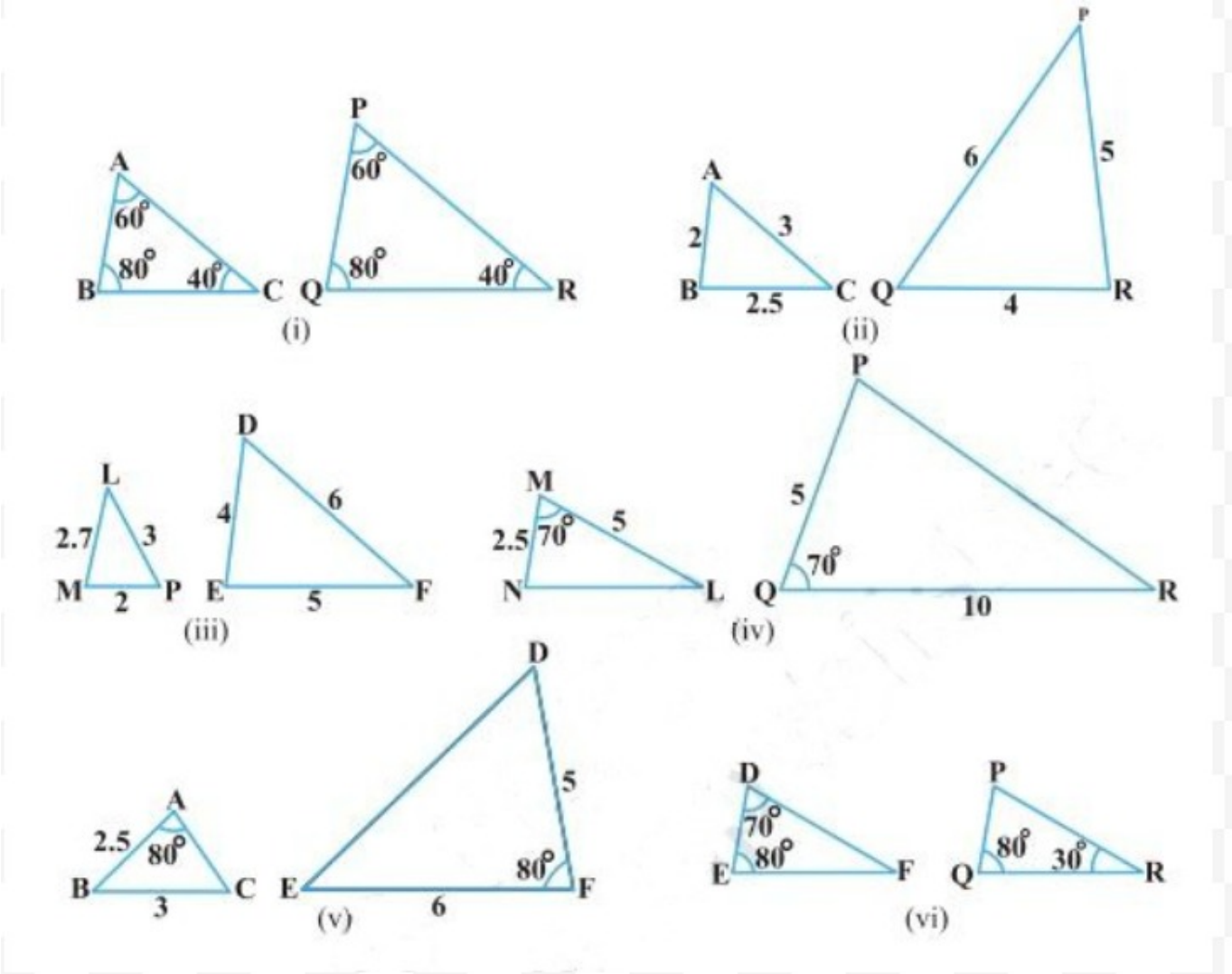
$\Rightarrow AB \| CD$

$\therefore ABCD$ is a trapezium.

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Exercise 6.3

Q1 State which pairs of triangles in Figure are similar. Write the similarity criterion used by you for answering the question and also write the pairs of similar triangles in the symbolic form :



Answer. (i) $\angle A = \angle P = 60^\circ$
$\angle B = \angle Q = 80^\circ$
$\angle C = \angle R = 40^\circ$
Therefore, $\triangle \mathrm{ABC} - \triangle \mathrm{PQR}[\text{By AAA similarity criterion }]$
$\frac{\mathrm{AB}}{\mathrm{QR}} = \frac{\mathrm{BC}}{\mathrm{RP}} = \frac{\mathrm{CA}}{\mathrm{PQ}}$

(ii) $\therefore \triangle \mathrm{ABC} - \Delta \mathrm{QRP} \quad [\text{ By SSS similarity criterion }]$

(iii) The given triangles are not similar as the corresponding sides are not proportional.

(iv) The given triangles are not similar as the corresponding sides are not proportional.

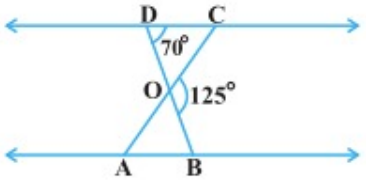
(v) THe given triangles are not similar as the corresponding sides are not proportional.

(vi) In $\Delta \mathrm{DEF}$
$\angle D + \angle E + \angle F = 180^\circ$
(Sum of the measures of the angles of a triangle is 180°.)
$70^\circ + 80^\circ + \angle F = 180^\circ$
$\angle F = 30^\circ$

Similarly, In $\triangle \mathrm{PQR}$
$\angle P+\angle Q+\angle R=180^{\circ}$
(Sum of the measures of the angles of a triangle is 180°.)
$\angle P+80^{\circ}+30^{\circ}=180^{\circ}$
$\angle P=70^{\circ}$
In $\triangle DEF$ and ΔPQR,
$\angle D=\angle P\left(Each 70^{\circ}\right)$
$\angle E=\angle Q\left(Each 80^{\circ}\right)$
$\angle F=\angle R$ (Each 30°)
$\therefore \triangle \mathrm{DEF} \sim \triangle \mathrm{PQR}[\mathrm{By\ AAA\ similarity\ criterion}\ ]$

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Q2 In Figure, Δ ODC ~ Δ OBA, ∠ BOC = 125° and ∠ CDO = 70°. Find ∠ DOC, ∠ DCO and ∠ OAB.

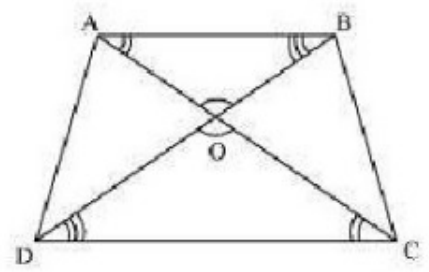


Answer. DOB is a straight line.
$\therefore \angle DOC+\angle COB=180^{\circ}$
$\Rightarrow \angle DOC=180^{\circ}-125^{\circ}$
$=55^{\circ}$
In ΔDOC
$\angle DCO+\angle CDO+\angle DOC=180^{\circ}$
(Sum of the measures of the angles of a triangle is 180°.)
$\Rightarrow \angle DCO+70^{\circ}+55^{\circ}=180^{\circ}$
$\Rightarrow \angle DCO=55^{\circ}$
It is given that $\triangle ODC-\triangle OBA$
$\therefore \angle OAB=\angle OCD[$ Corresponding angles are equal in similar triangles.]
$\Rightarrow \angle OAB=55^{\circ}$

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Q3 Diagonals AC and BD of a trapezium ABCD with AB || DC intersect each other at the point O. Using a similarity criterion for two triangles, show that $\frac{\mathrm{OA}}{\mathrm{OC}}=\frac{\mathrm{OB}}{\mathrm{OD}}$

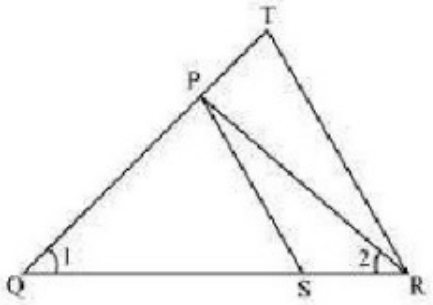
Answer.



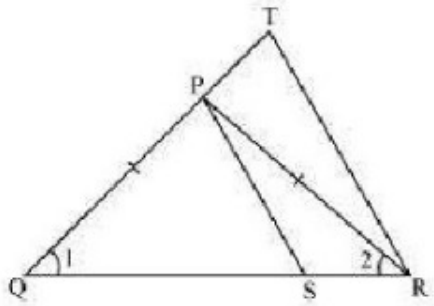
In $\triangle DOC$ and $\triangle BOA$,
$\angle \mathrm{CDO}=\angle \mathrm{ABO}[$ Alternate interior angles as $\mathrm{AB} \| \mathrm{CD}]$
$\angle \mathrm{DCO}=\angle \mathrm{BAO}[$ Alternate interior angles as $\mathrm{AB} \| \mathrm{CD}]$
$\angle DOC=\angle BOA[$ Vertically opposite angles $]$
$\therefore \triangle DOC-\triangle BOA$ [AAA similarity criterion]
$\therefore \frac{\mathrm{DO}}{\mathrm{BO}}=\frac{\mathrm{OC}}{\mathrm{OA}}$ [Corresponding sides are proportional]
$\Rightarrow \frac{\mathrm{OA}}{\mathrm{OC}}=\frac{\mathrm{OB}}{\mathrm{OD}}$

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Q4 In Figure, $\frac{\mathrm{QR}}{\mathrm{QS}}=\frac{\mathrm{QT}}{\mathrm{PR}}$ and $\angle 1=\angle 2$. Show that $\Delta \mathrm{PQS}-\Delta \mathrm{TQR}$.



Answer.



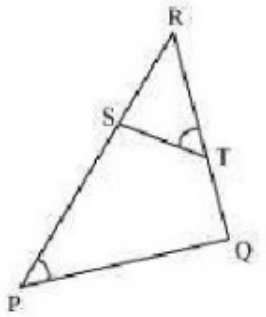
In $\Delta PQR, \angle PQR=\angle PRQ$
$\therefore \mathrm{PQ}=\mathrm{PR}(\mathrm{i})$
Given,
$\frac{\mathrm{QR}}{\mathrm{OS}}=\frac{\mathrm{QT}}{\mathrm{PR}}$
Using (i), we obtain
$\frac{\mathrm{QR}}{\mathrm{QS}}=\frac{\mathrm{QT}}{\mathrm{QP}}$ (ii)
In $\triangle \mathrm{PQS}$ and $\Delta \mathrm{TQR}$

$\frac{\mathrm{QR}}{\mathrm{QS}}=\frac{\mathrm{QT}}{\mathrm{QP}} \quad[\text{Using}(ii)]$
$\angle Q=\angle Q$
$\therefore \triangle \mathrm{PQS} \sim \triangle \mathrm{TQR}$ [SAS similarity criterion]

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Q5 S and T are points on sides PR and QR of Δ PQR such that ∠ P = ∠ RTS. Show that Δ RPQ ~ Δ RTS.

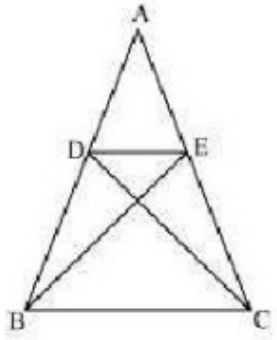
Answer.



In $\triangle \mathrm{RPQ}$ and $\Delta \mathrm{RST}$,
$\angle \mathrm{RTS}=\angle \mathrm{QPS}(\text{ Given })$
$\angle R=\angle R(\text{ Common angle })$
$\therefore \triangle RPQ-\Delta RTS(By \text{ AA similarity criterion})$

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Q6 In Figure, if Δ ABE ≅ Δ ACD, show that Δ ADE ~ Δ ABC.

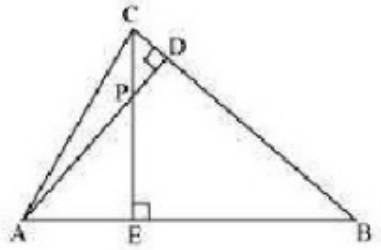


Answer. It is given that $\triangle \mathrm{ABE} \cong \triangle \mathrm{ACD}$
$\therefore AB=AC[ByCPCT](1)$
$\text{And, } AD=AE[BYCPCT](2)$
$\text{In } \triangle ADE \text{ and } \triangle ABC$
$\frac{\mathrm{AD}}{\mathrm{AB}}=\frac{\mathrm{AE}}{\mathrm{AC}}[\text{ Dividing equation } (2)\text{by}(1)]$
$\angle A=\angle A[\text{ Common angle }]$
$\therefore \triangle \mathrm{ADE}-\triangle \mathrm{ABC}[\text{By SAS similarity criterion}]$

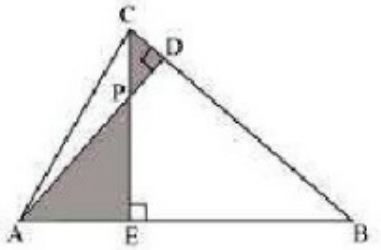
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Q7 In Figure, altitudes AD and CE of Δ ABC intersect each other at the point P. Show that:

(i) ΔAEP ~ Δ CDP
(ii) ΔABD ~ Δ CBE
(iii) ΔAEP ~ ΔADB
(iv) Δ PDC ~ Δ BEC

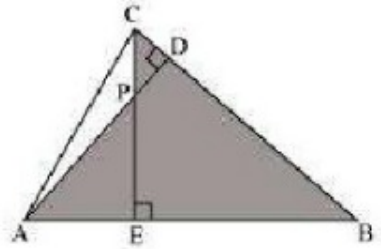


Answer. (i)



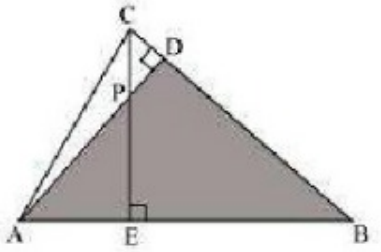
In $\triangle AEP$ and $\triangle CDP$.
$\angle AEP = \angle CDP\,(Each 90^{\circ})$
$\angle \mathrm{APE} = \angle \mathrm{CPD}(\text{ Vertically opposite angles })$
Hence, by using AA similarity criterion,
$\triangle \mathrm{AEP} \sim \Delta \mathrm{CDP}$

(ii)

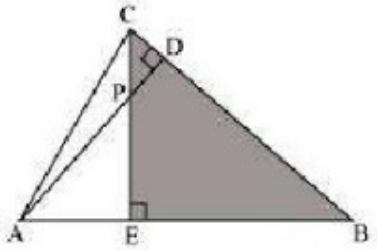


In $\triangle \mathrm{ABD}$ and $\Delta \mathrm{CBE}$
$\angle ADB = \angle CEB\,(Each 90^{\circ})$
$\angle ABD = \angle CBE(\text{ Common })$
Hence, by using AA similarity criterion,
$\triangle ABD - \Delta CBE$

(iii)



In $\triangle \mathrm{AEP}$ and $\triangle \mathrm{ADB}$,
$\angle AEP = \angle ADB\,(Each 90^\circ)$
$\angle PAE = \angle DAB(\text{ Common })$
Hence, y using AA similarity criterion,
$\triangle \mathrm{AEP} \sim \triangle \mathrm{ADB}$

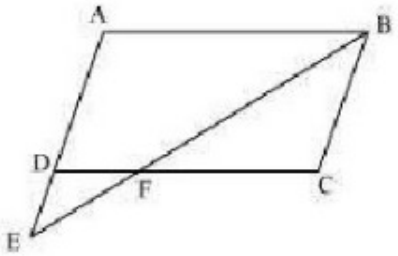


(iv)
In $\Delta \mathrm{PDC}$ and $\Delta \mathrm{BEC}$
$\angle \mathrm{PDC} = \angle \mathrm{BEC}\,(\text{ Each } 90^\circ)$
$\angle \mathrm{PCD} = \angle \mathrm{BCE}(\text{ Common angle })$
Hence, by using AA similarity criterion,
$\triangle PDC - \triangle BEC$

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Q8 E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. Show that Δ ABE ~ Δ CFB.

Answer.

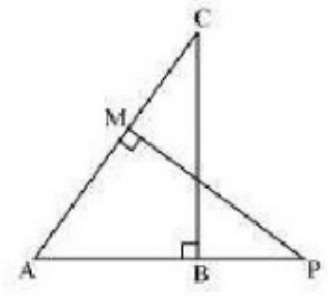


In $\triangle \mathrm{ABE}$ and $\Delta \mathrm{CFB}$,
$\angle \mathrm{A} = \angle \mathrm{C}(\text{ Opposite angles of a parallelogram})$
$\angle AEB = \angle CBF(\text{ Alternate interior angles as } AE \| BC)$
$\therefore \triangle \mathrm{ABE} - \Delta \mathrm{CFB}(\text{By AA similarity criterion})$

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Q9 In Figure, ABC and AMP are two right triangles, right angled at B and M respectively. Prove that:
(i) $\Delta \mathrm{ABC} \sim \Delta \mathrm{AMP}$

(ii) $\frac{\mathrm{CA}}{\mathrm{PA}}=\frac{\mathrm{BC}}{\mathrm{MP}}$

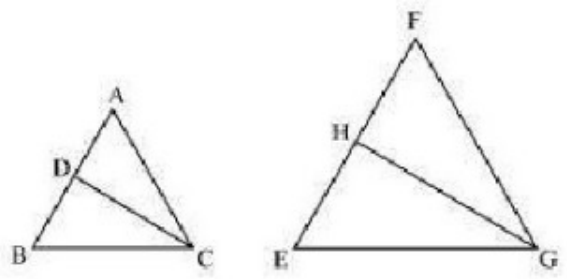


Answer. In $\triangle ABC$ and $\triangle AMP$,
$\angle ABC=\angle AMP\,(Each 90^{\circ})$
$\angle A=\angle A(\text{ Common })$
$\therefore \triangle \mathrm{ABC}-\triangle \mathrm{AMP}(\text{By AA similarity criterion})$
$\Rightarrow \frac{\mathrm{CA}}{\mathrm{PA}}=\frac{\mathrm{BC}}{\mathrm{MP}}$ (Corresponding sides of similar triangles are proportional)

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Q10 CD and GH are respectively the bisectors of ∠ACB and ∠ EGF such that D and H lie on sides AB and FE of Δ ABC and Δ EFG respectively. If ΔABC ~ Δ FEG, show that:
(i) $\frac{CD}{GH}=\frac{AC}{FG}$
(ii) $\Delta \mathrm{DCB}-\Delta \mathrm{HGE}$
(iii) $\Delta \mathrm{DCA}-\Delta \mathrm{HGF}$

Answer.

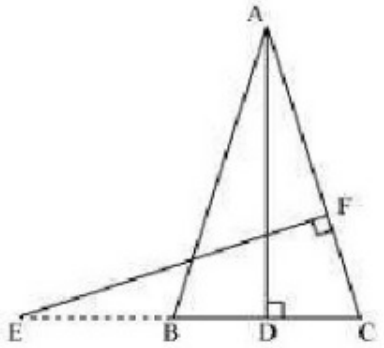


It is given that $\triangle \mathrm{ABC}-\Delta \mathrm{FEG}$
$\therefore \angle A=\angle F, \angle B=\angle E,$ and $\angle ACB=\angle FGE$
$\angle ACB=\angle FGE$
$\therefore \angle ACD=\angle FGH(\text{ Angle bisector })$
$\text{And, } \angle \mathrm{DCB}=\angle \mathrm{HGE}\,(\text{Angle bisector})$
$\text{In } \triangle \mathrm{ACD} \text{ and } \Delta \mathrm{FGH} \text{ ,}$
$\angle \mathrm{A}=\angle \mathrm{F}(\text{ Proved above})$
$\angle \mathrm{ACD}=\angle \mathrm{FGH}(\text{ Proved above })$
$\therefore \triangle ACD \sim \Delta FGH(By \text{ AA similarity criterion})$
$\Rightarrow \frac{\mathrm{CD}}{\mathrm{GH}}=\frac{\mathrm{AC}}{\mathrm{FG}}$
In $\triangle DCB$ and ΔHGE,
$\angle \mathrm{DCB}=\angle \mathrm{HGE}(\text{ Proved above })$
$\angle \mathrm{B}=\angle \mathrm{E}(\text{ Proved above })$
$\triangle \triangle DCB \sim \Delta HGE(By \text{ AA similarity criterion})$

$\text{In } \triangle DCA \text{ and } \Delta HGF$
$\angle ACD = \angle FGH(\text{ Proved above })$
$\angle A = \angle F(\text{ Proved above })$
$\therefore \triangle \mathrm{DCA} \sim \Delta \mathrm{HGF}(\text{ By AA similarity criterion})$

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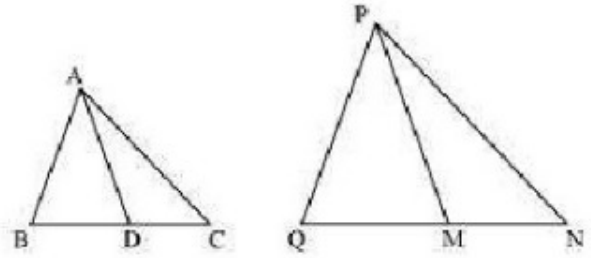
Q11 In Figure, E is a point on side CB produced of an isosceles triangle ABC with AB = AC. If AD ⊥ BC and EF ⊥AC, prove that Δ ABD ~ Δ ECF.



Answer. It is given that ABC is an isosceles triangle.
$\therefore \mathrm{AB} = \mathrm{AC}$
$\Rightarrow \angle \mathrm{ABD} = \angle \mathrm{ECF}$
$\text{In } \triangle \mathrm{ABD} \text{ and } \Delta \mathrm{ECF}$
$\angle ADB = \angle EFC\,(Each 90^{\circ})$
$\angle BAD = \angle CEF(\text{ Proved above })$
$\therefore \triangle \mathrm{ABD} \sim \Delta \mathrm{ECF}(\text{By using AA similarity criterion })$

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Q12 Sides AB and BC and median AD of a triangle ABC are respectively proportional to sides PQ and QR and median PM of Δ PQR (see Figure). Show that Δ ABC ~ Δ PQR.



Answer. Median divides the opposite side.
$\therefore \mathrm{BD} = \frac{\mathrm{BC}}{2} \text{ and } \mathrm{QM} = \frac{\mathrm{QR}}{2}$
Given that,
$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$

$\Rightarrow \frac{AB}{PQ}=\frac{\frac{1}{2}BC}{\frac{1}{2}QR}=\frac{AD}{PM}$

$\Rightarrow \frac{AB}{PQ}=\frac{BD}{QM}=\frac{AD}{PM}$

In $\triangle ABD$ and ΔPQM

$\frac{\mathrm{AB}}{\mathrm{PQ}}=\frac{\mathrm{BD}}{\mathrm{QM}}=\frac{\mathrm{AD}}{\mathrm{PM}}$ (Proved above)

$\triangle\triangle \mathrm{ABD} \sim \triangle \mathrm{PQM}$(By SSS similarity criterion)

$\Rightarrow \angle ABD=\angle PQM$(Corresponding angles of similar triangles)

In $\triangle ABC$ and ΔPQR ,

$\angle ABD=\angle PQM$ (Proved above)

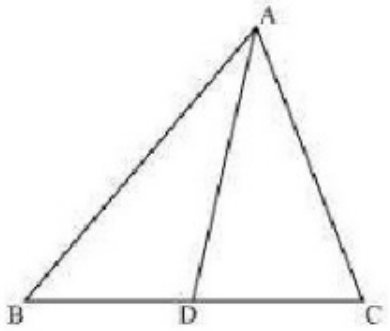
$\frac{AB}{PQ}=\frac{BC}{QR}$

$\therefore \triangle ABC \sim \Delta PQR(By$ SAS similarity criterion)

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Q13 D is a point on the side BC of a triangle ABC such that ∠ADC = ∠ BAC. Show that CA^2 = CB.CD.

Answer.



In $\triangle ADC$ and ΔBAC,

$\angle ADC=\angle BAC$(Given)

$\angle ACD=\angle BCA$(Common angle)

$\therefore \triangle ADC \sim \Delta BAC(By$ AA similarity criterion)

We know that corresponding sides of similar triangles are in proportion.

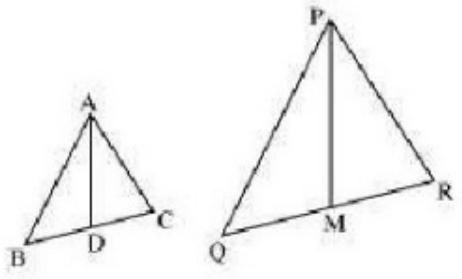
$\therefore \frac{\mathrm{CA}}{\mathrm{CB}}=\frac{\mathrm{CD}}{\mathrm{CA}}$

$\Rightarrow \mathrm{CA}^2=\mathrm{CB} \times \mathrm{CD}$

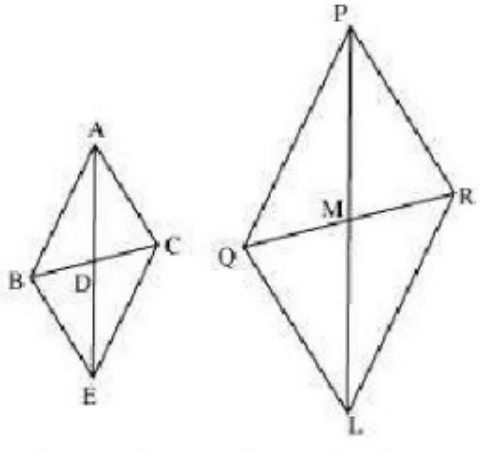
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Q14 Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that ΔABC ~ Δ PQR.

Answer.



Given that,
$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$
Let us extend AD and PM up to point E and L respectively, such that AD = DE and PM = ML. Then, join B to E, C to E, Q to L, and R to L.



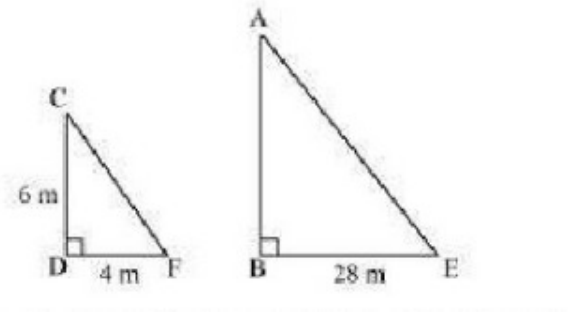
We know that medians divide opposite sides.
Therefore, BD = DC and QM = MR
Also, AD = DE (By construction)
And, PM = ML(By construction)
In quadrilateral ABEC, diagonals AE and BC bisect each other at point D.
Therefore, quadrilateral ABEC is a parallelogram.
$\therefore AC = BE$ and $AB = EC$ (Opposite sides of a parallelogram are equal)
Similarly, we can prove that quadrilateral PQLR is a parallelogram and PR = QL, PQ = LR
It was given that
$\frac{\mathrm{AB}}{\mathrm{PQ}} = \frac{\mathrm{AC}}{\mathrm{PR}} = \frac{\mathrm{AD}}{\mathrm{PM}}$
$\Rightarrow \frac{AB}{PQ} = \frac{BE}{QL} = \frac{2AD}{2PM}$
$\Rightarrow \frac{\mathrm{AB}}{\mathrm{PQ}} = \frac{\mathrm{BE}}{\mathrm{QL}} = \frac{\mathrm{AE}}{\mathrm{PL}}$
$\therefore \triangle \mathrm{ABE} \sim \triangle \mathrm{PQL}$ (By SSS similarity criterion)
We know that corresponding angles of similar triangles are equal.
$\therefore \angle BAE = \angle QPL \ldots (1)$
Similarly, it can be proved that $\triangle \mathrm{AEC} \sim \triangle \mathrm{PLR}$ and $\angle CAE = \angle RPL \ldots (2)$
Adding equation (1) and (2), we obtain
$\angle BAE + \angle CAE = \angle QPL + \angle RPL$
$\Rightarrow \angle CAB = \angle RPQ \ldots (3)$
In $\triangle ABC$ and ΔPQR,
$\frac{\mathrm{AB}}{\mathrm{PQ}} = \frac{\mathrm{AC}}{\mathrm{PR}}$ (Given)
$\angle CAB = \angle RPQ[U \text{ sing equation } (3)]$

$\therefore \triangle \mathrm{ABC} \sim \Delta \mathrm{PQR}(\mathrm{By\ SAS\ similarity\ criterion})$

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Q15 A vertical pole of length 6 m casts a shadow 4 m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower.

Answer.

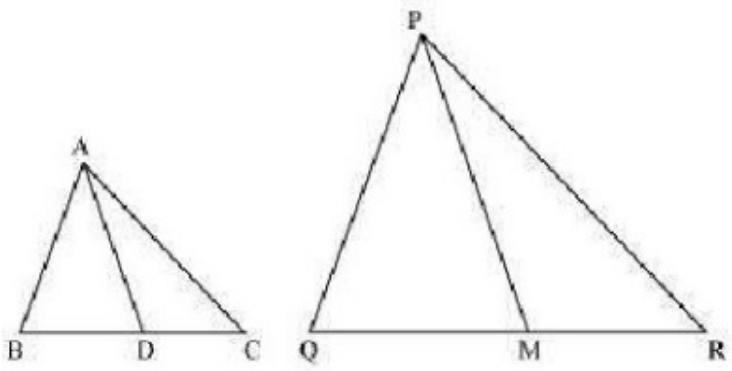


Let AB and CD be a tower and a pole respectively.
Let the shadow of BE and DF be the shadow of AB and CD respectively.
At the same time, the light rays from the sun will fall on the tower and the pole at the same angle.
Therefore, $\angle DCF = \angle BAE$
And, $\angle \mathrm{DFC} = \angle \mathrm{BEA}$
$\angle \mathrm{CDF} = \angle \mathrm{ABE}(\ \mathrm{Tower\ and\ pole\ are\ vertical\ to\ the\ ground})$
$\therefore \triangle \mathrm{ABE} \sim \triangle \mathrm{CDF}$ (AAA similarity criterion)
$\Rightarrow \frac{\mathrm{AB}}{\mathrm{CD}} = \frac{\mathrm{BE}}{\mathrm{DF}}$
$\Rightarrow \frac{\mathrm{AB}}{6\mathrm{m}} = \frac{28}{4}$
$\Rightarrow AB = 42\mathrm{m}$
Therefore, the height of the tower will be 42 metres.

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Q16 If AD and PM are medians of triangles ABC and PQR, respectively where Δ ABC ~ Δ PQR, prove that $\frac{\mathrm{AB}}{\mathrm{PQ}} = \frac{\mathrm{AD}}{\mathrm{PM}}$

Answer.



It is given that $\triangle \mathrm{ABC} \sim \triangle \mathrm{PQR}$
We know that the corresponding sides of similar triangles are in proportion.

$\therefore \frac{\mathrm{AB}}{\mathrm{PQ}}=\frac{\mathrm{AC}}{\mathrm{PR}}=\frac{\mathrm{BC}}{\mathrm{QR}}$ ...(1)
Since AD and PM are medians, they will divide their opposite sides.
$\mathrm{BD}=\frac{\mathrm{BC}}{2}$ and $\mathrm{QM}=\frac{\mathrm{QR}}{2}$ ...(3)
From equations (1) and (3), we obtain
$\frac{\mathrm{AB}}{\mathrm{PQ}}=\frac{\mathrm{BD}}{\mathrm{QM}}$ ...(4)
In $\triangle \mathrm{ABD}$ and $\Delta \mathrm{PQM}$
$\angle B=\angle Q[U \sin g \text{ equation } (2)]$
$\frac{AB}{PQ}=\frac{BD}{QM}$ [Using equation (4)]
$\therefore \triangle \mathrm{ABD} \sim \Delta \mathrm{PQM}$ (By SAS similarity criterion)
$\Rightarrow \frac{\mathrm{AB}}{\mathrm{PQ}}=\frac{\mathrm{BD}}{\mathrm{QM}}=\frac{\mathrm{AD}}{\mathrm{PM}}$

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Exercise 6.4

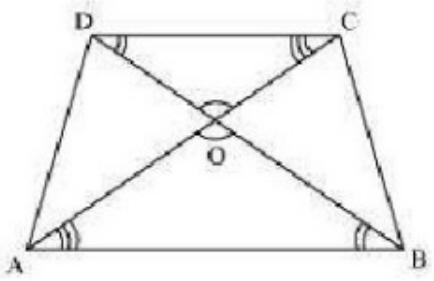
Q1 Let Δ ABC ~ Δ DEF and their areas be, respectively, $64 \mathrm{cm}^2$ and $121 \mathrm{cm}^2$. If EF = 15.4 cm, find BC.

Answer. It is given that $\triangle \mathrm{ABC}-\Delta \mathrm{DEF}$.
$\therefore \frac{\operatorname{ar}(\Delta \mathrm{ABC})}{\operatorname{ar}(\Delta \mathrm{DEF})}=\left(\frac{\mathrm{AB}}{\mathrm{DE}}\right)^2=\left(\frac{\mathrm{BC}}{\mathrm{EF}}\right)^2=\left(\frac{\mathrm{AC}}{\mathrm{DF}}\right)^2$
Given that,
$\mathrm{EF}=15.4 \mathrm{cm}$,
$\operatorname{ar}(\Delta \mathrm{ABC})=64 \mathrm{cm}^2$
$\operatorname{ar}(\Delta \mathrm{DEF})=121 \mathrm{cm}^2$
$\therefore \frac{\operatorname{ar}(\mathrm{ABC})}{\operatorname{ar}(\mathrm{DEF})}=\left(\frac{\mathrm{BC}}{\mathrm{EF}}\right)^2$
$\Rightarrow\left(\frac{64 \mathrm{cm}^2}{121 \mathrm{cm}^2}\right)=\frac{\mathrm{BC}^2}{(15.4 \mathrm{cm})^2}$
$\Rightarrow \frac{\mathrm{BC}}{15.4}=\left(\frac{8}{11}\right) \mathrm{cm}$
$\Rightarrow \mathrm{BC}=\left(\frac{8 \times 15.4}{11}\right) \mathrm{cm}=(8 \times 1.4) \mathrm{cm}=11.2 \mathrm{cm}$

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Q2 Diagonals of a trapezium ABCD with AB || DC intersect each other at the point O. If AB = 2 CD, find the ratio of the areas of triangles AOB and COD.

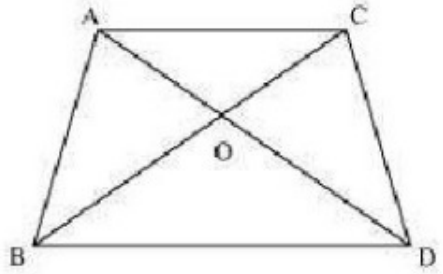
Answer.



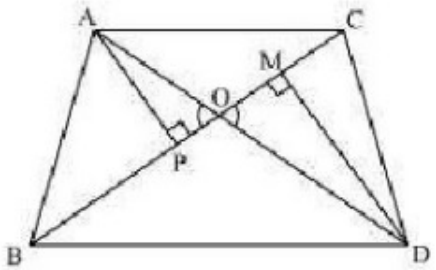
Since AB || CD,
$\therefore \angle OAB = \angle OCD$ and $\angle OBA = \angle ODC$(Alternate interior angles)
In $\triangle$AOB and ΔCOD,
$\angle$AOB = $\angle$COD (Vertically opposite angles)
$\angle OAB = \angle OCD$(Alternate interior angles)
$\angle OBA = \angle ODC$(Alternate interior angles)
$\therefore \triangle$AOB $\sim \Delta$COD(By AAA similarity criterion)
$\therefore \frac{\text{ar}(\Delta\text{AOB})}{\text{ar}(\Delta\text{COD})} = \left(\frac{\text{AB}}{\text{CD}}\right)^2$
Since $AB = 2CD$
$\therefore \frac{\text{ar}(\Delta\text{AOB})}{\text{ar}(\Delta\text{COD})} = \left(\frac{2\text{CD}}{\text{CD}}\right)^2 = \frac{4}{1} = 4:1$

Page : 143 , Block Name : Exercise 6.4

Q3 In Figure, ABC and DBC are two triangles on the same base BC. If AD intersects BC at O, show that
$\frac{\text{ar(ABC)}}{\text{ar(DBC)}} = \frac{\text{AO}}{\text{DO}}$



Answer. Let us draw two perpendicular AP and DM on line BC.



We know that area of a triangle = 1/2 x Base x Height
$\therefore \frac{\text{ar}(\Delta\text{ABC})}{\text{ar}(\Delta\text{DBC})} = \frac{\frac{1}{2}\text{BC}\times\text{AP}}{\frac{1}{2}\text{BC}\times\text{DM}} = \frac{\text{AP}}{\text{DM}}$
In $\triangle$APO and $\triangle$DMO

$\angle APO = \angle DMO\,(Each = 90^{\circ})$
$\angle \mathrm{AOP} = \angle \mathrm{DOM}(\text{ Vertically opposite angles})$
$\therefore \triangle \mathrm{APO} \sim \triangle \mathrm{DMO}(\text{By AA similarity criterion})$
$\therefore \frac{AP}{DM} = \frac{AO}{DO}$
$\Rightarrow \frac{\mathrm{ar}(\Delta \mathrm{ABC})}{\mathrm{ar}(\Delta \mathrm{DBC})} = \frac{\mathrm{AO}}{\mathrm{DO}}$

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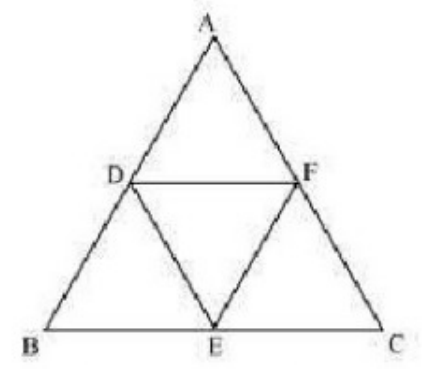
Q4 If the areas of two similar triangles are equal, prove that they are congruent.

Answer. Let us assume two similar triangles are $\triangle \mathrm{ABC} \sim \Delta \mathrm{PQR}$
$\frac{\mathrm{ar}(\Delta \mathrm{ABC})}{\mathrm{ar}(\Delta \mathrm{PQR})} = \left(\frac{\mathrm{AB}}{\mathrm{PQ}}\right)^2 = \left(\frac{\mathrm{BC}}{\mathrm{QR}}\right)^2 = \left(\frac{\mathrm{AC}}{\mathrm{PR}}\right)^2$ (1)
Given that, ar $(\Delta \mathrm{ABC}) =$ ar $(\Delta \mathrm{PQR})$
$\Rightarrow \frac{\mathrm{ar}(\Delta \mathrm{ABC})}{\mathrm{ar}(\Delta \mathrm{PQR})} = 1$
Putting this value in equation (1), we obtain
$\mathrm{I} = \left(\frac{\mathrm{AB}}{\mathrm{PQ}}\right)^2 = \left(\frac{\mathrm{BC}}{\mathrm{QR}}\right)^2 = \left(\frac{\mathrm{AC}}{\mathrm{PR}}\right)^2$
$\Rightarrow \mathrm{AB} = \mathrm{PQ}, \mathrm{BC} = \mathrm{QR}, \text{ and } \mathrm{AC} = \mathrm{PR}$
$\therefore \Delta \mathrm{ABC} \cong \Delta \mathrm{PQR} \quad (\text{ By SSS congruence criterion })$

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Q5 D, E and F are respectively the mid-points of sides AB, BC and CA of Δ ABC. Find the ratio of the areas of Δ DEF and Δ ABC.

Answer.



D and E are the mid-points of ΔABC.

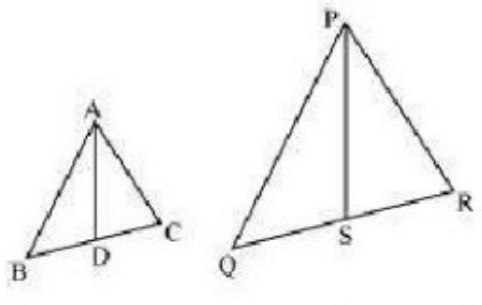
$\therefore \mathrm{DE} \| \mathrm{AC} \text{ and } \mathrm{DE} = \frac{1}{2}\mathrm{AC}$
In $\triangle BED$ and ΔBCA,
$\angle BED = \angle BCA$ (Corresponding angles)
$\angle \mathrm{BDE} = \angle \mathrm{BAC}$ (Corresponding angles)
$\angle \mathrm{EBD} = \angle \mathrm{CBA}$ (Common angles)
$\therefore \triangle BED \sim \Delta BCA$ (AAA similarity criterion)
$\Rightarrow \frac{\mathrm{ar}(\Delta \mathrm{BED})}{\mathrm{ar}(\Delta \mathrm{BCA})} = \frac{1}{4}$

$\Rightarrow \mathrm{ar}(\Delta \mathrm{BED})=\frac{1}{4} \mathrm{ar}(\Delta \mathrm{BCA})$
Similarly, $\mathrm{ar}(\Delta \mathrm{CFE})=\frac{1}{4} \mathrm{ar}(\mathrm{CBA})$ and ar $(\Delta \mathrm{ADF})=\frac{1}{4} \mathrm{ar}(\Delta \mathrm{ABC})$
Also $(\Delta \mathrm{DEF})=\mathrm{ar}(\Delta \mathrm{ABC})-[\mathrm{ar}(\Delta \mathrm{BED})+\mathrm{ar}(\Delta \mathrm{CFE})+\mathrm{ar}(\Delta \mathrm{ADF})]$
$\Rightarrow \mathrm{ar}(\Delta \mathrm{DEF})=\mathrm{ar}(\Delta \mathrm{ABC})-\frac{3}{4} \mathrm{ar}(\Delta \mathrm{ABC})=\frac{1}{4} \mathrm{ar}(\Delta \mathrm{ABC})$
$\Rightarrow \frac{\mathrm{ar}(\Delta \mathrm{DEF})}{\mathrm{ar}(\Delta \mathrm{ABC})}=\frac{1}{4}$

Page : 144 , Block Name : Exercise 6.4

Q6 Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding medians.

Answer.

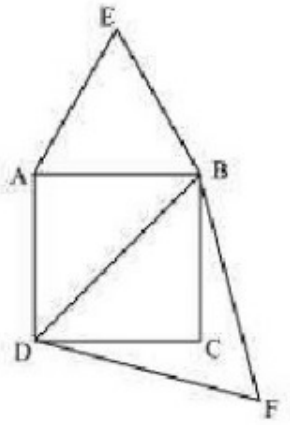


Let us assume two similar triangles as $\triangle \mathrm{ABC} \sim \Delta \mathrm{PQR}$. Let AD and PS be the medians of these triangles.
$\because \triangle ABC \sim \Delta PQR$
$\therefore \frac{\mathrm{AB}}{\mathrm{PQ}}=\frac{\mathrm{BC}}{\mathrm{QR}}=\frac{\mathrm{AC}}{\mathrm{PR}}$ ..(1)
$\angle A=\angle P, \angle B=\angle Q, \angle C=\angle R \ldots(2)$
Since AD and PS are medians,
$\therefore BD=DC=\frac{BC}{2}$
And, $QS=SR=\frac{QR}{2}$
Equation (1) becomes
$\frac{\mathrm{AB}}{\mathrm{PQ}}=\frac{\mathrm{BD}}{\mathrm{QS}}=\frac{\mathrm{AC}}{\mathrm{PR}}$ ...(3)
In $\triangle ABD$ and ΔPQS,
$\angle B=\angle Q[$ Using equation $(2)]$
And, $\frac{AB}{PQ}=\frac{BD}{QS}[Using$ equation $(3)]$
$\therefore \triangle \mathrm{ABD} \sim \Delta \mathrm{PQS}$ (SAS similarity criterion)
Therefore, it can be said that
$\frac{\mathrm{AB}}{\mathrm{PQ}}=\frac{\mathrm{BD}}{\mathrm{QS}}=\frac{\mathrm{AD}}{\mathrm{PS}}$ ...(4)
$\frac{\mathrm{ar}(\Delta \mathrm{ABC})}{\mathrm{ar}(\Delta \mathrm{PQR})}=\left(\frac{\mathrm{AB}}{\mathrm{PQ}}\right)^2=\left(\frac{\mathrm{BC}}{\mathrm{QR}}\right)^2=\left(\frac{\mathrm{AC}}{\mathrm{PR}}\right)^2$
From equations (1) and (4), we may find that
$\frac{AB}{PQ}=\frac{BC}{QR}=\frac{AC}{PR}=\frac{AD}{PS}$
And hence,
$\frac{\mathrm{ar}(\Delta \mathrm{ABC})}{\mathrm{ar}(\Delta \mathrm{PQR})}=\left(\frac{\mathrm{AD}}{\mathrm{PS}}\right)^2$

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Q7 Prove that the area of an equilateral triangle described on one side of a square is equal to half the area of the equilateral triangle described on one of its diagonals.

Answer.

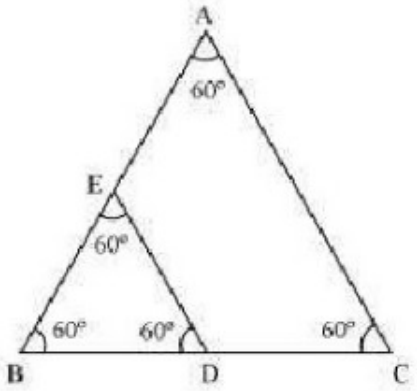


Let ABCD be a square of side a.
Therefore, its diagonal = $\sqrt{2}a$
Two desired equilateral triangles are formed as $\triangle \mathrm{ABE}$ and $\triangle \mathrm{DBF}$
Two desired equilateral triangles are formed as $\triangle \mathrm{ABE}$ and $\Delta \mathrm{DBF}$.
Side of an equilateral triangle, $\triangle \mathrm{ABE}$, described on one of its sides = a
Side of an equilateral triangle, $\triangle \mathrm{DBF}$, described on one of its diagonals $= \sqrt{2}a$
We know that equilateral triangles have all its angles as 60° and all its sides of the same length. Therefore, all equilateral triangles are similar to each other. Hence, the ratio between the areas of these triangles will be equal to the square of the ratio between the sides of these triangles.
$\frac{\text{Area of } \Delta \mathrm{ABE}}{\text{Area of } \Delta \mathrm{DBF}} = \left(\frac{a}{\sqrt{2}a}\right)^2 = \frac{1}{2}$

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Q8 Tick the correct answer and justify :
ABC and BDE are two equilateral triangles such that D is the mid-point of BC. Ratio of the areas of triangles ABC and BDE is
(A) 2 : 1
(B) 1 : 2
(C) 4 : 1
(D) 1 : 4

Answer.



We know that equilateral triangles have all its angles as 60° and all its sides of the same length. Therefore, all equilateral triangles are similar to each other. Hence, the ratio between the areas of these triangles will be equal to the square of the ratio between the sides of these triangles.
Let side of $\Delta \mathrm{ABC} = x$
Therefore, side of $\Delta BDE = \frac{x}{2}$
$\therefore \frac{\text{area}(\Delta \mathrm{ABC})}{\text{area}(\Delta \mathrm{BDE})} = \left(\frac{x}{\frac{x}{2}}\right)^2 = \frac{4}{1}$
Hence, the correct answer is (C).

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Q9 Tick the correct answer and justify :
Sides of two similar triangles are in the ratio 4 : 9. Areas of these triangles are in the ratio
(A) 2 : 3
(B) 4 : 9
(C) 81 : 16
(D) 16 : 81

Answer. If two triangles are similar to each other, then the ratio of the areas of these triangles will be equal to the square of the ratio of the corresponding sides of these triangles.
It is given that the sides are in the ratio 4:9.
Therefore, ratio between areas of these triangles $= \left(\frac{4}{9}\right)^2 = \frac{16}{81}$
Hence, the correct answer is (D).

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Exercise 6.5

Q1 Sides of triangles are given below. Determine which of them are right triangles. In case of a right triangle, write the length of its hypotenuse.
(i) 7 cm, 24 cm, 25 cm
(ii) 3 cm, 8 cm, 6 cm
(iii) 50 cm, 80 cm, 100 cm

(iv) 13 cm, 12 cm, 5 cm

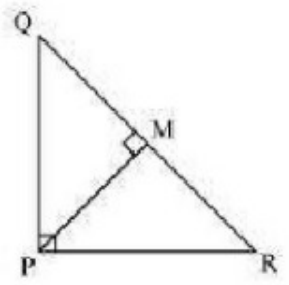
Answer. (i) It is given that the sides of the triangle are 7 cm, 24 cm, and 25 cm. Squaring the lengths of these sides, we will obtain 49, 576, and 625.
49 + 576 = 625
Or, $7^2 + 24^2 = 25^2$
The sides of the given triangle are satisfying Pythagoras theorem.
Therefore, it is a right triangle.
We know that the longest sides of a right triangle is the hypotenuse.
Therefore, the length of the hypotenuse of this triangle is 25 cm.

(ii) It is given that sides of the triangle are 3 cm, 8 cm, and 6 cm.
Squaring the lengths of these sides, we will obtain 9, 64, and 36.
However, 9 + 36 $\neq 64$
Or, $3^2 + 6^2 \neq 8^2$
Clearly, the sum of the squares of the lengths of two sides is not equal to the square of the length of the third side.
Hence, it is not a right triangle
(iii) Given that sides are 50 cm, 80 cm. And 100 cm.
Squaring the lengths of these sides, we will obtain 2500, 6400, and 10000.
However, 2500 + 6400 $\neq 10000$
Or, $50^2 + 80^2 \neq 100^2$
Clearly, the sum of the squares of the lengths of two sides is not equal to the square of the length of the third side.
Therefore, the given triangle is not satisfying Pythagoras theorem.
Hence, it is not a right triangle.
(iv) Given that sides are 13 cm, 12 cm, and 5 cm.
Squaring the lengths of these sides, we will obtain 169, 144, and 24.
Clearly, 144 + 25 = 169.
Or, $12^2 + 5^2 = 13^2$
The sides of the given triangle are satisfying Pythagoras theorem.
Therefore, it is a right triangle.
We know that the longest side of a right triangle is the hypotenuse.
Therefore, the length of the hypotenuse of this triangle is 13 cm.

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Q2 PQR is a triangle right angled at P and M is a point on QR such that PM $\perp$ QR. Show that PM^2 = QM . MR

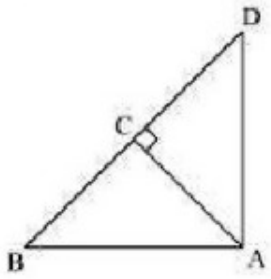
Answer.



Let $\angle \mathrm{MPR}=x$
In $\Delta \mathrm{MPR}$
$\angle MRP=180^{\circ}-90^{\circ}-x$
$\angle MRP=90^{\circ}-x$
Similarly, in $\Delta \mathrm{MPQ}$
$$\angle \mathrm{MPQ}=90^{\circ}-\angle \mathrm{MPR}$$
$$=90^{\circ}-x$$
$\angle \mathrm{MQP}=180^{\circ}-90^{\circ}-(90^{\circ}-x)$
$\angle \mathrm{MOP}=x$
In $\triangle \mathrm{QMP}$ and $\Delta \mathrm{PMR}$,
$\angle \mathrm{MPQ}=\angle \mathrm{MRP}$
$\angle \mathrm{PMQ}=\angle \mathrm{RMP}$
$\angle \mathrm{MQP}=\angle \mathrm{MPR}$
$\therefore \triangle \mathrm{QMP}-\mathrm{APMR} \quad (\text{ By AAA similarity criterion })$
$\Rightarrow \frac{\mathrm{QM}}{\mathrm{PM}}=\frac{\mathrm{MP}}{\mathrm{MR}}$
$\Rightarrow \mathrm{PM}^2=\mathrm{QM} \times \mathrm{MR}$

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Q3 In Figure, ABD is a triangle right angled at A and AC $\perp$ BD. Show that
(i) $\mathrm{AB}^2=\mathrm{BC}.\mathrm{BD}$
(ii) $\mathrm{AC}^2=\mathrm{BC} \cdot \mathrm{DC}$
(iii) $\mathrm{AD}^2=\mathrm{BD}.\mathrm{CD}$



Answer. (i) In $\triangle \mathrm{ADB}$ and $\triangle \mathrm{CAB}$,
$\angle DAB=\angle ACB \quad (\text{ Each } 90^{\circ})$
$\angle \mathrm{ABD}=\angle \mathrm{CBA} \quad (\text{ Common angle })$
$\therefore \triangle \mathrm{ADB}-\triangle \mathrm{CAB}$ (AA similarity criterion)
$\Rightarrow \frac{AB}{CB}=\frac{BD}{AB}$
$\Rightarrow AB^2=CB \times BD$

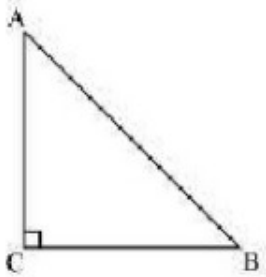
(ii) Let $\angle \mathrm{CAB}=x$
In $\Delta \mathrm{CBA}$,
$\angle CBA=180^{\circ}-90^{\circ}-x$
$\angle CBA=90^{\circ}-x$
Similarly, In $\triangle \mathrm{CAD}$,
$\angle \mathrm{CAD}=90^{\circ}-\angle \mathrm{CAB}$
$=90^{\circ}-x$
$\angle CDA=180^{\circ}-90^{\circ}-(90^{\circ}-x)$
$\angle CDA=x$
In $\triangle \mathrm{CBA}$ and $\Delta \mathrm{CAD}$
$\angle \mathrm{CBA}=\angle \mathrm{CAD}$
$\angle \mathrm{CAB}=\angle \mathrm{CDA}$
$\angle ACB=\angle DCA \quad (\text{Each } 90^{\circ})$
$\therefore \Delta \mathrm{CBA} \sim \Delta \mathrm{CAD} \quad (\text{ By AAA rule })$
$\Rightarrow \frac{\mathrm{AC}}{\mathrm{DC}}=\frac{\mathrm{BC}}{\mathrm{AC}}$
$\Rightarrow \mathrm{AC}^2=\mathrm{DC} \times \mathrm{BC}$

(iii) In $\triangle DCA$ and ΔDAB,
$\angle \mathrm{DCA}=\angle \mathrm{DAB}\,(\mathrm{Each} 90^{\circ})$
$\angle \mathrm{CDA}=\angle \mathrm{ADB}(\text{ Common angle })$
$\therefore \Delta \mathrm{DCA} \sim \Delta \mathrm{DAB}$ (AA similarity criterion)
$\Rightarrow \frac{\mathrm{DC}}{\mathrm{DA}}=\frac{\mathrm{DA}}{\mathrm{DB}}$
$\Rightarrow AD^2=BD \times CD$

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Q4 ABC is an isosceles triangle right angled at C. Prove that $\mathrm{AB}^2=2 \mathrm{AC}^2$.

Answer.

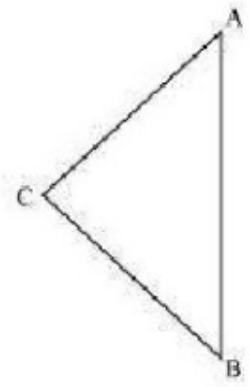


Given that $\triangle \mathrm{ABC}$ is an isosceles triangle.
$\therefore AC=CB$
Applying Pythagoras theorem in $\triangle \mathrm{ABC}$ (i.e., right-angled at point C), we obtain
$\mathrm{AC}^2+\mathrm{CB}^2=\mathrm{AB}^2$
$\Rightarrow \mathrm{AC}^2+\mathrm{AC}^2=\mathrm{AB}^2 \,(AC=CB)$
$\Rightarrow 2 \mathrm{AC}^2=\mathrm{AB}^2$

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Q5 ABC is an isosceles triangle with $\mathrm{AC} = \mathrm{BC}$. If $\mathrm{AB}^2 = 2\mathrm{AC}^2$. prove that ABC is a right triangle.

Answer.

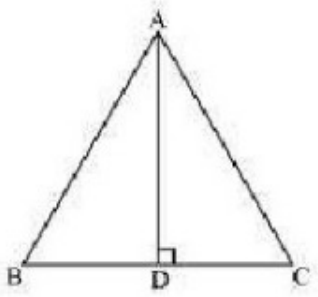


Given that,
$\mathrm{AB}^2 = 2\mathrm{AC}^2$
$\Rightarrow \mathrm{AB}^2 = \mathrm{AC}^2 + \mathrm{AC}^2$
$\Rightarrow \mathrm{AB}^2 = \mathrm{AC}^2 + \mathrm{BC}^2 \quad (\text{As } \mathrm{AC} = \mathrm{BC})$
The triangle is satisfying the pythagoras theorem.
Therefore, the given triangle is a right-angled triangle.

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Q6 ABC is an equilateral triangle of side 2a. Find each of its altitudes.

Answer.

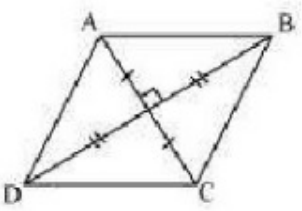


Let AD be the altitude in the given equilateral triangle, $\triangle \mathrm{ABC}$.
We know that altitude bisects the opposite side.
$\therefore BD = DC = a$
In $\triangle \mathrm{ADB}$
Applying pythagoras theorem, we obtain
$\mathrm{AD}^2 + \mathrm{DB}^2 = \mathrm{AB}^2$
$\Rightarrow \mathrm{AD}^2 + a^2 = (2a)^2$
$\Rightarrow \mathrm{AD}^2 + a^2 = 4a^2$
$\Rightarrow \mathrm{AD}^2 = 3a^2$
$\Rightarrow \mathrm{AD} = a\sqrt{3}$
In an equilateral triangle, all the altitudes are equal in length.
Therefore, the length of each altitude will be $\sqrt{3}a$.

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Q7 Prove that the sum of the squares of the sides of a rhombus is equal to the sum of the squares of its diagonals.

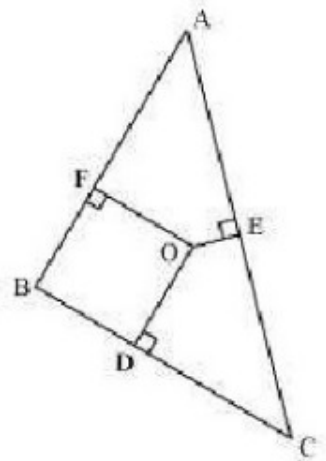
Answer.



In $\triangle \mathrm{AOB}, \Delta \mathrm{BOC}, \Delta \mathrm{COD}, \triangle \mathrm{AOD}$
Applying Pythagoras theorem, we obtain
$\mathrm{AB}^2 = \mathrm{AO}^2 + \mathrm{OB}^2$ ...(1)
$\mathrm{BC}^2 = \mathrm{BO}^2 + \mathrm{OC}^2$ ....(2)
$\mathrm{CD}^2 = \mathrm{CO}^2 + \mathrm{OD}^2$ ...(3)
$\mathrm{AD}^2 = \mathrm{AO}^2 + \mathrm{OD}^2$ ...(4)
Adding all these equations, we obtain
$\mathrm{AB}^2 + \mathrm{BC}^2 + \mathrm{CD}^2 + \mathrm{AD}^2 = 2\left(\mathrm{AO}^2 + \mathrm{OB}^2 + \mathrm{OC}^2 + \mathrm{OD}^2\right)$
$= 2\left(\left(\frac{\mathrm{AC}}{2}\right)^2 + \left(\frac{\mathrm{BD}}{2}\right)^2 + \left(\frac{\mathrm{AC}}{2}\right)^2 + \left(\frac{\mathrm{BD}}{2}\right)^2\right)$
((Diagonals bisect each other) \)
$= 2\left(\frac{(AC)^2}{2} + \frac{(BD)^2}{2}\right)$
$= (AC)^2 + (BD)^2$

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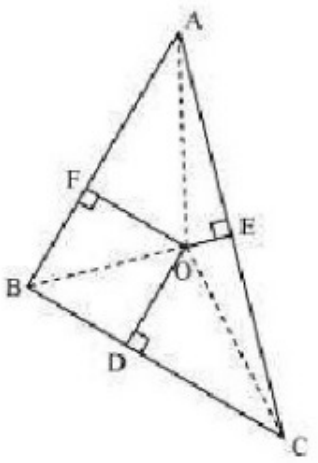
Q8 In Figure, O is a point in the interior of a triangle ABC, OD ⊥ BC, OE ⊥AC and OF ⊥AB. Show that



(i) $\mathrm{OA}^2 + \mathrm{OB}^2 + \mathrm{OC}^2 - \mathrm{OD}^2 - \mathrm{OE}^2 - \mathrm{OF}^2 = \mathrm{AF}^2 + \mathrm{BD}^2 + \mathrm{CE}^2$
(ii) $AF^2 + BD^2 + CE^2 = AE^2 + CD^2 + BF^2$

Answer.

Join OA, OB, and OC.



(i) Applying Pythagoras theorem in $\triangle \mathrm{AOF}$, we obtain

$\mathrm{OA}^2 = \mathrm{OF}^2 + \mathrm{AF}^2$

Similarly, in $\triangle BOD$,

$\mathrm{OB}^2 = \mathrm{OD}^2 + \mathrm{BD}^2$

Similarly, in $\Delta \mathrm{COE}$,

$\mathrm{OC}^2 = \mathrm{OE}^2 + \mathrm{EC}^2$

Adding these equations,

$\mathrm{OA}^2 + \mathrm{OB}^2 + \mathrm{OC}^2 = \mathrm{OF}^2 + \mathrm{AF}^2 + \mathrm{OD}^2 + \mathrm{BD}^2 + \mathrm{OE}^2 + \mathrm{EC}^2$

$\mathrm{OA}^2 + \mathrm{OB}^2 + \mathrm{OC}^2 - \mathrm{OD}^2 - \mathrm{OE}^2 - \mathrm{OF}^2 = \mathrm{AF}^2 + \mathrm{BD}^2 + \mathrm{EC}^2$

(ii) From the above result,

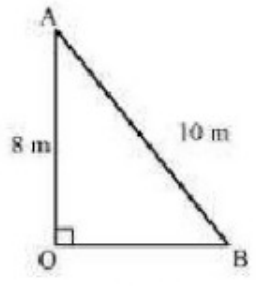
$\mathrm{AF}^2 + \mathrm{BD}^2 + \mathrm{EC}^2 = \left(\mathrm{OA}^2 - \mathrm{OE}^2\right) + \left(\mathrm{OC}^2 - \mathrm{OD}^2\right) + \left(\mathrm{OB}^2 - \mathrm{OF}^2\right)$

$\therefore \mathrm{AF}^2 + \mathrm{BD}^2 + \mathrm{EC}^2 = \mathrm{AE}^2 + \mathrm{CD}^2 + \mathrm{BF}^2$

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Q9 A ladder 10 m long reaches a window 8 m above the ground. Find the distance of the foot of the ladder from base of the wall.

Answer.



Let OA be the wall and AB be the ladder.

Therefore, by Pythagoras theorem,

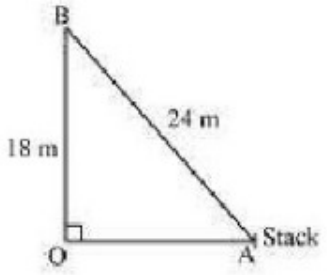
$\mathrm{AB}^2 = \mathrm{OA}^2 + \mathrm{BO}^2$

$(10\text{m})^2 = (8\text{m})^2 + \text{OB}^2$
$100\text{m}^2 = 64\text{m}^2 + \text{OB}^2$
$\text{OB}^2 = 36\text{m}^2$
$\text{OB} = 6\text{m}$
Therefore, the distance of the foot of the ladder from the base of the wall is 6 m.

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Q10 A guy wire attached to a vertical pole of height 18 m is 24 m long and has a stake attached to the other end. How far from the base of the pole should the stake be driven so that the wire will be taut?

Answer.

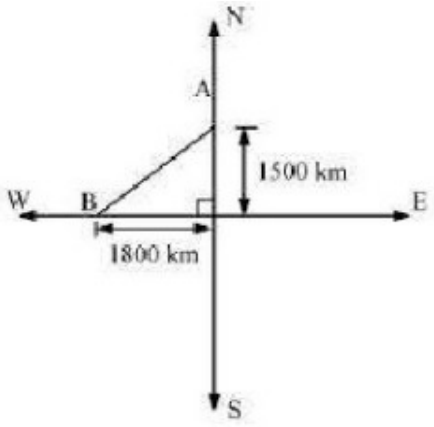


Let OB be the pole and AB be the wire.
By Pythagoras theorem,
$\text{AB}^2 = \text{OB}^2 + \text{OA}^2$
$(24\text{m})^2 = (18\text{m})^2 + \text{OA}^2$
$\text{OA}^2 = (576 - 324)\text{m}^2 = 252\text{m}^2$
$\text{OA} = \sqrt{252}\text{m} = \sqrt{6 \times 6 \times 7}\text{m} = 6\sqrt{7}\text{m}$
Therefore, the distance from the base is $6\sqrt{7}\text{m}$.

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Q11 An aeroplane leaves an airport and flies due north at a speed of 1000 km per hour. At the same time, another aeroplane leaves the same airport and flies due west at a speed of 1200 km per hour. How far apart will be the two planes after $1\frac{1}{2}$ hours?

Answer.



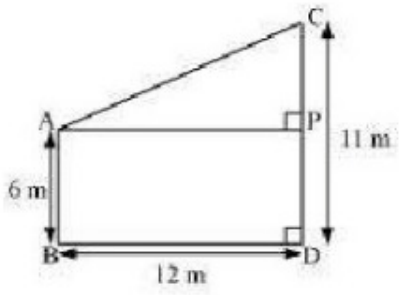
Distance travelled by the plane flying towards north in $1\frac{1}{2}\text{hrs}$
$= 1,000 \times 1\frac{1}{2} = 1,500\text{km}$

Similarly, distance travelled vu the plane flying towards west in $1\frac{1}{2}\text{hrs}$
$= 1,200 \times 1\frac{1}{2} = 1,800\text{km}$
Let these distances be represented by OA and OB respectively.
Applying Pythagoras as theorem,
Distance between these planes after $1\frac{1}{2}hrs$, $AB = \sqrt{OA^2 + OB^2}$
$= \left(\sqrt{(1,500)^2 + (1,800)^2}\right)\text{km} = (\sqrt{2250000 + 3240000})\text{km}$
$= (\sqrt{5490000})\text{km} = (\sqrt{9 \times 610000})\text{km} = 300\sqrt{61}\text{km}$
Therefore, the distance between these planes will be $300\sqrt{61}\text{km}$ after $1\frac{1}{2}\text{hrs}$

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Q12 Two poles of heights 6 m and 11 m stand on a plane ground. If the distance between the feet of the poles is 12 m, find the distance between their tops.

Answer.

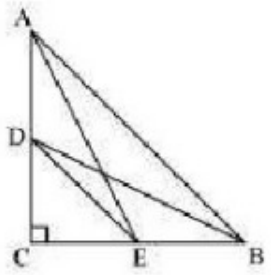


Let CD and AB be the poles of height 11 m and 6 m.
Therefore, Cp = 11 - 6 = 5 m
From the figure, it can be observed that AP = 12m
Applying Pythagoras theorem for $\triangle\text{APC}$, we obtain
$\text{AP}^2 + \text{PC}^2 = \text{AC}^2$
$(12\text{m})^2 + (5\text{m})^2 = \text{AC}^2$
$\text{AC}^2 = (144 + 25)\text{m}^2 = 169\text{m}^2$
$\text{AC} = 13\text{m}$
Therefore, the distance between their tops is 13 m.

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Q13 D and E are points on the sides CA and CB respectively of a triangle ABC right angled at C. Prove that $\text{AE}^2 + \text{BD}^2 = \text{AB}^2 + \text{DE}^2$.

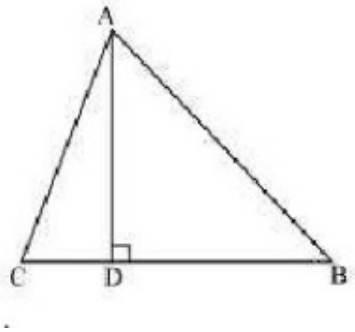
Answer.



Applying Pythagoras theorem in $\triangle \mathrm{ACE}$, we obtain
$\mathrm{AC}^2 + \mathrm{CE}^2 = \mathrm{AE}^2$ ...(1)
Applying Pythagoras theorem in $\triangle BCD$, we obtain
$\mathrm{BC}^2 + \mathrm{CD}^2 = \mathrm{BD}^2$ ..(2)
Using equation (1) and equation (2), we obtain
$\mathrm{AC}^2 + \mathrm{CE}^2 + \mathrm{BC}^2 + \mathrm{CD}^2 = \mathrm{AE}^2 + \mathrm{BD}^2$ ...(3)
Applying Pythagoras theorem in $\Delta \mathrm{CDE}$, we obtain
$\mathrm{AB}^2 = \mathrm{AC}^2 + \mathrm{CB}^2$
Putting the values in equation (3), we obtain
$\mathrm{DE}^2 + \mathrm{AB}^2 = \mathrm{AE}^2 + \mathrm{BD}^2$

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Q14 The perpendicular from A on side BC of a Δ ABC intersects BC at D such that DB = 3 CD (see Figure). Prove that $2\mathrm{AB}^2 = 2\mathrm{AC}^2 + \mathrm{BC}^2$.



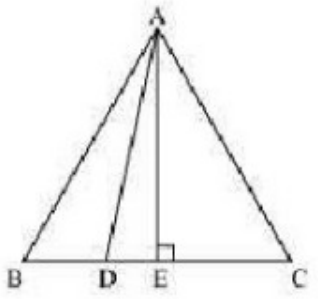
Answer. Applying Pythagoras theorem for $\triangle \mathrm{ACD}$, we obtain
$\mathrm{AC}^2 = \mathrm{AD}^2 + \mathrm{DC}^2$
$\mathrm{AD}^2 = \mathrm{AC}^2 - \mathrm{DC}^2$ ...(1)
Applying Pythagoras theorem in $\triangle \mathrm{ABD}$, we obtain
$\mathrm{AB}^2 = \mathrm{AD}^2 + \mathrm{DB}^2$
$\mathrm{AD}^2 = \mathrm{AB}^2 - \mathrm{DB}^2$ ...(2)
From equation (1) and equation (2), we obtain
$\mathrm{AC}^2 - \mathrm{DC}^2 = \mathrm{AB}^2 - \mathrm{DB}^2 \quad \ldots (3)$
It is given that 3DC = DB
$\therefore \mathrm{DC} = \frac{\mathrm{BC}}{4}$ and $\mathrm{DB} = \frac{3\mathrm{BC}}{4}$
Putting these values in equation (3), we obtain,
$\mathrm{AC}^2 - \left(\frac{\mathrm{BC}}{4}\right)^2 = \mathrm{AB}^2 - \left(\frac{3\mathrm{BC}}{4}\right)^2$
$\mathrm{AC}^2 - \frac{\mathrm{BC}^2}{16} = \mathrm{AB}^2 - \frac{9\mathrm{BC}^2}{16}$

$16\mathrm{AC}^2 - \mathrm{BC}^2 = 16\mathrm{AB}^2 - 9\mathrm{BC}^2$
$16\mathrm{AB}^2 - 16\mathrm{AC}^2 = 8\mathrm{BC}^2$
$2\mathrm{AB}^2 = 2\mathrm{AC}^2 + \mathrm{BC}^2$

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Q15 In an equilateral triangle ABC, D is a point on side BC such that BD = 1/3 BC. Prove that $9\mathrm{AD}^2 = 7\mathrm{AB}^2$.

Answer.

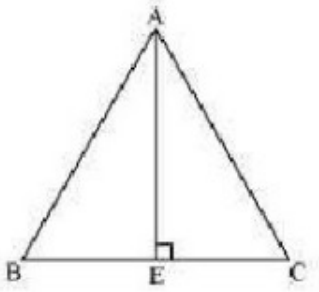


Let the side of the equilateral triangle be a, and AE be the altitude of $\triangle \mathrm{ABC}$
$\therefore BE = EC = \frac{BC}{2} = \frac{a}{2}$
And, $AE = \frac{a\sqrt{3}}{2}$
Given that, BD = 1/3 BC
$\therefore BD = \frac{a}{3}$
$\mathrm{DE} = \mathrm{BE} - \mathrm{BD} = \frac{a}{2} - \frac{a}{3} = \frac{a}{6}$
Applying Pythagoras theorem in $\triangle \mathrm{ADE}$, we obtain
$\mathrm{AD}^2 = \mathrm{AE}^2 + \mathrm{DE}^2$
$\mathrm{AD}^2 = \left(\frac{a\sqrt{3}}{2}\right)^2 + \left(\frac{a}{6}\right)^2$
$= \left(\frac{3a^2}{4}\right) + \left(\frac{a^2}{36}\right)$
$= \frac{28a^2}{36}$
$= \frac{7}{9}\mathrm{AB}^2$
$\Rightarrow 9\mathrm{AD}^2 = 7\mathrm{AB}^2$

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Q16 In an equilateral triangle, prove that three times the square of one side is equal to four times the square of one of its altitudes.

Answer.



Let the side of the equilateral triangle be a, and AE be the altitude of $\triangle \mathrm{ABC}$

$\therefore BE = EC = \frac{BC}{2} = \frac{a}{2}$

Applying Pythagoras theorem in $\triangle \mathrm{ABE}$, we obtain

$AB^2 = AE^2 + BE^2$

$a^2 = \mathrm{AE}^2 + \left(\frac{a}{2}\right)^2$

$\mathrm{AE}^2 = a^2 - \frac{a^2}{4}$

$\mathrm{AE}^2 = \frac{3a^2}{4}$

$4\mathrm{AE}^2 = 3\mathrm{a}^2$

$\Rightarrow 4 \times (\text{ Square of altitude }) = 3 \times$ (Square of one side)

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Q17 Tick the correct answer and justify : In ΔABC, AB = $6\sqrt{3}$ cm, AC = 12 cm and BC = 6 cm. The angle B is :

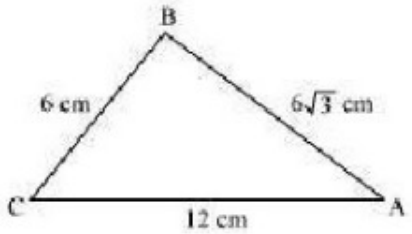
(A) 120°

(B) 60°

(C) 90°

(D) 45°

Answer.



Given that, AB = $= 6\sqrt{3}$cm, AC = 12 cm, and BC = 6 cm

It can be observed that

$AB^2 = 108$

$AC^2 = 144$

And, $BC^2 = 36$

$AB^2 + BC^2 = AC^2$

The given triangle, $\triangle \mathrm{ABC}$, is satisfying Pythagoras theorem.

Therefore, the triangle is a right triangle, right-angled at B.

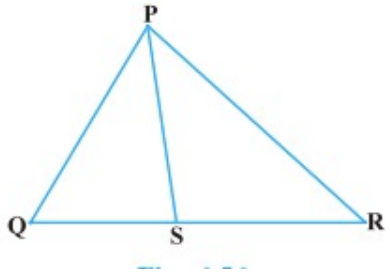
$\therefore \angle B = 90^\circ$

Hence, the correct answer is (C)

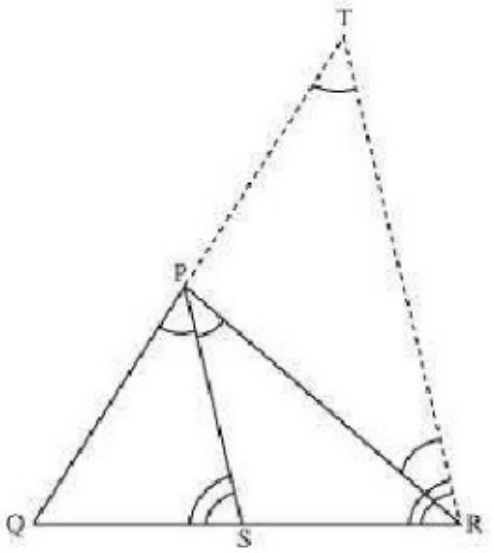
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Exercise 6.6

Q1 In Figure, PS is the bisector of ∠ QPR of Δ PQR. Prove that $\frac{\mathrm{QS}}{\mathrm{SR}} = \frac{\mathrm{PQ}}{\mathrm{PR}}$



Answer.



Let us draw a line segment RT parallel to SP which intersects extended line segment QP at point T,
Given that, PS is the angle bisector of $\angle \mathrm{QPR}$.

$\angle \mathrm{QPS} = \angle \mathrm{SPR} \ldots (1)$

By construction,

$\angle SPR = \angle PRT (ASPS || TR) \ldots (2)$

$\angle QPS = \angle QTR (ASPS || TR) \ldots (3)$

Using these equations, we obtain

$\angle \mathrm{PRT} = \angle \mathrm{QTR}$

$\therefore \mathrm{PT} = \mathrm{PR}$

By construction,

PS || TR

By using basic proportionality theorem for $\Delta \mathrm{QTR}$,

$\frac{\mathrm{QS}}{\mathrm{SR}} = \frac{\mathrm{QP}}{\mathrm{PT}}$

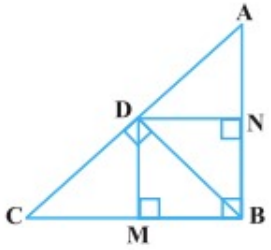
$\Rightarrow \frac{\mathrm{QS}}{\mathrm{SR}} = \frac{\mathrm{PQ}}{\mathrm{PR}}$ (PT =TR)

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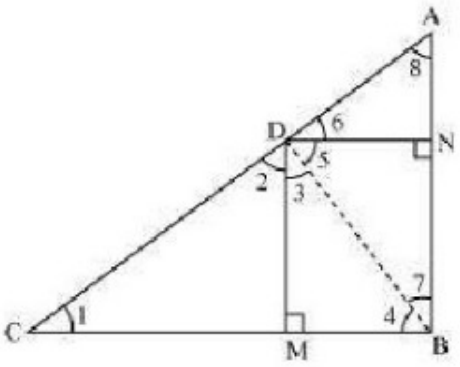
Q2 In Figure, D is a point on hypotenuse AC of ΔABC, such that BD ⊥AC, DM ⊥ BC and DN ⊥ AB. Prove that :

(i) $\mathrm{DM}^2 = \mathrm{DN} \cdot \mathrm{MC}$

(ii) $\mathrm{DN}^2 = \mathrm{DM}.\ \mathrm{AN}$



Answer. (i) Let us join DB.



We have, DN || CB, DM || AB, and $\angle B = 90^\circ$

$\therefore$ DMBN is a rectangle.

$\therefore \mathrm{DN} = \mathrm{MB}$ and $\mathrm{DM} = \mathrm{NB}$

The condition to be proved is the case when D is the foot of the perpendicular drawn from B to AC.

$\therefore \angle CDB = 90^\circ$

$\Rightarrow \angle 2 + \angle 3 = 90^\circ \ldots (1)$

In $\triangle$CDM,

$\angle 1 + \angle 2 + \angle \mathrm{DMC} = 180^\circ$

$\Rightarrow \angle 1 + \angle 2 = 90^\circ \ldots (2)$

In $\triangle$DMB,

$\angle 3 + \angle DMB + \angle 4 = 180^\circ$

$\Rightarrow \angle 3 + \angle 4 = 90^\circ \ldots (3)$

From equation (1) and (2), we obtain

$\angle 2 = \angle 4$

In $\triangle \mathrm{DCM}$ and $\triangle \mathrm{BDM}$,

$\angle 1 = \angle 3$ (Proved above)

$\angle 2 = \angle 4$(Proved above)

$\therefore \triangle \mathrm{DCM} \sim \triangle \mathrm{BDM}$(AA similarity criterion)

$\Rightarrow \frac{\mathrm{BM}}{\mathrm{DM}} = \frac{\mathrm{DM}}{\mathrm{MC}}$

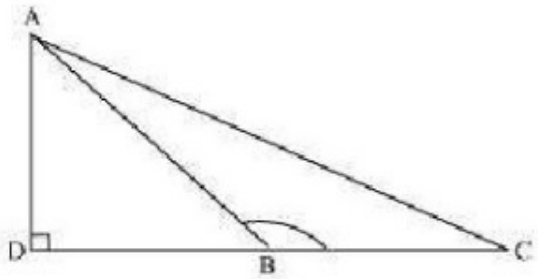
$\Rightarrow \frac{\mathrm{DN}}{\mathrm{DM}} = \frac{\mathrm{DM}}{\mathrm{MC}} \quad (\mathrm{BM} = \mathrm{DN})$

$\Rightarrow \mathrm{DM}^2 = \mathrm{DN} \times \mathrm{MC}$

(ii) In right triangle DBN,
$\angle 5 + \angle 7 = 90^\circ \ldots (4)$
$In right triangle DAN$,
$\angle 6 + \angle 8 = 90^\circ \ldots (5)$
D is the foot of the perpendicular drawn from B to AC.
$\therefore \angle ADB = 90^\circ$
$\Rightarrow \angle 5 + \angle 6 = 90^\circ \ldots (6)$
From equation (5) and (6), we obtain
$\angle 8 = \angle 5$
In $\triangle$DNA and $\triangle$BND,
$\angle 6 = \angle 7$ (Proved above)
$\angle 8 = \angle 5$ (Proved above)
$\therefore \triangle \mathrm{DNA} \sim \Delta \mathrm{BND}$ (AA similarity criterion)
$\Rightarrow \frac{\mathrm{AN}}{\mathrm{DN}} = \frac{\mathrm{DN}}{\mathrm{NB}}$
$\Rightarrow \mathrm{DN}^2 = \mathrm{AN} \times \mathrm{NB}$
$\Rightarrow \mathrm{DN}^2 = \mathrm{AN} \times \mathrm{DM} (\mathrm{ASNB} = \mathrm{DM})$

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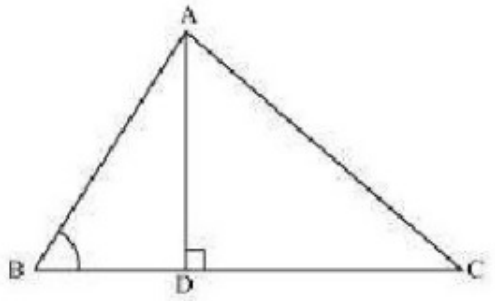
Q3 In Figure, ABC is a triangle in which ∠ABC > 90° and AD ⊥ CB produced. Prove that
$\mathrm{AC}^2 = \mathrm{AB}^2 + \mathrm{BC}^2 + 2\mathrm{BC}.\mathrm{BD}$



Answer. Applying Pythagoras theorem in $\triangle$ADB, we obtain
$AB^2 = AD^2 + DB^2 \ldots (1)$
Applying Pythagoras theorem in $\triangle$ACD, we obtain
$AC^2 = AD^2 + DC^2$
$AC^2 = AD^2 + (DB + BC)^2$
$AC^2 = AD^2 + DB^2 + BC^2 + 2DB \times BC$
$AC^2 = AB^2 + BC^2 + 2DB \times BC [\text{ Using equation } (1)]$

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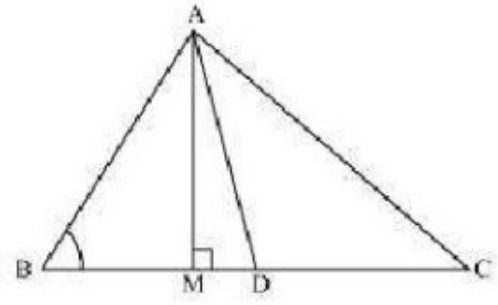
Q4 In Figure, ABC is a triangle in which ∠ ABC < 90° and AD ⊥ BC. Prove that
$\mathrm{AC}^2 = \mathrm{AB}^2 + \mathrm{BC}^2 - 2\mathrm{BC} \cdot \mathrm{BD}.$



Answer. Applying Pythagoras theorem in $\triangle \mathrm{ADB}$, we obtain
$AD^2 + DB^2 = AB^2$
$\Rightarrow AD^2 = AB^2 - DB^2 \ldots (1)$
Applying Pythagoras theorem in $\triangle \mathrm{ADC}$, we obtain
$\mathrm{AD}^2 + \mathrm{DC}^2 = \mathrm{AC}^2$
$\mathrm{AB}^2 - \mathrm{BD}^2 + \mathrm{DC}^2 = \mathrm{AC}^2 [\text{ Using equation } (1)]$
$\mathrm{AB}^2 - \mathrm{BD}^2 + (\mathrm{BC} - \mathrm{BD})^2 = \mathrm{AC}^2$
$\mathrm{AC}^2 = \mathrm{AB}^2 + (\mathrm{BC} - \mathrm{BD})^2 = \mathrm{AC}^2$
$\mathrm{AC}^2 = \mathrm{AB}^2 - \mathrm{BD}^2 + \mathrm{BC}^2 + \mathrm{BD}^2 - 2\mathrm{BC} \times \mathrm{BD}$
$= \mathrm{AB}^2 + \mathrm{BC}^2 - 2\mathrm{BC} \times \mathrm{BD}$

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Q5 In Figure, AD is a median of a triangle ABC and AM $\perp$ BC. Prove that :
(i) $\mathrm{AC}^2 = \mathrm{AD}^2 + \mathrm{BC} \cdot \mathrm{DM} + \left(\frac{\mathrm{BC}}{2}\right)^2$
(ii) $\mathrm{AB}^2 = \mathrm{AD}^2 - \mathrm{BC} \cdot \mathrm{DM} + \left(\frac{\mathrm{BC}}{2}\right)^2$
(iii) $\mathrm{AC}^2 + \mathrm{AB}^2 = 2\mathrm{AD}^2 + \frac{1}{2}\mathrm{BC}^2$



Answer. (i) Applying Pythagoras theorem in $\triangle \mathrm{AMD}$, we obtain
$AM^2 + MD^2 = AD^2 \ldots (1)$
Applying Pythagoras theorem in $\triangle \mathrm{AMC}$, we obtain
$\mathrm{AM}^2 + \mathrm{MC}^2 = \mathrm{AC}^2$
$\mathrm{AM}^2 + (\mathrm{MD} + \mathrm{DC})^2 = \mathrm{AC}^2$
$\left(\mathrm{AM}^2 + \mathrm{MD}^2\right) + \mathrm{DC}^2 + 2\mathrm{MD} \cdot \mathrm{DC} = \mathrm{AC}^2$
$AD^2 + DC^2 + 2MD.\, DC = AC^2 [\text{ Using equation } (1)]$

Using the result, $\mathrm{DC}=\frac{\mathrm{BC}}{2}$,, we obtain

$\mathrm{AD}^2+\left(\frac{\mathrm{BC}}{2}\right)^2+2\mathrm{MD}\cdot\left(\frac{\mathrm{BC}}{2}\right)=\mathrm{AC}^2$

$\mathrm{AD}^2+\left(\frac{\mathrm{BC}}{2}\right)^2+\mathrm{MD}\times\mathrm{BC}=\mathrm{AC}^2$

(ii) Applying Pythagoras theorem in $\triangle ABM$, we obtain

$AB^2=AM^2+MB^2$

$=\left(AD^2-DM^2\right)+MB^2$

$=\left(AD^2-DM^2\right)+(BD-MD)^2$

$=AD^2-DM^2+BD^2+MD^2-2BD\times MD$

$=AD^2+BD^2-2BD\times MD$

$=\mathrm{AD}^2+\left(\frac{\mathrm{BC}}{2}\right)^2-2\left(\frac{\mathrm{BC}}{2}\right)\times\mathrm{MD}$

$=\mathrm{AD}^2+\left(\frac{\mathrm{BC}}{2}\right)^2-\mathrm{BC}\times\mathrm{MD}$

(iii) Applying Pythagoras theorem in $\triangle\mathrm{ABM}$, we obtain

$AM^2+MB^2=AB^2\ldots(1)$

Applying Pythagoras theorem in $\triangle\mathrm{AMC}$, we obtain

$\mathrm{AM}^2+\mathrm{MC}^2=\mathrm{AC}^2\ldots(2)$

Adding equations (1) and (2), we obtain

$2\mathrm{AM}^2+\mathrm{MB}^2+\mathrm{MC}^2=\mathrm{AB}^2+\mathrm{AC}^2$

$2\mathrm{AM}^2+(\mathrm{BD}-\mathrm{DM})^2+(\mathrm{MD}+\mathrm{DC})^2=\mathrm{AB}^2+\mathrm{AC}^2$

$2\mathrm{AM}^2+\mathrm{BD}^2+\mathrm{DM}^2-2\mathrm{BD}\cdot\mathrm{DM}+\mathrm{MD}^2+\mathrm{DC}^2+\mathrm{AC}^2$

$2\mathrm{AM}^2+2\mathrm{MD}^2+\mathrm{BD}^2+\mathrm{DC}^2+2\mathrm{MD}(-\mathrm{BD}+\mathrm{DC})=\mathrm{AB}^2+\mathrm{AC}^2$

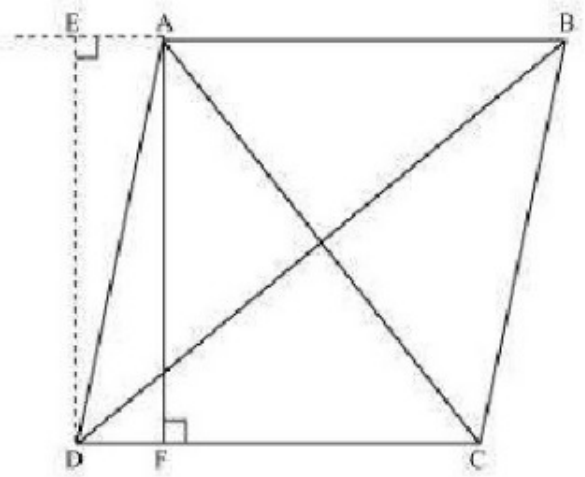
$2\left(\mathrm{AM}^2+\mathrm{MD}^2\right)+\left(\frac{\mathrm{BC}}{2}\right)^2+\left(\frac{\mathrm{BC}}{2}\right)^2+2\mathrm{MD}\left(-\frac{\mathrm{BC}}{2}+\frac{\mathrm{BC}}{2}\right)=\mathrm{AB}^2+\mathrm{AC}^2$

$2\mathrm{AD}^2+\frac{\mathrm{BC}^2}{2}=\mathrm{AB}^2+\mathrm{AC}^2$

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Q6 Prove that the sum of the squares of the diagonals of parallelogram is equal to the sum of the squares of its sides.

Answer.



Let ABCD be a parallelogram.
Let us draw perpendicular DE on extended side AB, and AF on side DC.
Applying Pythagoras theorem in $\triangle DEA$, we obtain
$\mathrm{DE}^2 + \mathrm{EA}^2 = \mathrm{DA}^2 \ldots (i)$
Applying Pythagoras theorem in $\triangle DEA$, we obtain
$\mathrm{DE}^2 + \mathrm{EA}^2 = \mathrm{DA}^2 \ldots$ (i)
Applying Pythagoras theorem in $\triangle \mathrm{DEB}$, we obtain
$\mathrm{DE}^2 + \mathrm{EB}^2 = \mathrm{DB}^2$
$\mathrm{DE}^2 + (\mathrm{EA} + \mathrm{AB})^2 = \mathrm{DB}^2$
$\left(\mathrm{DE}^2 + \mathrm{EA}^2\right) + \mathrm{AB}^2 + 2\mathrm{EA} \times \mathrm{AB} = \mathrm{DB}^2$
$\mathrm{DA}^2 + \mathrm{AB}^2 + 2\mathrm{EA} \times \mathrm{AB} = \mathrm{DB}^2 \ldots$ (ii)
Applying Pythagoras theorem in $\triangle \mathrm{ADF}$, we obtain
$\mathrm{AD}^2 = \mathrm{AF}^2 + \mathrm{FD}^2$
Applying Pythagoras theorem in $\triangle \mathrm{AFC}$, we obtain
$AC^2 = AF^2 + FC^2$
$= \mathrm{AF}^2 + (\mathrm{DC} - \mathrm{FD})^2$
$= \mathrm{AF}^2 + \mathrm{DC}^2 + \mathrm{FD}^2 - 2\mathrm{DC} \times \mathrm{FD}$
$= \left(\mathrm{AF}^2 + \mathrm{FD}^2\right) + \mathrm{DC}^2 - 2\mathrm{DC} \times \mathrm{FD}$
$\mathrm{AC}^2 = \mathrm{AD}^2 + \mathrm{DC}^2 - 2\mathrm{DC} \times \mathrm{FD} \ldots (ii)$
Since ABCD is a parallelogram,
$AB = CD \ldots (iv)$
And, $BC = AD \ldots (v)$
In $\triangle \mathrm{DEA}$ and $\triangle \mathrm{ADF}$
$\angle DEA = \angle AFD\,(Both 90^\circ)$
$\angle EAD = \angle ADF (EA \| DF)$
$AD = AD(\text{ Common })$
$\therefore \Delta EAD \cong \triangle FDA$ (AAS congruence criterion)
$\Rightarrow EA = DF \ldots (vi)$
Adding equations (i) and (iii), we obtain

$\mathrm{DA}^2 + \mathrm{AB}^2 + 2\mathrm{EA} \times \mathrm{AB} + \mathrm{AD}^2 + \mathrm{DC}^2 - 2\mathrm{DC} \times \mathrm{FD} = \mathrm{DB}^2 + \mathrm{AC}^2$

$\mathrm{DA}^2 + \mathrm{AB}^2 + \mathrm{AD}^2 + \mathrm{DC}^2 + 2\mathrm{EA} \times \mathrm{AB} - 2\mathrm{DC} \times \mathrm{FD} = \mathrm{DB}^2 + \mathrm{AC}^2$

$\mathrm{BC}^2 + \mathrm{AB}^2 + \mathrm{AD}^2 + \mathrm{DC}^2 + 2\mathrm{EA} \times \mathrm{AB} - 2\mathrm{AB} \times \mathrm{EA} = \mathrm{DB}^2 + \mathrm{AC}^2$

[Using equations (iv) and (vi)]

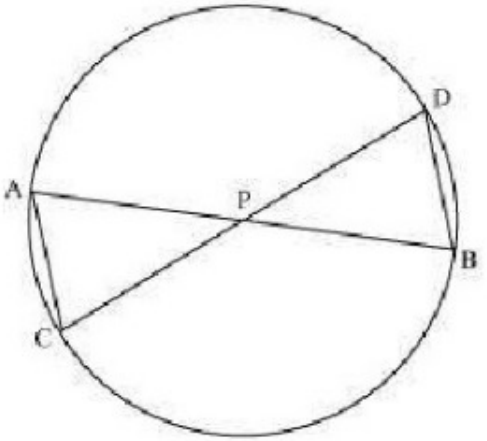
$AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2$

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Q7 In Figure, two chords AB and CD intersect each other at the point P. Prove that :

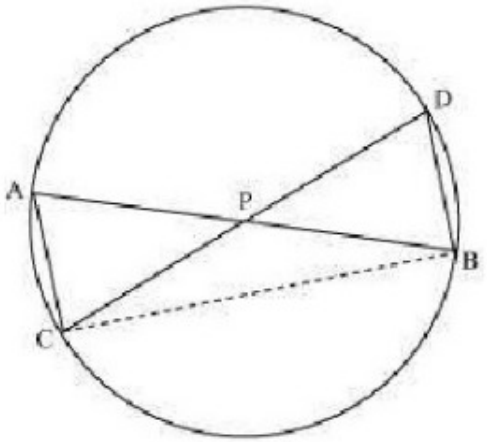
(i) ΔAPC ~ Δ DPB

(ii) AP . PB = CP . DP



Answer.

Let us join CB.



(i) In $\triangle APC$ and ΔDPB,

$\angle \mathrm{APC} = \angle \mathrm{DPB}$ (Vertically opposite angles)

$\angle \mathrm{CAP} = \angle \mathrm{BDP}$ (Angles in the same segment for chord CB)

$\triangle \mathrm{APC} \sim \Delta \mathrm{DPB}$ (By AA similarity criterion)

(ii) We have already proved that

$\triangle \mathrm{APC} \sim \Delta \mathrm{DPB}$

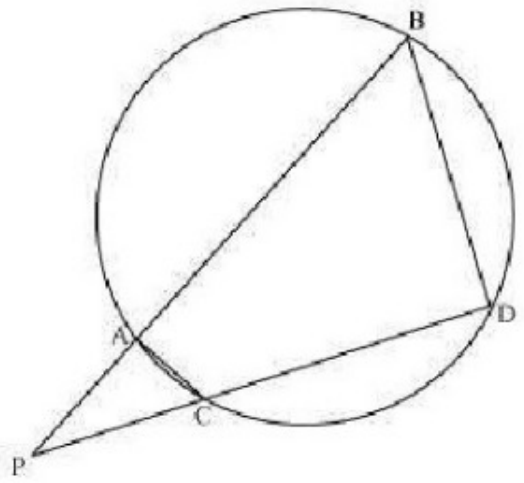
We know that the corresponding sides of similar triangles are proportional.

$\therefore \frac{\mathrm{AP}}{\mathrm{DP}} = \frac{\mathrm{PC}}{\mathrm{PB}} = \frac{\mathrm{CA}}{\mathrm{BD}}$

$\Rightarrow \frac{AP}{DP} = \frac{PC}{PB}$
$\therefore \mathrm{AP.PB} = \mathrm{PC.DP}$

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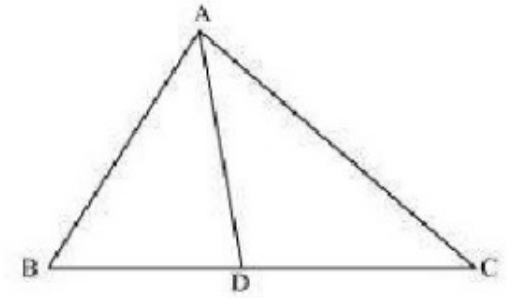
Q8 In Figure, two chords AB and CD of a circle intersect each other at the point P (when produced) outside the circle. Prove that
(i) Δ PAC ~ Δ PDB
(ii) PA . PB = PC . PD



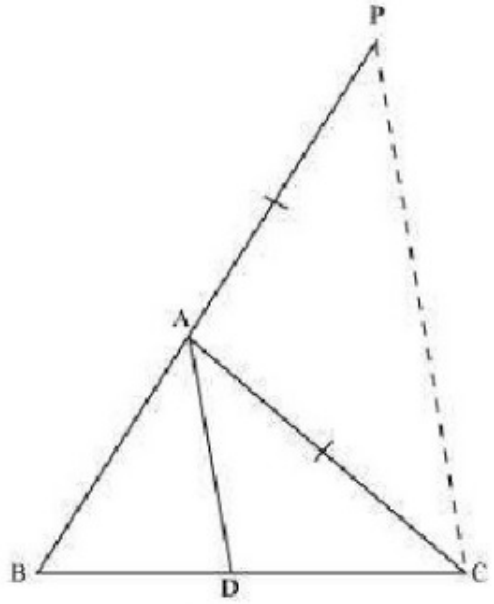
Answer. (i) In $\triangle \mathrm{PAC}$ and $\Delta \mathrm{PDB}$,
$\angle P = \angle P$(Common)
$\angle \mathrm{PAC} = \angle \mathrm{PDB}$ (Exterior angle of a cyclic quadrilateral is $\angle \mathrm{PCA} = \angle \mathrm{PBD}$ equal to the opposite interior angle)
$\therefore \triangle \mathrm{PAC} \sim \Delta \mathrm{PDB}$
(ii) We know that the corresponding sides of similar triangles are proportional.
$\therefore \frac{\mathrm{PA}}{\mathrm{PD}} = \frac{\mathrm{AC}}{\mathrm{DB}} = \frac{\mathrm{PC}}{\mathrm{PB}}$
$\Rightarrow \frac{\mathrm{PA}}{\mathrm{PD}} = \frac{\mathrm{PC}}{\mathrm{PB}}$
$\therefore \mathrm{PA.PB} = \mathrm{PC.PD}$

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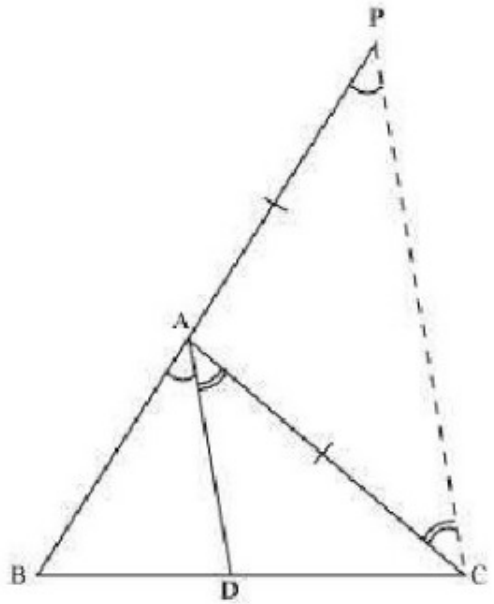
Q9 In Figure, D is a point on side BC of Δ ABC such that $\frac{\mathrm{BD}}{\mathrm{CD}} = \frac{\mathrm{AB}}{\mathrm{AC}}$ Prove that AD is the bisector of ∠ BAC.



Answer. Let us extend BA to P such that AP =AC, Join PC.



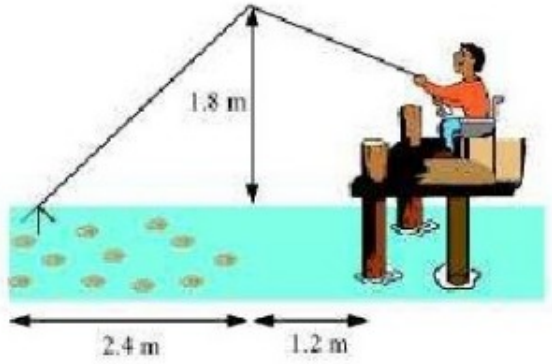
It is given that,
$\frac{\mathrm{BD}}{\mathrm{CD}}=\frac{\mathrm{AB}}{\mathrm{AC}}$
$\Rightarrow \frac{\mathrm{BD}}{\mathrm{CD}}=\frac{\mathrm{AP}}{\mathrm{AC}}$
By using the converse of basic proportionality theorem, we obtain AD || PC
$\mathrm{AD} \| \mathrm{PC}$
$\Rightarrow \angle BAD=\angle APC(\text{ Corresponding angles }) \ldots$ (1)
And $\angle DAC=\angle ACP$ (Alternate interior angles)...(2)



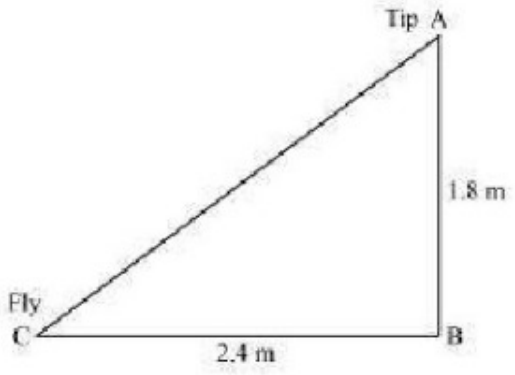
By construction, we have
$AP=AC$
$\Rightarrow \angle APC=\angle ACP \ldots(3)$
On comparing equations (1), (2), and (3), we obtain
$\angle BAD=\angle APC$
$\Rightarrow \mathrm{AD}$ is the bisector of the angle BAC.

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Q10 Nazima is fly fishing in a stream. The tip of her fishing rod is 1.8 m above the surface of the water and the fly at the end of the string rests on the water 3.6 m away and 2.4 m from a point directly under the tip of the rod. Assuming that her string (from the tip of her rod to the fly) is taut, how much string does she have out (see Figure)? If she pulls in the string at the rate of 5 cm per second, what will be the horizontal distance of the fly from her after 12 seconds?



Answer.



Let AB be the height of the tip of the fishing rod from the water surface. Let BC be the horizontal distance of the fly from the tip of the fishing rod.
Then, AC is the length of the string.
AC can be found by applying Pythagoras theorem in $\triangle \mathrm{ABC}$

$AC^2 = AB^2 + BC^2$

$AB^2 = (1.8m)^2 + (2.4m)^2$

$AB^2 = (3.24 + 5.76)m^2$

$AB^2 = 9.00m^2$

$\Rightarrow AB = \sqrt{9}\mathrm{m} = 3\mathrm{m}$

Thus, the length of the string out is 3 m.
She pulls the string at the rate of 5 cm per second.
Therefore, string pulled in 12 seconds = 12 x 5 = 60 cm = 0.6 m.

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