NCERT SOLUTIONS

CLASS - 10th



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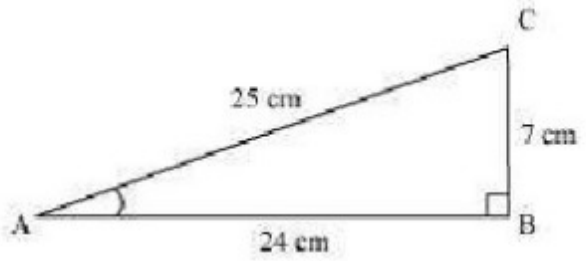
Class : 10th
Subject : Maths
Chapter : 8
Chapter Name : Introduction to Trigonometry

Exercise 8.1

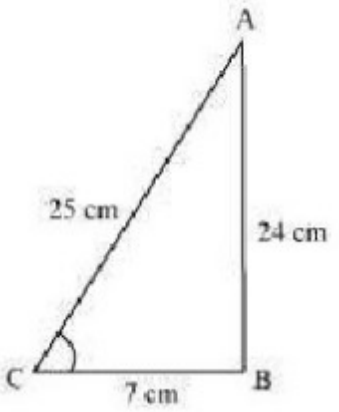
Q1 In $\triangle$ ABC right angled at B, AB=24 cm, BC=7 m. Determine
(i) $\sin A, \cos A$
(ii) $\sin C, \cos C$

Answer.

$$\text{Applying Pythagoras theorem for } \Delta ABC, \text{ we obtain}$$
$$AC^2 = AB^2 + BC^2$$
$$= (24\text{cm})^2 + (7\text{cm})^2$$
$$= (576 + 49)\text{cm}^2$$
$$= 625\text{cm}^2$$
$$\therefore AC = \sqrt{625}\text{cm} = 25\text{cm}$$



$$\sin A = \frac{\text{Side opposite to } \angle A}{\text{Hypotenuse}} = \frac{BC}{AC}$$
$$= \frac{7}{25}$$
$$\cos A = \frac{\text{Side adjacent to } \angle A}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{24}{25}$$



(ii)

$\sin \mathrm{C}=\frac{\text{Side opposite to } \angle \mathrm{C}}{\text{Hypotenuse}}=\frac{\mathrm{AB}}{\mathrm{AC}}$

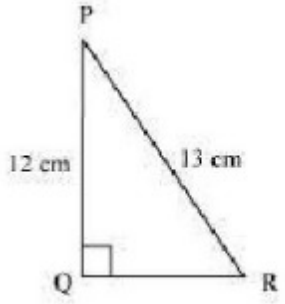
$=\frac{24}{25}$

$\cos \mathrm{C}=\frac{\text{Side adjacent to } \angle \mathrm{C}}{\text{Hypotenuse}}=\frac{\mathrm{BC}}{\mathrm{AC}}$

$=\frac{7}{25}$

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Q2 In the given figure find tan P - cot R.



Answer.

Applying Pythagoras theorem for $\Delta PQR,$ we obtain

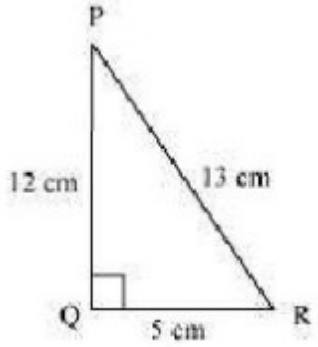
$PR^2 = PQ^2 + QR^2$

$(13\text{cm})^2 = (12\text{cm})^2 + QR^2$

$169\text{cm}^2 = 144\text{cm}^2 + QR^2$

$25\text{cm}^2 = QR^2$

$QR = 5\text{cm}$

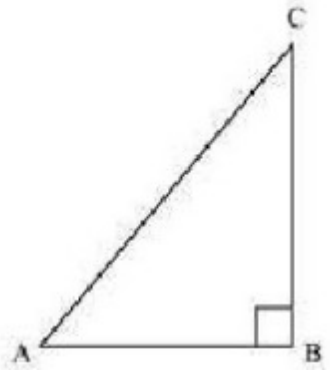


$$\tan \mathbf{P} = \frac{\text{Side opposite to } \angle P}{\text{Side adjacent to } \angle P} = \frac{QR}{PQ}$$
$$= \frac{5}{12}$$
$$\cot R = \frac{\text{Side adjacent to } \angle R}{\text{Side opposite to } \angle R} = \frac{QR}{PQ}$$
$$= \frac{5}{12}$$
$\tan \mathrm{P} - \cot \mathrm{R} = \frac{5}{12} - \frac{5}{12} = 0$

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Q3 If $\sin A = \frac{3}{4}, \text{ calculate } \cos A \text{ and } \tan A.$

Answer.LetΔABC be a right-angled triangle, right-angled at point B



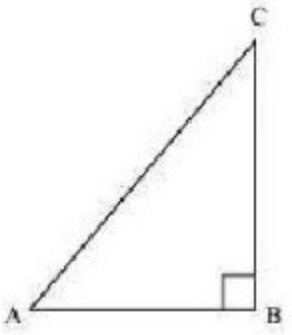
Given that,
$\sin A = \frac{3}{4}$
$\frac{BC}{AC} = \frac{3}{4}$
Let BC be 3k . Therefore, AC will be 4k , where k is a positive integer.
Applying Pythagoras theorem in $\triangle ABC$, we obtain
$AC^2 = AB^2 + BC^2$
$(4k)^2 = AB^2 + (3k)^2$
$16k^2 - 9k^2 = AB^2$
$7k^2 = AB^2$
$AB = \sqrt{7}k$

$$\cos A = \frac{\text{Side adjacent to } \angle A}{\text{Hypotent } t0 \angle A}$$
$$= \frac{AB}{AC} = \frac{\sqrt{7k}}{4k} = \frac{\sqrt{7}}{4}$$
$$\tan A = \frac{\text{Side opposite to } \angle A}{\text{Side adjacent to } \angle A}$$
$$= \frac{BC}{AB} = \frac{3k}{\sqrt{7k}} = \frac{3}{\sqrt{7}}$$

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Q4 Given $15 \cot A = 8.$ $\text{Find } \sin A \text{ and } \sec A.$

Answer. Consider a right-angled triangle, right-angled at B.



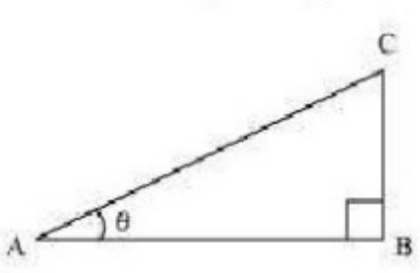
Given that,
$\sin A = \frac{3}{4}$
$\frac{BC}{AC} = \frac{3}{4}$
Let BC be 3k. Therefore, AC will be 4k, where k is a positive integer.
Applying Pythagoras theorem in $\triangle \mathrm{ABC}$, we obtain
$\mathrm{AC}^2 = \mathrm{AB}^2 + \mathrm{BC}^2$
$(4k)^2 = \mathrm{AB}^2 + (3\mathrm{k})^2$
$16\mathrm{k}^2 - \mathrm{gk}^2 = \mathrm{AB}^2$
$7k^2 = \mathrm{AB}^2$
$\mathrm{AB} = \sqrt{7}k$
$$\cos A = \frac{\text{Side adjacent to } \angle A}{\text{Hypotenuse}}$$
$$= \frac{AB}{AC} = \frac{\sqrt{7k}}{4k} = \frac{\sqrt{7}}{4}$$
$$\tan A = \frac{\text{Side opposite to } \angle A}{\text{Side adjacent to } \angle A}$$
$$= \frac{BC}{AB} = \frac{3k}{\sqrt{7k}} = \frac{3}{\sqrt{7}}$$

$$\cot A = \frac{\text{Side adjacent to } \angle A}{\text{Side opposite to } \angle A}$$
$$= \frac{AB}{BC}$$
It is given that,
$\cot A = \frac{8}{15}$
$\frac{AB}{BC} = \frac{8}{15}$
Let AB be 8k.Therefore, 3C will be 15k, where k is a positive integer.
Applying Pythagoras theorem in $\triangle$ABC, we obtain
$\mathrm{AC}^2 = \mathrm{AB}^2 + \mathrm{BC}^2$
$= (8k)^2 + (15k)^2$
$= 64k^2 + 225\mathrm{k}^2$
$= 289\mathrm{k}^2$
$\mathrm{AC} = 17k$
$$\sin A = \frac{\text{Side opposite to } \angle A}{\text{Hypotenuse}} = \frac{BC}{AC}$$
$$= \frac{15k}{17k} = \frac{15}{17}$$
$$\sec A = \frac{\text{Hypotenuse}}{\text{Side adjacent to } \angle A}$$
$$= \frac{AC}{AB} = \frac{17}{8}$$

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Q5 Given $\sec\theta = \frac{13}{12}$, calculate all other trigonometric ratios.

Answer. Consider a right-angle $\triangle\mathrm{ABC},$ right-angled at point B



$$\sec\theta = \frac{\text{Hypotenuse}}{\text{Side adjacent to } \angle\theta}$$
$\frac{13}{12} = \frac{\mathrm{AC}}{\mathrm{AB}}$
If AC is 13k, AB will be 12k, where k is a positive integer.
Applying Pythagoras theorem in AABC, we obtain

$(AC)^2 = (AB)^2 + (BC)^2$

$(13k)^2 = (12k)^2 + (BC)^2$

$169k^2 = 144k^2 + BC^2$

$25k^2 = BC^2$

BC = 5k

$\sin\theta = \frac{\text{Side opposite to } \angle\theta}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{5k}{13k} = \frac{5}{13}$

$\cos\theta = \frac{\text{Side adjacent to } \angle\theta}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{12k}{13k} = \frac{12}{13}$

$\tan\theta = \frac{\text{Side opposite to } \angle\theta}{\text{Side adjacent to } \angle\theta} = \frac{BC}{AB} = \frac{5k}{12k} = \frac{5}{12}$

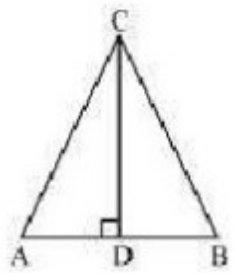
$\cot\theta = \frac{\text{Side adjacent to } \angle\theta}{\text{Side opposite to } \angle\theta} = \frac{AB}{BC} = \frac{12k}{5k} = \frac{12}{5}$

$cosec\,\theta = \frac{\text{Hypotenuse}}{\text{Side opposite to } \angle\theta} = \frac{AC}{BC} = \frac{13k}{5k} = \frac{13}{5}$

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Q6 If $\angle A$ and $\angle B$ are acute angles such that $\cos A = \cos B$, then show that $\angle A = \angle B$

Answer. Let us consider a triangle ABC in which $\mathrm{CD} \perp \mathrm{AB}$

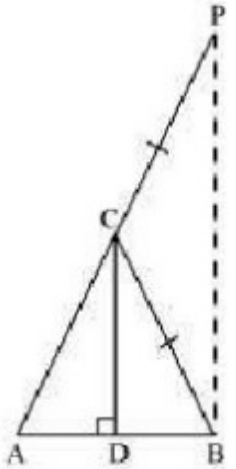


It is given that,

$\cos A = \cos B$

$\Rightarrow \frac{AD}{AC} = \frac{BD}{BC} \ldots$

We have to prove $\angle A = \angle B$. To prove this, let us extend AC to P such that BC = CP



From equation (1) we obtain,

$\frac{\mathrm{AD}}{\mathrm{BD}}=\frac{\mathrm{AC}}{\mathrm{BC}}$

$\Rightarrow \frac{\mathrm{AD}}{\mathrm{BD}}=\frac{\mathrm{AC}}{\mathrm{CP}}$ (By construction, we have $\mathrm{BC}=\mathrm{CP}$)

By using the converse of B.P.T,

$\mathrm{CD}\|\mathrm{BP}$

$\Rightarrow \angle \mathrm{ACD}=\angle \mathrm{CPB}(\text{ Corresponding angles }) \ldots \ (3)$

And, $\angle \mathrm{BCD}=\angle \mathrm{CBP}$ (Alternate interior angles)... (4)

By construction, we have $\mathrm{BC}=\mathrm{CP}$.

$\therefore \angle \mathrm{CBP}=\angle \mathrm{CPB}$ (Angle opposite to equal sides of a triangle)... (5)

From equations $(3),(4),$ and $(5),$ we obtain

$\angle \mathrm{ACD}=\angle \mathrm{BCD} \ldots (6)$

$\angle CDA=\angle CDB\,[Both 90^{\circ}]$

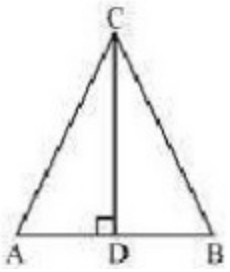
Therefore, the remaining angles should be equal.

$\therefore \angle CAD=\angle CBD$

$\Rightarrow \angle A=\angle B$

Alternatively,

Let us consider triangle ABC in which $CD \perp AB$



It is given that,

Cos A = Cos B

$\Rightarrow \frac{AD}{AC}=\frac{BD}{BC}$

$\Rightarrow \frac{AD}{BD}=\frac{AC}{BC}$

Let $\frac{AD}{BD}=\frac{AC}{BC}=k$

$\Rightarrow AD=kBD \ldots (1)$

And, $AC=kBC \ldots (2)$

using Pythagoras theorem for triangles CAD and CBD, we obtain

$\mathrm{CD}^2=\mathrm{AC}^2-\mathrm{AD}^2 \ldots (3)$

And, $\mathrm{CD}^2=\mathrm{BC}^2-\mathrm{BD}^2 \ldots \ (4)$

From equations (3) and $(4),$ we obtain

$\mathrm{AC}^2-\mathrm{AD}^2=\mathrm{BC}^2-\mathrm{BD}^2$

$\Rightarrow (kBC)^2-(kBD)^2=BC^2-BD^2$

$\Rightarrow k^2\left(BC^2-BD^2\right)=BC^2-BD^2$

$\Rightarrow k^2=1$

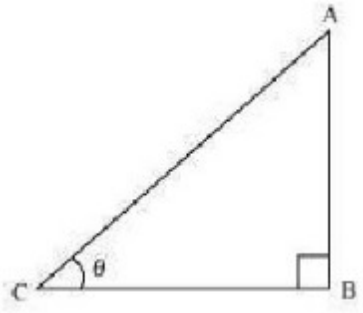
$\Rightarrow k=1$

Putting this value in equation (2), we obtain
AC = BC
$\Rightarrow \angle A = \angle B$(Angles opposite to equal sides of a angle)

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Q7 If $\cot\theta = \frac{7}{8}$, evaluate
$(i)\ \frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$ ($(\mathrm{ii})\ \cot^2\theta$

Answer. Let us consider a right triangle ABC, right-angled at point B.



$$\cot\theta = \frac{\text{Side adjacent to } \angle\theta}{\text{Side opposite to } \angle\theta} = \frac{\text{BC}}{\text{AB}}$$
$$= \frac{7}{8}$$

If BC is 7k, then AB will be 8k, where k is a positive integer.
Applying Pythagoras theorem in $\triangle$ABC, we obtain

$$AC^2 = AB^2 + BC^2$$
$$= (8k)^2 + (7k)^2$$
$$= 64k^2 + 49k^2$$
$$= 113k^2$$
$$AC = \sqrt{113}k$$
$$\sin\theta = \frac{\text{Side opposite to } \angle\theta}{\text{Hypotenuse}} = \frac{\text{AB}}{\text{AC}}$$
$$= \frac{8k}{\sqrt{113}k} = \frac{8}{\sqrt{113}}$$
$$\cos\theta = \frac{\text{Side adjacent to } \angle\theta}{\text{Hypotenuse}} = \frac{\text{BC}}{\text{AC}}$$
$$= \frac{7k}{\sqrt{113}k} = \frac{7}{\sqrt{113}}$$

(i) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \frac{(1-\sin^2\theta)}{(1-\cos^2\theta)}$
$= \frac{1-\left(\frac{8}{\sqrt{113}}\right)^2}{1-\left(\frac{7}{\sqrt{113}}\right)^2} = \frac{1-\frac{64}{113}}{1-\frac{49}{113}}$

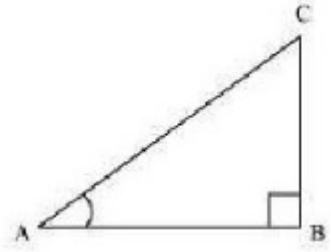
$= \frac{\frac{49}{113}}{\frac{64}{113}} = \frac{49}{64}$

(ii) $\cot^2\theta = (\cot\theta)^2 = \left(\frac{7}{8}\right)^2 = \frac{49}{64}$

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Q8 If $3\cot A = 4$, Check whether $\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$ or not.

Answer. It is given that 3 Cot A = 4
Or, Cot A = $\frac{4}{3}$
Consider a right triangle ABC, right-angled at point B.



$\cot A = \frac{\text{Side adjacent to } \angle A}{\text{Side opposite to } \angle A}$
$\frac{AB}{BC} = \frac{4}{3}$
If AB is 4k , then BC will be 3k, where k is a positive integer.
In $\triangle$ABC,
$(AC)^2 = (AB)^2 + (BC)^2$
$= (4k)^2 + (3k)^2$
$= 16k^2 + 9k^2$
$= 25k^2$
$AC = 5k$

$$\cos A = \frac{\text{Side adjacent to } \angle A}{\text{Hypotenuse}} = \frac{AB}{AC}$$
$$= \frac{4k}{5k} = \frac{4}{5}$$
$$\sin A = \frac{\text{Side opposite to } \angle A}{\text{Hypotenuse}} = \frac{BC}{AC}$$
$$= \frac{3k}{5k} = \frac{3}{5}$$
$$\tan A = \frac{\text{Side opposite to } \angle A}{\text{Hypotenuse}} = \frac{BC}{AB}$$
$$= \frac{3k}{4k} = \frac{3}{4}$$

$\frac{1-\tan^2 A}{1+\tan^2 A} = \frac{1-\left(\frac{3}{4}\right)^2}{1+\left(\frac{3}{4}\right)^2} = \frac{1-\frac{9}{16}}{1+\frac{9}{16}}$

$= \frac{\frac{7}{16}}{\frac{25}{16}} = \frac{7}{25}$

$\cos^2 A - \sin^2 A = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$

$= \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$

$\therefore \quad \frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$

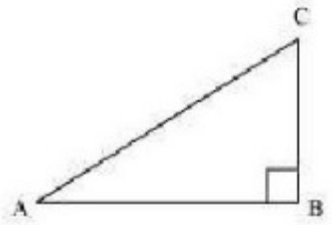
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Q9 In $\triangle ABC,$ right angled at B . If $A = \frac{1}{\sqrt{3}},$ find the value of

(i) $\sin A \cos C + \cos A \sin C$

(ii) $\cos A \cos C - \sin A \sin C$

Answer.



$\tan A = \frac{1}{\sqrt{3}}$

$\frac{BC}{AB} = \frac{1}{\sqrt{3}}$

If BC is k , then AB will be , where k is a positive integer.

In $\triangle$ABC,

$\mathrm{AC}^2 = \mathrm{AB}^2 + \mathrm{BC}^2$

$= (\sqrt{3}k)^2 + (k)^2$

$= 3\mathrm{k}^2 + \mathrm{k}^2 = 4\mathrm{k}^2$

$\therefore \mathrm{AC} = 2\mathrm{k}$

$\sin A = \frac{\text{Side opposite to } \angle A}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{k}{2k} = \frac{1}{2}$

$\cos A = \frac{\text{Side adjacent to } \angle A}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$

$\sin C = \frac{\text{Side opposite to } \angle C}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$

$\cos C = \frac{\text{Side adjacent to } \angle C}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{k}{2k} = \frac{1}{2}$

(i) sin A cos C + cos A sin C

$= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{1}{4} + \frac{3}{4}$

$= \frac{4}{4} = 1$

(ii) cos A cos C - sin A sin C

$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0$

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Q10

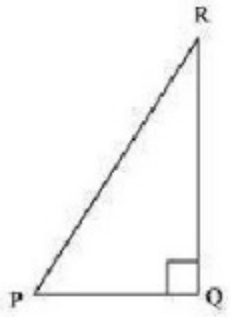
$\text{In } \Delta\text{PQR, right angled at Q, PR} + \text{QR} = 25\text{cm and PQ} = 5\text{cm. Determine the values of } \sin\text{P}, \cos\text{P and } \tan\text{P}$

Answer. Given that, PR + QR = 25

PQ = 5

Let PR be x.

Therefore, QR = 25 -x



Applying Pythagoras theorem in ΔPQR, we obtain

$\text{PR}^2 = \text{PQ}^2 + \text{QR}^2$

$x^2 = (5)^2 + (25 - x)^2$

$x^2 = 25 + 625 + x^2 - 50x$

$50x = 650$

$x = 13$

$\text{Therefore, PR} = 13\text{cm}$

$\text{QR} = (25 - 13)\text{cm} = 12\text{cm}$

$\sin\text{P} = \frac{\text{Side opposite to } \angle\text{P}}{\text{Hypotenuse}} = \frac{\text{QR}}{\text{PR}} = \frac{12}{13}$

$\cos P = \frac{\text{Side adjacent to } \angle P}{\text{Hypotenuse}} = \frac{PQ}{PR} = \frac{5}{13}$

$\tan P = \frac{\text{Side opposite to } \angle P}{\text{Side adjacent to } \angle P} = \frac{QR}{PQ} = \frac{12}{5}$

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Q11

State whether the following are true or false. Justify your answer.

(1) The value of tan A is always less than 1 .

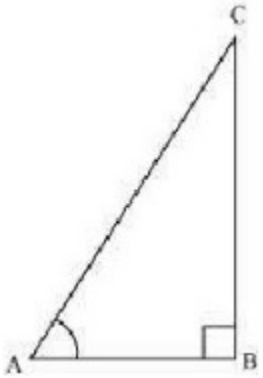
(ii) $\sec A = \frac{12}{5}$ for some value of angle A.

(iii) $\cos A$ is the abbreviation used for the cosecant of angle A.

(iv) cot A is the product of cot and A

$(v) \sin\theta = \frac{4}{3},$ for some angle θ

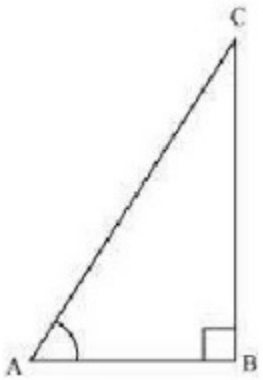
Answer. (i) Consider a ΔABC, right-angled at B.



$$\tan A = \frac{\text{Side opposite to } \angle A}{\text{Side adjacent to } \angle A}$$
$$= \frac{12}{5}$$
$\text{But } \frac{12}{5} > 1$
$\therefore \tan A > 1$
$\text{So, } \tan A < 1 \text{ is not always true.}$
$\text{Hence, the given statement is false.}$

(ii)

$\sec A = \frac{12}{5}$



$$\frac{\text{Hypotenuse}}{\text{Side adjacent to } \angle A} = \frac{12}{5}$$
$$\frac{\text{AC}}{\text{AB}} = \frac{12}{5}$$

Let AC be 12k, AB will be 5k, where k is a positive integer.

Applying Pythagoras theorem in Δ ABC, we obtain

$$AC^2 = AB^2 + BC^2$$
$$(12k)^2 = (5k)^2 + BC^2$$
$$144k^2 = 25k^2 + BC^2$$
$$BC^2 = 119k^2$$
$$BC = 10.9k$$

It can be observed that for given two sides AC = 12k and AB = 5k,
BC should be such that,

AC - AB < BC < AC + AB
12k - 5k < BC < 12k + 5k
7k < BC < 17k
However, BC = 10.9k. Clearly, such a triangle is possible and hence, such value of sec A is possible.
Hence, the given statement is true.
(iii) Abbreviation used for cosecant of angle A is cosec A. And cos A is the abbreviation used for cosine of angle A.
Hence, the given statement is false.
(iv) cot A is not the product of cot not A. It is the cotangent of $\angle$A.
Hence, the given statement is false.
(v) $\sin\theta = \frac{4}{3}$
We know that in a right-angled triangle,
$\sin\theta = \frac{\text{Side opposite to } \angle\theta}{\text{Hypotenuse}}$
In a right-angled triangle, hypotenuse is always greater than the remaining two sides.
Therefore, such value of sin θ is not possible.
Hence, the given statement is false.

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Exercise 8.2

Q1 Evaluate the following
(i) $\sin 60^\circ \cos 30^\circ + \sin 30^\circ \cos 60^\circ$
(ii) $2\tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$
(iii) $\frac{\cos 45^\circ}{\sec 30^\circ + \csc 30^\circ}$
(iv) $\frac{\sin 30^\circ + \tan 45^\circ - \csc 60^\circ}{\sec 30^\circ + \cos 60^\circ + \cot 45^\circ}$
(v) $\frac{5\cos^2 60^\circ + 4\sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ}$

Answer. (i) $\sin 60^\circ \cos 30^\circ + \sin 30^\circ \cos 60^\circ$
$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$
$= \frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1$
(ii) $2\tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$
$= 2(1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$
$= 2 + \frac{3}{4} - \frac{3}{4} = 2$
(iii) $\frac{\cos 45^\circ}{\sec 30^\circ + cosec 30^\circ}$

$= \frac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}}+2} = \frac{\frac{1}{\sqrt{2}}}{\frac{2+2\sqrt{3}}{\sqrt{3}}}$

$= \frac{\sqrt{3}}{\sqrt{2}(2+2\sqrt{3})} = \frac{\sqrt{3}}{2\sqrt{2}+2\sqrt{6}}$

$= \frac{\sqrt{3}(2\sqrt{6}-2\sqrt{2})}{(2\sqrt{6}+2\sqrt{2})(2\sqrt{6}-2\sqrt{2})}$

$= \frac{2\sqrt{3}(\sqrt{6}-\sqrt{2})}{(2\sqrt{6})^2-(2\sqrt{2})^2} = \frac{2\sqrt{3}(\sqrt{6}-\sqrt{2})}{24-8} = \frac{2\sqrt{3}(\sqrt{6}-\sqrt{2})}{16}$

$= \frac{\sqrt{18}-\sqrt{6}}{8} = \frac{3\sqrt{2}-\sqrt{6}}{8}$

(iv) $\frac{\sin 30^\circ+\tan 45^\circ-cosec 60^\circ}{\sec 30^\circ+\cos 60^\circ+\cot 45^\circ}$

$= \frac{\frac{1}{2}+1-\frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}}+\frac{1}{2}+1} = \frac{\frac{3}{2}-\frac{2}{\sqrt{3}}}{\frac{3}{2}+\frac{2}{\sqrt{3}}}$

$= \frac{3\sqrt{3}-4}{\frac{3\sqrt{3}-4}{2\sqrt{3}+4}} = \frac{(3\sqrt{3}-4)}{(3\sqrt{3}+4)}$

$= \frac{(3\sqrt{3}-4)(3\sqrt{3}-4)}{(3\sqrt{3}+4)(3\sqrt{3}-4)} = \frac{(3\sqrt{3}-4)^2}{(3\sqrt{3})^2-(4)^2}$

$= \frac{27+16-24\sqrt{3}}{27-16} = \frac{43-24\sqrt{3}}{11}$

(v) $\frac{5\cos^2 60^\circ+4\sec^2 30^\circ-\tan^2 45^\circ}{\sin^2 30^\circ+\cos^2 30^\circ}$

$= \frac{5\left(\frac{1}{2}\right)^2+4\left(\frac{2}{\sqrt{3}}\right)^2-(1)^2}{\left(\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}$

$= \frac{5\left(\frac{1}{4}\right)+\left(\frac{16}{3}\right)-1}{\frac{1}{4}+\frac{3}{4}}$

$= \frac{15+64-12}{\frac{4}{4}} = \frac{67}{12}$

Page : 187 , Block Name : Exercise 8.2

Q2 Choose the correct option and justify your choice:

$(i) \frac{2\tan 30^\circ}{1+\tan^2 30^\circ} =$

(A) $\sin 60^\circ$ (B) $\cos 60^\circ$ (C) $\tan 60^\circ$ (D) $\sin 30^\circ$

$(ii) \frac{1-\tan^2 45^\circ}{1+\tan^2 45^\circ} =$

(A) $\tan 90^\circ$ (B) 1 (C) $\sin 45^\circ$ (D) 0

$(iii) \sin 2A = 2\sin A$ is true when $A =$

(A) 0° (B) 30° (C) 45° (D) 60°

$(iv) \frac{2\tan 30^\circ}{1-\tan^2 30^\circ} =$

(A) $\cos 60^\circ$ (B) $\sin 60^\circ$ (C) $\tan 60^\circ$ (D) $\sin 30^\circ$

Answer. (i) $\frac{2\tan 30^\circ}{1+\tan^2 30^\circ}$

$= \frac{2\left(\frac{1}{\sqrt{3}}\right)}{1+\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1+\frac{1}{3}} = \frac{\sqrt{3}}{\frac{4}{3}}$

$= \frac{6}{4\sqrt{3}} = \frac{\sqrt{3}}{2}$

Out of the given alternatives, only $\sin 60^\circ = \frac{\sqrt{3}}{2}$

Hence, (A) is correct.

(ii) $\frac{1-\tan^2 45^\circ}{1+\tan^2 45^\circ}$

$= \frac{1-(1)^2}{1+(1)^2} = \frac{1-1}{1+1} = \frac{0}{2} = 0$

Hence, (D) is correct.

(iii) Out of the given alternatives, only A = 0° is correct.

As sin 2A = sin $^\circ$ = 0

2 sin A = 2 sin $^\circ$ = 2(0) = 0

Hence, (A) is correct.

(iv) $\frac{2\tan 30^\circ}{1-\tan^2 30^\circ}$

$= \frac{2\left(\frac{1}{\sqrt{3}}\right)}{1-\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1-\frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}}$

$= \sqrt{3}$

Out of the given alternatives, only $\tan 60^\circ = \sqrt{3}$

Hence, (C) is correct.

Page : 187 , Block Name : Exercise 8.2

Q3 If $\tan(A+B) = \sqrt{3}$ and $(A-B) = \frac{1}{\sqrt{3}}$ $0^\circ < A+B \leq 90^\circ, A > B$ find A and B .

Answer.

$\tan(\mathrm{A}+\mathrm{B}) = \sqrt{3}$

$\Rightarrow \tan(\mathrm{A}+\mathrm{B}) = \tan 60$

$\Rightarrow \mathrm{A}+\mathrm{B} = 60 \ldots (1)$

$\tan(\mathrm{A}-\mathrm{B}) = \frac{1}{\sqrt{3}}$

$\Rightarrow \tan(\mathrm{A}-\mathrm{B}) = \frac{1}{\sqrt{3}}$

$\Rightarrow \mathrm{A}-\mathrm{B} = 30 \ldots (2)$

$\Rightarrow \mathrm{A}-\mathrm{B} = 30 \ldots (2)$

On adding both equations, we obtain

$2A = 90$

$\Rightarrow A = 45$

From equation (1), we obtain

$45 + B = 60$

$B = 15$

Therefore, $\angle A = 45^\circ$ and $\angle B = 15^\circ$

Page : 187 , Block Name : Exercise 8.2

Q4 State whether the following are true or false. Justify your answer.

(i) $\sin(A + B) = \sin A + \sin B$

(ii) The value of $\sin\theta$ increases as θ increases

(iii) The value of $\cos\theta$ increases as θ increases

(iv) $\sin\theta = \cos\theta$ for all values of θ

(v) $\cot A$ is not defined for $A = 0^\circ$

Answer. (i) $\sin(A + B) = \sin A + \sin B$

Let $A = 30^\circ$ and $B = 60^\circ$

$\sin(A + B) = \sin(30^\circ + 60^\circ)$

$= \sin 90^\circ$

$= 1$

$\sin A + \sin B = \sin 30^\circ + \sin 60^\circ$

$= \frac{1}{2} + \frac{\sqrt{3}}{2} = \frac{1+\sqrt{3}}{2}$

Clearly, $\sin(A + B) \neq \sin A + \sin B$

Hence, the given statement is false.

(ii) The value of $\sin\theta$ increases as θ increases in the interval of $0^\circ < \theta < 90^\circ$ as

$\sin 0^\circ = 0$

$\sin 30^\circ = \frac{1}{2} = 0.5$

$\sin 45^\circ = \frac{1}{\sqrt{2}} = 0.707$

$\sin 60^\circ = \frac{\sqrt{3}}{2} = 0.866$

$\sin 90^\circ = \frac{\sqrt{3}}{2} = 0.866$

Hence, the given statement is true.

(iii) $\cos 0^\circ = 1$

$\cos 30^\circ = \frac{\sqrt{3}}{2} = 0.866$

$\cos 45^\circ = \frac{1}{\sqrt{2}} = 0.707$

$\cos 60^\circ = \frac{1}{2} = 0.5$

$\cos 90^\circ = 0$

It can be observed that the value of $\cos\theta$ does not increase in the interval of $0^\circ < \theta$

Hence, the given statement is false.

(iv) $\sin\theta = \cos\theta$ for all values of θ

This is true when $\theta = 45^\circ$

$\sin 45^\circ = \frac{1}{\sqrt{2}}$

$\cos 45^\circ = \frac{1}{\sqrt{2}}$

It is not true for all other values of θ .

Hence, the given statement is false.

(v) cot A is not defined for $A = 0^\circ$

$\cot A = \frac{\cos A}{\sin A}$

$\cot 0^\circ = \frac{\cos 0^\circ}{\sin 0^\circ} = \frac{1}{0} =$ undefined

Hence, the given statement is true.

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Exercise 8.3

Q1 Evaluate:

(i) $\frac{\sin 18^\circ}{\cos 72^\circ}$ (ii) $\frac{\tan 26^\circ}{\cot 64^\circ}$ (iii) $\cos 48^\circ - \sin 42^\circ$ (iv) $cosec 31^\circ - \sec 59^\circ$

Answer. (i) $\frac{\sin 18^\circ}{\cos 72^\circ} = \frac{\sin(90^\circ - 72^\circ)}{\cos 72^\circ}$

$= \frac{\cos 72^\circ}{\cos 72^\circ} = 1$

$(ii) \frac{\tan 26^\circ}{\cot 64^\circ} = \frac{\tan(90^\circ - 64^\circ)}{\cot 64^\circ}$

$= \frac{\cot 64^\circ}{\cot 64^\circ} = 1$

(iii) $\cos 48^\circ - \sin 42^\circ = \cos(90^\circ - 42^\circ) - \sin 42^\circ$

$= \sin 42^\circ - \sin 42^\circ$

$= 0$

$(\text{iv}) \cos \text{ec} 31^\circ - \sec 59^\circ = cosec\,(90^\circ - 59^\circ) - \sec 59^\circ$
$= \sec 59^\circ - \sec 59^\circ$
$= 0$

Page : 189 , Block Name : Exercise 8.3

Q2 Show that:
(i) $\tan 48^\circ \tan 23^\circ \tan 42^\circ \tan 67^\circ = 1$
(ii) $\cos 38^\circ \cos 52^\circ - \sin 38^\circ \sin 52^\circ = 0$

Answer.
(i) $\tan 48^\circ \tan 23^\circ \tan 42^\circ \tan 67^\circ$
$= \tan(90^\circ - 42^\circ) \tan(90^\circ - 67^\circ) \tan 42^\circ \tan 67^\circ$
$= \cot 42^\circ \cot 67^\circ \tan 42^\circ \tan 67^\circ$
$= (\cot 42^\circ \tan 42^\circ)\,(\cot 67^\circ \tan 67^\circ)$
$= (1)(1)$
$= 1$
(ii) $\cos 38^\circ \cos 52^\circ - \sin 38^\circ \sin 52^\circ$
$= \cos(90^\circ - 52^\circ) \cos(90^\circ - 38^\circ) - \sin 38^\circ \sin 52^\circ$
$= \sin 52^\circ \sin 38^\circ - \sin 38^\circ \sin 52^\circ$
$= 0$

Page : 189 , Block Name : Exercise 8.3

Q3 If $\tan 2A = \cot(A - 18^\circ),$ where $2A$ is an acute angle, find the value of A.

Answer.
Given that,
$\tan 2A = \cot(A - 18^\circ)$
$\cot(90^\circ - 2A) = \cot(A - 18^\circ)$
$90^\circ - 2A = A - 18^\circ$
$108^\circ = 3A$
$A = 36^\circ$

Page : 189 , Block Name : Exercise 8.3

Q4 If $\tan A = \cot B,$ prove that $A + B = 90^\circ$

Given that,

$\tan A = \cot B$

Answer. $\tan A = \tan(90^\circ - B)$

$A = 90^\circ - B$

$A + B = 90^\circ$

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Q5 If $\sec 4A = cosec\,(A - 20^\circ)$, where $4A$ is an acute angle, find the value of A.

Given that,

$\sec 4A = cosec\,(A - 20^\circ)$

Answer. $cosec\,(90^\circ - 4A) = cosec\,(A - 20^\circ)$

$90^\circ - 4A = A - 20^\circ$

$110^\circ = 5A$

$A = 22^\circ$

Page : 189 , Block Name : Exercise 8.3

Q6 If A, B and C are interior angles of a triangle ABC then show that $\sin\left(\frac{B+C}{2}\right) = \cos\frac{A}{2}$

Answer.

We know that for a triangle ABC

$\angle A + \angle B + \angle C = 180^\circ$

$\angle B + \angle C = 180^\circ - \angle A$

$\frac{\angle B + \angle C}{2} = 90^\circ - \frac{\angle A}{2}$

$\sin\left(\frac{B+C}{2}\right) = \sin\left(90^\circ - \frac{A}{2}\right)$

$= \cos\left(\frac{A}{2}\right)$

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Q7

Express $\sin 67^\circ + \cos 75^\circ$ in terms of trigonometric ratios of angles between 0° and 45°.

$\sin 67^\circ + \cos 75^\circ$

Answer. $= \sin(90^\circ - 23^\circ) + \cos(90^\circ - 15^\circ)$

$= \cos 23^\circ + \sin 15^\circ$

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Exercise 8.4

Q1 Express the trigonometric ratios $\sin A$, $\sec A$ and $\tan A$ in terms of $\cot A$.

Answer. We know that,

$cosec^2 A = 1 + \cot^2 A$

$\frac{1}{cosec^2 A} = \frac{1}{1+\cot^2 A}$

$\sin^2 A = \frac{1}{1+\cot^2 A}$

$\sin A = \pm\frac{1}{\sqrt{1+\cot^2 A}}$

$\sqrt{1 + \cot^2 A}$ will always be positive as we are adding two positive quantities.

Therefore, $\sin A = \frac{1}{\sqrt{1+\cot^2 A}}$

We know that, $\tan A = \frac{\sin A}{\cos A}$

However, $\cot A = \frac{\cos A}{\sin A}$

Therefore, $\tan A = \frac{1}{\cot A}$

Also, $\sec^2 A = 1 + \tan^2 A$

$= 1 + \frac{1}{\cot^2 A}$

$= \frac{\cot^2 A+1}{\cot^2 A}$

$\sec A = \frac{\sqrt{\cot^2 A+1}}{\cot A}$

Page : 193 , Block Name : Exercise 8.4

Q2 Write all the other trigonometric ratios of$\angle$A in terms of $\sec A$.

$$\cos A = \frac{1}{\sec A}$$

$$A|so, \sin^2 A + \cos^2 A = 1$$

$$\sin^2 A = 1 - \cos^2 A$$

$$\sin A = \sqrt{1 - \left(\frac{1}{\sec^2 A}\right)^2}$$

Answer. We know that,

$$= \sqrt{\frac{\sec^2 A - 1}{\sec^2 A}} = \frac{\sqrt{\sec^2 A - 1}}{\sec A}$$

$$= \sqrt{\frac{\sec^2 A}{\sec^2 A}} = \frac{\sqrt{\sec^2 A - 1}}{\sec A}$$

$$\tan^2 A = \sec^2 A - 1$$

$\tan A = \sqrt{\sec^2 A - 1}$

$\cot A = \frac{\cos A}{\sin A} = \frac{\sec A}{\sec A}$

$= \frac{1}{\sqrt{\sec^2 A - 1}}$

$cosecA = \frac{1}{\sin A} = \frac{\sec A}{\sqrt{\sec^2 A - 1}}$

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Q3 Evaluate

(i) $\frac{\sin^2 63^\circ + \sin^2 27^\circ}{\cos^2 17^\circ + \cos^2 73^\circ}$

(ii) $\sin 25^\circ \cos 65^\circ + \cos 25^\circ \sin 65^\circ$

Answer. $\frac{\sin^2 63^\circ + \sin^2 27^\circ}{\cos^2 17^\circ + \cos^2 73^\circ}$

$= \frac{[\sin(90^\circ - 27^\circ)]^2 + \sin^2 27^\circ}{[\cos(90^\circ - 73^\circ)]^2 + \cos^2 73^\circ}$

$= \frac{[\cos 27^\circ]^2 + \sin^2 27^\circ}{[\sin 73^\circ]^2 + \cos^2 73^\circ}$

$= \frac{\cos^2 27^\circ + \sin^2 27^\circ}{\sin^2 73^\circ + \cos^2 73^\circ}$

$= \frac{1}{1} \left(\text{As } \sin^2 \text{A} + \cos^2 \text{A} = 1\right)$

$= 1$

(ii) $\sin 25^\circ \cos 65^\circ + \cos 25^\circ \sin 65^\circ$

$= (\sin 25^\circ) \{\cos(90^\circ - 25^\circ)\} + \cos 25^\circ \{\sin(90^\circ - 25^\circ)\}$

$= (\sin 25^\circ) (\sin 25^\circ) + (\cos 25^\circ) (\cos 25^\circ)$

$= \sin^2 25^\circ + \cos^2 25^\circ$

$= 1 \left(\text{As } \sin^2 A + \cos^2 A = 1\right)$

Page : 193 , Block Name : Exercise 8.4

Q4 Choose the correct option. Justify your choice.

(i) $9\sec^2 A - 9\tan^2 A =$

(A) 1 (B) 9 (C) 8 (D) 0

(ii) $(1+\tan\theta+\sec\theta)(1+\cot\theta-cosec\theta) =$

(A) 0 (B) 1 (C) 2 (D) -1

(iii) $(\sec A+\tan A)(1-\sin A) =$

(A) $\sec A$ (B) $\sin A$ (C) $cosecA$ (D) $\cos A$

$(iv)\frac{1+\tan^2 A}{1+\cot^2 A} =$

$(A)\sec^2 A$ $(B)-1$

$(C)\cot^2 A$ (D) $\tan^2 A$

Answer.

$$\begin{aligned}&\text{(i) } 9\sec^2 A - 9\tan^2 A\\&= 9\left(\sec^2 A - \tan^2 A\right)\\&= 9(1)\left[\text{As } \sec^2 A - \tan^2 A = 1\right]\\&= 9\end{aligned}$$

Hence, alternatives (B) is correct.

(ii) $(1+\tan\theta+\sec\theta)(1+\cot\theta-cosec\theta)$

$$\begin{aligned}&= \left(1+\frac{\sin\theta}{\cos\theta}+\frac{1}{\cos\theta}\right)\left(1+\frac{\cos\theta}{\sin\theta}-\frac{1}{\sin\theta}\right)\\&= \left(\frac{\cos\theta+\sin\theta+1}{\cos\theta}\right)\left(\frac{\sin\theta+\cos\theta-1}{\sin\theta}\right)\\&= \frac{(\sin\theta+\cos\theta)^2-(1)^2}{\sin\theta\cos\theta}\\&= \frac{\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta-1}{\sin\theta\cos\theta}\\&= \frac{2\sin\theta+\cos\theta+2\sin\theta\cos\theta-1}{\sin\theta\cos\theta} = 2\\&= \frac{2\sin\theta\cos\theta}{\sin\theta\cos\theta} = 2\end{aligned}$$

Hence, alternative (C) is incorrect.

(iii) (secA + tanA)(1-sinA)

$$\begin{aligned}&= \left(\frac{1}{\cos A}+\frac{\sin A}{\cos A}\right)(1-\sin A)\\&= \left(\frac{1+\sin A}{\cos A}\right)(1-\sin A)\\&= \frac{1-\sin^2 A}{\cos A} = \frac{\cos^2 A}{\cos A}\\&= \cos A\end{aligned}$$

Hence, alternative (D) is correct.

(iv) $\frac{1+\tan^2 A}{1+\cot^2 A} = \frac{1+\frac{\sin^2 A}{\cos^2 A}}{1+\frac{\cos^2 A}{\sin^2 A}}$

$= \frac{\frac{\cos^2 A+\sin^2 A}{\cos^2 A}}{\frac{\sin^2 A+\cos^2 A}{\sin^2 A}} = \frac{\frac{1}{\cos^2 A}}{\frac{1}{\sin^2 A}}$

$= \frac{\sin^2 A}{\cos^2 A} = \tan^2 A$

Hence, alternative (D) is correct.

Page : 193 , Block Name : Exercise 8.4

Q5

Prove the following identities, where the angles involved are acute angles for which the expressions are defined.

$(i)(cosec\theta - \cot\theta)^2 = \frac{1-\cos\theta}{1+\cos\theta}$ (ii) $\frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A} = 2\sec A$

$(iii)\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta cosec\theta$

[Hint : Write the expression in terms of $\sin\theta$ and $\cos\theta$]

$(iv)\frac{1+\sec A}{\sec A} = \frac{\sin^2 A}{1-\cos A}$ [Hint : Simplify LHS and RHS separately]

$(v)\frac{\cos A-\sin A+1}{\cos A+\sin A-1} = cosecA + \cot A,$ using the identity cosec $^2 A = 1 + \cot^2 A$

$(vi)\sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A$

$(vii)\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$

$(viii)(\sin A + cosecA)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$

(ix) $(cosecA - \sin A)(\sec A - \cos A) = \frac{1}{\tan A+\cot A}$

[Hint : Simplify LHS and RHS separately]

$(x)\left(\frac{1+\tan^2 A}{1+\cot^2 A}\right) = \left(\frac{1-\tan A}{1-\cot A}\right)^2 = \tan^2 A$

Answer. $(i)(cosec\theta - \cot\theta)^2 = \frac{1-\cos\theta}{1+\cos\theta}$

L.H.S. $= (cosec\theta - \cot\theta)^2$

$= \left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)^2$

$= \frac{(1-\cos\theta)^2}{(\sin\theta)^2} = \frac{(1-\cos\theta)^2}{\sin^2\theta}$

$= \frac{(1-\cos\theta)^2}{1-\cos^2\theta} = \frac{(1-\cos\theta)^2}{(1-\cos\theta)(1+\cos\theta)} = \frac{1-\cos\theta}{1+\cos\theta}$

$= R.H.S.$

$(ii)\frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A} = 2\sec A$

$$\text{L.H.S.} = \frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A}$$
$$= \frac{\cos^2 A + (1+\sin A)^2}{(1+\sin A)(\cos A)^2}$$
$$= \frac{\cos^2 A + 1 + \sin^2 A + 2\sin A}{(1+\sin A)(\cos A)}$$
$$= \frac{\sin^2 A + \cos^2 A + 1 + 2\sin A}{(1+\sin A)(\cos A)}$$
$= \frac{1+1+2\sin A}{(1+\sin A)(\cos A)} = \frac{2+2\sin A}{(1+\sin A)(\cos A)}$
$$= \frac{2(1+\sin A)}{(1+\sin A)(\cos A)} = \frac{2}{\cos A} = 2\sec A$$
$= R.H.S.$

(iii) $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta cosec\theta$

L.H.S.$= \frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta}$

$= \frac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\sin\theta}{\cos\theta}}$

$= \frac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta-\cos\theta}{\sin\theta}+\frac{\cos\theta}{\cos\theta}}$

$$= \frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)} + \frac{\cos^2\theta}{\sin\theta(\sin\theta-\cos\theta)}$$
$$= \frac{1}{(\sin\theta-\cos\theta)}\left[\frac{\sin^2\theta}{\cos\theta} - \frac{\cos^2\theta}{\sin\theta}\right]$$
$$= \left(\frac{1}{\sin\theta-\cos\theta}\right)\left[\frac{\sin^3\theta-\cos^3\theta}{\sin\theta\cos\theta}\right]$$
$$= \left(\frac{1}{\sin\theta-\cos\theta}\right)\left[\frac{(\sin\theta-\cos\theta)\left(\sin^2\theta+\cos^2\theta+\sin\theta\cos\theta\right)}{\sin\theta\cos\theta}\right]$$
$$= \frac{(1+\sin\theta\cos\theta)}{(\sin\theta\cos\theta)}$$
$= sec\theta cosec\theta$
$= R.H.S$

(iv) $\frac{1+\sec A}{\sec A} = \frac{\sin^2 A}{1-\cos A}$

$\text{L.H.S.} = \frac{1+\sec A}{\sec A} = \frac{1+\frac{1}{\cos A}}{\frac{1}{\cos A}}$

$$= \frac{\cos A + 1}{\cos A} = (\cos A + 1)$$
$$= \frac{(1-\cos A)(1+\cos A)}{(1-\cos A)}$$
$$= \frac{1-\cos^2 A}{1-\cos A} = \frac{\sin^2 A}{1-\cos A}$$

= R.H.S.

(v) $\frac{\cos A-\sin A+1}{\cos A+\sin A-1} = cosecA + \cot A$

Using the identity $cosec^2 A = 1 + \cot^2 A$

L.H.S. $= \frac{\cos A-\sin A+1}{\cos A+\sin A-1}$

$= \frac{\frac{\cos A}{\sin A}-\frac{\sin A}{\sin A}+\frac{1}{\sin A}}{\frac{\cos A}{\sin A}+\frac{\sin A}{\sin A}+\frac{1}{\sin A}}$

$= \frac{\cot A-1+cosecA}{\cot A+1-cosecA}$

$= \frac{\{(\cot A)-(1-cosecA)\}\{(\cot A)-(1-cosecA)\}}{\{(\cot A)+(1-cosecA)\}\{(\cot A)-(1-cosecA)\}}$

$= \frac{(\cot A - 1 + cosecA)^2}{(\cot A)^2 - (1 - cosecA)^2}$

$= \frac{\cot^2 A + 1 + cosec^2 A - 2\cot A - 2cosecA + 2\cot A cosecA}{\cot^2 A - (1 + cosec^2 A - 2cosecA)}$

$= \frac{2\,\mathrm{cosec}^2 A + 2\cot A\cos \epsilon cA - 2\cot A - 2\,\mathrm{cosec}\, A}{\cot^2 A - 1 - cosec^2 A + 2cosecA}$

$= \frac{2cosecA(cosecA + \cot A) - 2(\cot A + cosecA)}{\cot^2 A - cosec^2 A - 1 + 2cosecA}$

$= \frac{(cosecA + \cot A)(2cosecA - 2)}{-1 - 1 + 2cosecA}$

$= \frac{(cosecA + \cot A)(2cosecA - 2)}{(2cosecA - 2)}$

$= cosecA + \cot A$

$= R.H.S$

(vi) $\sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A$

L.H.S. $= \sqrt{\frac{1 + \sin A}{1 - \sin A}}$

$= \sqrt{\frac{(1 + \sin A)(1 + \sin A)}{(1 - \sin A)(1 + \sin A)}}$

$= \frac{(1+\sin A)}{\sqrt{1-\sin^2 A}} = \frac{1+\sin A}{\sqrt{\cos^2 A}}$

$= \frac{1+\sin A}{\cos A} \quad = \sec A + \tan A$

= R.H.S.

(vii) $\frac{\sin\theta-2\sin^3\theta}{2\cos\theta\cos\theta} = \tan\theta$

L.H.S. $= \frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta}$

$= \frac{\sin\theta\left(1 - 2\sin^2\theta\right)}{\cos\theta\left(2\cos^2\theta - 1\right)}$

$$= \frac{\sin\theta \times \left(1 - 2\sin^2\theta\right)}{\cos\theta \times \left\{2\left(1 - \sin^2\theta\right) - 1\right\}}$$

$$= \frac{\sin\theta \times \left(1 - 2\sin^2\theta\right)}{\cos\theta \times \left(1 - 2\sin^2\theta\right)}$$

$$= \tan\theta = R.H.S$$

$(viii) (\sin A + cosecA)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$

L.H.S$= (\sin A + cosecA)^2 + (\cos A + \sec A)^2$

$$= \sin^2 A + \text{cosec}^2 A + 2\sin A cosecA + \cos^2 A + \sec^2 A + 2\cos A \sec A$$

$$= \left(\sin^2 A + \cos^2 A\right) + \left(cosec^2 A + \sec^2 A\right) + 2\sin A\left(\frac{1}{\sin A}\right) + 2\cos A\left(\frac{1}{\cos A}\right)$$

$$= (1) + \left(1 + \cot^2 A + 1 + \tan^2 A\right) + (2) + (2)$$

$$= 7 + \tan^2 A + \cot^2 A$$

$$= R.H.S$$

$(ix) (cosecA - \sin A)(\sec A - \cos A) = \frac{1}{\tan A + \cot A}$

L.H.S $= (cosecA - \sin A)(\sec A - \cos A)$

$$= \left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right)$$

$$= \left(\frac{1 - \sin^2 A}{\sin A}\right)\left(\frac{1 - \cos^2 A}{\cos A}\right)$$

$$= \frac{\left(\cos^2 A\right)\left(\sin^2 A\right)}{\sin A \cos A}$$

$$= \sin A \cos A$$

R.H.S$= \frac{1}{\tan A + \cot A}$

$$= \frac{1}{\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}} = \frac{1}{\frac{\sin^2 A + \cos^2 A}{\sin A \cos A}}$$

$$= \frac{\sin A \cos A}{\sin^2 A + \cos^2 A} = \sin A \cos A$$

Hence, L.H.S = R.H.S

$(x) \left(\frac{1 + \tan^2 A}{1 + \cot^2 A}\right) = \left(\frac{1 - \tan A}{1 - \cot A}\right)^2 = \tan^2 A$

$$\frac{1 + \tan^2 A}{1 + \cot^2 A} = \frac{1 + \frac{\sin^2 A}{\cos^2 A}}{1 + \frac{\cos^2 A}{\sin^2 A}} = \frac{\frac{\cos^2 A + \sin^2 A}{\cos^2 A}}{\frac{\cos^2 A}{\sin^2 A}}$$

$$= \frac{\frac{1}{\cos^2 A}}{\frac{1}{\sin^2 A}} = \frac{\sin^2 A}{\cos^2 A}$$

$$= \tan^2 A$$

$$\left(\frac{1-\tan A}{1-\cot A}\right)^2 = \frac{1+\tan^2 A-2\tan A}{1+\cot^2 A-2\cot A}$$

$$= \frac{\sec^2 A-2\tan A}{\operatorname{cosec}^2 A-2\cot A}$$

$$= \frac{\frac{1}{\cos^2 A}-\frac{2\sin A}{\cos A}}{\frac{1}{\sin^2 A}-\frac{2\cos A}{\sin A}} = \frac{\frac{1-2\sin A\cos A}{\cos^2 A}}{\frac{1-2\sin^2 A}{\sin^2 A}}$$

$$= \frac{\sin^2 A}{\cos^2 A} = \tan^2 A$$

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