NCERT SOLUTIONS

CLASS - 12th

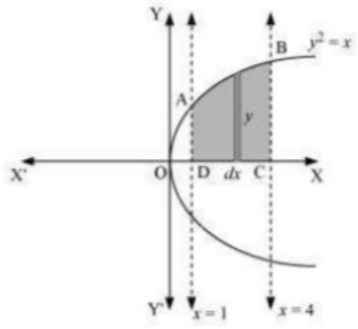


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Class : 12th
Subject : Maths
Chapter : 8
Chapter Name : Application of integrals

Q1 Find the area of the region bounded by the curve $y^2 = x$ and the lines x = 1, x = 4 and the x-axis in the first quadrant.

Answer.



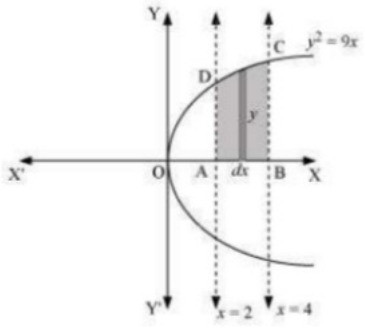
The area of the region bounded by the curve, y^2 = x, the lines, x = 1 and x = 4, and the x-axis is the area ABCD.

$$\text{Area of ABCD} = \int_1^4 y dx$$
$$= \int \sqrt{x} dx$$
$$= \left[\frac{\frac{3}{2}}{\frac{3}{2}}\right]_1$$
$$= \frac{2}{3}\left[(4)^{\frac{3}{2}} - (1)^{\frac{3}{2}}\right]$$
$$= \frac{2}{3}[8 - 1]$$
$$= \frac{14}{3} \text{ units}$$

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Q2 Find the area of the region bounded by $y^2 = 9x$, x = 2, x = 4 and the x-axis in the first quadrant.

Answer.



The area of the region bounded by the curve, y^2 = 9x, x = 2, and x = 4, and the x-axis is the area ABCD.

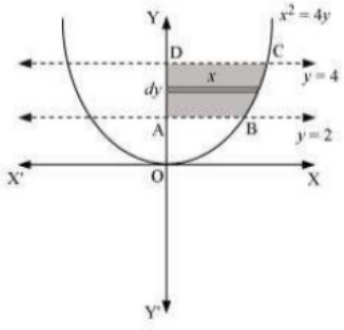
$$\text{Area of ABCD} = \int_2^4 y dx$$
$$= \int_2^4 3\sqrt{x} dx$$
$$= 3\left[\frac{3}{2}\right]_2^4$$
$$= 2\left[x^{\frac{3}{2}}\right]_2^4$$

$= 2\left[(4)^{\frac{3}{2}} - (2)^{\frac{3}{2}}\right]$
$= 2[8 - 2\sqrt{2}]$
$= (16 - 4\sqrt{2})$ units

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Q3 Find the area of the region bounded by $x^2 = 4y$, y = 2, y = 4 and the y-axis in the first quadrant.

Answer.



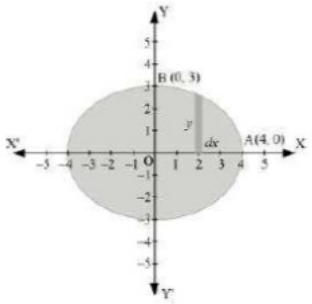
the area of the region bounded by x^2 = 4y, y = 2, y = 4 and the y-axis is the area ABCD.

$$\text{Area of ABCD} = \int_2^4 x\,dy$$
$$= \int_2^4 2\sqrt{y}\,dy$$
$$= 2\int_2^4 \sqrt{y}\,dy$$
$$= 2\left[\frac{\frac{3}{2}}{2}\right]_2$$
$= \frac{4}{3}\left[(4)^{\frac{3}{2}} - (2)^{\frac{3}{2}}\right]$
$= \frac{4}{3}[8 - 2\sqrt{2}]$
$= \left(\frac{32 - 8\sqrt{2}}{3}\right)$ units

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Q4 Find the area of the region bounded by the ellipse
$\frac{x^2}{16} + \frac{y^2}{9} = 1$

Answer.
The given equation of the ellipse, $\frac{x^2}{16} + \frac{y^2}{9} = 1$, can be represented as



It can be observed that the ellipse is symmetrical about x-axis and y-axis.
∴ Area bounded by ellipse = 4 × Area of OAB

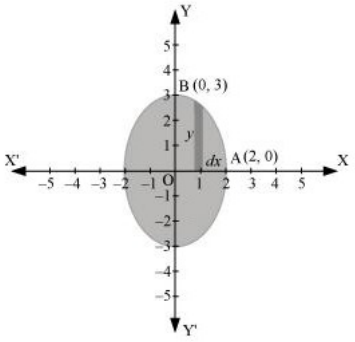
$$\text{Area of OAB} = \int_0^4 y dx$$
$$= \int_0^4 \sqrt{1 - \frac{x^2}{16}} dx$$
$$= \frac{3}{4} \int_0^4 \sqrt{16 - x^2} dx$$
$$= \frac{3}{4} \left[\frac{x}{2} \sqrt{16 - x^2} + \frac{16}{2} \sin^{-1} \frac{x}{4} \right]_0^4$$
$$= \frac{3}{4} \left[2\sqrt{16 - 16} + 8 \sin^{-1}(1) - 0 - 8 \sin^{-1}(0) \right]$$
$$= \frac{3}{4} \left[\frac{8\pi}{2} \right]$$
$$= \frac{3}{4} [4\pi]$$
$$= 3\pi$$

Therefore, area bounded by the ellipse = 4 × 3π = 12π units

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Q5 Find the area of the region bounded by the ellipse
$\frac{x^2}{4} + \frac{y^2}{9} = 1$

Answer.
The given equation of the ellipse can be represented as



$\frac{x^2}{4} + \frac{y^2}{9} = 1$
$\Rightarrow y = 3\sqrt{1 - \frac{x^2}{4}}$

It can be observed that the ellipse is symmetrical about x-axis and y-axis.
∴ Area bounded by ellipse = 4 × Area OAB

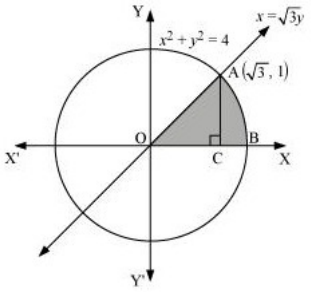
$$\therefore \text{ Area of OAB } = \int_0^2 y dx$$
$$= \int_0^2 \sqrt[3]{1 - \frac{x^2}{4}} dx \quad [\text{ Using } (1)]$$
$$= \frac{3}{2} \int_0^2 \sqrt{4 - x^2} dx$$
$$= \frac{3}{2} \left[\frac{x}{2} \sqrt{4 - x^2} + \frac{4}{2} \sin^{-} \frac{x}{2} \right]_0^2$$
$$= \frac{3}{2} \left[\frac{2\pi}{2} \right]$$
$$= \frac{3\pi}{2}$$

$$\text{Therefore, area bounded by the ellipse } = 4 \times \frac{3\pi}{2} = 6\pi \text{ units}$$

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Q6 Find the area of the region in the first quadrant enclosed by x-axis, line $x = \sqrt{3}y$ and the circle $x^2 + y^2 = 4$

Answer.
The area of the region bounded by the circle,$x^2 + y^2 = 4, x = \sqrt{3}y$, and the x-axis is the area OAB.



The point of intersection of the line and the circle in the first quadrant is $(\sqrt{3}, 1)$.
Area OAB = Area ΔOCA + Area ACB

$$\text{Area of OAC} \quad = \frac{1}{2} \times \text{OC} \times \text{AC} = \frac{1}{2} \times \sqrt{3} \times 1 = \frac{\sqrt{3}}{2} \ldots (i)$$

$$\text{Area of ABC} = \int_{\sqrt{3}}^{2} y dx$$

$$= \int_{\sqrt{3}}^{2} \sqrt{4 - x^2} dx$$

$$= \left[\frac{x}{2} \sqrt{4 - x^2} + \frac{4}{2} \sin^{-1} \frac{x}{2} \right]_{\sqrt{5}}^{2}$$

$$= \left[2 \times \frac{\pi}{2} - \frac{\sqrt{3}}{2} \sqrt{4 - 3} - 2 \sin^{-1} \left(\frac{\sqrt{3}}{2} \right) \right]$$

$$= \left[\pi - \frac{\sqrt{3}}{2} - 2 \left(\frac{1}{3} \right) \right]$$

$$= \left[\pi - \frac{\sqrt{3}}{2} - \frac{2\pi}{3} \right]$$

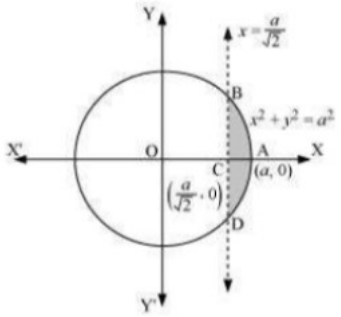
$$= \left[\frac{\pi}{3} - \frac{\sqrt{3}}{2} \right] \ldots (ii)$$

Therefore, area enclosed by x -axis, the line $x = \sqrt{3}y,$ and the circle $x^2 + y^2 = 4$ in the first
$\frac{\sqrt{3}\pi}{2} + \frac{3\sqrt{2}}{2} = \frac{\pi}{3}$ units

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Q7 Find the area of the smaller part of the circle $x^2 + y^2 = a^2$ cut off by the line
$x = \frac{a}{\sqrt{2}}$

Answer.
The area of the smaller part of the circle, $x^2 + y^2 = a^2,$ cut off by the line,
area ABCDA.



It can be observed that the area ABCD is symmetrical about x-axis.
∴ Area ABCD = 2 × Area ABC

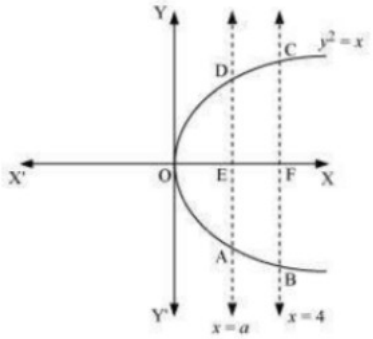
$$\text{Area of } ABC = \int_a^{\pi} ydx$$
$$= \int_{\sqrt{2}}^{\pi} \sqrt{a^2 - x^2}dx$$
$$= \left[\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_{\frac{a}{\sqrt{2}}}^{a}$$
$$= \left[\frac{a^2}{2}\left(\frac{\pi}{2}\right) - \frac{a}{2\sqrt{2}}\sqrt{a^2 - \frac{a^2}{2}} - \frac{a^2}{2}\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)\right]$$
$$= \frac{a^2\pi}{4} - \frac{a}{2\sqrt{2}} \cdot \frac{a}{\sqrt{2}} - \frac{a^2}{2}\left(\frac{\pi}{4}\right)$$
$$= \frac{a^2\pi}{4} - \frac{a^2}{4} - \frac{a^2\pi}{8}$$
$$= \frac{a^2}{4}\left[\pi - 1 - \frac{\pi}{2}\right]$$
$$= \frac{a^2}{4}\left[\frac{\pi}{2} - 1\right]$$
$$\Rightarrow \text{Area ABCD} = 2\left[\frac{a^2}{2}\left(\frac{\pi}{2} - 1\right)\right] = \frac{a^2}{2}\left(\frac{\pi}{2} - 1\right)$$
$\text{Therefore, the area of smaller part of the circle, } x^2 + y^2 = a^2, \text{ cut off by the line, } x = \frac{a}{\sqrt{2}}$
$\frac{a^2}{2}\left(\frac{\pi}{2} - 1\right) \text{ units.}$

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Q8 The area between x = y^2 and x = 4 is divided into two equal parts by the line x = a, find the value of a.

Answer.

{ The line, } x=a \text { , divides the area bounded by the parabola and } x=4 \text { into two equal }} \\ {\text { parts. }} \\ {\therefore \text { Area OAD



It can be observed that the area ABCD is symmetrical about x-axis.
∴ Area ABCD = 2 × Area ABC
Area OED=Area EFCD
$$\text{Area } OED = \int_0^{t} ydx$$
$$= \int_0^{\pi} \sqrt{x}dx$$
$$= \left[\frac{3}{2}\right]_0^{a}$$
$$= \frac{2}{3}(a)^{\frac{3}{2}} \quad \ldots(i)$$
$$\text{Area of } EFCD = \int_0^{4} \sqrt{x}dx$$
$$= \left[\begin{matrix}\frac{3}{2} \\ \frac{3}{2}\end{matrix}\right]_0$$
$$= \frac{2}{3}\left[8 - a^{\frac{3}{2}}\right] \quad \ldots(ii)$$

From (1) and (2), we obtain

$\frac{2}{3}(a)^{\frac{3}{2}}=\frac{2}{3}\left[8-(a)^{\frac{3}{2}}\right]$

$\Rightarrow 2\cdot(a)^{\frac{3}{2}}=8$

$\Rightarrow (a)^{\frac{3}{2}}=4$

$\Rightarrow a=(4)^{\frac{2}{3}}$

Therefore, the value of a is $(4)^{\frac{2}{3}}$

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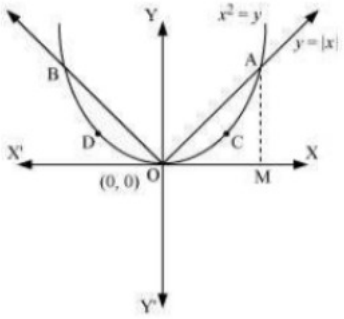
Q9 Find the area of the region bounded by the parabola y = x^2 and $y=|x|$

Answer.

Find the area of the region bounded by the parabola $y=x^2$ and $y=|x|$

Answer

The area bounded by the parabola, $x^2=y,$ and the line, $y=|x|,$ can be represented as



The given area is symmetrical about y -axis.

$\therefore$ Area OACO = Area ODBO

The point of intersection of parabola, $x^2=y,$ and line, $y=x,$ is A $(1,1)$.

Area of OACO = Area $\Delta OAB-$ Area OBACO

$\therefore$ Area of ΔOAB $=\frac{1}{2}\times \mathrm{OB}\times \mathrm{AB}=\frac{1}{2}\times 1\times 1=\frac{1}{2}$

Area of OBACO $=\int_0 ydx=\int x^2dx=\left[\frac{x^3}{3}\right]_0^1=\frac{1}{3}$

$\Rightarrow$ Area of OACO = Area of $\Delta OAB-$ Area of OBACO Therefore,required area

$=\frac{1}{2}-\frac{1}{3}$

$=\frac{1}{6}$

$=2\left[\frac{1}{6}\right]=\frac{1}{3}$

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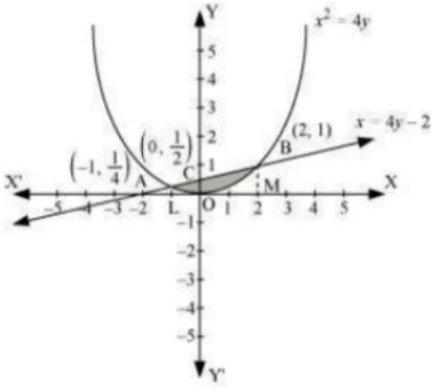
Q10 Find the area bounded by the curve x^2 = 4y and the line x = 4y – 2

Answer.

Find the area bounded by the curve $x^2=4y$ and the line $x=4y-2$

Answer

The area bounded by the curve, $x^2=4y,$ and line, $x=4y-2,$ is represented by the shaded area OBAO.



Let A and B be the points of intersection of the line and parabola.

A are $\left(-1, \frac{1}{4}\right)$

Coordinates of point

Coordinates of point B are $(2, 1)$

We draw AL and BM perpendicular to x -axis.

It can be observed that,

$\text{Area OBAO} = \text{Area OBCO} + \text{Area OACO} \ldots \text{(i)}$

Then, $\text{Area OBCO} =$ Area OMBC - Area OMBO

$$= \int_0^2 \frac{x+2}{4} dx - \int_0^2 \frac{x^2}{4} dx$$

$$= \frac{1}{4}\left[\frac{x^2}{2} + 2x\right]_0^2 - \frac{1}{4}\left[\frac{x^3}{3}\right]_0^2$$

$= \frac{1}{4}[2+4] - \frac{1}{4}\left[\frac{8}{3}\right]$

$= \frac{3}{2} - \frac{2}{3}$

$= \frac{5}{6}$

Similarly, Area OACO = Area OLAC - Area OLAO

$= \int_{-1}^{\infty} \frac{x+2}{4} dx - \int_{-1} \frac{x^2}{4} dx$

$= \frac{1}{4}\left[\frac{x^2}{2} + 2x\right]_{-1}^{0} - \frac{1}{4}\left[\frac{x^3}{3}\right]^0$

$= -\frac{1}{4}\left[\frac{(-1)^2}{2} + 2(-1)\right] - \left[-\frac{1}{4}\left(\frac{(-1)^3}{3}\right)\right]$

$= -\frac{1}{4}\left[\frac{1}{2} - 2\right] - \frac{1}{12}$

$= \frac{1}{2} - \frac{1}{8} - \frac{1}{12}$

$= \frac{7}{24}$

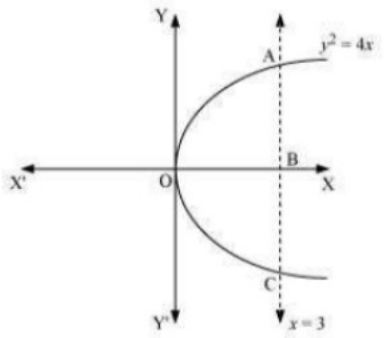
Therefore, required area $= \left(\frac{5}{6} + \frac{7}{24}\right) = \frac{9}{8}$

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Q11 Find the area of the region bounded by the curve y^2 = 4x and the line x = 3

Answer.

The region bounded by the parabola, $y^2 = 4x,$ and the line, $x = 3,$ is the area OACO.



The area OACO is symmetrical about x -axis.

$\therefore$ Area of $OACO = 2$ (Area of OAB)

$$\text{Area OACO} = 2\left[\int_0^3 y dx\right]$$

$$= 2\int 2\sqrt{x} dx$$

$$= 4\left[\frac{x^3}{2}\right]_0^3$$

$$= \frac{8}{3}\left[(3)^{\frac{3}{2}}\right]$$

$$= 8\sqrt{3}$$

Therefore,the reqiured area is $8\sqrt{3}$

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Q12 Area lying in the first quadrant and bounded by the circle $x^2 + y^2 = 4$ and the lines x = 0 and x = 2 is

A. π

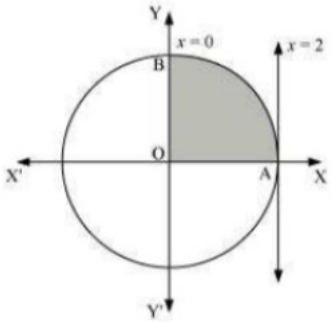
B. $\frac{\pi}{2}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{4}$

Answer.

The area bounded by the circle and the lines, $x = 0$ and $x = 2,$ in the first quadrant is represented as



$$\therefore \text{ Area OAB} = \int_0^2 y dx$$

$$= \int_0^2 \sqrt{4 - x^2} dx$$

$$= \left[\frac{x}{2}\sqrt{4 - x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2}\right]_0^2$$

$$= 2\left(\frac{\pi}{2}\right)$$

$$= \pi \text{ units}$$

Thus,the correct answer is A.

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Q13 Area of the region bounded by the curve $y^2 = 4x$,y-axis and the line y = 3 is

A. 2

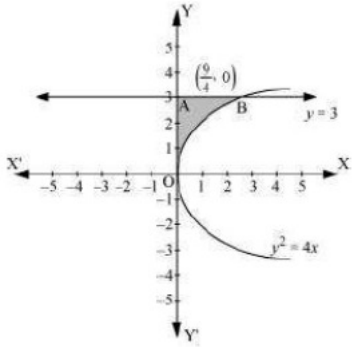
B. $\frac{9}{4}$

C. $\frac{9}{3}$

D. $\frac{9}{2}$

Answer.

The bounded by the curve, $y^2 = 4x, y$ -axis, and $y = 3$ is represented as



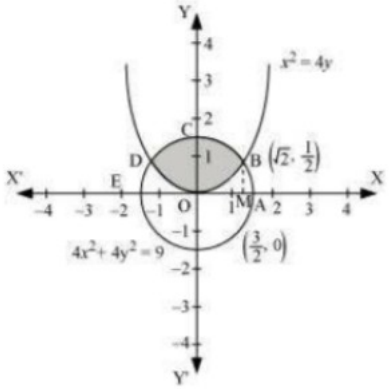
$$\therefore \text{ Area OAB} = \int_0^3 x dy$$
$$= \int_0^3 \frac{y^2}{4} dy$$
$$= \frac{1}{4}\left[\frac{y^3}{3}\right]_0^3$$
$$= \frac{1}{12}(27)$$
$$= \frac{9}{4} \text{ units}$$

Thus the correct answer is B.

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Q1 Find the area of the circle $4x^2 + 4y^2 = 9$ which is interior of the parabola $x^2 = 4y$

Answer. The required area is represented by the shaded area OBCDO.



Solving the given equation of circle, $4x^2 + 4y^2 = 9,$ and parabola, $x^2 = 4y,$ we obtain the point of intersection as $\left(\sqrt{2}, \frac{1}{2}\right)$ and D $\left(-\sqrt{2}, \frac{1}{2}\right)$

It can be observed that the required area is symmetrical about y-axis.

$\therefore$ Area OBCDO $= 2 \times$ Area OBCO

We draw BM perpendicular to OA.

Therefore, the coordinates of M are $(\sqrt{2}, 0)$.

Therefore, Area OBCO = Area OMBCO - Area OMBO

$$= \int_0^{\sqrt{2}} \sqrt{\frac{(9 - 4x^2)}{4}} dx - \int \sqrt{\frac{x^2}{4}} dx$$
$$= \frac{1}{2}\int_0^{\sqrt{2}} \sqrt{9 - 4x^2} dx - \frac{1}{4}\int_0^{\sqrt{2}} x^2 dx$$
$$= \frac{1}{4}\left[x\sqrt{9 - 4x^2} + \frac{9}{2}\sin^{-1}\frac{2x}{3}\right]_0^{\sqrt{2}} - \frac{1}{4}\left[\frac{x^3}{3}\right]_0^{\sqrt{2}}$$
$$= \frac{1}{4}\left[\sqrt{2}\sqrt{9 - 8} + \frac{9}{2}\sin^{-1}\frac{2\sqrt{2}}{3}\right] - \frac{1}{12}(\sqrt{2})^3$$
$$= \frac{\sqrt{2}}{4} + \frac{9}{8}\sin^{-1}\frac{2\sqrt{2}}{3} - \frac{\sqrt{2}}{6}$$
$$= \frac{\sqrt{2}}{12} + \frac{9}{8}\sin^{-1}\frac{2\sqrt{2}}{3}$$
$$= \frac{1}{2}\left(\frac{\sqrt{2}}{6} + \frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3}\right)$$

Therefore, the required area OBCDO is

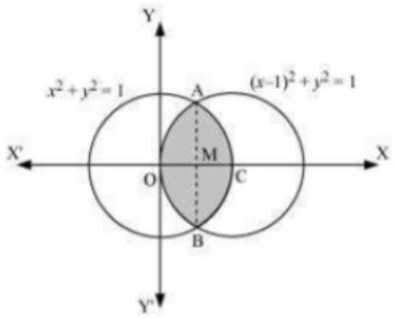
$\left(2 \times \frac{1}{2}\left[\frac{\sqrt{2}}{6} + \frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3}\right]\right) = \left[\frac{\sqrt{2}}{6} + \frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3}\right]$ units

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Q2 Find the area bounded by curves $(x - 1)^2 + y^2 = 1$ and $x^2 + y^2 = 1$

Answer.

The area bounded by the curves, $(x-1)^2+y^2=1$ and $x^2+y^2=1$, is represented by the shaded area as



On solving the equations, $(x-1)^2+y^2=1$ and $x^2+y^2=1$, we obtain the point of intersection as $A\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

It can be observed that the required area is symmetrical about x-axis.

$\therefore$ Area OBCAO $= 2 \times$ Area OCAO

We join AB, which intersects oC at M, such that AM is perpendicular to oc.

The coordinates of M are $\left(\frac{1}{2}, 0\right)$

$\Rightarrow$ Area $OCAO =$ Area OMAO+Area MCAM

$$= \left[\int_0^{\frac{1}{2}} \sqrt{1-(x-1)^2}dx + \int_{\frac{1}{2}}^{1} \sqrt{1-x^2}dx\right]$$

$$= \left[\frac{x-1}{2}\sqrt{1-(x-1)^2} + \frac{1}{2}\sin^{-1}(x-1)\right]_0^{\frac{1}{2}} + \left[\frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\sin^{-1}x\right]_{\frac{1}{2}}^{1}$$

$$= \left[-\frac{1}{4}\sqrt{1-\left(-\frac{1}{2}\right)^2} + \frac{1}{2}\sin^{-1}\left(\frac{1}{2}-1\right) - \frac{1}{2}\sin^{-1}(-1)\right] +$$

$$\left[\frac{1}{2}\sin^{-1}(1) - \frac{1}{4}\sqrt{1-\left(\frac{1}{2}\right)^2} - \frac{1}{2}\sin^{-1}\left(\frac{1}{2}\right)\right]$$

$$= \left[-\frac{\sqrt{3}}{8} + \frac{1}{2}\left(-\frac{\pi}{6}\right) - \frac{1}{2}\left(-\frac{\pi}{2}\right)\right] + \left[\frac{1}{2}\left(\frac{\pi}{2}\right) - \frac{\sqrt{3}}{8} - \frac{1}{2}\left(\frac{\pi}{6}\right)\right]$$

$$= \left[-\frac{\sqrt{3}}{4} - \frac{\pi}{12} + \frac{\pi}{4} + \frac{\pi}{4} - \frac{\pi}{12}\right] + \left[\frac{1}{2}\left(\frac{\pi}{2}\right) - \frac{\sqrt{3}}{8} - \frac{1}{2}\left(\frac{\pi}{6}\right)\right]$$

$$= \left[-\frac{\sqrt{3}}{4} - \frac{\pi}{6} + \frac{\pi}{2}\right]$$

$$= \left[\frac{2\pi}{6} - \frac{\sqrt{3}}{4}\right]$$

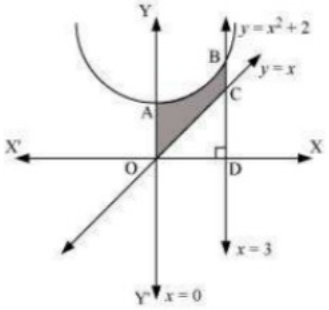
Therefore ,required area OBCAO $= 2 \times \left(\frac{2\pi}{6} - \frac{\sqrt{3}}{4}\right) = \left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right)$ units

Page : 371 , Block Name : Exercise 8.2

Q3 Find the area of the region bounded by the curves $y=x^2+2, y=x, x=0$ and $x=3$

Answer.

The area bounded by the curves, $y=x^2+2, y=x, x=0,$ and $x=3,$ is represented by the shaded area OCBAO as



Then, Area OCBAO = Area ODBAO - Area ODCO

$= \int_0^3 \left(x^2+2\right) dx - \int_0^3 x dx$

$= \left[\frac{x^3}{3}+2x\right]_0^3 - \left[\frac{x^2}{2}\right]_0^3$

$= [9+6] - \left[\frac{9}{2}\right]$

$= 15 - \frac{9}{2}$

$= \frac{21}{2}$ units

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Q4

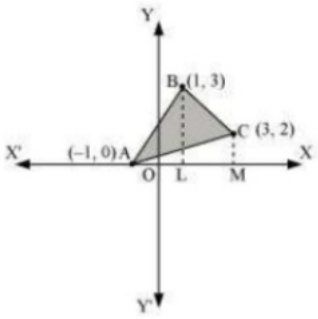
Using integration finds the area of the region bounded by the triangle whose vertices are $(-1,0),(1,3)$ and $(3,2)$

Answer.

BL and CM are drawn perpendicular to x -axis.

It can be observed in the following figure that,

Area (ΔACB) = Area $(ALBA)$ + Area $(BLMCB)$ − Area $(AMCA) \ldots (1)$



Equation of line segment AB is

$y - 0 = \frac{3-0}{1+1}(x+1)$

$y = \frac{3}{2}(x+1)$

$\therefore$ Area (ALBA) $= \int_{-1}^{+} \frac{3}{2}(x+1)dx = \frac{3}{2}\left[\frac{x^2}{2}+x\right]_{-1}^{1} = \frac{3}{2}\left[\frac{1}{2}+1-\frac{1}{2}+1\right] = 3$ units

Equation of line segment BC is

$y - 3 = \frac{2-3}{3-1}(x-1)$

$y = \frac{1}{2}(-x+7)$

$\therefore$ Area (BLMCB) $= \int_1^3 \frac{1}{2}(-x+7)dx = \frac{1}{2}\left[-\frac{x^2}{2}+7x\right]_1^3 = \frac{1}{2}\left[-\frac{9}{2}+21+\frac{1}{2}-7\right] = 5$ units

Equation of line segment AC is

$y - 0 = \frac{2-0}{3+1}(x+1)$

$y = \frac{1}{2}(x+1)$

$\therefore$ Area (AMCA) $= \frac{1}{2}\int_1^3 (x+1)dx = \frac{1}{2}\left[\frac{x^2}{2}+x\right]_{-1}^{3} = \frac{1}{2}\left[\frac{9}{2}+3-\frac{1}{2}+1\right] = 4$ units

Therefore, from equation (1), we obtain

Area $(\Delta ABC) = (3+5-4) = 4$ units

Page : 371 , Block Name : Exercise 8.2

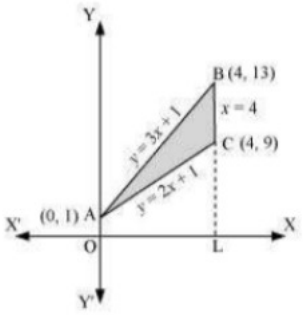
Q5

Using integration find the area of the triangular region whose sides have the equations $y = 2x+1, y = 3x+1$ and $x = 4$.

Answer.

The equations of sides of the triangle are $y = 2x+1, y = 3x+1,$ and $x = 4$.

On solving these equations, we obtain the vertices of triangle as $A(0,1), B(4,13),$ and $C(4,9)$.



It can be observed that,

Area $(\Delta ACB) =$ Area (OLBAO) -Area (OLCAO)

$= \int_0^4 (3x+1)dx - \int_0^4 (2x+1)dx$

$= \left[\frac{3x^2}{2} + x\right]_0^4 - \left(\frac{2x^2}{2} + x\right]_0^4$

$= 28 - 20$

$= 8$ units

Page : 371 , Block Name : Exercise 8.2

Q6

Smaller area enclosed by the circle $x^2 + y^2 = 4$ and the line $x + y = 2$ is

A. $2(n-2)$

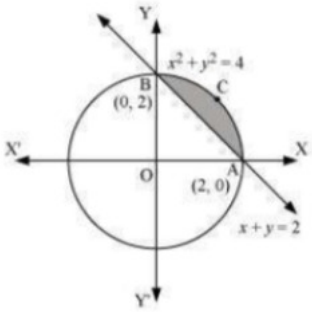
B. $\pi - 2$

C. $2\pi - 1$

D. $2(n+2)$

Answer.

The smaller area enclosed by the circle, $x^2 + y^2 = 4,$ and the line, $x + y = 2,$ is represented by the shaded area ACBA as



It can be observed that,

Area $ACBA =$ Area OACBO - Area (\DeltaOAB)

$= \int_0^2 \sqrt{4-x^2}dx - \int_0^2 (2-x)dx$

$= \left[\frac{x}{2}\sqrt{4-x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2}\right]_0^2 - \left[2x - \frac{x^2}{2}\right]_0^2$

$= \left[2 \cdot \frac{\pi}{2}\right] - [4-2]$

$= (\pi - 2) - [4-2]$

$= (\pi - 2)$ units

Thus, the correct answer is B.

Page : 372 , Block Name : Exercise 8.2

Q7 Area lying between the curve $y^2 = 4x$ and $y = 2x$ is

A. $\frac{2}{3}$

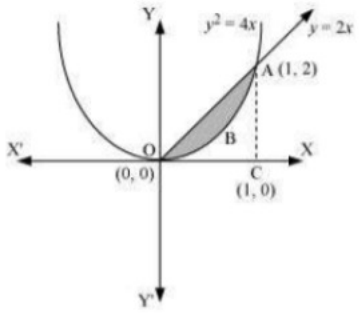
B. $\frac{1}{3}$

C. $\frac{1}{4}$

D. $\frac{3}{4}$

Answer.

The area lying between the curve, $y^2 = 4x$ and $y = 2x,$ is represented by the shaded area OBAO as



The points of intersection of these curves are $0(0, 0)$ and $A(1, 2)$.

We draw AC perpendicular to x -axis such that the coordinates of C are $(1, 0)$

$\therefore$ Area OBAO $=$ Area $(\Delta \mathrm{OCA}) -$ Area(OCABO)

$= \int_0^4 2xdx - \int_0^2 2\sqrt{x}dx$

$= 2\left[\frac{x^2}{2}\right]_0^1 - 2\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^1$

$= \left|1 - \frac{4}{3}\right|$

$= \left|-\frac{1}{3}\right|$

$= \frac{1}{3}$ units

Thus, the correct answer is B.

Page : 372 , Block Name : Exercise 8.2

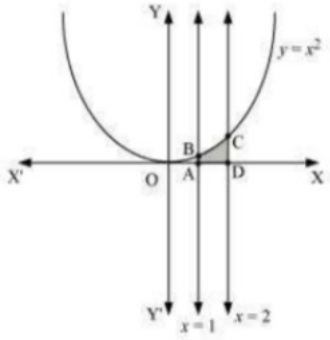
Q1

Find the area under the given curves and given lines:

(i) $y = x^2, x = 1, x = 2$ and x -axis

(ii) $y = x^4, x = 1, x = 5$ and x -axis

Answer. (i) The required area is represented by the shaded area ADCBA as



$$\text{Area } ADCBA = \int^2 ydx$$

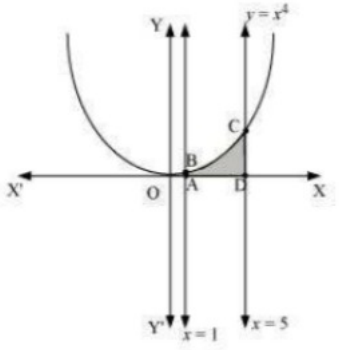
$$= \int x^2 dx$$

$$= \left[\frac{x^3}{3}\right]_1^2$$

$$= \frac{8}{3} - \frac{1}{3}$$

$$= \frac{7}{3} \text{ units}$$

(ii) The required area is represented by the shaded area ADCBA as

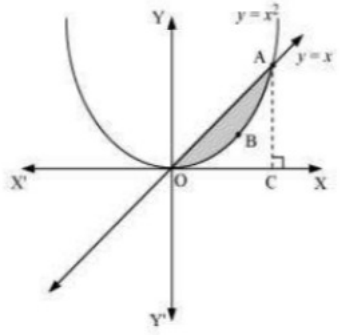


$$\text{Area ADCBA} = \int^5 x^4 dx$$
$$= \left[\frac{x^5}{5}\right]_1^5$$
$$= \frac{(5)^5}{5} - \frac{1}{5}$$
$$= (5)^4 - \frac{1}{5}$$
$$= 625 - \frac{1}{5}$$
$$= 624.8 \text{ units}$$

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Q2 Find the area between the curve $y = x$ and $y = x^2$

Answer. The required area is represented by the shaded area OBAO as

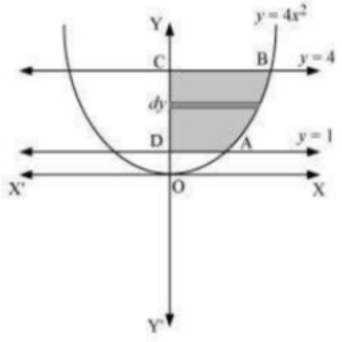


$\text{The points of intersection of the curves, } y = x \text{ and } y = x^2, \text{ is A}(1, 1)$.
We draw AC perpendicular to x -axis.
$\therefore$ Area (OBAO) = Area (ΔOCA) − Area (OCABO) $\ldots (i)$
$= \int x dx - \int_0^t x^2 dx$
$= \left[\frac{x^2}{2}\right]_0^1 - \left[\frac{x^3}{3}\right]_0^1$
$= \frac{1}{2} - \frac{1}{3}$
$= \frac{1}{6}$ units

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Q3
Find the area of the region lying in the first quadrant and bounded by $y = 4x^2, x = 0, y = 1$ and $y = 4$

Answer. The area in the first quadrant bounded by $y = 4x^2, x = 0, y = 1,$ and $y = 4$ is represented by the shaded area ABCDA as



$$\therefore \text{ Area ABCD} = \int_x^1 x dx$$

$$= \int \frac{\sqrt{y}}{2} dx$$

$$= \frac{1}{2}\left[\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right]^4$$

$$= \frac{1}{3}\left[(4)^{\frac{3}{2}} - 1\right]$$

$$= \frac{1}{3}[8 - 1]$$

$$= \frac{7}{3} \text{ units}$$

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Q4 Sketch the graph of $y = |x + 3|$ and evaluate $\int_{-6}^{0} |x + 3| dx$

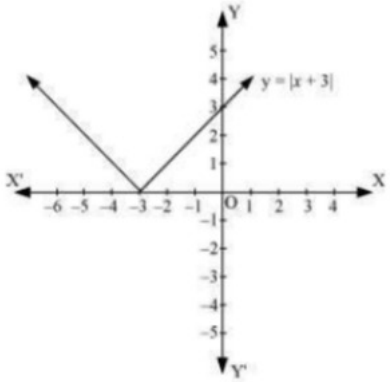
Answer.

The given equation is $y = |x + 3|$

The corresponding values of x and y are given in the following table.

x	- 6	- 5	- 4	- 3	- 2	- 1	0
y	3	2	1	0	1	2	3

On plotting these points, we obtain the graph of $y = |x + 3|$ as follows.



It is known that, $(x + 3) \leq 0$ for $-6 \leq x \leq -3$ and $(x + 3) \geq 0$ for $-3 \leq x \leq 0$

$$\therefore \int_{-6}^{0} (x + 3) dx = -\int_{-6}^{-3} (x + 3) dx + \int_{-3}^{0} (x + 3) dx$$

$$= -\left[\frac{x^2}{2} + 3x\right]_{-6}^{-3} + \left[\frac{x^2}{2} + 3x\right]_{-3}^{0}$$

$$= -\left[\left(\frac{(-3)^2}{2} + 3(-3)\right) - \left(\frac{(-6)^2}{2} + 3(-6)\right)\right] + \left[0 - \left(\frac{(-3)^2}{2} + 3(-3)\right)\right]$$

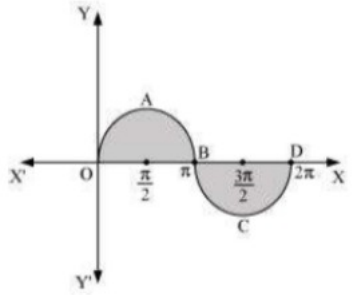
$$= -\left[-\frac{9}{2}\right] - \left[-\frac{9}{2}\right]$$

$$= 9$$

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Q5 Find the area bounded by curve $y = \sin x$ between $x = 0$ and $x = 2\pi$

Answer. The graph of y = sin x can be drawn as



∴ Required area = Area OABO + Area BCDB

$= \int_0^{\pi} \sin x dx + \left| \int_{\pi}^{2\pi} \sin x dx \right|$

$= [-\cos x]_0^x + |-\cos x]_{\pi}^{2\pi}|$

$= [-\cos \pi + \cos 0] + |-\cos 2\pi + \cos \pi|$

$= 1 + 1 + |(-1 - 1)|$

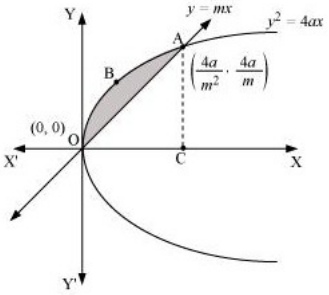
$= 2 + |-2|$

$= 2 + 2 = 4$ units

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Q6 Find the area enclosed between the parabola y^2 = 4ax and the line y = mx

Answer. The area enclosed between the parabola, \(y^{2} = 4ax, and the line, y = mx, is represented by the shaded area OABO as



The points of intersection of both the curves are $(0, 0)$ and $\left(\frac{4d}{m^2}, \frac{4a}{m}\right)$

we draw AC perpendicular to x -axis.

∴ Area OABO = Area OCABO − Area (ΔOCA)

$= \int_0^{\frac{4a}{m^2}} 2\sqrt{ax}\, dx - \int_0^{\frac{4a}{m^2}} mx\, dx$

$= 2\sqrt{a}\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^{\frac{4a}{m^2}} - m\left[\frac{x^2}{2}\right]_0^{\frac{4a}{m^2}}$

$= \frac{4}{3}\sqrt{a}\left(\frac{4a}{m^2}\right)^{\frac{3}{2}} - \frac{m}{2}\left[\left(\frac{4a}{m^2}\right)^2\right]$

$= \frac{32a^2}{3m^3} - \frac{m}{2}\left(\frac{16a^2}{m^4}\right)$

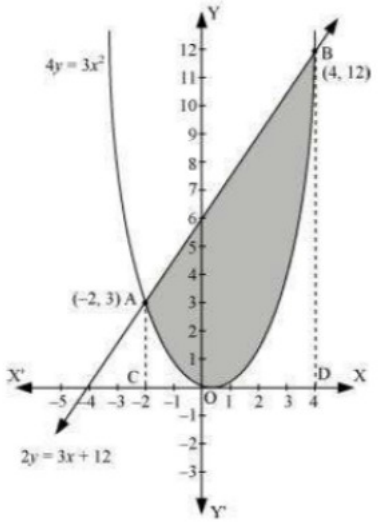
$= \frac{32a^2}{3m^3} - \frac{8a^2}{m^3}$

$= \frac{8a^2}{3m^3}$ units

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Q7 Find the area enclosed by the parabola $4y = 3x^2$ and the line $2y = 3x + 12$

Answer. The area enclosed between the parabola $4y = 3x^2$ and the line $2y = 3x + 12$ is represented by the shaded area OBAO as



The points of intersection of the given curves are A (–2, 3) and (4, 12).
We draw AC and BD perpendicular to x-axis.
∴ Area OBAO = Area CDBA – (Area ODBO + Area OACO)

$$= \int_{-2}^{4} \frac{1}{2}(3x + 12)dx - \int_{-2}^{1} \frac{3x^2}{4}dx$$

$$= \frac{1}{2}\left[\frac{3x^2}{2} + 12x\right]_{-2}^{4} - \frac{3}{4}\left[\frac{x^3}{3}\right]_{-2}^{4}$$

$= \frac{1}{2}[24 + 48 - 6 + 24] - \frac{1}{4}[64 + 8]$

$= \frac{1}{2}[90] - \frac{1}{4}[72]$

$= 45 - 18$

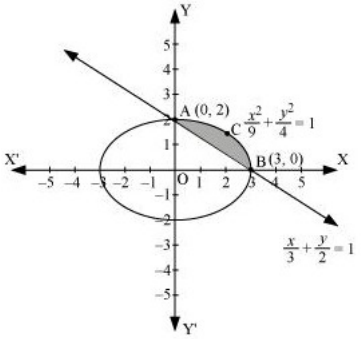
$= 27$ units

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Q8

Find the area of the smaller region bounded by the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and the line $\frac{x}{3} + \frac{y}{2} = 1$

Answer. The area of the smaller region bounded by the ellipse, $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and the line, $\frac{x}{3} + \frac{y}{2} = 1$ is represented by the shaded region BCAB as



∴ Area BCAB = Area (OBCAO) – Area (OBAO)

$$= \int_0^3 2\sqrt{1 - \frac{x^2}{9}}dx - \int_0^3 2\left(1 - \frac{x}{3}\right)dx$$

$$= \frac{2}{3}\left[\int_0^3 \sqrt{9 - x^2}dx\right] - \frac{2}{3}\int_0^3 (3 - x)dx$$

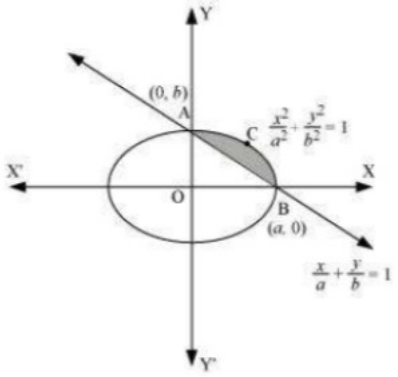
$$= \frac{2}{3}\left[\frac{x}{2}\sqrt{9 - x^2} + \frac{9}{2}\sin^{-1}\frac{x}{3}\right]_0^3 - \frac{2}{3}\left[3x - \frac{x^2}{2}\right]_0^3$$

$$= \frac{2}{3}\left[\frac{9}{2}\left(\frac{\pi}{2}\right)\right] - \frac{2}{3}\left[9 - \frac{9}{2}\right]$$
$$= \frac{2}{3}\left[\frac{9\pi}{4} - \frac{9}{2}\right]$$
$$= \frac{2}{3} \times \frac{9}{4}(\pi - 2)$$
$$= \frac{3}{2}(\pi - 2) \text{ units}$$

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Q9 Find the area of the smaller region bounded by the Find the area of the smaller region bounded by the ellipse
$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the line $\frac{x}{a} + \frac{y}{b} = 1$

Answer. The area of the smaller region bounded by the ellipse,
$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the line $\frac{x}{a} + \frac{y}{b} = 1$ is represented by the shaded region BCAB as

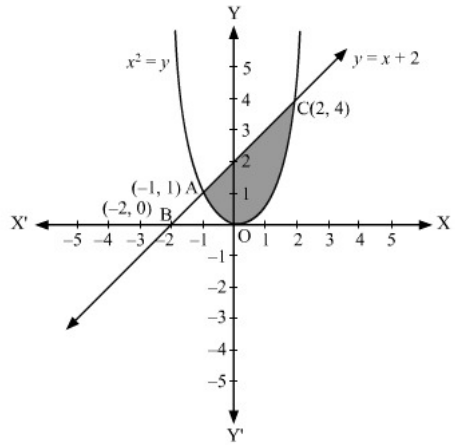


∴ Area BCAB = Area (OBCAO) − Area (OBAO)
$$= \int_0^a b\sqrt{1 - \frac{x^2}{a^2}}dx - \int_b^a b\left(1 - \frac{x}{a}\right)dx$$
$$= \frac{b}{a}\int_0^0 \sqrt{a^2 - x^2}dx - \frac{b}{a}\int_0^a (a - x)dx$$
$$= \frac{b}{a}\left[\left\{\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right\}_0^a - \left\{ax - \frac{x^2}{2}\right\}_0^a\right]$$
$$= \frac{b}{a}\left[\left\{\frac{a^2}{2}\left(\frac{\pi}{2}\right)\right\} - \left\{a^2 - \frac{a^2}{2}\right\}\right]$$
$$= \frac{b}{a}\left[\frac{a^2\pi}{4} - \frac{a^2}{2}\right]$$

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Q10 Find the area of the region enclosed by the parabola x^2 = y, the line y = x + 2 and x-axis

Answer. The area of the region enclosed by the parabola, x^2 = y, the line, y = x + 2, and x-axis is represented by the shaded region OACO as



The point of intersection of the parabola, $x^2 = y$, and the line, $y = x + 2$, is $A(-1, 1)$.

$\therefore$ Area OABCO = Area (BCA) + Area COAC

$= \int_2^1 (x+2)dx + \int_{-1}^0 x^2 dx$

$= \left[\frac{x^2}{2} + 2x\right]_{-2}^{-1} + \left[\frac{x^3}{3}\right]_{-1}^{0}$

$= \left[\frac{(-1)^2}{2} + 2(-1) - \frac{(-2)^2}{2} - 2(-2)\right] + \left[-\frac{(-1)^3}{3}\right]$

$= \left[\frac{1}{2} - 2 - 2 + 4 + \frac{1}{3}\right]$

$= \frac{5}{6}$ units

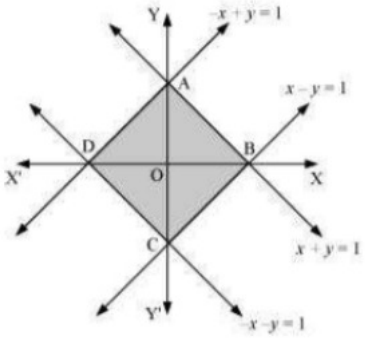
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Q11

Using the method of integration find the area bounded by the curve $|x| + |y| = 1$

[Hint: the required region is bounded by lines $x + y = 1, x - y = 1, -x + y = 1$ and $-x - y = 11$]

Answer. The area bounded by the curve, $|x| + |y| = 1$, is represented by the shaded region $ADCB$ as



The curve intersects the axes at points A (0, 1), B (1, 0), C (0, −1), and D (−1, 0).

It can be observed that the given curve is symmetrical about x-axis and y-axis.

∴ Area ADCB = 4 × Area OBAO

$= 4\int_0^d (1-x)dx$

$= 4\left(x - \frac{x^2}{2}\right)^1$

$= 4\left[1 - \frac{1}{2}\right]_0$

$= 4\left(\frac{1}{2}\right)$

$= 2$ units

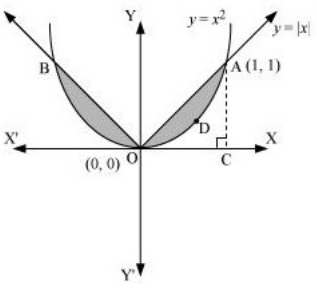
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Q12 Find the area bounded by curves

$\{(x, y) : y \geq x^2 \text{ and } y = |x|\}$

Answer. The area bounded by the curves,

$\{(x, y) : y \geq x^2 \text{ and } y = |x|\}$, is represented by the shaded region as



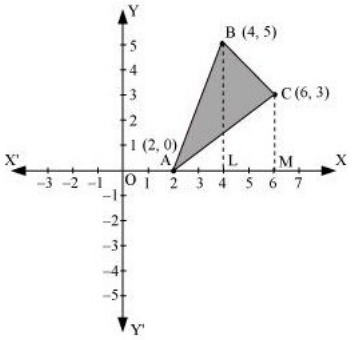
It can be observed that the required area is symmetrical about y-axis.

$$\begin{aligned}\text{Required area } &= 2[\text{ Area (OCAO)} - \text{ Area (OCADO)}]\\ &= 2\left[\int x dx - \int x^2 dx\right]\\ &= 2\left[\left[\frac{x^2}{2}\right]_0^1 - \left[\frac{x^3}{3}\right]_0^1\right]\\ &= 2\left[\frac{1}{2} - \frac{1}{3}\right]\\ &= 2\left[\frac{1}{6}\right] = \frac{1}{3}\text{ units}\end{aligned}$$

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Q13 Using the method of integration find the area of the triangle ABC, coordinates of whose vertices are A (2, 0), B (4, 5) and C (6, 3)

Answer. The vertices of ΔABC are A (2, 0), B (4, 5), and C (6, 3).



Equation of line segment AB is

$\text{Equation of line segment AB is}$

$y - 0 = \frac{5-0}{4-2}(x - 2)$

$2y = 5x - 10$

$y = \frac{5}{2}(x - 2) \quad \ldots(i)$

$\text{Equation of line segment BC is}$

$y - 5 = \frac{3-5}{6-4}(x - 4)$

$2y - 10 = -2x + 8$

$2y = -2x + 18$

$y = -x + 9 \quad \ldots(ii)$

$\text{Equation of line segment ca is}$

$y - 3 = \frac{0-3}{2-6}(x - 6)$

$-4y + 12 = -3x + 18$

$y = \frac{3}{4}(x - 2) \quad \ldots(iii)$

Area (ΔABC) = Area (ABLA) + Area (BLMCB) − Area (ACMA)

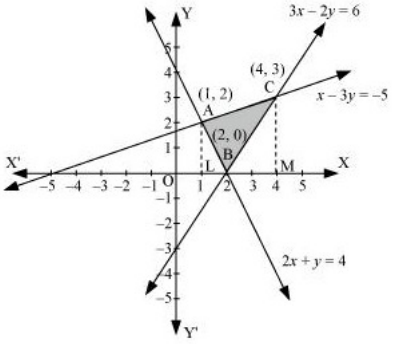
$$\begin{aligned}&= \int_2^4 \frac{5}{2}(x-2)dx + \int_4^6 (-x+9)dx - \int_2^6 \frac{3}{4}(x-2)dx\\ &= \frac{5}{2}\left[\frac{x^2}{2} - 2x\right]_2^4 + \left[\frac{-x^2}{2} + 9x\right]_4^6 - \frac{3}{4}\left[\frac{x^2}{2} - 2x\right]_2^6\\ &= \frac{5}{2}[8 - 8 - 2 + 4] + [-18 + 54 + 8 - 36] - \frac{3}{4}[18 - 12 - 2 + 4]\\ &= 5 + 8 - \frac{3}{4}(8)\\ &= 13 - 6\\ &= 7\text{ units}\end{aligned}$$

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Q14 Using the method of integration find the area of the region bounded by lines:
2x + y = 4, 3x − 2y = 6 and x − 3y + 5 = 0

Answer. The given equations of lines are
2x + y = 4 ... (i)
3x − 2y = 6 ... (ii)

And, x – 3y + 5 = 0 ... (iii)



The area of the region bounded by the lines is the area of ΔABC. AL and CM are the perpendiculars on x-axis.
Area (ΔABC) = Area (ALMCA) – Area (ALB) – Area (CMB)

$$= \int^{4} \left(\frac{x+5}{3}\right) dx - \int^{2} (4-2x)dx - \int_{2}^{4} \left(\frac{3x-6}{2}\right) dx$$

$$= \frac{1}{3}\left[\frac{x^2}{2}+5x\right]_1^4 - \left[4x-x^2\right]_1^2 - \frac{1}{2}\left[\frac{3x^2}{2}-6x\right]_2^4$$

$$= \frac{1}{3}\left[8+20-\frac{1}{2}-5\right] - [8-4-4+1] - \frac{1}{2}[24-24-6+12]$$

$$= \left(\frac{1}{3}\times\frac{45}{2}\right) - (1) - \frac{1}{2}(6)$$

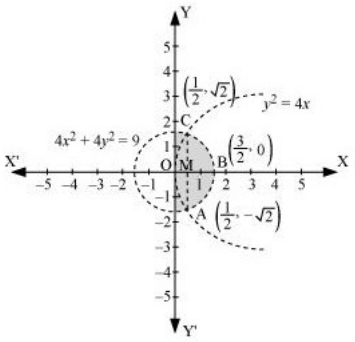
$$= \frac{15}{2} - 1 - 3$$

$$= \frac{15}{2} - 4 = \frac{15-8}{2} = \frac{7}{2} \text{ units}$$

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Q15 Find the area of the region $\left\{(x, y) : y^2 \leq 4x, 4x^2 + 4y^2 \leq 9\right\}$

Answer. The area bounded by the curves, $\left\{(x, y) : y^2 \leq 4x, 4x^2 + 4y^2 \leq 9\right\}$ is represented as



The points of intersection of both the curves are
$\left(\frac{1}{2}, \sqrt{2}\right)$ and $\left(\frac{1}{2}, -\sqrt{2}\right)$
The required area is given by OABCO.
It can be observed that area OABCO is symmetrical about x-axis.
∴ Area OABCO = 2 × Area OBC
Area OBCO = Area OMC + Area MBC

$$= \int_0^1 2\sqrt{x}dx + \int_{\frac{1}{2}}^{\frac{3}{2}} \frac{1}{2}\sqrt{9-4x^2}dx$$

$$= \int_0^1 2\sqrt{x}dx + \int_{\frac{1}{2}}^{\frac{3}{2}} \frac{1}{2}\sqrt{(3)^2-(2x)^2}dx$$

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Q16 Area bounded by the curve y = x3, the x-axis and the ordinates x = –2 and x = 1 is

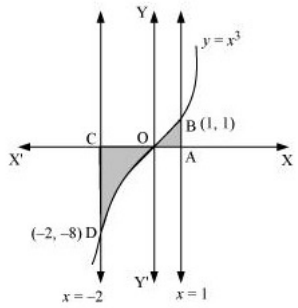
A. -9

B. $-\frac{15}{4}$

B. $\frac{15}{4}$

D. $\frac{17}{4}$

Answer.



Required area $= \int_2^1 ydx$

$= \int_2^1 x^3 dx$

$= \left[\frac{x^4}{4}\right]_{-2}^{1}$

$= \left[\frac{1}{4} - \frac{(-2)^4}{4}\right]$

$= \left(\frac{1}{4} - 4\right) = -\frac{15}{4}$ units

$= \left(\frac{1}{4} - 4\right) = -\frac{15}{4}$ units

Thus, the correct answer is B.

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Q17

The area bounded by the curve $y = x|x|$, x -axis and the ordinates $x = -1$ and $x = 1$ is given by

[Hint: $y = x^2$ if $x > 0$ and $y = -x^2$ if $x < 0$]

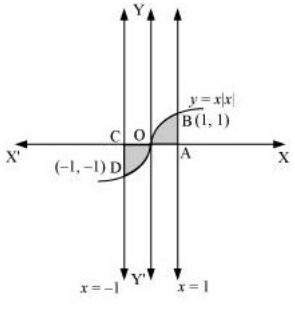
A. 0

B. $\frac{1}{3}$

C. $\frac{2}{3}$

D. $\frac{4}{3}$

Answer.



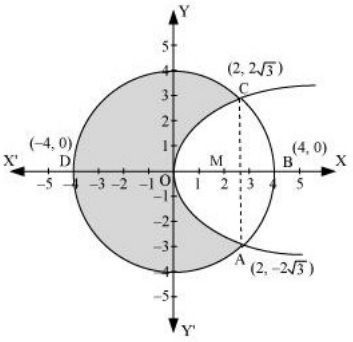
$$\text{Required area } = \int_{-1}^{1} y dx$$
$$= \int_{-1}^{\infty} x|x| dx$$
$$= \int_{-1}^{0} x^2 dx + \int_{0}^{1} x^2 dx$$
$$= \left[\frac{x^3}{3}\right]_{-1}^{0} + \left[\frac{x^3}{3}\right]_{0}^{1}$$
$$= -\frac{2}{3} \text{ units}$$
$$= \frac{2}{3} \text{ units}$$

Thus, the correct answer is C.

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Q18 The area of the circle $x^2 + y^2 = 16$ exterior to the parabola $y^2 = 6x$
$(A)\ \frac{4}{3}(4\pi - \sqrt{3})$ (B) $\frac{4}{3}(4\pi + \sqrt{3})$ (C) $\frac{4}{3}(8\pi - \sqrt{3})$ (D) $\frac{4}{3}(8\pi + \sqrt{3})$

Answer. The given equations are
$x^2 + y^2 = 16 \ldots (i)$
$y^2 = 6x \ldots (ii)$



Area bounded by the circle and parabola
$= 2[\text{Arca(OADO)} + \text{Area(ADBA)}]$
$= 2\left[\int_0^2 \sqrt{16x} dx + \int_2^{+} \sqrt{16 - x^2} dx\right]$
$= 2\left[\sqrt{6}\left\{\frac{x^{\frac{3}{2}}}{2}\right\}_0^2\right] + 2\left[\frac{x}{2}\sqrt{16 - x^2} + \frac{16}{2}\sin^{-1}\frac{x}{4}\right]_2^4$
$= 2\sqrt{6} \times \frac{2}{3}\left[x^{\frac{3}{2}}\right]_0^2 + 2\left[8 \cdot \frac{\pi}{2} - \sqrt{16 - 4} - 8\sin^{-1}\left(\frac{1}{2}\right)\right]$
$= \frac{4\sqrt{6}}{3}(2\sqrt{2}) + 2\left[4\pi - \sqrt{12} - 8\frac{\pi}{6}\right]$
$= \frac{16\sqrt{3}}{3} + 8\pi - 4\sqrt{3} - \frac{8}{3}\pi$
$= \frac{4}{3}[4\sqrt{3} + 6\pi - 3\sqrt{3} - 2\pi]$
$= \frac{4}{3}[\sqrt{3} + 4\pi]$
$\text{Area of circle } = n(r)^2$
$= 16n \text{ units}$

$$\therefore \text{ Required area } = 16\pi - \frac{4}{3}[4\pi + \sqrt{3}]$$
$$= \frac{4}{3}[4 \times 3\pi - 4\pi - \sqrt{3}]$$
$$= \frac{4}{3}(8\pi - \sqrt{3}) \text{ units}$$

Thus, the correct answer is C

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Q19 The area bounded by the y-axis, y = cos x and y = sin x
When

$0 \leq x \leq \frac{\pi}{2}$.

A. $2(\sqrt{2}-1)$

B. $\sqrt{2}-1$

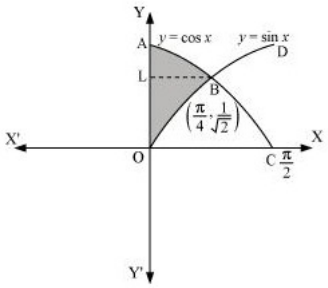
C. $\sqrt{2}+1$

D. $\sqrt{2}$

Answer. The given equations are

y = cos x ... (i)

And, y = sin x ... (ii)



Required area = Area (ABLA) + area (OBLO)

$$= \int_1^4 x dy + \int_0^1 x dy$$

$$= \int_1^d \cos^{-1} y dy + \int_0^{\frac{1}{2}} \sin^{-1} x dy$$

Integrating by parts, we obtain

$= \left[y \cos^{-1} y - \sqrt{1-y^2}\right]_{\frac{1}{\sqrt{2}}}^{1} + \left[x \sin^{-1} x + \sqrt{1-x^2}\right]_0^{\frac{1}{\sqrt{2}}}$

$= \left[\cos^{-1}(1) - \frac{1}{\sqrt{2}} \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) + \sqrt{1-\frac{1}{2}}\right] + \left[\frac{1}{\sqrt{2}} \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) + \sqrt{1-\frac{1}{2}} - 1\right]$

$= \frac{-\pi}{4\sqrt{2}} + \frac{1}{\sqrt{2}} + \frac{\pi}{4\sqrt{2}} + \frac{1}{\sqrt{2}} - 1$

$= \frac{2}{\sqrt{2}} - 1$

$= \sqrt{2} - 1$ units

Thus, the correct answer is B.

Put $2x = t \Rightarrow dx = \frac{dt}{2}$

When $x = \frac{3}{2}, t = 3$ and when $x = \frac{1}{2}, t = 1$

$= \int_0^{\frac{1}{2}} 2\sqrt{x} dx + \frac{1}{4} \int_1^3 \sqrt{(3)^2 - (t)^2} dt$

$= 2\left[\frac{x^{\frac{3}{2}}}{2}\right]^{\frac{1}{2}} + \frac{1}{4}\left[\frac{t}{2}\sqrt{9-t^2} + \frac{9}{2}\sin^{-1}\left(\frac{t}{3}\right)\right]_1^3$

$= 2\left[\frac{2}{3}\left(\frac{1}{2}\right)^{\frac{3}{2}}\right] + \frac{1}{4}\left[\left\{\frac{3}{2}\sqrt{9-(3)^2} + \frac{9}{2}\sin^{-1}\left(\frac{3}{3}\right)\right\} - \left\{\frac{1}{2}\sqrt{9-(1)^2} + \frac{9}{2}\sin^{-1}\left(\frac{1}{3}\right)\right\}\right]$

$= \frac{2}{3\sqrt{2}} + \frac{1}{4}\left[\left\{0 + \frac{9}{2}\sin^{-1}(1)\right\} - \left\{\frac{1}{2}\sqrt{8} + \frac{9}{2}\sin^{-1}\left(\frac{1}{3}\right)\right\}\right]$

$= \frac{\sqrt{2}}{3} + \frac{1}{4}\left[\frac{9\pi}{4} - \sqrt{2} - \frac{9}{2}\sin^{-1}\left(\frac{1}{3}\right)\right]$

$= \frac{\sqrt{2}}{3} + \frac{9\pi}{16} - \frac{\sqrt{2}}{4} - \frac{9}{8}\sin^{-1}\left(\frac{1}{3}\right)$

$= \frac{9\pi}{16} - \frac{9}{8}\sin^{-1}\left(\frac{1}{3}\right) + \frac{\sqrt{2}}{12}$

Therefore , the required area is $\left[2 \times \left(\frac{9\pi}{16} - \frac{9}{8}\sin^{-1}\left(\frac{1}{3}\right) + \frac{\sqrt{2}}{12}\right)\right] = \frac{9\pi}{8} - \frac{9}{4}\sin^{-1}\left(\frac{1}{3}\right) + \frac{1}{3\sqrt{2}}$ units

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