NCERT SOLUTIONS

CLASS - 10th



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Class : 10th
Subject : Maths
Chapter : 5
Chapter Name : Arithmetic Progressions

Exercise 5.1

Q1 In which of the following situations, does the list of numbers involved make an arithmetic progression, and why?
(i) The taxi fare after each km when the fare is ₽ 15 for the first km and ₽ 8 for each additional km.
(ii) The amount of air present in a cylinder when a vacuum pump removes $\frac{1}{4}$ of the air remaining in the cylinder at a time.
(iii) The cost of digging a well after every metre of digging, when it costs ₽ 150 for the first metre and rises by ₽ 50 for each subsequent metre.
(iv) The amount of money in the account every year, when ₽ 10000 is deposited at compound interest at 8 % per annum.

Answer. (i) It can be observe that
Taxi fare for 1^{st} km = 15
Taxi fare for first 2 km = 15 + 8 = 23
Taxi fare for first 3 km = 23 + 8 = 31
Taxi fare for first 4 km = 31 + 8 = 39
Clearly 15, 23, 31, 39.... forms an A.P. because every term is 8 more than the preceding term
(ii) Let the initial volume of air in a cylinder be V lit. In each stroke, the vacuum pump removes $\frac{1}{4}$ of air remaining in the cylinder at a time.
In other words, after every stroke ,only $1 - \frac{1}{4} = \frac{3}{4}th$ part of air will remain.
Therefore, volumes will be $V\left(\frac{3V}{4}\right) \cdot \left(\frac{3}{4}V\right)^2 \cdot \left(\frac{3}{4}V\right)^3 \ldots$
Clearly, it can be observed that the adjacent terms of this series do
not have the same difference between them. Therefore, this is not an A.P
(iii) Cost of digging for first metre = 150
Cost of digging for first 2 metres = 150 + 50 = 200
Cost of digging for first 3 metres = 200 + 50 = 250
Cost of digging for first 4 metres = 250 + 50 = 300
Clearly, 150, 200, 250, 300 forms an A.P. because every term is 50 more than the preceding term.

(iv) We know that if Rs P is deposited at r% compound interest per annum for n years, our money will be $\mathbf{P}\left(1+\frac{r}{100}\right)^{n}$ after n years.

Therefore, after every year, our money will be

$10000\left(1+\frac{8}{100}\right), 10000\left(1+\frac{8}{100}\right)^{2}, 10000\left(1+\frac{8}{100}\right)^{3}, 10000\left(1+\frac{8}{100}\right)^{4}$

Clearly, adjacent terms of this series do not have the same difference between them. Therefore, this is not an A.P.

Page : 99 , Block Name : Exercise 5.1

Q2 Write first four terms of the AP, when the first term a and the common difference d are given as follows:

(i) a = 10, d = 10

(ii) a = −2, d = 0

(iii) a = 4, d = – 3

(iv) a = – 1, d = 1/2

(v) a = – 1.25, d = – 0.25

Answer. $a=10, d=10$

Let the series $a_1, a_2, a_3, a_4, a_5 \ldots$

$a_1 = a = 10$

$a_2 = a_1 + d = 10 + 10 = 20$

$a_3 = a_2 + d = 20 + 10 = 30$

$a_4 = a_3 + d = 30 + 10 = 40$

$a_5 = a_4 + d = 40 + 10 = 50$

Therefore , the series will be 10,20,30,40,50..

First four terms of this A.P. will be 10, 20, 30, and 40.

(ii) $a=-2, d=0$

Let the series $a_{1,} a_2, a_3, a_4 \ldots$

$a_1 = a = -2$

$a_2 = a_1 + d = -2 + 0 = -2$

$a_3 = a_2 + d = -2 + 0 = -2$

$a_4 = a_3 + d = -2 + 0 = -2$

Therefore the series will be $-2, -2, -2, -2 \ldots$

First four terms of this AP. Will be $-2, -2, -2 \text{ and } -2$

(iii) $a=4, d=-3$

Let the series be $a_1, a_2, a_{3r} a_4 \ldots$

$a_1 = a = 4$

$a_2 = a_1 + d = 4 - 3 = 1$

$a_3 = a_2 + d = 1 - 3 = -2$

$a_4 = a_3 + d = -2 - 3 = -5$

(iv) $a = -1, d = \frac{1}{2}$
Let the series be $a_1, a_2, a_3, a_4 \ldots$
$a_1 = a = -1$
$a_2 = a_1 + d = -1 + \frac{1}{2} = -\frac{1}{2}$
$a_3 = a_2 + d = -\frac{1}{2} + \frac{1}{2} = 0$
$a_4 = a_3 + d = 0 + \frac{1}{2} = \frac{1}{2}$
Clearly the series will be
$-1, -\frac{1}{2}, 0, \frac{1}{2}$
Four terms of this A.P. will be $-1, -\frac{1}{2}, 0$ and $\frac{1}{2}$

(v) $a = -1.25, d = -0.25$
Let the series be $a_1, a_2, a_{3r} a_4 \ldots$
$a_1 = a = -1.25$
$a_2 = a_1 + d = -1.25 - 0.25 = -1.50$
$a_3 = a_2 + d = -1.50 - 0.25 = -1.75$
$a_4 = a_3 + d = -1.75 - 0.25 = -2.00$
Clearly the series will be $1.25, -1.50, -1.75, -2.00 \ldots \ldots$
First four terms of this A.P will be $-1.25, -1.50, -1.75$ and -2.00

Page : 99 , Block Name : Exercise 5.1

Q3 For the following APs, write the first term and the common difference:
(i) $3, 1, -1, -3 \ldots$
(ii) $-5, -1, 3, 7 \ldots$
(iii) $3, \frac{5}{3}, \frac{9}{3}, \frac{13}{3} \ldots$
(iv) $0.6, 1.7, 2.8, 3.9 \ldots$

Answer. (i) $3, 1, -1, -3 \ldots$
Here, first term, a = 3
Common difference, d = Second term — First term
= 1-3=-2

(ii) $-5, -1, 3, 7 \ldots$
Here, first term, a = -5
Common difference, d = Second term — First term
$= (-1) - (-5) = -1 + 5 = 4$

(iii) $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3} \ldots$
Here, first term $a = \frac{1}{3}$
Common difference, d = Second term — First term

$= \frac{5}{3} - \frac{1}{3} = \frac{4}{3}$

(iv) $0.6, 1.7, 2.8, 3.9 \ldots$
Here, first term , a=0.6
Common difference, d = second term - First term
$= 1.7 - 0.6$
$= 1.1$

Page : 99 , Block Name : Exercise 5.1

Q4 Which of the following are APs? If they form an AP, find the common difference d and write three more terms.

(i) $2, 4, 8, 16, \ldots$
(ii) $2, \frac{5}{2}, 3, \frac{7}{2}, \ldots$
(iii) $-1.2, -3.2, -5.2, -7.2, \ldots$
(iv) $-10, -6, -2, 2, \ldots$
(v) $3, 3+\sqrt{2}, 3+2\sqrt{2}, 3+3\sqrt{2}, \ldots$
(vi) $0.2, 0.22, 0.222, 0.2222$
(vii) $0, -4, -8, -12, \ldots$
(viii) $-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \ldots$
(ix) $1, 3, 9, 27, \ldots$
(x) $a, 2a, 3a, 4a, \ldots$
(xi) $a, a^2, a^3, a^4, \ldots$
(xii) $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}, \ldots$
(xiii) $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}, \ldots$
(xiv) $1^2, 3^2, 5^2, 7^2, \ldots$
(xv) $1^2, 5^2, 7^2, 73, \ldots$

Answer. Missing

Page : 99 , Block Name : Exercise 5.1

Exercise 5.2

Q1 Fill in the blanks in the following table, given that a is the first term, d the common difference and an the nth term of the AP

	a	d	n	a_n
I	7	3	8	.....
II	− 18	.....	10	0
III	.....	− 3	18	− 5
IV	− 18.9	2.5	.....	3.6
V	3.5	0	105	.....

Answer. (I) $a = 7, d = 3, n = 8, a_n = ?$
We know that ,
For an A.P $a_n = a + (n-1)d$
$= 7 + (8-1)3$
$= 7 + (7)3$
$= 7 + 21 = 28$
Hence $a_n = 28$

(II) Given that
$a = -18, n = 10, a_n = 0, d = ?$
We know that
$a = -18, n = 10, a_n = 0, d = ?$
We know that
$a_n = a + (n-1)d$
$0 = -18 + (10-1)d$
$18 = 9d$
$d = \frac{18}{9} = 2$
Hence common difference , d=2

(III) Given that
$d = -3, n = 18, a_n = -5$
We know that

$a_n = a + (n-1)d$
$-5 = a + (18-1)(-3)$
$-5 = a + (17)(-3)$
$-5 = a - 51$
$a = 51 - 5 = 46$
Hence a=46

(IV) $a = -18.9, d = 2.5, a_n = 3.6, n = ?$
We know that
$a_n = a + (n-1)d$
$3.6 = -18.9 + (n-1)2.5$
$3.6 + 18.9 = (n-1)2.5$
$22.5 = (n-1)2.5$
$(n-1) = \frac{22.5}{2.5}$
$n - 1 = 9$
$n = 10$
Hence, n=10

(V) $a = 3.5, d = 0, n = 105, a_n = ?$
We know that
$a_n = a + (n-1)d$
$a_n = 3.5 + (105-1)0$
$a_n = 3.5 + 104 \times 0$
$a_n = 3.5$
Hence, $a_n = 3.5$

Page : 105 , Block Name : Exercise 5.2

Q2 Choose the correct choice in the following and justify
(I) 30th term of the A.P: 10, 7, 4, , is
A. 97 B. 77 C. -77 D. -87
(II) 11^{th} term of the A.P. $^{-3,-\frac{1}{2},2}$ is
A 28B.22C. -38D. $^{-48\frac{1}{2}}$

Answer. Given that
A.P. 10,7,4....
First term a=10
Common difference ,$d = a_2 - a_1 = 7 - 10$
=-3

$\text{We know that, } a_n = a + (n-1)d$

$a_{30} = 10 + (30-1)(-3)$

$a_{30} = 10 + (29)(-3)$

$a_{30} = 10 - 87 = -77$

Hence the correct answer is C

II. Given that A.P. $-3, -\frac{1}{2}, 2, \ldots$

First term a= -3

Common difference $d = a_2 - a_1$

$= -\frac{1}{2} - (-3)$

$= -\frac{1}{2} + 3 = \frac{5}{2}$

We know that

$a_n = a + (n-1)d$

$a_{11} = -3 + (11-1)\left(\frac{5}{2}\right)$

$a_{11} = -3 + (10)\left(\frac{5}{2}\right)$

$a_{11} = -3 + 25$

$a_{11} = 22$

Hence, the answer is B.

Page : 106 , Block Name : Exercise 5.2

Q3 In the following APs find the missing term in the boxes

Answer. I. $2, \square, 26$

For this A.P.,

a=2

$a_3 = 26$

We know that $a_n = a + (n-1)d$

$a_3 = 2 + (3-1)d$

$26 = 2 + 2d$

$24 = 2d$

$d = 12$

$a_2 = 2 + (2-1)12$

$= 14$

Therefore 14 is the missing term

II. $\square, 13, \square, 3$

For this A.P.

$a_2 = 13 \text{ and}$

$a_4 = 3$

We know that $a_n = a + (n-1)d$

$a_2 = a + (2-1)d$

$13 = a + d(1)$

$a_4 = a + (4-1)d$

$3 = a + 3d(II)$

On subtracting (I) from (II) we obtain

$-10 = 2d$

$d = -5$

From equation (I) we obtain

$13 = a + (-5)$

$a = 18$

$a_3 = 18 + (3-1)(-5)$

$= 18 + 2(-5) = 18 - 10 = 8$

Therefore the missing terms are 18 and 7 respectively .

III. $5, \square, \square, 9\frac{1}{2}$

For this A.P.

$a = 5$

$a_4 = 9\frac{1}{2} = \frac{19}{2}$

We know that

$a_n = a + (n-1)d$

$a_4 = a + (4-1)d$

$\frac{19}{2} = 5 + 3d$

$\frac{19}{2} - 5 = 3d$

$\frac{9}{2} = 3d$

$d = \frac{3}{2}$

$a_2 = a + d = 5 + \frac{3}{2} = \frac{13}{2}$

$a_3 = a + 2d = 5 + 2\left(\frac{3}{2}\right) = 8$

Therefore the missing terms are 13/2 and 8 respectively

IV. $-4, \square. \square. \square. \square.6$

For this A.P.,

$a = -4$ and

$a_6 = 6$

We know that

$a_n = a + (n-1)d$
$a_6 = a + (6-1)d$
$6 = -4 + 5d$
$10 = 5d$
$d = 2$
$a_2 = a + d = -4 + 2 = -2$
$a_3 = a + 2d = -4 + 2(2) = 0$
$a_4 = a + 3d = -4 + 3(2) = 2$
$a_5 = a + 4d = -4 + 4(2) = 4$
Therefore ,the missing terms are -2,0,2 and 4 respectively .

V. $\square, 38, \square, \square, \square, -22$
For this A.P.
$a_2 = 38$
$a_6 = -22$
We know that
$a_n = a + (n-1)d$
$a_2 = a + (2-1)d$
$38 = a + d(1)$
$a_6 = a + (6-1)d$
$-22 = a + 5d(2)$
On subtracting equation (1) from (2) we obtain
$-22 - 38 = 4d$
$-60 = 4d$
$d = -15$
$a = a_2 - d = 38 - (-15) = 53$
$a_3 = a + 2d = 53 + 2(-15) = 23$
$a_4 = a + 3d = 53 + 3(-15) = 8$
$a_5 = a + 4d = 53 + 4(-15) = -7$
Therefore the missing terms are 53,23, 8 and -7 respectively.

Page : 106 , Block Name : Exercise 5.2

Q4 Which term of the AP : 3, 8, 13, 18, . . . ,is 78?

Answer. $3, 8, 13, 18, \ldots$
For this A.P.,
a=3
$d = a_2 - a_1 = 8 - 3 = 5$
Let nth term of this A.P. be 78

$a_n = a + (n-1)d$
$78 = 3 + (n-1)5$
$75 = (n-1)5$
$(n-1) = 15$
$n = 16$
Hence, 16tn term of this A.P. is 78.

Page : 106 , Block Name : Exercise 5.2

Q5 Find the number of terms in each of the following APs
I. $7, 13, 19, \ldots, 205$
II. $18, 15\frac{1}{2}, 13, \ldots, -47$

Answer. I. $7, 13, 19, \ldots, 205$
For this A.P. ,
$a = 7$
$d = a_2 - a_1 = 13 - 7 = 6$
Let there are n terms in this A.P.
$a_n = 205$
We know that
$a_n = a + (n-1)d$
Therefore $205 = 7 + (n-1)6$
$198 = (n-1)6$
$33 = (n-1)$
$n = 34$
Therefore this given series has 34 terms in it.

II. $18, 15\frac{1}{2}, 13, \ldots, -47$
For this A.P.,
$a = 18$
$d = a_2 - a_1 = 15\frac{1}{2} - 18$
$d = \frac{31 - 36}{2} = -\frac{5}{2}$
Let there are n terms in this A.P.
Therefore $a_n = -47$ and we know that

$a_n = a + (n-1)d$

$-47 = 18 + (n-1)\left(-\frac{5}{2}\right)$

$-47 - 18 = (n-1)\left(-\frac{5}{2}\right)$

$-65 = (n-1)\left(-\frac{5}{2}\right)$

$(n-1) = \frac{-130}{-5}$

$(n-1) = 26$

$n = 27$

Therefore, this given A.P. has 27 terms in it.

Page : 106 , Block Name : Exercise 5.2

Q6 Check whether -150 is a term of hw A,.P. $11, 8, 5, 2, \ldots$

Answer. For this A.P.,

$a = 11$

$d = a_2 - a_1 = 8 - 11 = -3$

Let -150 be the n^{th} term of this A.P.

We know that

$a_n = a + (n-1)d$

$-150 = 11 + (n-1)(-3)$

$-150 = 11 - 3n + 3$

$-164 = -3n$

$n = \frac{164}{3}$

Clearly , n is not an integer

Therefore -150 is not a term of this A.P.

Page : 106 , Block Name : Exercise 5.2

Q7 Find the 31^{st} term of an AP whose 11th term is 38 and the 16th term is 73.

Answer. Given that,

$a_{11} = 38$

$a_{16} = 73$

We know that,

$a_n = a + (n-1)d$

$a_{11} = a + (11-1)d$

$38 = a + 10d (1)$

Similarly,

$a_{16} = a + (16 - 1)d$

$73 = a + 15d(2)$

On subtracting (1) from (2) we obtain

$35 = 5d$

$d = 7$

From an equation (1)

$38 = a + 10 \times (7)$

$38 - 70 = a$

$a = -32$

$a_{31} = a + (31 - 1)d$

$= -32 + 30(7)$

$= -32 + 210$

=178

Hence 31^{st} term is 178

Page : 106 , Block Name : Exercise 5.2

Q8 An A.P. consists of 50 terms of which 3rd term is 12 and the last term is 106. Find the 29th term

Answer. Given that,

$a_3 = 12$

$a_{50} = 106$

We know that,

$a_n = a + (n - 1)d$

$a_3 = a + (3 - 1)d$

$12 = a + 2d(1)$

Similarly $a_{50} = a + (50 - 1)d$

$106 = a + 49d(11)$

On subtracting (I) from (II) we obtain

$94 = 47d$

$d = 2$

From equation (I) we obtain

$12 = a + 2(2)$

$a = 12 - 4 = 8$

$a_{29} = a + (29 - 1)d$

$a_{29} = 8 + (28)2$

$a_{29} = 8 + 56 = 64$

Therefore 29^{th} term is 64.

Page : 106 , Block Name : Exercise 5.2

Q9 If the 3^{rd} and the 9th terms of an A.P. are 4 and Which term of this A.P. is zero. -8 respectively.

Answer. Given that,
$a_3 = 4$
$a_9 = -8$
We know that
$a_n = a + (n-1)d$
$a_3 = a + (3-1)d$
$4 = a + 2d(1)$
$a_9 = a + (9-1)d$
$-8 = a + 8d(11)$
On subtracting equation $(I) from (II)$, we obtain.
$-12 = 6d$
$d = -2$
$\text{From equation (I), we obtain}$
$4 = a + 2(-2)$
$4 = a - 4$
$a = 8$
$\text{Let } n^{\text{th}} \text{ term of this A.P. be zero.}$
$a_n = a + (n-1)d$
$0 = 8 + (n-1)(-2)$
$0 = 8 - 2n + 2$
$2n = 10$
$n = 5$
Hence 5^{th} term of this A.P. is 0.

Page : 106 , Block Name : Exercise 5.2

Q10 If the 3rd and the 9th terms of an AP are 4 and – 8 respectively, which term of this AP is zero?

Answer. We know that,
$\text{For an } A.P., a_n = a + (n-1)d$
$a_{17} = a + (17-1)d$
$a_{17} = a + 16d$
Similarly $a_{10} = a + 9d$

It is given that
$a_{17} - a_{10} = 7$
$(a + 16d) - (a + 9d) = 7$
$7d = 7$
$d = 1$
Therefore the common difference is 1

Page : 106 , Block Name : Exercise 5.2

Q11 Which term of the A.P. 3, 15, 27, 39, . will be 132 more than its 54th term?

Answer. Given A.P. is $3, 15, 27, 39, \ldots$
$a = 3$
$d = a_2 - a_1 = 15 - 3 = 12$
$a_{54} = a + (54 - 1)d$
$= 3 + (53)(12)$
$= 3 + 636 = 639$
$132 + 639 = 771$
We have to find out the term of this A.P. which is 771.
$\text{Let } n^{\text{th}} \text{ term be } 771.$
$a_n = a + (n - 1)d$
$771 = 3 + (n - 1)12$
$768 = (n - 1)12$
$(n - 1) = 64$
$n = 65$
$\text{Therefore, } 65^{\text{th}} \text{ term was } 132 \text{ more than } 54^{\text{th}} \text{ term.}$
Alternatively,
$\text{Let } n^{\text{th}} \text{ term be } 132 \text{ more than } 54^{\text{th}} \text{ term.}$
$n = 54 + \frac{132}{12}$
$= 54 + 11 = 65^{\text{th}} \text{ term}$

Page : 106 , Block Name : Exercise 5.2

Q12 Two APs have the same common difference. The difference between their 100th terms is 100, what is the difference between their 1000th terms?

Answer. Let the first term of these A.P.s be al and a2 respectively and the common difference of these A.P.s be d.
For first A.P.

$a_{100} = a_1 + (100 - 1)d$

$= a_1 + 99d$

$a_{1000} = a_1 + (1000 - 1)d$

$a_{1000} = a_1 + 999d$

For second A.P.

$a_{100} = a_2 + (100 - 1)d$

$= a_2 + 99d$

$a_{1000} = a_2 + (1000 - 1)d$

$= a_2 + 999d$

Given that,difference between

100^{th} term of these A.P.s $= 100$

Therefore, $(a_1 + 99d) - (a_2 + 99d) = 100$

$a_1 - a_2 = 100(1)$

Difference between 1000^{th} terms of these A.P.s

$(a_1 + 999d) - (a_2 + 999d) = a_1 - a_2$

From equation (1)

This difference, $a_1 - a_2 = 100$

Hence the difference between 1000^{th} terms of these A.P. will be 100.

Page : 106 , Block Name : Exercise 5.2

Q13 How many three-digit numbers are divisible by 7?

Answer. First three-digit number that is divisible by 7= 105

Next number $= 105 + 7 = 112$

Therefore $105, 112, 119, \ldots$

All are three digit numbers which are divisible by 7 and thus, all these are terms of an A.P. having first term as 105 and common difference as 7. The maximum possible three-digit number is 999. When we divide it

by 7, the remainder will be 5. Clearly, 999 — 5 = 994 is the maximum possible three-digit number that is divisible by 7.

The series is as follows.

105, 112, 119, …., 994

Let 994 be the nth term of this A.P.

$a = 105$
$d = 7$
$a_n = 994$
$n =?$
$a_n = a + (n-1)d$
$994 = 105 + (n-1)7$
$889 = (n-1)7$
$n = 128$
Therefore 128 three digit numbers are divisible by 7.

Page : 106 , Block Name : Exercise 5.2

Q14 How many multiples of 4 lie between 10 and 250?
Answer. First multiple of 4 that is greater than 10 is 12. Next will be 16.
Therefore, 12, 16, 20, 24,
All these are divisible by 4 and thus, all these are terms of an A.P. with first term as 12 and common difference as 4.
When we divide 250 by 4, the remainder will be 2. Therefore, 250 — 2 = 248 is divisible by 4.
The series is as follows.
12, 16, 20, 24, …., 248
Let 248 be the nth term of this AP.
$a = 12$
$d = 4$
$a_n = 248$
$a_n = a + (n-1)d$
$248 = 12 + (n-1)4$
$\frac{236}{4} = n - 1$
$n = 60$
Therefore there are 60 multiples of 4 between 10 and 250

Page : 106 , Block Name : Exercise 5.2

Q15 For what value of n, are the nth terms of two APs: 63, 65, 67, . . . and 3, 10, 17, . . . equal?

Answer.
$63, 65, 67, \ldots$
$a = 63$
$d = a_2 - a_1 = 65 - 63 = 2$
n^{th} term of this A.P. $= a_n = a + (n-1)d$
$a_n = 63 + (n-1)2 = 63 + 2n - 2$

$a_n = 61 + 2n(1)$

$3, 10, 17, \ldots$

$a = 3$

$d = a_2 - a_1 = 10 - 3 = 7$

n^{th} term of this A.P. $= 3 + (n-1)7$

$a_n = 3 + 7n - 7$

$a_n = 7n - 4(2)$

It is given that, nth term of these A.P.s are equal to each other. Equating both these equations, we obtain

$61 + 2n = 7n - 4$

$61 + 4 = 5n$

$5n = 65$

$n = 13$

Therefore 13^{th} terms of both these A.P.s are equal to each other.

Page : 106 , Block Name : Exercise 5.2

Q16 Determine the AP whose third term is 16 and the 7th term exceeds the 5th term by 12

$= a_3 = 16$

$a + (3-1)d = 16$

$a + 2d = 16(1)$

Answer. $a_7 - a_5 = 12$

$[a + (7-1)d] - [a + (5-1)d] = 12$

$(a + 6d) - (a + 4d) = 12$

$2d = 12$

$d = 6$

From equation (1), we obta in

$a + 2(6) = 16$

$a + 12 = 16$

$a = 4$

Therefore, A.P. will be

$4, 10, 16, 22, \ldots$

Page : 106 , Block Name : Exercise 5.2

Q17 Find the 20th term from the last term of the AP : 3, 8, 13, . . ., 253.

Answer. Given A.P is

$a = 253$
$d = 248 - 253 = -5$
$n = 20$
$a_{20} = a + (20 - 1)d$
$a_{20} = 253 + (19)(-5)$
$a_{20} = 253 - 95$
$a = 158$
Therefore 20th term from last term is 158

Page : 107 , Block Name : Exercise 5.2

Q18 The sum of the 4th and 8th terms of an AP is 24 and the sum of the 6th and 10th terms is 44. Find the first three terms of the AP.

Answer.
We know that,
$a_n = a + (n - 1)d$
$a_4 = a + (4 - 1)d$
$a_4 = a + 3d$
Similarly,
$a_8 = a + 7d$
$a_6 = a + 5d$
$a_{10} = a + 9d$

$\text{Given that, } a_4 + a_8 = 24$
$a + 3d + a + 7d = 24$
$2a + 10d = 24$
$a + 5d = 12 (1)$
$a_6 + a_{10} = 44$
$a + 5d + a + 9d = 44$
$2a + 14d = 44$
$a + 7d = 22 (2)$
$\text{On subtracting equation (1) from (2), we obtain}$
$2d = 22 - 12$
$2d = 10$
$d = 5$
$\text{From equation (1), we obtain}$

$a + 5d = 12$
$a + 5(5) = 12$
$a + 25 = 12$
$a = -13$
$a_2 = a + d = -13 + 5 = -3$
$a_3 = a_2 + d = -8 + 5 = -3$
Therefore, the first three terms of this A.P. are $-13, -8,$ and -3.

Page : 107 , Block Name : Exercise 5.2

Q19 Subba Rao started work in 1995 at an annual salary of
5000 and $receivedan \in crementof$ 200 each year. In which year did his income reach ` 7000?

Answer. It can be observed that the incomes that Subba Rao obtained in various years are in A.P. as every year, his salary is increased by Rs 200.
Therefore, the salaries of each year after 1995 are 5000, 5200, 5400,
Here, a = 5000
d = 200
Let after nth year, his salary be Rs 7000.
Therefore, an $a_n = a + (n - 1)d$
7000 = 5000 + (n - 1) 200
200(n - 1) = 2000
(n - 1) = 10
n = 11
Therefore, in 11th year, his salary will be Rs 7000.

Page : 107 , Block Name : Exercise 5.2

Q20 Ramkali saved ₽ 5 in the first week of a year and then increased her weekly savings by ₽1.75. If in the nth week, her weekly savings become ₽20.75, find n.

Answer. Given that,
$a = 5$
$d = 1.75$
$a_n = 20.75$
$n = ?$
$a_n = a + (n - 1)d$

$20.75 = 5 + (n-1)1.75$
$15.75 = (n-1)1.75$
$(n-1) = \frac{15.75}{1.75} = \frac{1575}{175}$
$= \frac{63}{7} = 9$
$n - 1 = 9$
$n = 10$
Hence, n is 10.

Page : 107 , Block Name : Exercise 5.2

Exercise 5.3

Q1 Find the sum of the following A.P.
(i) 2, 7, 12 ,. ..., to 10 terms.
(ii) - 37, 33 — 29 ,..., to 12 terms
(iii) 0.6, 1.7 28 ,.... to 100 terms
(iv) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}, \ldots\ldots,$ to 11 terms

Answer. (i) $2, 7, 12, \ldots,$ to 10 terms
For this A.P

$$a = 2$$
$$d = a_2 - a_1 = 7 - 2 = 5$$
$$n = 10$$

We know that,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$
$$S_{19} = \frac{10}{2}[2(2) + (10-1)5]$$
$$= 5[4 + (9) \times (5)]$$
$$= 5 \times 49 = 245$$

(ii) $-37, -33, -29, \ldots,$ to 12 terms
For this A.P.
$a = -37$
$d = a_2 - a_1 = (-33) - (-37)$
$= -33 + 37 = 4$

$n = 12$

We know that,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\begin{aligned} S_{12} &= \frac{12}{2}[2(-37) + (12-1)4] \\ &= 6[-74 + 11 \times 4] \\ &= 6[-74 + 44] \\ &= 6(-30) = -180 \end{aligned}$$

(iii) 0.6,1.7,2.8,... to 100 terms

For this A.P.

$a = 0.6$

$d = a_2 - a_1 = 1.7 - 0.6 = 1.1$

$n = 100$

We know that,

$$\begin{aligned} S_n &= \frac{n}{2}[2a + (n-1)d] \\ S_{100} &= \frac{100}{2}[2(0.6) + (100-1)1.1] \\ &= 50[1.2 + (99) \times (1.1)] \\ &= 50[1.2 + 108.9] \\ &= 50[110.1] \\ &= 5505 \end{aligned}$$

(iv) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}, \ldots\ldots\ldots,$ to 11 terms

For this A.P.

$$a = \frac{1}{15}$$

$$n = 11$$

$$\begin{aligned} d = a_2 - a_1 &= \frac{1}{12} - \frac{1}{15} \\ &= \frac{5-4}{60} = \frac{1}{60} \end{aligned}$$

We know that,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{11} = \frac{11}{2}\left[2\left(\frac{1}{15}\right) + (11-1)\frac{1}{60}\right]$$

$$= \frac{11}{2}\left[\frac{2}{15} + \frac{10}{60}\right] = \frac{11}{2}\left[\frac{4+5}{30}\right]$$

$$= \frac{11}{2}\left[\frac{2}{15} + \frac{1}{6}\right] = \frac{11}{2}\left[\frac{4+5}{30}\right]$$

$$= \frac{11}{2}\left[\frac{2}{15} + \frac{1}{6}\right] = \frac{11}{2}\left[\frac{4+5}{30}\right]$$

$$= \left(\frac{11}{2}\right)\left(\frac{9}{30}\right) = \frac{33}{20}$$

Page : 112 , Block Name : Exercise 5.3

Q2 Find the sums given below :

(i) $7 + 10\frac{1}{2} + 14 + \ldots + 84$

(ii) $54 + 32 + 30 + \ldots + 10$

(iii) $-5 + (-8) + (-11) + \ldots + (-230)$

Answer. (i) $7 + ^{10\frac{1}{2}} + 14 + \ldots\ldots\ldots + 84$

For this A.P

$a = 7$

$l = 84$

$d = a_2 - a_1 = 10\frac{1}{2} - 7 = \frac{21}{2} - 7 = \frac{7}{2}$

Let 84 be the n^{th} term of this A.P.

$I = a + (n-1)d$

$84 = 7 + (n-1)\frac{7}{2}$

$77 = (n-1)\frac{7}{2}$

$22 = n - 1$

$n = 23$

We know that

$$S_n = \frac{n}{2}(a + l)$$

$$S_n = \frac{23}{2}[7 + 84]$$

$$= \frac{23 \times 91}{2} = \frac{2093}{2}$$

$$= 1046\frac{1}{2}$$

(ii) $34+32+30+\ldots\ldots\ldots+10$
For this A.P.
a=34
$d=a_2-a_1=32-34=-2$
$I=10$
Let 10 be the n^{th} term of this A.P.
$I=a+(n-1)d$
$10=34+(n-1)(-2)$
$-24=(n-1)(-2)$
$12=n-1$
$n=13$
$S_n=\frac{n}{2}(a+l)$
$=\frac{13}{2}(34+10)$
$=\frac{13\times44}{2}=13\times22$
$=\frac{13\times44}{2}=13\times22$
$=286$

(iii) $(-5)+(-8)+(-11)+\ldots\ldots\ldots+(-230)$
For this A.P.
$a=-5$
$l=-230$
$d=a_2-a_1=(-8)-(-5)$
$=-8+5=-3$
Let -230 be the n^{th} term of this A.P.
$I=a+(n-1)d$
$-230=-5+(n-1)(-3)$
$-225=(n-1)(-3)$
$(n-1)=75$
$n=76$
And, $S_n=\frac{n}{2}(a+l)$
$=\frac{76}{2}[(-5)+(-230)]$
$=38(-235)$
$=-8930$

Page : 112 , Block Name : Exercise 5.3

Q3 In an AP :
(i) Given $a=5, d=3, a_n=50,$ find n and S_n

(ii) Given $a = 7, a_{13} = 35,$ find d and S_{13}
(iii) Given $a_{12} = 37, d = 3,$ find a and S_{12}
(iv) Given $a_3 = 15, S_{10} = 125,$ find d and a_{10}
(v) Given $d = 5, S_9 = 75,$ find a and a_9
(vi) Given $a = 2, d = 8, S_n = 90,$ find n and a_n
(vii) Given $a = 8, a_n = 62, s_n = 210,$ find n and d
(viii) Given $a_n = 4, d = 2, S_n = -14,$ find n and a
(ix) Given $a = 3, n = 8, S = 192,$ find d
(x) Given $I = 28, S = 144$ and there are total 9 find a

Answer. (i) Given that $a = 5, d = 3, a_n = 50$

$$\text{As } a_n = a + (n-1)d$$
$$\therefore 50 = 5 + (n-1)3$$
$$45 = (n-1)3$$
$$15 = n - 1$$
$$n = 16$$
$$S_n = \frac{n}{2}[a + a_n]$$
$$S_{16} = \frac{16}{2}[5 + 50]$$
$$= 8 \times 55$$
$$= 440$$

(ii) Given that $a = 7, a_{13} = 35$

$$\text{As } a_n = a + (n-1)d_r$$
$$\therefore a_{13} = a + (13-1)d$$
$$35 = 7 + 12d$$
$$35 - 7 = 12d$$
$$28 = 12d$$
$$d = \tfrac{7}{3}$$
$$S_n = \frac{n}{2}[a + a_n]$$
$$S_{13} = \frac{n}{2}[a + a_{13}]$$
$$= \frac{13}{2}[7 + 35]$$
$$= \frac{13 \times 42}{2} = 13 \times 21$$
$$= 273$$

(iii) Given that $a_{12} = 37, d = 3$

$\text{As } a_n = a + (n-1)d$

$a_{12} = a + (12-1)3$

$37 = a + 33$

$a = 4$

$S_n = \frac{n}{2}[a + a_n]$

$S_n = \frac{12}{2}[4 + 37]$

$S_n = 6(41)$

$S_n = 246$

(iv) Given that $a_3 = 15, S_{10} = 125$

$\text{As } a_n = a + (n-1)d$

$a_3 = a + (3-1)d$

$15 = a + 2d(i)$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{19} = \frac{10}{2}[2a + (10-1)d]$

$125 = 5(2a + 9d)$

$25 = 2a + 9d$

On multiplying equation (I) by 2, we obtain

30 = 2a + 4d ...(iii)

on subtracting equation (iii) from (ii), we obtain

$-5 = 5d$

$d = -1$

From equation (

$15 = a + 2(-1)$

$15 = a - 2$

$a = 17$

$a_{10} = a + (10-1)d$

$a_{10} = 17 + (9)(-1)$

$a_{10} = 17 - 9 = 8$

(v) Given that $d = 5, S_9 = 75$

$\text{As } S_n = \frac{n}{2}[2a + (n-1)d]$

$S_0 = \frac{9}{2}[2a + (9-1)5]$

$75 = \frac{9}{2}(2a + 40)$

$25 = 3(a + 20)$

$25 = 3a + 60$

$3a = 25 - 60$

$a = \frac{-35}{3}$

$a_n = a + (n-1)d$

$a_9 = a + (9-1)(5)$

$= \frac{-35}{3} + 8(5)$

$= \frac{-35}{3} + 40$

$= \frac{-35+120}{3} = \frac{85}{3}$

(vi) Given that $a = 2, d = 8, S_n = 90$

As $S_n = \frac{n}{2}[2a + (n-1)d]$

$90 = \frac{n}{2}[4 + (n-1)8]$

$90 = n[2 + (n-1)4]$

$90 = n[2 + 4n - 4]$

$90 = n(4n-2) = 4n^2 - 2n$

$4n^2 - 2n - 90 = 0$

$4n^2 - 20n + 18n - 90 = 0$

$4n(n-5) + 18(n-5) = 0$

$(n-5)(4n+18) = 0$

Either $n - 5 = 0$ or $4n + 18 = 0$

$n = 5$ or $^{n=-\frac{18}{4}} = \frac{-9}{2}$

However, n can neither be negative nor fractional.

Therefore, $n = 5$

$a_n = a + (n-1)d$

$a_5 = 2 + (5-1)8$

$= 2 + (4)(8)$

$= 2 + 32 = 34$

(vii) Given that $a = 8, a_n = 62, S_n = 210$

$S_n = \frac{n}{2}[a + a_n]$

$210 = \frac{n}{2}[8 + 62]$

$210 = \frac{n}{2}(70)$

$n = 6$

$a_n = a + (n-1)d$

$62 = 8 + (6-1)d$

$62 - 8 = 5d$

$54 = 5d$

$a = \frac{54}{5}$

(viii) Given that $a_n = 4, d = 2, S_n = -14$

$a_n = a + (n-1)d$

$4 = a + (n-1)2$

$4 = a + 2n - 2$

$a + 2n = 6$

$a = 6 - 2n(i)$

$S_n = \frac{n}{2}[a + a_n]$

$-14 = \frac{n}{2}[a + 4]$

$-28 = n(a + 4)$

$-28 = n(6 - 2n + 4)\{$ From equation $(i)\}$

$-28 = n(-2n + 10)$

$-28 = -2n^2 + 10n$

$2n^2 - 10n - 28 = 0$

$n^2 - 5n - 14 = 0$

$n^2 - 7n + 14 = 0$

$n^2 - 7n + 2n - 14 = 0$

$n(n-7)(n+2) = 0$

Either $n - 7 = 0$ or $n + 2 = 0$

$n = 7$ or $n = -2$

However, n can neither be negative nor fractional.

Therefore, $n = 7$

From equation (i), we obtain

$a = 6 - 2n$

$a = 6 - 2(7)$

$= 6 - 14$

$= -8$

(ix) Given that $a = 3, n = 8, S = 192$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$192 = \frac{8}{2}[2 \times 3 + (8-1)d]$

$192 = 4[6 + 7d]$

$48 = 6 + 7d$

$42 = 7d$

$d = 6$

(x) Given that $I = 28, S = 144$ and there are total of 9 terms

$S_n = \frac{n}{2}(a + l)$
$144 = \frac{9}{2}(a + 28)$
$(16) \times (2) = a + 28$
$32 = a + 28$
$a = 4$

Page : 112 , Block Name : Exercise 5.3

Q4 How many terms of the AP : 9, 17, 25, . . . must be taken to give a sum of 636?

Answer. Let there be n terms of this A.P.
$\text{For this A.P., } a = 9$
$d = a_2 - a_1 = 17 - 9 = 8$
$S_n = \frac{n}{2}[2a + (n - 1)d]$
$636 = \frac{n}{2}[2 \times a + (n - 1)8]$
$636 = \frac{n}{2}[18 + (n - 1)8]$

$636 = n[9 + 4n - 4]$
$636 = n(4n + 5)$
$4n^2 + 5n - 636 = 0$
$4n^2 + 53n - 48n - 636 = 0$
$n(4n + 53) - 12(4n + 53) = 0$
$(4n + 53)(n - 12) = 0$
$\text{Either } 4n + 53 = 0 \text{ or } n - 12 = 0$
$n = \frac{-53}{4} \text{ or } n = 12$
n cannot be 4 . As the number of terms can neither be negative nor fractional, therefore, n = 12 only.

Page : 113 , Block Name : Exercise 5.3

Q5 The first term of an AP is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference.

Answer. Given that ,
$a = 5$
$I = 45$
$S_n = 400$
$S_n = \frac{n}{2}(a + l)$
$400 = \frac{n}{2}(5 + 45)$
$400 = \frac{n}{2}(50)$

$n = 16$
$I = a + (n-1)d$
$45 = 5 + (16-1)d$
$40 = 15d$
$d = \frac{40}{15} = \frac{8}{3}$

Page : 113 , Block Name : Exercise 5.3

Q6 The first and the last term of an AP are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum?

Answer. Given that
$a = 17$
$l = 350$
$d = 9$
$\text{Let there be } n \text{ terms in the A.P.}$
$I = a + (n-1)d$
$350 = 17 + (n-1)9$
$333 = (n-1)9$
$(n-1) = 37$
$n = 38$

$S_n = \frac{n}{2}(a + l)$
$\Rightarrow S_n = \frac{38}{2}(17 + 350) = 19(367) = 6973$
Thus this A.P. contains 38 terms and the terms of this A.P. is 6973

Page : 113 , Block Name : Exercise 5.3

Q7 Find the sum of first 22 terms of an AP in which d = 7 and 22nd term is 149

Answer.
$d = 7$
$a_{22} = 149$
$S_{22} = ?$
$a_n = a + (n-1)d$
$a_{22} = a + (22-1)d$
$149 = a + 21 \times 7$
$149 = a + 147$
$a = 2$

$$
\begin{aligned}
S_n &= \frac{n}{2}(a + a_n) \\
&= \frac{22}{2}(2 + 149) \\
&= 11(151) = 1661
\end{aligned}
$$

Page : 113 , Block Name : Exercise 5.3

Q8 Find the sum of first 51 terms of an AP whose second and third terms are 14 and 18 respectively

Answer. Given that

$$
\begin{aligned}
a_2 &= 14 \\
a_3 &= 18 \\
d &= a_3 - a_2 = 18 - 14 = 4 \\
a_2 &= a + d \\
14 &= a + 4 \\
a &= 10 \\
S_n &= \frac{n}{2}[2a + (n-1)d] \\
S_{51} &= \frac{51}{2}[2 \times 10 + (51-1)4] \\
&= \frac{51}{2}[20 + (50)(4)] \\
&= \frac{51(220)}{2} = 51(110) \\
&= 5610
\end{aligned}
$$

Page : 113 , Block Name : Exercise 5.3

Q9 If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of first n terms.

Answer. Given that,

$$
\begin{aligned}
S_7 &= 49 \\
S_{17} &= 289 \\
S_n &= \tfrac{n}{2}[2a + (n-1)d] \\
S_7 &= \tfrac{7}{2}[2a + (7-1)d] \\
79 &= (a + 3d) \\
a + 3d &= 7(i)
\end{aligned}
$$

$S_{17} = \frac{17}{2}[2a + (17-1)d]$

$289 = \frac{17}{2}[2a + 16d]$

$17 = (a + 8d)$

$a + 8d = 17 (ii)$

$\text{Subtracting equation (i) from equation (ii),}$

$5d = 10$

$d = 2$

$\text{From equation } (i)$

$a + 3(2) = 7$

$a + 6 = 7$

$a = 1$

$$\begin{aligned} S_n &= \frac{n}{2}[2a + (n-1)d] \\ &= \frac{n}{2}[2(1) + (n-1)(2)] \\ &= \frac{n}{2}(2 + 2n - 2) \\ &= \frac{n}{2}(2n) \\ &= n^2 \end{aligned}$$

Page : 113 , Block Name : Exercise 5.3

Q10 Show that $a_1, a_2 \ldots, a_n, \ldots$ form an AP where an is defined as below
(i) $a_n = 3 + 4n$
(li) $a_n = 9 - 5n$
Also find the sum of the first 15 terms in each case.

Answer.

$$\begin{aligned} &(\text{i}) a_n = 3 + 4n \\ &a_1 = 3 + 4(1) = 7 \\ &a_2 = 3 + 4(2) = 3 + 8 = 11 \\ &a_3 = 3 + 4(3) = 3 + 12 = 15 \\ &a_4 = 3 + 4(4) = 3 + 16 = 19 \end{aligned}$$

$\text{It can be observed that}$

$a_2 - a_1 = 11 - 7 = 4$

$a_3 - a_2 = 15 - 11 = 4$

$a_4 - a_3 = 19 - 15 = 4$

, $a_{k+1} - a_k$ is same every time. Therefore, this is an AP with common difference as 4 and first term as 7.

$$S_n = \frac{n}{2}[2a + (n-1)d]$$
$$S_{15} = \frac{15}{2}[2(7) + (15-1)4]$$
$$= \frac{15}{2}[(14) + 56]$$
$$= \frac{15}{2}(70)$$
$$=15 \times 35$$
$$=525$$

(ii) $a_n = 9 - 5n$

$a_1 = 9 - 5 \times 1 = 9 - 5 = 4$

$a_2 = 9 - 5 \times 2 = 9 - 10 = -1$

$a_3 = 9 - 5 \times 3 = 9 - 15 = -6$

(ii) $a_4 = 9 - 5 \times 4 = 9 - 20 = -11$

It can be observed that

$a_2 - a_1 = -1 - 4 = -5$

$a_3 - a_2 = -1 - 4 = -5$

$a_3 - a_2 = -11 - (-1) = -5$

i$e.$, $a_{k+1} - a_k$ is same every time. Therefore, this is an A.P. with common difference as -5 and first term as 4.

$$S_n = \frac{n}{2}[2a + (n-1)d]$$
$$S_{15} = \frac{15}{2}[2(4) + (15-1)(-5)]$$
$$= \frac{15}{2}[8 + 14(-5)]$$
$$= \frac{15}{2}(8 - 70)$$
$$= \frac{15}{2}(-62) = 15(-31)$$
$$= -465$$

Page : 113 , Block Name : Exercise 5.3

Q11 If the sum of the first n terms of an AP is $4n - n^2$, what is the first term (that is S_1)? What is the sum of first two terms? What is the second term? Similarly find the 3rd, the 10^{th} and the nth terms.

Answer. Given that

$S_n = 4n - n^2$

$\text{First term, } a = S_1 = 4(1) - (1)^2 = 4 - 1 = 3$

$\text{Sum of first two terms } = S_2$

$= 4(2) - (2)^2 = 8 - 4 = 4$

$\text{Second term, } a_2 = S_2 - S_1 = 4 - 3 = 1$

$d = a_2 - a = 1 - 3 = -2$

$a_n = a + (n - 1)d$

$= 3 + (n - 1)(-2)$

$= 3 - 2n + 2$

$= 5 - 2n$

$\text{Therefore, } a_3 = 5 - 2(3) = 5 - 6 = -1$

$a_{10} = 5 - 2(10) = 5 - 20 = -15$

$\text{Hence, the sum of first two terms is 4. The second term is } 1.3^{\text{th}}, 10^{\text{th}}$

$\text{and } n^{\text{th}} \text{ terms are } -1, -15, \text{ and } 5 - 2n \text{ respectively.}$

Page : 113 , Block Name : Exercise 5.3

Q12 Find the sum of the first 40 positive integers divisible by 6.

Answer. The positive integers that are divisible by 6 are
6, 12, 18, 24
It can be observed that these are making an AP. whose first term is 6 and common difference is 6.

$a = 6$

$d = 6$

$S_{40} = ?$

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$S_{40} = \frac{40}{2}[2(6) + (40 - 1)6]$

$= 20[12 + (39)(6)]$

$= 20(12 + 234)$

$= 20 \times 246$

$= 4920$

Page : 113 , Block Name : Exercise 5.3

Q13 Find the sum of the first 15 multiples of 8.

Answer. The multiples of 8 are
8, 16, 24, 32..
These are in an A.P., having first term as 8 and common difference as 8.

Therefore, a = 8
d = 8
S_{15} =?

$$S_n = \frac{n}{2}[2a + (n-1)d]$$
$$= \frac{15}{2}[2(8) + (15-1)8]$$
$$= \frac{15}{2}[16 + 14(8)]$$
$$= \frac{15}{2}(16 + 14(8)]$$
$$= \frac{15(128)}{2} = 15 \times 64$$
$$=960$$

Page : 113 , Block Name : Exercise 5.3

Q14 Find the sum of the odd numbers between 0 and 50.

Answer. The odd numbers between 0 and 50 are
1, 3, 5, 7, 9..... 49
Therefore, it can be observed that these odd numbers are In an A.P.

$$a = 1$$
$$d = 2$$
$$I = 49$$

$$I = a + (n-1)d$$
$$49 = 1 + (n-1)2$$
$$48 = 2(n-1)$$
$$n - 1 = 24$$
$$n = 25$$

$$S_n = \frac{n}{2}(a + l)$$
$$S_{25} = \frac{25}{2}(1 + 49)$$
$$= \frac{25(50)}{2} = (25)(25)$$
$$=625$$

Page : 113 , Block Name : Exercise 5.3

Q15 A contract on construction job specifies a penalty for delay of completion beyond a certain date as follows: rs 200 for the first day, rs 250 for the second day, rs 300 for the third day, etc.,

the penalty for each succeeding day being rs 50 more than for the preceding day. How much money the contractor has to pay as penalty, if he has delayed the work by 30 days?

Answer. It can be observed that these penalties are in an A.P. having first term as 200 and common difference as 50.
a = 200
d = 50
Penalty that has to be paid if he has delayed the work by 30 days =
S_{30}
$= \frac{30}{2}[2(200) + (30 - 1)50]$
$= 15[400 + 1450]$
$= 15(180)$
$= 27750$
Therefore , the contractor has to pay Rs 27750 as penalty

Page : 113 , Block Name : Exercise 5.3

Q16 A sum of ₱700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If each prize is ₱20 less than its preceding prize, find the value of each of the prizes

Answer. Let the cost of 1^{st} prize be P.
Cost of 2nd prize = P — 20
And cost of 3rd prize = P — 40
It can be observed that the cost of these prizes are in an A.P. having common difference as — 20 and first term as P.
a = P
d = —20
Given that $S_7 = 700$

$\frac{7}{2}[2a + (7 - 1)d] = 700$
$\frac{[2a+(6)(-20)]}{2} = 100$
$a + 3(-20) = 100$
$a - 60 = 100$
$a = 160$
Therefore, the value of each of the prizes was Rs 160, Rs 140, Rs 120, Rs 100, Rs 80, Rs 60, and Rs 40.

Page : 113 , Block Name : Exercise 5.3

Q17 In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant,

will be the same as the class, in which they are studying, e.g., a section of Class I will plant 1 tree, a section of Class II will plant 2 trees and so on till Class XII. There are three sections of each class. How many trees will be planted by the students?

Answer. It can be observed that the number of trees planted by the students is in an AP.
1,2,3,4,5,.....12
First term, a = 1
common difference, d = 2 — 1 =1

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{12} = \frac{12}{2}[2(1) + (12-1)(1)]$

$= 6(2 + 11)$

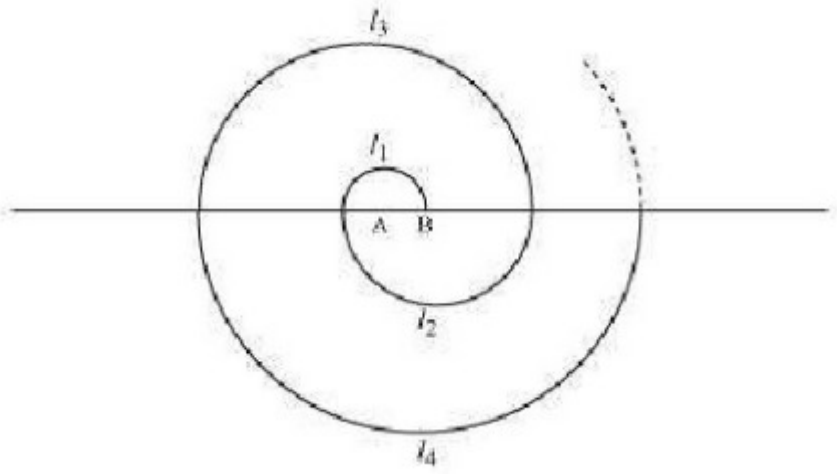
$= 6(13)$

$= 78$

Therefore, number of trees planted by I section of the classes = 78
Number of trees planted by 3 sections of the classes = 3 x 78 = 234
Therefore, 234 trees will be planted by the students.

Page : 113 , Block Name : Exercise 5.3

Q18 A spiral is made up of successive semicircles, with centres alternately at A and B, starting with centre at A, of radii 0.5 cm, 1.0 cm, 1.5 cm, 2.0 cm, . . . as shown in Fig. 5.4. What is the total length of such a spiral made up of thirteen consecutive semicircles$\left(\text{ Take } \pi = \frac{22}{7}\right)$



Answer.

$$\text{Semi-perimeter of circle} = \text{nr}$$

$$\text{I}_1 = \text{n}(0.5) = \frac{\pi}{2}\text{cm}$$

$$\text{I}_2 = \text{n}(1) = \text{ncm}$$

$$\text{I}_3 = \text{n}(1.5) = \frac{3\pi}{2}\text{cm}$$

Therefore, $\text{I}_1, \text{I}_2, \text{I}_3,$, i.e. the lengths of the semi-circles are in an A.P.,

$\frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi, \ldots\ldots\ldots$

$a = \frac{\pi}{2}$

$d = \pi - \frac{\pi}{2} = \frac{\pi}{2}$

$S_{13} = ?$

We know that the sum of n terms of an a A.P. is given by

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{13}{2}\left[2\left(\frac{\pi}{2}\right) + (13-1)\left(\frac{\pi}{2}\right)\right]$$

$$= \frac{13}{2}\left[\pi + \frac{12\pi}{2}\right]$$

$$= \left(\frac{13}{2}\right)(7\pi)$$

$$= \frac{91\pi}{2}$$

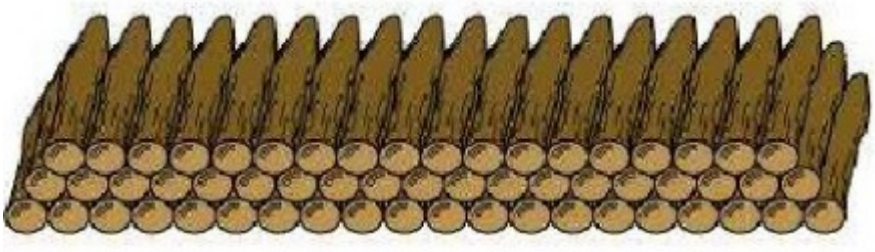
$$= \frac{91 \times 22}{2 \times 7} = 13 \times 11$$

=143

Therefore, the length of such spiral of thirteen consecutive semi-circles will be 143 cm.

Page : 113 , Block Name : Exercise 5.3

Q19 200 logs are stacked in the following manner: 20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on (see Fig. 5.5). In how many rows are the 200 logs placed and how many logs are in the top row?



Answer. It can be observed that the numbers of logs in rows are in an A.P.

$20, 19, 18 \ldots$

For this A.P.

$a = 20$

$d = a_2 - a_1 = 19 - 20 = -1$

Let a total of 200 logs placed in n rows.

$S_n = 200$
$S_n = \frac{n}{2}[2a + (n-1)d]$
$200 = \frac{n}{2}[2(20) + (n-1)(-1)]$
$400 = n(40 - n + 1)$
$400 = n(41 - n)$
$400 = 41n - n^2$
$n^2 - 41n + 400 = 0$
$n^2 - 16n - 25n + 400 = 0$
$n(n-16) - 25(n-16) = 0$
$(n-16)(n-25) = 0$
Either $(n-16) = 0$ or $n - 25 = 0$
$n = 16$ or $n = 25$
$a_n = a + (n-1)d$
$a_{16} = 20 + (16-1)(-1)$
$a_{16} = 20 - 15$
$a_{16} = 5$
Similarly,
$a_{25} = 20 + (25-1)(-1)$
$a_{25} = 20 - 24$
$= -4$

Clearly, the number of logs in 16th row is 5. However, the number of logs in 25th row is negative, which is not possible.
Therefore, 200 logs can be placed in 16 rows and the number of logs in the 16th row is 5.

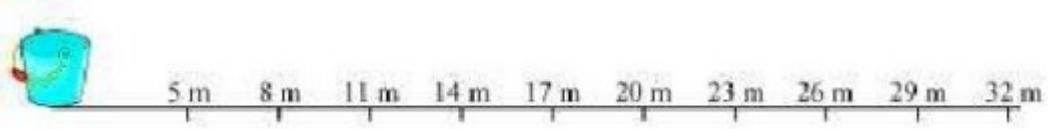
Page : 114 , Block Name : Exercise 5.3

Q20 In a potato race, a bucket is placed at the starting point, which is 5 m from the first potato, and the other potatoes are placed 3 m apart in a straight line. There are ten potatoes in the line (see Fig



A competitor starts from the bucket, picks up the nearest potato, runs back with it, drops it in the bucket, runs back to pick up the next potato, runs to the bucket to drop it in, and she continues in the same way until all the potatoes are in the bucket. What is the total distance the competitor has to run?
[Hint : To pick up the first potato and the second potato, the total distance (in metres) run by a competitor is 2 × 5 + 2 × (5 + 3)]

Answer.



The distances of potatoes are as follows.
5, 8, 11, 14...
It can be observed that these distances are in A.P.
d = 8-5=3

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{10} = \frac{10}{2}[2(5) + (10-1)3]$

$= 5[10 + 9 \times 3]$

$= 5(10 + 27) = 5(37)$

$= 185$

As every time she has to run back to the bucket, therefore, the total distance that the competitor has to run will be two times of it. Therefore, total distance that the competitor will run = 2 x 185 = 370 m

Page : 114 , Block Name : Exercise 5.3

Exercise 5.4

Q1 Which term of the AP : 121, 117, 113, . . ., is its first negative term?
[Hint : Find n for a_n < 0]

Answer.

Given A.P. $121, 117, 113 \ldots$

$a = 121$

$d = 117 - 121 = -4$

$a_n = a + (n-1)d$

$= 121 + (n-1)(-4)$

$= 121 - 4n + 4$

$= 125 - 4n$

We have to find the first negative term of this A.P.

$\text{Therefore, } a_n < 0$

$125 - 4n < 0$

$125 < 4n$

$n > \frac{125}{4}$

$n > 31.25$

Therefore, 32^{nd} term will be the first negative term of this A.P.

Page : 115 , Block Name : Exercise 5.4 (Optional)

Q2 The sum of the third and the seventh terms of an AP is 6 and their product is 8. Find the sum of first sixteen terms of the AP.

Answer. We know that

$a_n = a + (n-1)d$

$a_3 = a + (3-1)d$

$a_3 = a + 2d$

$\text{Similarly, } a_7 = a + 6d$

$\text{Given that, } a_3 + a_7 = 6$

$(a + 2d) + (a + 6d) = 6$

$2a + 8d = 6$

$a + 4d = 3$

$a = 3 - 4d(i)$

$\text{Also, it is given that } (a_3) \times (a_7) = 8$

$(a + 2d) \times (a + 6d) = 8$

$\text{From equation } (i)$

$(3 - 4d + 2d) \times (3 - 4d + 6d) = 8$

$(3 - 2d) \times (3 + 2d) = 8$

$9 - 4d^2 = 8$

$4d^2 = 9 - 8 = 1$

$d^2 = \frac{1}{4}$

$d = \pm\frac{1}{2}$

$d = \frac{1}{2} \text{ or } -\frac{1}{2}$

$\text{From equation (} i$

$\left(\text{When } d \text{ is } \frac{1}{2} \right)$

$a = 3 - 4d$

$a = 3 - 4\left(\frac{1}{2}\right)$

$= 3 - 2 = 1$

$\left(\text{ When } d \text{ is } -\frac{1}{2}\right)$

$a = 3 - 4\left(-\frac{1}{2}\right)$

$a = 3 - 4\left(-\frac{1}{2}\right)$

$a = 3 + 2 = 5$

$S_n = \frac{n}{2}[2a(n-1)d]$

$\left(\text{ When } a \text{ is } 1 \text{ and } d \text{ is } \frac{1}{2}\right)$

$S_{16} = \frac{16}{2}\left[2(1) + (16-1)\left(\frac{1}{2}\right)\right]$

$= 8\left[2 + \frac{15}{2}\right]$

$= 4(19) = 76$

$\left(\text{ When } a \text{ is } 5 \text{ and } d \text{ is } -\frac{1}{2}\right)$

$S_{16} = \frac{16}{2}\left[2(5) + (16-1)\left(-\frac{1}{2}\right)\right]$

$= 8\left[10 + (15)\left(-\frac{1}{2}\right)\right]$

$= 8\left(\frac{5}{2}\right)$

=20

Page : 115 , Block Name : Exercise 5.4 (Optional)

Q3 A ladder has rungs 25 cm apart. (See figure). The rungs decrease uniformly in length from 45 cm at the bottom to 25 cm at the top. If the top and bottom rungs are $2\frac{1}{2}\text{m}$ apart, what is the length of the wood required for the rungs?
[Hint: number of rungs $= \frac{250}{25}$]

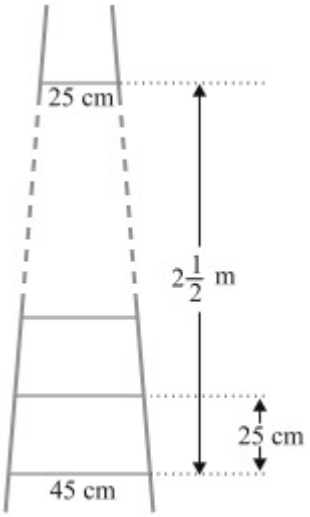


Fig. 5.7

Answer. It is given that the rungs are 25 cm apart and the top and bottom rungs are $2\frac{1}{2}\mathrm{m}$ apart.
Total number of rungs
$= \frac{2\frac{1}{2}\times 100}{25} + 1 = \frac{250}{25} + 1 = 11$
Now, as the lengths of the rungs decrease uniformly, they will be in an A.P
The length of the wood required for the rungs equals the sum of all the terms of this A.P.
$\text{terms of this A.P.}$
$\text{First term, } a = 45$
$\text{Last term, } I = 25$
$n = 11$
$S_n = \frac{n}{2}(a + l)$
$\therefore S_{10} = \frac{11}{2}(45 + 25) = \frac{11}{2}(70) = 385\text{cm}$
Therefore the length of the wood required for the rungs is 385.

Page : 115 , Block Name : Exercise 5.4 (Optional)

Q4 The houses of a row are number consecutively from 1 to 49. Show that there is a value of x such that the sum of numbers of the houses preceding the house numbered x is equal to the sum of the number of houses following it.
Find this value of x.
$[\text{Hint} S_{x-1} = S_{49} - S_x]$

Answer. The number of houses was 1,2,3 49
It can be observed that the number of houses are in A.P having a as 1 and d also as 1
Let us assume that the number of x^{th} house was like this.
We know that,

$\text{Sum of } n \text{ terms in an A.P. } = \frac{n}{2}[2a + (n-1)d]$

$\text{Sum of number of houses preceding } x^{\text{th}} \text{ house } = S_{x-1}$

$= \frac{(x-1)}{2}[2a + (x-1-1)d]$

$= \frac{x-1}{2}[2(1) + (x-2)(1)]$

$= \frac{x-1}{2}[2 + x - 2]$

$= \frac{(x)(x-1)}{2}$

$\text{Sum of number of houses following } x^{\text{th}} \text{ house } = S_{49} - S_x$

$= \frac{49}{2}[2(1) + (49-1)(1)] - \frac{x}{2}[2(1) + (x-1)(1)]$

$= \frac{49}{2}(2 + 49 - 1) - \frac{x}{2}(2 + x - 1)$

$= \left(\frac{49}{2}\right)(50) - \frac{x}{2}(x+1)$

$= 25(49) - \frac{x(x+1)}{2}$

$\text{It is given that these sums are equal to each other.}$

$\frac{x(x-1)}{2} = 25(49) - x\left(\frac{x+1}{2}\right)$

$\frac{x^2}{2} - \frac{x}{2} = 1225 - \frac{x^2}{2} - \frac{x}{2}$

$x^2 = 1225$

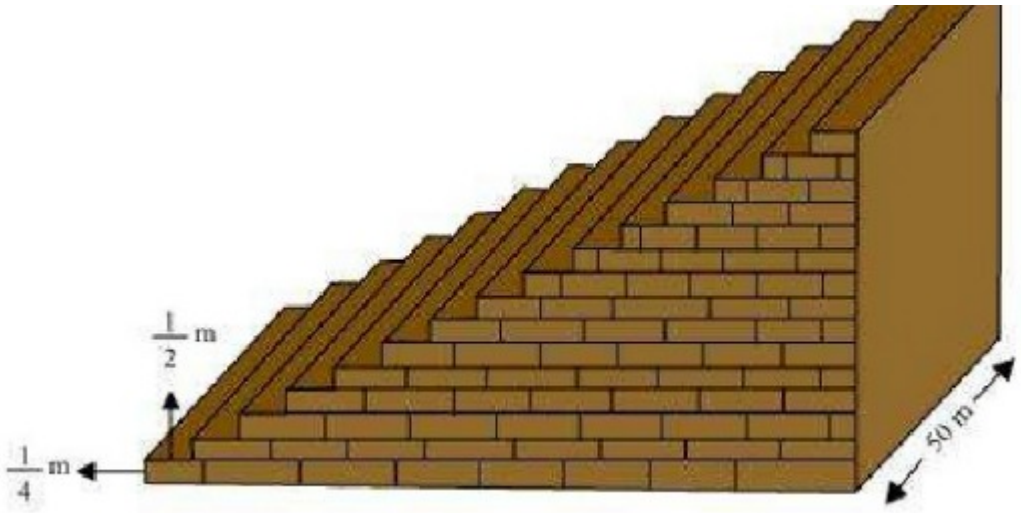
$x = \pm 35$

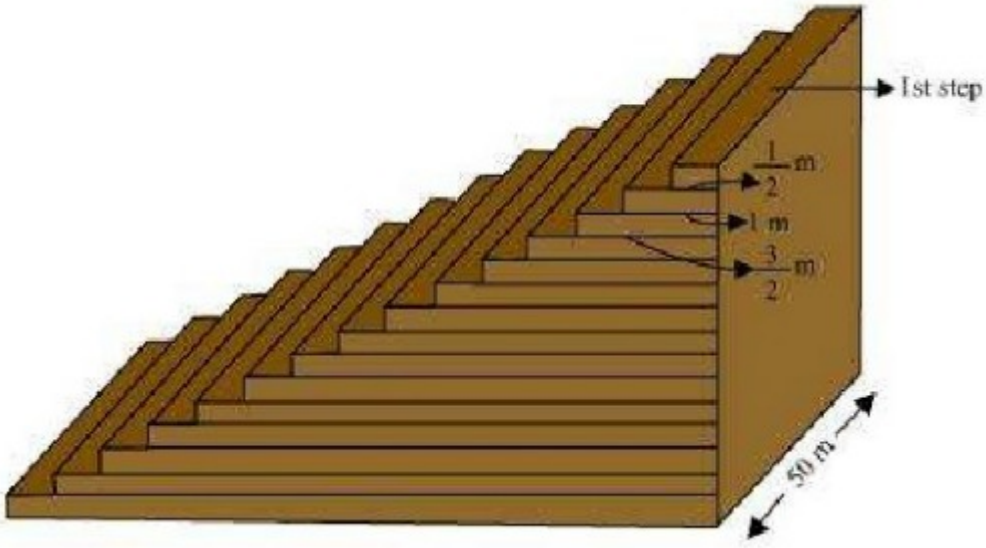
$\text{However, the house numbers are positive integers.}$

The value of x will be 35 only Therefore, house number 35 is such that the sum of the numbers of houses preceding the house numbered 35 is equal to the sum of the numbers of the houses following it.

Page : 115 , Block Name : Exercise 5.4 (Optional)

Q5 A small terrace at a football ground comprises of 15 steps each of which is 50 m long and built of solid concrete.
Each step has a rise of ¼ m and a tread of ½ m (See figure) calculate the total volume of concrete required to build the terrace.





Answer.

From the figure it can be observed that

1^{st} step is 2m wide,

2^{nd} step is 1m wide,

3^{rd} step is $\frac{3}{2}$m wide.

Therefore, the width of each step is increasing by 1/2m each time whereas their height 4m and length 50 m remains the same.

Therefore, the Widths of these steps are $\frac{1}{2}, 1, \frac{3}{2}, 2, \ldots$

Volume of concrete in 1^{st} step $= \frac{1}{4} \times \frac{1}{2} \times 50 = \frac{25}{4}$

Volume of concrete in 2^{nd} step $= \frac{1}{4} \times 1 \times 50 = \frac{25}{2}$

Volume of concrete in 3rd step 3^{rd} step $= \frac{1}{4} \times \frac{3}{2} \times 50 = \frac{75}{4}$

It can be observed that the volumes of concrete in these steps are in an A.P.

$\frac{25}{4}, \frac{25}{2}, \frac{75}{4}, \ldots$

$a = \frac{25}{4}$

$d = \frac{25}{2} - \frac{25}{4} = \frac{25}{4}$

$\text{and } S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{15} = \frac{15}{2}\left[2\left(\frac{25}{4}\right) + (15-1)\frac{25}{4}\right]$

$= \frac{15}{2}\left[\frac{25}{2} + \frac{(14)25}{4}\right]$

$= \frac{15}{2}\left[\frac{25}{2} + \frac{175}{2}\right]$

$= \frac{15}{2}(100) = 750$

Volume of concrete required to build the terrace is 750m^3

Page : 115 , Block Name : Exercise 5.4 (Optional)