NCERT SOLUTIONS

CLASS - 10th



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Class : 10th
Subject : Maths
Chapter : 10
Chapter Name : Circles

Exercise 10.1

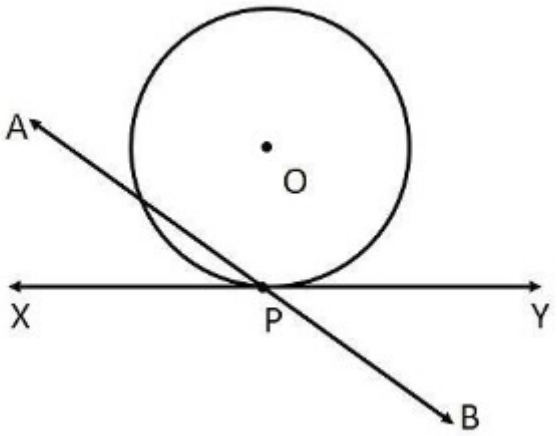
Q1 How many tangents can a circle have?

Answer. Tangent is a line that intersects the circle at one point There are infinite number of points on circle At every point, there is one tangent
Hence, there are infinite number of tangents in a circle.

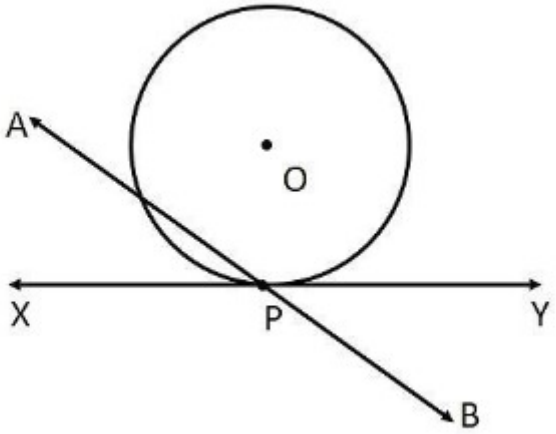
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Q2 Fill in the blanks :
(i) A tangent to a circle intersects it in point (s).
(ii) A line intersecting a circle in two points is called a .
(iii) A circle can have parallel tangents at the most.
(iv) The common point of a tangent to a circle and the circle is called .

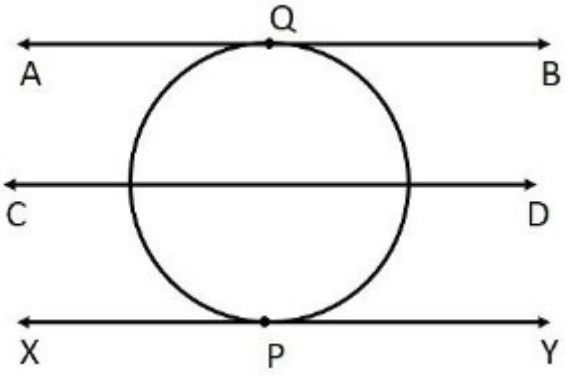
Answer. (i)



Note only there can be one tangent at point P i.e. tangent XY If we try to make more than one line at point P example AB,it becomes a secant (as it intersects at more than one point)
(ii)

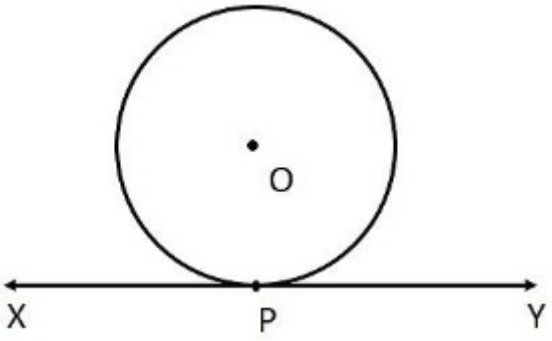


(iii)



Note only two tangents are possible AB and XYCD is not a tangent it is a secant as it intersects at 2 points

(iv)



Here P is the point of contact.

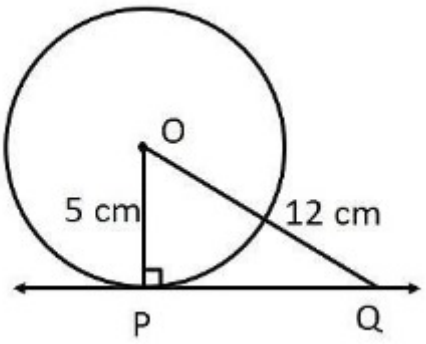
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Q3 A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q so that OQ = 12 cm. Length PQ is :

(A) 12 cm

(B) 13 cm
(C) 8.5 cm
(D) $\sqrt{119}$cm

Answer. Given OP radius 5 cm & OQ = 12 cm



Since PQ is a tangent,
$\mathrm{OP} \perp \mathrm{PQ}$
$\text{So, } \angle \mathrm{OPQ} = 90^{\circ}$
$\text{Hence, } \Delta \mathrm{OPQ} \text{ is a right triangle}$
$\Delta \mathrm{OPQ}$
Using Pythagoras theorem
$(\text{Hypotenuse})^2 = (\text{Height})^2 + (\text{Base})^2$
$\mathrm{OQ}^2 = (\mathrm{OP})^2 + (\mathrm{PQ})^2$
$12^2 = 5^2 + (\mathrm{PQ})^2$
$144 = 25 + (\mathrm{PQ})^2$
$144 - 25 = \mathrm{PQ}^2$
$\mathrm{PQ} = \sqrt{119}$
So (D) is correct

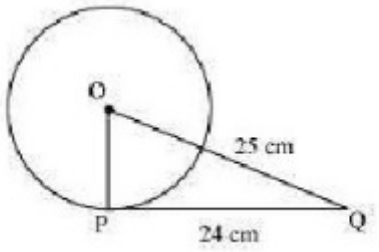
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Exercise 10.2

Q1 From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. The radius of the circle is
(A) 7 cm
(B) 12 cm
(C) 15 cm

(D) 24.5 cm

Answer.



Let O be the centre of the circle.
Given that,
OQ=25cm and PQ=24 cm
As the radius is perpendicular to the tangent at the point the contact
Therefore ,$OP \perp PQ$
Applying pythagoras theorem in triangle OPQ, we obtain
$\mathrm{OP}^2 + \mathrm{PQ}^2 = \mathrm{OQ}^2$
$\mathrm{OP}^2 + 24^2 = 25^2$
$\mathrm{OP}^2 = 625 - 576$
$\mathrm{OP}^2 = 49$
OP=7
Therefore , the radius of circle is 7 cm.
Hence ,alternative (A) is correct.

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Q2 In Fig. 10.11, if TP and TQ are the two tangents to a circle with centre O so that $\angle \mathrm{PTQ}$= 110°, then $\angle \mathrm{PTQ}$ is equal to
(A) 60°
(B) 70°
(C) 80°
(D) 90°

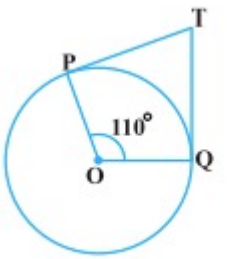


Fig. 10.11

Answer. It is given that TP and TQ are tangents.
Therefore, radius drawn to these tangents will be perpendicular to the tangents.
Thus, $OP \perp TP$ and $OQ \perp TO$
Angle OPT= 90°

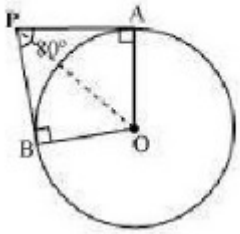
Angle OQT = 90°
In quadrilateral POQT,
Sum of all interior angles = 360°
$\angle OPT + \angle POQ + \angle OQT + \angle PTQ = 360^\circ$
$90^\circ + 110^\circ + 90^\circ + \angle PTQ = 360^\circ$
$\angle \mathrm{PTQ} = 70^\circ$

Hence ,alternative (B) is correct.

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Q3 If tangents PA and PB from a point P to a circle with centre O are inclined to each other at angle of 80°, then $\angle \mathrm{POA}$ is equal to
(A) 50°
(B) 60°
(C) 70°
(D) 80°

Answer. It is given that PA and PB are tangents

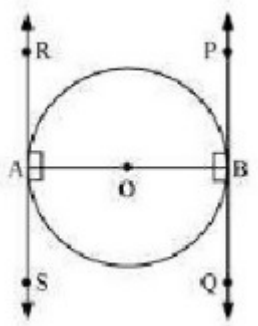


Therefore, the radius drawn to these tangents will be perpendicular to the tangents.
Thus, $OA \perp PA$ and $0B \perp PB$
$\angle \mathrm{OBP} = 90^\circ$
$\angle \mathrm{OAP} = 90^\circ$
In AOBP,
Sum of all interior angles = 360°
$\angle OAP + \angle APB + \angle PBO + \angle BOA = 360^\circ$
$90^\circ + 80^\circ + 90^\circ + \angle BOA = 360^\circ$
$\angle BOA = 100^\circ$
$\triangle OPB$ and ΔOPA,
AP=BP(tangents from a point)
OA=OB(radii of the circle)
OP=OP(common side)
Therefore ,$\triangle OPB \approx \triangle OPA$ (SSS congruence criterion)
$A \mapsto B, P \rightarrow P, O \rightarrow O$
And thus, $\angle \mathrm{POB} = \angle \mathrm{POA}$
$\angle \mathrm{POA} = \frac{1}{2} \angle \mathrm{AOB} = \frac{100^\circ}{2} = 50^\circ$
Hence , alternative (A) is correct.

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Q4 Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

Answer.

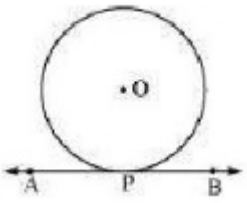


Let AB be a diameter of the circle. Two tangents PQ and RS are drawn at points A and B respectively.
Radius drawn to these tangents will be perpendiculæ to the tangents.
Thus, $OA \perp RS$ and $0B \perp PQ$
$\angle OAR = 90^{\circ}$
$\angle OAS = 90^{\circ}$
$\angle OBP = 90^{\circ}$
$\angle OBQ = 90^{\circ}$
I can be observed that ,
$\angle OAR = \angle OBQ$ (Alternate interior angle)
$\angle OAS = \angle OBP$ (Alternate interior angle)
Since alternate interior angles equal, lines PQ and RS will be parallel.

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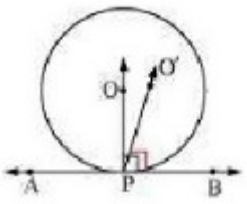
Q5 Prove that the perpendicular at the point of contact to the tangent to a circle passes through the centre.

Answer. Let us consider a circle With centre O. Let AB be a tangent which touches the circle at P.



We have to prove that the line to AB at p passes through centre O. We shallprove this by contradiction method.

Let us assume that the perpendicular to AB at P does not pass through centre O. Let it pass through another point O'. Join OP and O'P.

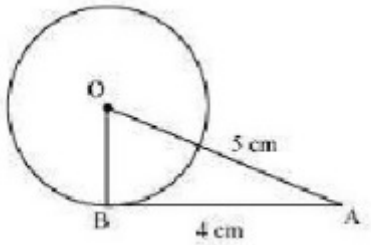


As perpendicular to AB at P passes through O', therefore,
$\angle O'PB = 90^\circ$ .....(1)
O is tie centre of the circle and P is the point of contact. We know the line joining the centre and the point Of contact to the tangent Of the circle are perpendicular to each other.
$\angle OPB = 90^\circ$.....(2)
Comparing equations (1) and (2), we obtain
$\angle O'PB = \angle OPB$.....(3)
From the figure, it can be observed that,
$\angle O'PB < \angle OPB$.....(4)
Therefore , $\angle O'PB < \angle OPB$ is not possible .it is only possible , when the line $\mathrm{O'P}$ coincides with OP.
Therefore the perpendicular to AB through P passes through centre O.

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Q6 The length of a tangent from a point A at distance 5 cm from the centre of the circle is 4 cm. Find the radius of the circle.

Answer.



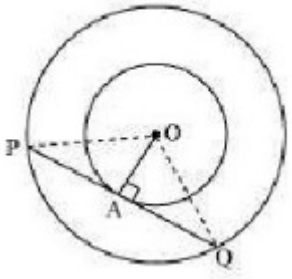
Let us consider a circle at point O.
AB is a tangent drawn on this circle from point A.
Given that,
OA = 5cm and AB = 4 cm
In triangle ABO,
$OB \perp AB$
Applying the pythagoras theorem in triangle ABO , we obtain
$AB^2 + BO^2 = OA^2$
$4^2 + \mathrm{BO}^2 = 5^2$

$16 + BO^2 = 25$
$\mathrm{BO}^2 = 9$
BO=3 cm
Hence , the radius of the circle is 3 cm

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Q7 Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

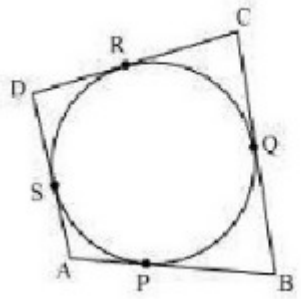
Answer.



Let the two concentric circles be centered at point O. And let PQ be the chord of the larger circle which touches the smaller circle at point A, Therefore, PQ is tangent to the smaller- circle,
$OA \perp PQ$ (As OA is the radius Of the circle)
Applying Pythagoras theorem in triangle OAP, we obtain
$\mathrm{OA}^2 + \mathrm{AP}^2 = \mathrm{OP}^2$
$3^2 + AP^2 = 5^2$
$9 + AP^2 = 25$
$\mathrm{AP}^2 = 16$
AP=4
Since $OA \perp PQ$
AP = AQ (Perpendicular from the center Of the circle bisects the chord)
$\therefore PQ = 2AP = 2 \times 4 = 8$
Therefore, the length of the chord of the larger circle is 8 cm.

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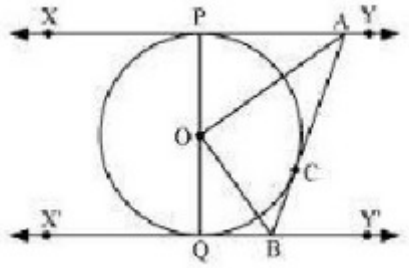
Q8 A quadrilateral ABCD is drawn to circumscribe a circle (see Fig. 10.12). Prove that AB + CD = AD + BC.



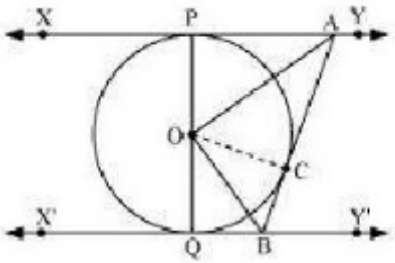
Answer. It can be observed that
DR = DS (Tangents on the circle from point D) ..... (1)
CR = CQ (Tangents on the circle from point C) ..... (2)
BP = BQ (Tangents on the circle from point B) ..... (3)
AP = AS on the circle from point A) ..... (4)
Adding all these equations, we obtain
$\mathrm{DR} + \mathrm{CR} + \mathrm{BP} + \mathrm{AP} = \mathrm{DS} + \mathrm{CQ} + \mathrm{BQ} + \mathrm{AS}$
$(DR + CR) + (BP + AP) = (DS + AS) + (CQ + BQ)$
$CD + AB = AD + BC$

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Q9 In Fig. 10.13, XY and $X'Y'$ are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and $X'Y'$ at B. Prove that $\angle \mathrm{AOB} = 90^\circ$.



Answer. Let us joint point O to C.



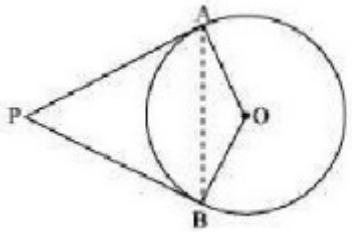
$\triangle \mathrm{OPA}$ and $\triangle \mathrm{OCA}_{\mathrm{r}}$
OP = OC (Radii of the same circle)
AP=AC (Tangents from point A)
AO = AO (Common side)

$\Delta OPA \cong \triangle OCA$ (SSS congruence criterion)
Therefore, $\mathrm{P} \mapsto \mathrm{C}, \mathrm{A} \mapsto \mathrm{A}, \mathrm{O} \mapsto \mathrm{O}$
$\angle \mathrm{POA} = \angle \mathrm{COA}$.....(1)
Similarly $\triangle OQB \cong \triangle OCB$
$\angle QOB = (\angle COS$.....(2)
Since P O Q is a diameter of the circle, it is a straight line.
Therefore $\angle POA + \angle COA + \angle COB + \angle QOB = 180^\circ$
From equations (1) and (2), it can be observed that
$2\angle COA + 2\angle COB = 180^\circ$
$\angle \mathrm{COA} + \angle \mathrm{COB} = 90^\circ$
$\angle AOB = 90^\circ$

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Q10 Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre.

Answer.

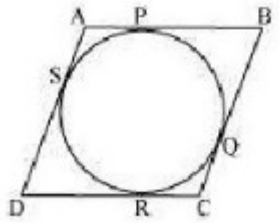


Let us consider a circle centered at point O. Let P be external point from which two tangents PA and PB are drawn to the circle which are touching the circle at point
A and B respectively and AB is the line segment, joining point Of contacts A and 3 together such that it subtends Angle AOB at centre- O of the circle.
It can be observed that
$OA(radius) \perp PA(tangent)$
Therefore ,$\angle OAP = 90^\circ$
Similarly , $OB \perp PB(tangent)$
$\angle \mathrm{OBP} = 90^\circ$
In quadrilateral OAPB,
Sum of the all interior angles
$\angle OAP + \angle APB + \angle PBO + \angle BOA = 360^\circ$
$900 + \angle APB + 90^\circ + \angle BOA = 360^\circ$
$\angle APB + \angle BOA = 180^\circ$
Hence, it can be observed that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre.

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Q11 Prove that the parallelogram circumscribing a circle is a rhombus.

Answer. Since ABCO is a parallelogram,
AB=AD.....(1)
BC=AD.....(2)



It can be observed that
DR = DS (Tangents on the circle from point D)
CR = CQ (Tangents on the circle from point C)
BP=BQ (Tangents on the circle from point B)
AP= AS (Tangents on the circle from point A)
Adding all these equations, we obtain
OR + CR + BP + AP DS + CQ + BQ + AS
(DR + CR) + (BP + AD) = (DS + AS) + (CQ + BQ)
CD + AB = AD + BC
On putting the values of equations (1) and (2) in this equation, we obtain
2A8 =2BC
AB=BC.....(3)
Comparing equations (1), (2), (3), we obtain
AS = BC = CD = DA
Hence, ABCD is a rhombus.

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Q12 A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 8 cm and 6 cm respectively (see Fig. 10.14). Find the sides AB and AC.

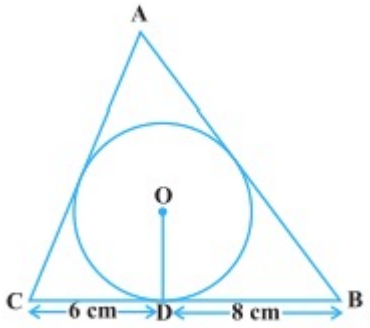
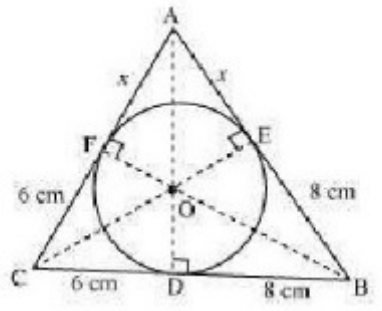


Fig. 10.14

Answer.



Let the given circle touch the sides AB and AC of the triangle at point E and F respectively and the Length of the line segment AF be x.
In triangle ABC,
CF=CD =6cm (Tangents on the circle from point C)
BE= BD = 8cm (Tangents on the circle from point B)
AE=AF =x cm (Tangents on the circle from point A)
AB=AE+EB=x + 8
BC=BD+ DC = 8+6=14
CA=CF+FA=6+x
2s=AB + BC + CA
=x+8+14+6+x
=28+2x
s=14+x

$\Delta \mathrm{ABC}=\sqrt{s(s-a)(s-b)(s-c)}$
$=\sqrt{\{14+x\}\{(14+x)-14\}\{(14+x)-(6+x)\}(14+x)-(8+x)\}}$
$=\sqrt{(14+x)(x)(8)(6)}$
$=4 \sqrt{3\left(14 x+x^{2}\right)}$

Area of $\Delta \mathrm{OBC}=\frac{1}{2} \times \mathrm{OD} \times \mathrm{BC}=\frac{1}{2} \times 4 \times 14=28$
Area of $\Delta \mathrm{OCA}=\frac{1}{2} \times \mathrm{OF} \times \mathrm{AC}=\frac{1}{2} \times 4 \times(6+x)=12+2 x$
Area of $\frac{1}{2} \times \mathrm{OE} \times \mathrm{AB}=\frac{1}{2} \times 4 \times(8+x)=16+2 x$
$\Delta \mathrm{ABC}=$ Area of $\Delta \mathrm{OBC}+$ Area of $\Delta \mathrm{OCA}+$ Area of $\triangle \mathrm{OAB}$
$4 \sqrt{3\left(14 x+x^{2}\right)}=28+12+2 x+16+2 x$
$\Rightarrow 4 \sqrt{3\left(14 x+x^{2}\right)}=56+4 x$
$\Rightarrow \sqrt{3\left(14 x+x^{2}\right)}=14+x$
$\Rightarrow 3\left(14 x+x^{2}\right)=(14+x)^{2}$
$\Rightarrow 42 x+3 x^{2}=196+x^{2}+28 x$
$\Rightarrow 2 x^{2}+14 x-196=0$
$\Rightarrow x^{2}+7 x-98=0$
$\Rightarrow x^{2}+14 x-7 x-98=0$
$\Rightarrow x(x+14)-7(x+14)=0$
$\Rightarrow (x+14)(x-7)=0$
Either x+14=0 or x-7=0

Therefore ,x = -1 and 7
However, x = -14 is not possible as the length of the sides will be negative.
Therefore, x=7
Hence, AB =x+8 = 7+8=15 cm
CA = 6 + x=6 + 7 =13 cm

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Q13 Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

Answer. Let ABCD be a quadrilateral circumscribing a circle centered at O such that it touches the circle at point P, Q, R, S. Let us join the v«tices of the quadrilateral ABCD to the of the circle.
Consider triangle OAP and triangle OAS,
AP = AS (Tangents from the sarne point)
OP=OS (Radii Of the sane circle)
OA=OA (Common side)
$\triangle OAP = \triangle OAS$ (SSS congruence criterion)
Therefore ,A <-> A,P <-> S ,O <-> O
And thus $\angle \mathrm{POA} = \angle \mathrm{AOS}$
$\angle 1 = \angle 8$,
Similarly,
$\angle 2 = \angle 3$,
$\angle 4 = \angle 5$,
$\angle 6 = \angle 7$,
$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360^\circ$
$(\angle 1 + \angle 8) + (\angle 2 + \angle 3) + (\angle 4 + \angle 5) + (\angle 6 + \angle 7) = 360^\circ$
$2\angle 1 + 2\angle 2 + 2\angle 5 + 2\angle 6 = 360^\circ$
$2(\angle 1 + \angle 2) + 2(\angle 5 + \angle 6) = 360^\circ$
$(\angle 1 + \angle 2) + (\angle 5 + \angle 6) = 180^\circ$
Similarly , we can prove that $\angle BOC + \angle DOA = 180^\circ$
Hence, opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle .

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