NCERT SOLUTIONS

CLASS - 12th



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Class : 12th
Subject : Maths
Chapter : 4
Chapter Name : Determinants

Q1 Evaluate the determinants In Exercises 1 and 2

$$\begin{vmatrix} 2 & 4 \\ -5 & -1 \end{vmatrix}$$

Answer.

$$\begin{vmatrix} 2 & 4 \\ -5 & -1 \end{vmatrix}$$

= 2(-1) -4(-5) = -2 + 20 = 18

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Q2

(i) $\begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$

(ii) $\begin{vmatrix} x^2 - x + 1 & x - 1 \\ x + 1 & x + 1 \end{vmatrix}$

Answer.

$$\begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$$

$(\cos\theta)(\cos\theta) - (-\sin\theta)(\sin\theta) = \cos^2\theta + \sin^2\theta = 1$

(ii) $\begin{vmatrix} x^2 - x + 1 & x - 1 \\ x + 1 & x + 1 \end{vmatrix}$

$= \left(x^2 - x + 1\right)(x + 1) - (x - 1)(x + 1)$

$= x^3 - x^2 + x + x^2 - x + 1 - \left(x^2 - 1\right)$

$= x^3 + 1 - x^2 + 1$

$= x^3 - x^2 + 2$

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Q3 *If* $A = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$, then show that $|2A| = 4|A|$

Answer. The given matrix is

$A = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$

$\therefore 2A = 2\begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 8 & 4 \end{bmatrix}$

$\therefore$ L. L. H. S. $= |2A| = \begin{vmatrix} 2 & 4 \\ 8 & 4 \end{vmatrix} = 2 \times 4 - 4 \times 8 = 8 - 32 = -24$

Now $|A| = \begin{vmatrix} 1 & 2 \\ 4 & 2 \end{vmatrix} = 1 \times 2 - 2 \times 4 = 2 - 8 = -6$

$\therefore$ R. H. S. $= 4|A| = 4 \times (-6) = -24$

$\therefore$ L. H. S. = R. H. S.

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Q4 If $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix}$, then show that $|3A| = 27|A|$

Answer. The given matrix is $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix}$

It can be observed that in the first column, two entries are zero.
Thus,we expand along the first column (C1) for easier calculation.

$|A| = 1\begin{vmatrix} 1 & 2 \\ 0 & 4 \end{vmatrix} - 0\begin{vmatrix} 0 & 1 \\ 0 & 4 \end{vmatrix} + 0\begin{vmatrix} 0 & 1 \\ 1 & 2 \end{vmatrix} = 1(4 - 0) - 0 + 0 = 4$

$\therefore 27|A| = 27(4) = 108 \quad \ldots(i)$

Now $3A = 3\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 3 \\ 0 & 3 & 6 \\ 0 & 0 & 12 \end{bmatrix}$

$$\therefore |3A| = 3\begin{vmatrix} 3 & 6 \\ 0 & 12 \end{vmatrix} - 0\begin{vmatrix} 0 & 3 \\ 0 & 12 \end{vmatrix} + 0\begin{vmatrix} 0 & 3 \\ 3 & 6 \end{vmatrix}$$

$$= 3(36 - 0) = 3(36) = 108 \ldots (ii)$$

From equation (i) and (ii), we have :

$|3A| = 27|A|$

Hence, the given result is proved.

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Q5 Evaluate the determinants

(i) $A = \begin{vmatrix} 3 & -1 & -2 \\ 0 & 0 & -1 \\ 3 & -5 & 0 \end{vmatrix}$

(ii) $\begin{vmatrix} 3 & -4 & 5 \\ 1 & 1 & -2 \\ 2 & 3 & 1 \end{vmatrix}$

(iii) $\begin{vmatrix} 0 & 1 & 2 \\ -1 & 0 & -3 \\ -2 & 3 & 0 \end{vmatrix}$

(iv) $\begin{vmatrix} 2 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{vmatrix}$

Answer.

(i) Let $A = \begin{vmatrix} 3 & -1 & -2 \\ 0 & 0 & -1 \\ 3 & -5 & 0 \end{vmatrix}$

It can be observed that in the second row,two entries are zero.

Thus, we expand along the second row for easier calculation.

$$|A| = -0\begin{vmatrix} -1 & -2 \\ -5 & 0 \end{vmatrix} + 0\begin{vmatrix} 3 & -2 \\ 3 & 0 \end{vmatrix} - (-1)\begin{vmatrix} 3 & -1 \\ 3 & -5 \end{vmatrix} = (-15 + 3) = -12$$

(ii) Let $\begin{vmatrix} 3 & -4 & 5 \\ 1 & 1 & -2 \\ 2 & 3 & 1 \end{vmatrix}$

By expanding along the first row, we have :

$$|A| = 3\begin{vmatrix}1 & -2\\ 3 & 1\end{vmatrix} + 4\begin{vmatrix}1 & -2\\ 2 & 1\end{vmatrix} + 5\begin{vmatrix}1 & 1\\ 2 & 3\end{vmatrix}$$

$= 3(1 + 6) + 4(1 + 4) + 5(3 - 2)$

$= 3(7) + 4(5) + 5(1)$

$= 21 + 20 + 5 = 46$

(iii) Let $\begin{vmatrix}0 & 1 & 2\\ -1 & 0 & -3\\ -2 & 3 & 0\end{vmatrix}$

By expanding along the first column, we have :

$$|A| = 0\begin{vmatrix}0 & -3\\ 3 & 0\end{vmatrix} - 1\begin{vmatrix}-1 & -3\\ -2 & 0\end{vmatrix} + 2\begin{vmatrix}-1 & 0\\ -2 & 3\end{vmatrix}$$

$= -1(-6) + 2(-3)$

$= 6 - 6 = 0$

(iv) Let $\begin{vmatrix}2 & -1 & -2\\ 0 & 2 & -1\\ 3 & -5 & 0\end{vmatrix}$

By expanding along the first row,we have :

$$|A| = 2\begin{vmatrix}2 & -1\\ -5 & 0\end{vmatrix} - 0\begin{vmatrix}-1 & -2\\ -5 & 0\end{vmatrix} + 3\begin{vmatrix}-1 & -2\\ 2 & -1\end{vmatrix}$$

$= 2(0 - 5) - 0 + 3(1 + 4)$

$= -10 + 15 = 5$

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Q6 If $A = \begin{bmatrix}1 & 1 & -2\\ 2 & 1 & -3\\ 5 & 4 & -9\end{bmatrix}$, find $|A|$

Answer. Let $A = \begin{bmatrix}1 & 1 & -2\\ 2 & 1 & -3\\ 5 & 4 & -9\end{bmatrix}$, find $|A|$

By expanding along the first row,we have :

$$|A| = 1\begin{vmatrix}1 & -3\\ 4 & -9\end{vmatrix} - 1\begin{vmatrix}2 & -3\\ 5 & -9\end{vmatrix} - 2\begin{vmatrix}2 & 1\\ 5 & 4\end{vmatrix}$$

$= 1(-9+12) - 1(-18+15) - 2(8-5)$

$= 1(3) - 1(-3) - 2(3)$

$= 3 + 3 - 6$

$= 6 - 6$

$= 0$

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Q7 Find values of x, if :

(i) $\begin{vmatrix}2 & 4\\ 5 & 1\end{vmatrix} = \begin{vmatrix}2x & 4\\ 6 & x\end{vmatrix}$

(ii) $\begin{vmatrix}2 & 3\\ 4 & 5\end{vmatrix} = \begin{vmatrix}x & 3\\ 2x & 5\end{vmatrix}$

Answer.

(i) $\begin{vmatrix}2 & 4\\ 5 & 1\end{vmatrix} = \begin{vmatrix}2x & 4\\ 6 & x\end{vmatrix}$

$\Rightarrow 2 \times 1 - 5 \times 4 = 2x \times x - 6 \times 4$

$\Rightarrow 2 - 20 = 2x^2 - 24$

$\Rightarrow 2x^2 = 6$

$\Rightarrow x^2 = 3$

$\Rightarrow x = \pm\sqrt{3}$

(ii) $\begin{vmatrix}2 & 3\\ 4 & 5\end{vmatrix} = \begin{vmatrix}x & 3\\ 2x & 5\end{vmatrix}$

$\Rightarrow 2 \times 5 - 3 \times 4 = x \times 5 - 3 \times 2x$

$\Rightarrow 10 - 12 = 5x - 6x$

$\Rightarrow -2 = -x$

$\Rightarrow x = 2$

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Q8 If $\begin{vmatrix}x & 2\\ 18 & x\end{vmatrix} = \begin{vmatrix}6 & 2\\ 18 & 6\end{vmatrix}$, then x is equal to

(A) 6 (B) = 6 (C) −6 (D) 0

Answer. $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$

$\Rightarrow x^2 - 36 = 36 - 36$

$\Rightarrow x^2 - 36 = 0$

$\Rightarrow x^2 = 36$

$\Rightarrow x = \pm 6$

Hence, the correct answer is B.

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Q1 Using the property of determinants and without expanding, prove that :

$$\begin{vmatrix} x & a & x+a \\ y & b & y+b \\ z & c & z+c \end{vmatrix} = 0$$

Answer.

$$\begin{vmatrix} x & a & x+a & a & x \\ y & b & y+b & & \\ z & c & z+c & & \end{vmatrix} = \begin{vmatrix} x & a & x \\ y & b & y \\ z & c & z \end{vmatrix} + \begin{vmatrix} x & a & a \\ y & b & b \\ z & c & c \end{vmatrix} = 0 + 0 = 0$$

[Here, the two columns of the determinants are identical]

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Q2 Using the property of determinants and without expanding,prove that :

$$\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix} = 0$$

Answer.

$$\Delta = \begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$$

Applying $R_1 \rightarrow R_1 + R_2$, we have :

$$\Delta = \begin{vmatrix} a-c & b-a & c-b \\ b-c & c-a & a-b \\ -(a-c) & -(b-a) & -(c-b) \end{vmatrix}$$

$$= -\begin{vmatrix} a-c & b-a & c-b \\ b-c & c-a & a-b \\ a-c & b-a & c-b \end{vmatrix}$$

Here, the two rows R_1 and R_3 are identical.

$\therefore \Delta = 0$

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Q3 Using the property of determinants and without expanding, prove that :

$$\begin{vmatrix} 2 & 7 & 65 \\ 3 & 8 & 75 \\ 5 & 9 & 86 \end{vmatrix} = 0$$

Answer.

$$\begin{vmatrix} 2 & 7 & 65 \\ 3 & 8 & 75 \\ 5 & 9 & 86 \end{vmatrix} = \begin{vmatrix} 2 & 7 & 63+2 \\ 3 & 8 & 72+3 \\ 5 & 9 & 81+5 \end{vmatrix}$$

$$= \begin{vmatrix} 2 & 7 & 63 \\ 3 & 8 & 72 \\ 5 & 9 & 81 \end{vmatrix} + \begin{vmatrix} 2 & 7 & 2 \\ 3 & 8 & 3 \\ 5 & 9 & 5 \end{vmatrix}$$

$$= \begin{vmatrix} 2 & 7 & 9(7) \\ 3 & 8 & 9(8) \\ 5 & 9 & 9(9) \end{vmatrix} + 0 \text{.....[Two column are identical]}$$

$$= 9\begin{vmatrix} 2 & 7 & 7 \\ 3 & 8 & 8 \\ 5 & 9 & 9 \end{vmatrix} \text{.....[Two column are identical]}$$

$= 0$

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Q4 Using the property of determinants and without expanding, prove that :

$$\begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix} = 0$$

Answer.

$$\Delta = \begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix}$$

By applying $C_3 \rightarrow C_3 + C_{2,}$, we have :

$$\Delta = \begin{vmatrix} 1 & bc & ab+bc+ca \\ 1 & ca & ab+bc+ca \\ 1 & ab & ab+bc+ca \end{vmatrix}$$

Here,two columns C1 and C3 are proportional.

$\therefore \Delta = 0$

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Q5 Using the property of determinant and without expanding, prove that :

$$\begin{vmatrix} b+c & q+r & y+z \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix} = 2\begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix}$$

Answer.

$$\Delta = \begin{vmatrix} b+c & q+r & y+z \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix}$$

$$= \begin{vmatrix} b+c & q+r & y+z \\ c+a & r+p & z+x \\ a & p & x \end{vmatrix} + \begin{vmatrix} b+c & q+r & y+z \\ c+a & r+p & z+x \\ b & q & y \end{vmatrix}$$

$= \Delta_1 + \Delta_2$(say) ...(i)

Now, $\Delta_1 = \begin{vmatrix} b+c & q+r & y+z \\ c+a & r+p & z+x \\ a & p & x \end{vmatrix}$

Applying $R_2 \rightarrow R_2 - R_3$, we have :

$$\Delta_1 = \begin{vmatrix} b+c & q+r & y+z \\ c & r & z \\ a & p & x \end{vmatrix}$$

Applying $R_1 \rightarrow R_1 - R_2$, we have

$$\Delta_1 = \begin{vmatrix} b & q & y \\ c & r & z \\ a & p & x \end{vmatrix}$$

Applying $R_1 \rightarrow R_3$ and $R_2 \leftrightarrow R_3$,we have

$$\Delta_1 = (-1)^2 \begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix} = \begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix} \quad \ldots (ii)$$

$$\Delta_2 = \begin{vmatrix} b+c & q+r & y+z \\ c+a & r+p & z+x \\ b & q & y \end{vmatrix}$$

Applying $R_2 \rightarrow R_2 - R_1$, we have

$$\Delta_2 = \begin{vmatrix} c & r & z \\ c+a & r+p & z+x \\ b & q & y \end{vmatrix}$$

Applying $R_1 \rightarrow R_1 - R_3$, we have :

$$\Delta_2 = \begin{vmatrix} c & r & z \\ a & p & x \\ b & q & y \end{vmatrix}$$

Applying $R_1 \mapsto R_2$ and $R_2 \leftrightarrow R_3$, we have

$$\Delta_2 = (-1)^2 \begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix} = \begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix} \quad \ldots (iii)$$

From, (i),(ii) and (iii), we have :

$$\Delta = 2 \begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix}$$

Hance, the give result is proved.

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Q6 By using properties of determinants, show that :

$$\begin{vmatrix} 0 & a & -b \\ -a & 0 & -c \\ b & c & 0 \end{vmatrix} = 0$$

Answer. We have

$$\Delta = \begin{vmatrix} 0 & a & -b \\ -a & 0 & -c \\ b & c & 0 \end{vmatrix}$$

Applying $R_1 \rightarrow cR_1$, we have :

$$\Delta = \frac{1}{c}\begin{vmatrix} 0 & ac & -bc \\ -a & 0 & -c \\ b & c & 0 \end{vmatrix}$$

Applying $R_1 \rightarrow R_1 - bR_2$, we have:

$$\Delta = \frac{1}{c}\begin{vmatrix} ab & ac & 0 \\ -a & 0 & -c \\ b & c & 0 \end{vmatrix}$$

$$= \frac{a}{c}\begin{vmatrix} b & c & 0 \\ -a & 0 & -c \\ b & c & 0 \end{vmatrix}$$

Here, the two rows R1 and R3 are identical.
$\therefore \Delta = 0$

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Q7 By using properties of determinants, show that :

$$\begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ca & cb & -c^2 \end{vmatrix} = 4a^2b^2c^2$$

Answer.

$$\Delta = \begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ca & cb & -c^2 \end{vmatrix}$$

$$= abc\begin{vmatrix} -a & b & c \\ a & -b & c \\ a & b & -c \end{vmatrix}$$ .....[Taking out factors a,b,c, from R1,R2andR3]

$$= a^2b^2c^2\begin{vmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix}$$ .....[Taking out factors a,b,c from C1,C2and C3]

Applying $R_2 \rightarrow R_2 + R_1$ and $R_a \rightarrow R_a + R_1$, we have :

$$\Delta = a^2b^2c^2\begin{vmatrix} -1 & 1 & 1 \\ 0 & 0 & 2 \\ 0 & 2 & 0 \end{vmatrix}$$

$$= a^2b^2c^2(-1)\begin{vmatrix} 0 & 2 \\ 2 & 0 \end{vmatrix}$$

$$= -a^2b^2c^2(0-4) = 4a^2b^2c^2$$

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Q8 (i) By using properties of determinants, show that :

$$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$$

(ii) By using properties of determinants, show that :

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$$

Answer. (i) Let :

$$\Delta = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

Applying $R_1 \rightarrow R_1 - R_3$ and $R_2 \rightarrow R_2 - R_3$, we have :

$$\Delta = \begin{vmatrix} 0 & a-c & a^2-c^2 \\ 0 & b-c & b^2-c^2 \\ 1 & c & c^2 \end{vmatrix}$$

$$= (c-a)(b-c)\begin{vmatrix} 0 & -1 & -a-c \\ 0 & 1 & b+c \\ 1 & c & c^2 \end{vmatrix}$$

Applying $R_1 \rightarrow R_1 + R_2$, we have

$$\Delta = (b-c)(c-a)\begin{vmatrix} 0 & 0 & -a+b \\ 0 & 1 & b+c \\ 1 & c & c^2 \end{vmatrix}$$

$$= (a-b)(b-c)(c-a)\begin{vmatrix} 0 & 0 & -1 \\ 0 & 1 & b+c \\ 1 & c & c^2 \end{vmatrix}$$

Expanding along C1 , we have :

$$\Delta = (a-b)(b-c)(c-a)\begin{vmatrix} 0 & -1 \\ 1 & b+c \end{vmatrix} = (a-b)(b-c)(c-a)$$

Hence, the given result is proved.

(ii) Let :

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix}$$

Applying $C_1 \rightarrow C_1 - C_3$ and $C_2 \rightarrow C_2 - C_3$

$$\Delta = \begin{vmatrix} 0 & 0 & 1 \\ a-c & b-c & c \\ a^3-c^3 & b^3-c^3 & c^3 \end{vmatrix}$$

$$=(c-a)(b-c)\begin{vmatrix} 0 & 0 & 1 \\ -1 & 1 & c \\ -\left(a^2+ac+c^2\right) & \left(b^2+bc+c^2\right) & c^3 \end{vmatrix}$$

Applying $C_1 \rightarrow C_1 + C_2$, we have

$$\Delta=(c-a)(b-c)\begin{vmatrix}0 & 0 & 1\\ 0 & 1 & c\\ \left(b^2-a^2\right)+(bc-ac) & \left(b^2+bc+c^2\right) & c^3\end{vmatrix}$$

$$=\begin{vmatrix}0 & 0 & 1\\ a-c & b-c & c\\ (a-c)\left(a^2+ac+c^2\right) & (b-c)\left(b^2+bc+c^2\right) & c^3\end{vmatrix}$$

$$=(b-c)(c-a)(a-b)\begin{vmatrix}0 & 0 & 1\\ 0 & 1 & c\\ -(a+b+c) & \left(b^2+bc+c^2\right) & c^3\end{vmatrix}$$

$$=(a-b)(b-c)(c-a)(a+b+c)\begin{vmatrix}0 & 0 & 1\\ 0 & 1 & c\\ -1 & \left(b^2+bc+c^2\right) & c^3\end{vmatrix}$$

Expanding along C1, we have :

$$\Delta=(a-b)(b-c)(c-a)(a+b+c)(-1)\begin{vmatrix}0 & 1\\ 1 & c\end{vmatrix}$$

$$=(a-b)(b-c)(c-a)(a+b+c)$$

Hence, the given result is proved.

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Q9 By using properties of determinants,show that:

$$\begin{vmatrix}x & x^2 & yz\\ y & y^2 & zx\\ z & z^2 & xy\end{vmatrix}=(x-y)(y-z)(z-x)(xy+yz+zx)$$

Answer. Let :

$$\Delta=\begin{vmatrix}x & x^2 & yz\\ y & y^2 & zx\\ z & z^2 & xy\end{vmatrix}$$

Applying $R_2 \rightarrow R_2 - R_1$ and $R_3 \rightarrow R_3 - R_1$,we have:

$$\Delta = \begin{vmatrix} x & x^2 & yz \\ y-x & y^2-x^2 & zx-yz \\ z-x & z^2-x^2 & xy-yz \end{vmatrix}$$

$$= (x-y)(z-x)\begin{vmatrix} x & x^2 & yz \\ -1 & -x-y & z \\ 1 & z+x & -y \end{vmatrix}$$

Applying $R_3 \rightarrow R_3 + R_2$,we have :

$$\Delta = (x-y)(z-x)\begin{vmatrix} x & x^2 & yz \\ -1 & -x-y & z \\ 0 & z-y & z-y \end{vmatrix}$$

$$= (x-y)(z-x)(z-y)\begin{vmatrix} x & x^2 & yz \\ -1 & -x-y & z \\ 0 & 1 & 1 \end{vmatrix}$$

Expanding along R3, we have :

$$\Delta = [(x-y)(z-x)(z-y)]\left[(-1)\begin{vmatrix} x & yz \\ -1 & z \end{vmatrix} + 1\begin{vmatrix} x & x^2 \\ -1 & -x-y \end{vmatrix}\right]$$

$$= (x-y)(z-x)(z-y)\left[(-xz-yz) + \left(-x^2-xy+x^2\right)\right]$$

$$= -(x-y)(z-x)(z-y)(xy+yz+zx)$$

$$= (x-y)(y-z)(z-x)(xy+yz+zx)$$

Hence, the given result is proved.

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Q10 By using properties of determinants, show that :

(i) $\begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} = (5x+4)(4-x)^2$

(ii) $\begin{vmatrix} y+k & y & y \\ y & y+k & y \\ y & y & y+k \end{vmatrix} = k^2(3y+k)$

Answer.

(i) $\Delta = \begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix}$

Applying $R_1 \rightarrow R_1 + R_2 + R_3$,we have :

$$\Delta = \begin{vmatrix} 5x+4 & 5x+4 & 5x+4 \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix}$$

$$= (5x+4)\begin{vmatrix} 1 & 1 & 1 \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix}$$

Applying $C_2 \rightarrow C_2 - C_1, C_3 \rightarrow C_3 - C_1$, we have:

$$\Delta = (5x+4)\begin{vmatrix} 1 & 0 & 0 \\ 2x & -x+4 & 0 \\ 2x & 0 & -x+4 \end{vmatrix}$$

$$= (5x+4)\begin{vmatrix} 1 & 1 & 1 \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix}$$

Applying $C_2 \rightarrow C_2 - C_1, C_3 \rightarrow C_3 - C_1$, we have:

$$\Delta = (5x+4)\begin{vmatrix} 1 & 0 & 0 \\ 2x & -x+4 & 0 \\ 2x & 0 & -x+4 \end{vmatrix}$$

$$= (5x+4)(4-x)(4-x)\begin{vmatrix} 1 & 0 & 0 \\ 2x & 1 & 0 \\ 2x & 0 & 1 \end{vmatrix}$$

Expanding along C3, we have :

$$\Delta = (5x+4)(4-x)^2\begin{vmatrix} 1 & 0 \\ 2x & 1 \end{vmatrix}$$

$$= (5x+4)(4-x)^2$$

Hence, the given result is proved.

(ii) Let :

$$\Delta = \begin{vmatrix} y+k & y & y \\ y & y+k & y \\ y & y & y+k \end{vmatrix}$$

Applying $R_1 \rightarrow R_1 + R_2 + R_3$,we have :

$$\Delta = \begin{vmatrix} 3y+k & 3y+k & 3y+k \\ y & y+k & y \\ y & y & y+k \end{vmatrix}$$

$$= (3y+k)\begin{vmatrix} 1 & 1 & 1 \\ y & y+k & y \\ y & y & y+k \end{vmatrix}$$

Applying $C_2 \rightarrow C_2 - C_1$ and $C_3 \rightarrow C_3 - C_1$, we have:

$$\Delta = (3y+k)\begin{vmatrix} 1 & 0 & 0 \\ y & k & 0 \\ y & 0 & k \end{vmatrix}$$

$$= k^2(3y+k)\begin{vmatrix} 1 & 0 & 0 \\ y & 1 & 0 \\ y & 0 & 1 \end{vmatrix}$$

Expanding along C3 , we have :

$$\Delta = k^2(3y+k)\begin{vmatrix} 1 & 0 \\ y & 1 \end{vmatrix} = k^2(3y+k)$$

Hence, the given result is proved.

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Q11 By using properties of determinants, show that :

(i) $$\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$$

(ii) $$\begin{vmatrix} x+y+2z & x & y \\ z & y+z+2x & y \\ z & x & z+x+2y \end{vmatrix} = 2(x+y+z)^3$$

Answer. (i) Let :

$$\Delta = \begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

Applying $R_1 \rightarrow R_1 + R_2 + R_3$, we have :

$$\Delta = \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

$$= (a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

Applying $C_2 \rightarrow C_2 - C_1, C_3 \rightarrow C_3 - C_1$, we have:

$$\Delta = (a+b+c)\begin{vmatrix} 1 & 0 & 0 \\ 2b & -(a+b+c) & 0 \\ 2c & 0 & -(a+b+c) \end{vmatrix}$$

$$= (a+b+c)^3\begin{vmatrix} 1 & 0 & 0 \\ 2b & -1 & 0 \\ 2c & 0 & -1 \end{vmatrix}$$

Expanding along C3, we have :

$\Delta = (a+b+c)^3(-1)(-1) = (a+b+c)^3$

Hence, the given result is proved.

(ii) $$\Delta = \begin{vmatrix} x+y+2z & x & y \\ z & y+z+2x & y \\ z & x & z+x+2y \end{vmatrix}$$

Applying $C_1 \rightarrow C_1 + C_2 + C_3$, we have :

$$\Delta = \begin{vmatrix} 2(x+y+z) & x & y \\ 2(x+y+z) & y+z+2x & y \\ 2(x+y+z) & x & z+x+2y \end{vmatrix}$$

$$= 2(x+y+z)\begin{vmatrix} 1 & x & y \\ 1 & y+z+2x & y \\ 1 & x & z+x+2y \end{vmatrix}$$

Applying $R_2 \rightarrow R_2 - R_1$ and $R_3 \rightarrow R_3 - R_1$, we have:

$$\Delta = 2(x+y+z)\begin{vmatrix} 1 & x & y \\ 0 & x+y+z & 0 \\ 0 & 0 & x+y+z \end{vmatrix}$$

$$= 2(x+y+z)^3\begin{vmatrix} 1 & x & y \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

Expanding along R3, we have :

$\Delta = 2(x+y+z)^3(1)(1-0) = 2(x+y+z)^3$

Hence, the given result is proved.

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Q12 By Using properties of determinants,show that :

$$\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = \left(1-x^3\right)^2$$

Answer.

$$\Delta = \begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix}$$

Applying $R_1 \rightarrow R_1 + R_2 + R_3$, we have :

$$\Delta = \begin{vmatrix} 1+x+x^2 & 1+x+x^2 & 1+x+x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix}$$

$$= \left(1+x+x^2\right)\begin{vmatrix} 1 & 1 & 1 \\ x^2 & 1 & x \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix}$$

Applying $C_2 \rightarrow C_2 - C_1$ and $C_3 \rightarrow C_3 - C_1$, we have:

$$\Delta = \left(1 + x + x^2\right)\begin{vmatrix} 1 & 0 & 0 \\ x^2 & 1 - x^2 & x - x^2 \\ x & x^2 - x & 1 - x \end{vmatrix}$$

$$= \left(1 + x + x^2\right)(1 - x)(1 - x)\begin{vmatrix} 1 & 0 & 0 \\ x^2 & 1 + x & x \\ x & -x & 1 \end{vmatrix}$$

$$= \left(1 - x^3\right)(1 - x)\begin{vmatrix} 1 & 0 & 0 \\ x^2 & 1 + x & x \\ x & -x & 1 \end{vmatrix}$$

Expanding along R_1, we have:

$$\Delta = \left(1 - x^3\right)(1 - x)(1)\begin{vmatrix} 1 + x & x \\ -x & 1 \end{vmatrix}$$

$$= \left(1 - x^3\right)(1 - x)\left(1 + x + x^2\right)$$

$$= \left(1 - x^3\right)\left(1 - x^3\right)$$

$$= \left(1 - x^3\right)^2$$

Hence, the given result is proved.

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Q13 By using properties of determinants, show that :

$$\begin{vmatrix} 1 + a^2 - b^2 & 2ab & -2b \\ 2ab & 1 - a^2 + b^2 & 2a \\ 2b & -2a & 1 - a^2 - b^2 \end{vmatrix} = \left(1 + a^2 + b^2\right)^3$$

Answer.

$$\Delta = \begin{vmatrix} 1 + a^2 - b^2 & 2ab & -2b \\ 2ab & 1 - a^2 + b^2 & 2a \\ 2b & -2a & 1 - a^2 - b^2 \end{vmatrix}$$

Applying $R_1 \rightarrow R_1 + bR_3$ and $R_2 \rightarrow R_2 - aR_3$,we have :

$$\Delta = \begin{vmatrix} 1+a^2+b^2 & 0 & -b\left(1+a^2+b^2\right) \\ 0 & 1+a^2+b^2 & a\left(1+a^2+b^2\right) \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$$

$$= \left(1+a^2+b^2\right)^2 \begin{vmatrix} 1 & 0 & -b \\ 0 & 1 & a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$$

Expanding along R_1, we have:

$$\Delta = \left(1+a^2+b^2\right)^2 \left[(1)\begin{vmatrix} 1 & a \\ -2a & 1-a^2-b^2 \end{vmatrix} - b\begin{vmatrix} 0 & 1 \\ 2b & -2a \end{vmatrix}\right]$$

$$= \left(1+a^2+b^2\right)^2 \left[1-a^2-b^2+2a^2-b(-2b)\right]$$

$$= \left(1+a^2+b^2\right)^2 \left(1+a^2+b^2\right)$$

$$= \left(1+a^2+b^2\right)^3$$

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Q14 By using properties of determinants,show that :

$$\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = 1+a^2+b^2+c^2$$

Answer.

$$\Delta = \begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix}$$

Taking out common factors a,b and c from R1,R2 and R3 respectively, we have :

$$\Delta = abc \begin{vmatrix} a + \frac{1}{a} & b & c \\ a & b + \frac{1}{b} & c \\ a & b & c + \frac{1}{c} \end{vmatrix}$$

Applying $R_2 \rightarrow R_2 - R_1$ and $R_3 \rightarrow R_3 - R_1$, we have :

$$\Delta = abc \begin{vmatrix} a + \frac{1}{a} & b & c \\ -\frac{1}{a} & \frac{1}{b} & 0 \\ -\frac{1}{a} & 0 & \frac{1}{c} \end{vmatrix}$$

Applying $C_1 \rightarrow ac_1, C_2 \rightarrow bC_2,$ and $C_3 \rightarrow cC_3$,we have :

$$\Delta = abc \times \frac{1}{abc} \begin{vmatrix} a^2 + 1 & b^2 & c^2 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} a^2 + 1 & b^2 & c^2 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix}$$

Expanding along R3, we have :

$$\Delta = -1 \begin{vmatrix} b^2 & c^2 \\ 1 & 0 \end{vmatrix} + 1 \begin{vmatrix} a^2 + 1 & b^2 \\ -1 & 1 \end{vmatrix}$$

$$= -1\left(-c^2\right) + \left(a^2 + 1 + b^2\right) = 1 + a^2 + b^2 + c^2$$

Hence, the given result is proved.

Page : 121 , Block Name : Exercise 4.2

Q15 Let A be a square matrix of order 3×3, then $|kA|$ is equal to
(A) $k|A|$ (B) $k^2|A|$ (C) $k^3|A|$ (D) $3k|A|$

Answer. Answer is C :
A is a Square matrix of order 3×3.

Let $A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$

$A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$

Then, $kA = \begin{bmatrix} ka_1 & kb_1 & kc_1 \\ ka_2 & kb_2 & kc_2 \\ ka_3 & kb_3 & kc_3 \end{bmatrix}$

$\therefore |kA| = \begin{vmatrix} ka_1 & kb_1 & kc_1 \\ ka_2 & kb_2 & kc_2 \\ ka_3 & kb_3 & kc_3 \end{vmatrix}$

$= k^3 \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ .....[Taking out common factors k from each row]

$= k^3|A|$

$\therefore |kA| = k^3|A|$

Hence, the correct answer is C.

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Q16 Which of the following is correct
Determinant is a square matrix.
Determinant is a number associated to a matrix.
Determinant is a number associated to a square matrix.
None of these

Answer. We know that to every square matrix, A=[aij] of order n.
We can associate a number called the determinant of square matrix A,
where aij=(i,j)th element of A.
Hence, the correct answer is C.

Page : 121 , Block Name : Exercise 4.2

Q1 Find area of the triangle with vertices at the point given in each of the following :
(i) (1,0),(6,0),(4,3)
(ii) (2,7,(1,1),(10,8)
(iii) (-2,-3),(3,2),(-1,-8)

Answer. (i) the are of the triangle with vertices (1,0),(6,0),(4,3) is given by the relation,

$$\Delta = \frac{1}{2}\begin{vmatrix} 1 & 0 & 1 \\ 6 & 0 & 1 \\ 4 & 3 & 1 \end{vmatrix}$$

$$= \frac{1}{2}[1(0-3) - 0(6-4) + 1(18-0)]$$

$$= \frac{1}{2}[-3+18] = \frac{15}{2} \text{ square units}$$

(ii) The area of the triangle with vertices (2,7),(1,1),(10,8) is given by the relation,

$$\Delta = \frac{1}{2}\begin{vmatrix} 2 & 7 & 1 \\ 1 & 1 & 1 \\ 10 & 8 & 1 \end{vmatrix}$$

$$= \frac{1}{2}[2(1-8) - 7(1-10) + 1(8-10)]$$

$$= \frac{1}{2}[-14+63-2] = \frac{1}{2}[-16+63]$$

$$= \frac{47}{2} \text{ square units}$$

(iii) The area of the triangle with vertices (-2,-3),(3,2),(-1,-8) is given by the relation,

$$\Delta = \frac{1}{2}\begin{vmatrix} -2 & -3 & 1 \\ 3 & 2 & 1 \\ -1 & -8 & 1 \end{vmatrix}$$

$$= \frac{1}{2}[-2(2+8) + 3(3+1) + 1(-24+2)]$$

$$= \frac{1}{2}[-2(10) + 3(4) + 1(-22)]$$

$$= \frac{1}{2}[-20+12-22]$$

$$= -\frac{30}{2} = -15$$

Hence, the area of triangle is $|-15| = 15$ square units.

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Q2 Show that points
A(a, $b + c$), B(b, $c + a$), C(c, $a + b$) are collinear.

Answer. Area of Δ ABC is given by the relation,

$$\Delta = \frac{1}{2}\begin{vmatrix} a & b+c & 1 \\ b & c+a & 1 \\ c & a+b & 1 \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} a & b+c & 1 \\ b-a & a-b & 0 \\ c-a & a-c & 0 \end{vmatrix} \text{ (Applying } R_2 \rightarrow R_2 - R_1 \text{ and } R_3 \rightarrow R_3 - R_1\text{)}$$

$$= \frac{1}{2}(a-b)(c-a)\begin{vmatrix} a & b+c & 1 \\ -1 & 1 & 0 \\ 1 & -1 & 0 \end{vmatrix}$$

$$= \frac{1}{2}(a-b)(c-a)\begin{vmatrix} a & b+c & 1 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{vmatrix} \left(\text{ Applying } R_3 \rightarrow R_3 + R_2\right)$$

= 0 (All elements of R3 are 0)
Thus, the area of the triangle formed by point A,B, and c is zero.
Hence, the points A, B, and C are collinear.

Page : 123 , Block Name : Exercise 4.3

Q3 Find values of k if area of triangle is 4sq. units and vertices are
(i) (k, 0),(4,0),(0,2)
(ii) (-2,0),(0,4),(0,k)

Answer. We know that the area of a triangle whose vertices are (x1, y1), (x2, y2), and

(x3, y3) is the absolute value of the determinant (Δ), where

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

It is given that the area of triangle is 4 square units.
$\therefore \Delta = \pm 4$
(i) The area of the triangle with vertices (k, 0), (4, 0), (0, 2) is given by the relation,

$$\Delta = \frac{1}{2}\begin{vmatrix} k & 0 & 1 \\ 4 & 0 & 1 \\ 0 & 2 & 1 \end{vmatrix}$$

$= \frac{1}{2}[k(0-2) - 0(4-0) + 1(8-0)]$

$= \frac{1}{2}[-2k + 8] = -k + 4$

$\therefore -k + 4 = \pm 4$

When $-k + 4 = -4$, $k = 8$

When $-k + 4 = 4$, $k = 0$

Hence, $k = 0, 8$

(ii) The area of the triangle with vertices (−2, 0), (0, 4), (0, k) is given by the relation,

$$\Delta = \frac{1}{2}\begin{vmatrix} -2 & 0 & 1 \\ 0 & 4 & 1 \\ 0 & k & 1 \end{vmatrix}$$

$= \frac{1}{2}[-2(4-k)]$

$= k - 4$

$\therefore k - 4 = \pm 4$

When $k - 4 = -4$, $k = 0$

When $k - 4 = 4$, $k = 8$

Hence, $k = 0, 8$

Page : 123 , Block Name : Exercise 4.3

Q4 (i) Find equation of line joining (1, 2) and (3, 6) using determinants

(ii) Find equation of line joining (3, 1) and (9, 3) using determinants

Answer. (i) Let P (x, y) be any point on the line joining points A (1, 2) and B (3, 6).
Then, the points A, B, and P are collinear. Therefore, the area of triangle ABP will be zero.

$$\therefore \frac{1}{2}\begin{vmatrix} 1 & 2 & 1 \\ 3 & 6 & 1 \\ x & y & 1 \end{vmatrix} = 0$$

$\Rightarrow \frac{1}{2}[1(6-y) - 2(3-x) + 1(3y - 6x)] = 0$

$\Rightarrow 6 - y - 6 + 2x + 3y - 6x = 0$

$\Rightarrow 2y - 4x = 0$

$\Rightarrow y = 2x$

Hence, the equation of the line joining the given points is y = 2x.

(ii) Let P (x, y) be any point on the line joining points A (3, 1) and
B (9, 3). Then, the points A, B, and P are collinear. Therefore, the area of triangle ABP will be zero.

$$\therefore \frac{1}{2}\begin{vmatrix} 3 & 1 & 1 \\ 9 & 3 & 1 \\ x & y & 1 \end{vmatrix} = 0$$

$$\Rightarrow \frac{1}{2}[3(3-y) - 1(9-x) + 1(9y - 3x)] = 0$$

$$\Rightarrow 9 - 3y - 9 + x + 9y - 3x = 0$$

$$\Rightarrow 6y - 2x = 0$$

$$\Rightarrow x - 3y = 0$$

Hence, the equation of the line joining the given points is x − 3y = 0

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Q5 If area of triangle is 35 sq units with vertices (2, – 6), (5, 4) and (k, 4). Then k is
(A) 12 (B) −2 (C) −12, −2 (D) 12, −2

Answer. (D),
The area of the triangle with vertices (2, −6), (5, 4), and (k, 4) is given by the relation,

$$\Delta = \frac{1}{2}\begin{vmatrix} 2 & -6 & 1 \\ 5 & 4 & 1 \\ k & 4 & 1 \end{vmatrix}$$

$$= \frac{1}{2}[2(4-4) + 6(5-k) + 1(20-4k)]$$

$$= \frac{1}{2}[30 - 6k + 20 - 4k]$$

$$= \frac{1}{2}[50 - 10k]$$

$$= 25 - 5k$$

It is given that the area of the triangle is ±35.
Therefore, we have:

$$\Rightarrow 25 - 5k = \pm 35$$

$$\Rightarrow 5(5-k) = \pm 35$$

$$\Rightarrow 5 - k = \pm 7$$

When $5 - k = -7$, $k = 5 + 7 = 12$

When $5 - k = 7$, $k = 5 - 7 = -2$

Hence, $k = 12, -2$

So The correct answer is D.

Page : 123 , Block Name : Exercise 4.3

Q1 Write Minors and Cofactors of the elements of following determinants:

(*i*) $\begin{vmatrix} 2 & -4 \\ 0 & 3 \end{vmatrix}$ (ii) $\begin{vmatrix} a & c \\ b & d \end{vmatrix}$

Answer.(i)The given determinant is $\begin{vmatrix} 2 & -4 \\ 0 & 3 \end{vmatrix}$

Minor of element aij is Mij.

∴M11 = minor of element a11 = 3

M12 = minor of element a12 = 0

M21 = minor of element a21 = −4

M22 = minor of element a22 = 2

Cofactor of aij is Aij = (−1)i+j ij.

$\therefore A_{11} = (-1)^{1+1}M_{11} = (-1)^2(3) = 3$

$A_{12} = (-1)^{1+2}M_{12} = (-1)^3(0) = 0$

$A_{21} = (-1)^{2+1}M_{21} = (-1)^3(-4) = 4$

$A_{22} = (-1)^{2+2}M_{22} = (-1)^4(2) = 2$

(ii) The given determinant is $\begin{vmatrix} a & c \\ b & d \end{vmatrix}$.

Minor of element a_{ij} is M_{ij}

$\therefore M_{11}$ = minor of element $a_{11} = d$

M_{12} = minor of element $a_{12} = b$

M_{21} = minor of element $a_{21} = c$

M_{22} = minor of element $a_{22} = a$

cofactor of a_{ij} is $A_{ij} = (-1)^{i+j}M_{ij}$

$\therefore A_{11} = (-1)^{1+1}M_{11} = (-1)^2(d) = d$

$A_{12} = (-1)^{1+2}M_{12} = (-1)^3(b) = -b$

$A_{21} = (-1)^{2+1}M_{21} = (-1)^3(c) = -b$

$A_{22} = (-1)^{2+2}M_{22} = (-1)^4(a) = a$

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Q2 (*i*) $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$ (ii) $\begin{vmatrix} 1 & 0 & 4 \\ 3 & 5 & -1 \\ 0 & 1 & 2 \end{vmatrix}$

Answer. (i) The given determinant is $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$.

By the definition of minors and cofactors, we have:

$M_{11} =$ minor of $a_{11} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$

$M_{12} =$ minor of $a_{13} = \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} = 0$

$M_{13} =$ minor of $a_{13} = \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} = 0$

$\mathrm{M}_{21} =$ minor of $a_{21} = \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} = 0$

$\mathrm{M}_{22} =$ minor of $a_{22} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$

$\mathrm{M}_{23} =$ minor of $a_{23} = \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} = 0$

$\mathrm{M}_{31} =$ minor of $a_{31} = \begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix} = 0$

$\mathrm{M}_{32} =$ minor of $a_{32} = \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} = 0$

$M_{33} =$ minor of $a_{33} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$

$A_{11} =$ cofactor of $a_{11} = (-1)^{1+1} M_{11} = 1$

$A_{12} =$ cofactor of $a_{12} = (-1)^{1+2} M_{12} = 0$

$A_{13} =$ cofactor of $a_{13} = (-1)^{1+3} M_{13} = 0$

$\mathrm{A}_{21} =$ cofactor of $a_{21} = (-1)^{2+1} \mathrm{M}_{21} = 0$

$\mathrm{A}_{22} =$ cofactor of $a_{22} = (-1)^{2+2} \mathrm{M}_{22} = 1$

$\mathrm{A}_{23} =$ cofactor of $a_{23} = (-1)^{2+3} \mathrm{M}_{23} = 0$

$A_{31} =$ cofactor of $a_{31} = (-1)^{3+1} M_{31} = 0$

$A_{32} =$ cofactor of $a_{32} = (-1)^{3+2} M_{32} = 0$

$A_{33} =$ cofactor of $a_{33} = (-1)^{3+3} M_{33} = 1$

(ii) The given determinant is $\begin{vmatrix} 1 & 0 & 4 \\ 3 & 5 & -1 \\ 0 & 1 & 2 \end{vmatrix}$

By definition of minors and cofactors, we have:

$M_{11} =$ minor of $a_{11} = \begin{vmatrix} 5 & -1 \\ 1 & 2 \end{vmatrix} = 10 + 1 = 11$

$M_{12} =$ minor of $a_{12} = \begin{vmatrix} 3 & -1 \\ 0 & 2 \end{vmatrix} = 6 - 0 = 6$

$M_{13} =$ minor of $a_{13} = \begin{vmatrix} 3 & 5 \\ 0 & 1 \end{vmatrix} = 3 - 0 = 3$

$M_{21} =$ minor of $a_{21} = \begin{vmatrix} 0 & 4 \\ 1 & 2 \end{vmatrix} = 0 - 4 = -4$

$M_{22} =$ minor of $a_{22} = \begin{vmatrix} 1 & 4 \\ 0 & 2 \end{vmatrix} = 2 - 0 = 2$

$M_{23} =$ minor of $a_{23} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 - 0 = 1$

$M_{31} =$ minor of $a_{31} = \begin{vmatrix} 0 & 4 \\ 5 & -1 \end{vmatrix} = 0 - 20 = -20$

$M_{32} =$ minor of $a_{32} = \begin{vmatrix} 1 & 4 \\ 3 & -1 \end{vmatrix} = -1 - 12 = -13$

$M_{33} =$ minor of $a_{33} = \begin{vmatrix} 1 & 0 \\ 3 & 5 \end{vmatrix} = 5 - 0 = 5$

$A_{11} =$ cofactor of $a_{11} = (-1)^{1+1} M_{11} = 11$

$A_{12} =$ cofactor of $a_{12} = (-1)^{1+2} M_{12} = -6$

$A_{13} =$ cofactor of $a_{13} = (-1)^{1+3} M_{13} = 3$

$A_{21} =$ cofactor of $a_{21} = (-1)^{2+1} M_{21} = 4$

$A_{22} =$ cofactor of $a_{22} = (-1)^{2+2} M_{22} = 2$

$A_{23} =$ cofactor of $a_{23} = (-1)^{2+3} M_{23} = -1$

A_{31} = cofactor of $a_{31} = (-1)^{3+1}M_{31} = -20$

A_{32} = cofactor of $a_{32} = (-1)^{3+2}M_{32} = 13$

A_{33} = cofactor of $a_{33} = (-1)^{3+3}M_{33} = 5$

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Q3 Using Cofactors of elements of second row, evaluate $\Delta = \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$

Answer. The given determinant is $\begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$

We have:

$\begin{vmatrix} 3 & 8 \\ 2 & 3 \end{vmatrix} = 9 - 16 = -7$

$\therefore A_{21}$ = cofactor of $a_{21} = (-1)^{2+1}M_{21} = 7$

$M_{23} = \begin{vmatrix} 5 & 3 \\ 1 & 2 \end{vmatrix} = 10 - 3 = 7$

We know that Δ is equal to the sum of the product of the elements of the second row with their corresponding cofactors.

$\therefore \Delta = a_{21}A_{21} + a_{22}A_{22} + a_{23}A_{23} = 2(7) + 0(7) + 1(-7) = 14 - 7 = 7$

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Q4 Using Cofactors of elements of third column, evaluate

$\Delta = \begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix}$

Answer. The given determinant is $\begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix}$

We have:

$M_{13} = \begin{vmatrix} 1 & y \\ 1 & z \end{vmatrix} = z - y$

$M_{23} = \begin{vmatrix} 1 & x \\ 1 & z \end{vmatrix} = z - x$

$M_{33} = \begin{vmatrix} 1 & x \\ 1 & y \end{vmatrix} = y - x$

$\therefore A_{13} =$ cofactor of $a_{13} = (-1)^{1+3} M_{13} = (z - y)$

$A_{23} =$ cofactor of $a_{23} = (-1)^{2+3} M_{23} = -(z - x) = (x - z)$

$A_{33} =$ cofactor of $a_{33} = (-1)^{3+3} M_{33} = (y - x)$

We know that Δ is equal to the sum of the product of the elements of the second row with their corresponding cofactors.

$\therefore \Delta = a_{13}A_{13} + a_{23}A_{23} + a_{33}A_{33}$

$= yz(z - y) + zx(x - z) + xy(y - x)$

$= yz^2 - y^2z + x^2z - xz^2 + xy^2 - x^2y$

$= \left(x^2z - y^2z\right) + \left(yz^2 - xz^2\right) + \left(xy^2 - x^2y\right)$

$= z\left(x^2 - y\right) + z^2(y - x) + xy(y - x)$

$= z(x - y)(x + y) + z^2(y - x) + xy(y - x)$

$= (x - y)\left(z(x + z) + z^2 - xy\right]$

$= (x - y)(z(x - z) + y(z - x)]$

$= (x - y)(z - x)[-z + y]$

$= (x - y)(y - z)(z - x)$

Hence, $\Delta = (x - y)(y - z)(z - x)$.

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Q5 If $\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ and A_{ij} is Cofactors of a_{ij} , then value of Δ

(A) $a_{11}A_{31} + a_{12}A_{32} + a_{13}A_{33}$ (B) $a_{11}A_{11} + a_{12}A_{21} + a_{13}A_{31}$

(C) $a_{21}A_{11} + a_{22}A_{12} + a_{23}A_{13}$ (D) $a_{11}A_{11} + a_{21}A_{21} + a_{31}A_{31}$

Answer.

Answer is : D

We know that:

Δ = Sum of the product of the elements of a column (or a row) with their corresponding cofactors

$\therefore \Delta = a_{11}A_{11} + a_{21}A_{21} + a_{31}A_{31}$

Hence, the value of Δ is given by the expression given in alternative D.

The correct answer is D.

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Q1 Find adjoint of the matrix.

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

Answer.

Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

We have,

$A_{11} = 4, A_{12} = -3, A_{21} = -2, A_{22} = 1$

$$\therefore adjA = \begin{bmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{bmatrix} = \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$$

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Q2 Find adjoint of the matrix.

$$\begin{bmatrix} 1 & -1 & 2 \\ 2 & 3 & 5 \\ -2 & 0 & 1 \end{bmatrix}$$

Answer. $A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 3 & 5 \\ -2 & 0 & 1 \end{bmatrix}$

$$A_{11} = \begin{vmatrix} 3 & 5 \\ 0 & 1 \end{vmatrix} = 3 - 0 = 3$$

We have, $A_{12} = -\begin{vmatrix} 2 & 5 \\ -2 & 1 \end{vmatrix} = -(2 + 10) = -12$

$$A_{13} = \begin{vmatrix} 2 & 3 \\ -2 & 0 \end{vmatrix} = 0 + 6 = 6$$

$A_{21} = -\begin{vmatrix} -1 & 2 \\ 0 & 1 \end{vmatrix} = -(-1-0) = 1$

$A_{22} = \begin{vmatrix} 1 & 2 \\ -2 & 1 \end{vmatrix} = 1 + 4 = 5$

$A_{23} = -\begin{vmatrix} 1 & -1 \\ -2 & 0 \end{vmatrix} = -(0-2) = 2$

$A_{31} = \begin{vmatrix} -1 & 2 \\ 3 & 5 \end{vmatrix} = -5-6 = -11$

$A_{32} = -\begin{vmatrix} 1 & 2 \\ 2 & 5 \end{vmatrix} = -(5-4) = -1$

$A_{33} = \begin{vmatrix} 1 & -1 \\ 2 & 3 \end{vmatrix} = 3 + 2 = 5$

Hence, $adjA = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix} = \begin{bmatrix} 3 & 1 & -11 \\ -12 & 5 & -1 \\ 6 & 2 & 5 \end{bmatrix}$

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Q3 $A(adjA) = (adjA)A = |A|I$

$\begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix}$

Answer. $A = \begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix}$ we have,

$|A| = -12-(-12) = -12+12 = 0$

$\therefore |A|I = 0\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

now, $A_{11} = -6, A_{12} = 4, A_{21} = -3, A_{22} = 2$

$\therefore adjA = \begin{bmatrix} -6 & -3 \\ 4 & 2 \end{bmatrix}$

$$A(adjA) = \begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix}\begin{bmatrix} -6 & -3 \\ 4 & 2 \end{bmatrix}$$

Now,

$$= \begin{bmatrix} -12+12 & -6+6 \\ 24-24 & 12-12 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$(adjA)A = \begin{bmatrix} -6 & -3 \\ 4 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix}$$

Also,

$$= \begin{bmatrix} -12+12 & -18+18 \\ 8-8 & 12-12 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Hence, $A(adjA) = (adjA)A = |A|I$

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Q4 $A(adjA) = (adjA)A = |A|I$

$$\begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$$

Answer. $A=\begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$

$|A| = 1(0-0) + 1(9+2) + 2(0-0) = 11$

$$\therefore |A|I = 11\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$

Now,

$A_{11} = 0, A_{12} = -(9+2) = -11, A_{13} = 0$

$A_{21} = -(-3-0) = 3, A_{22} = 3-2 = 1, A_{23} = -(0+1) = -1$

$A_{31} = 2-0 = 2, A_{32} = -(-2-6) = 8, A_{33} = 0+3 = 3$

$$\therefore adjA = \begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix}$$

$$A(adjA) = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}\begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 0+9+2 & 0+0+0 & 0-6+6 \\ -11+3+8 & 11+0+0 & -22-2+24 \\ 0-3+3 & 0+0+0 & 0+2+9 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$

Hence, $A(adjA) = (adjA)A = |A|I$

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Q5 Find the inverse of the matrix

$$\begin{bmatrix} 2 & -2 \\ 4 & 3 \end{bmatrix}$$

Answer. Let, A= $\begin{bmatrix} 2 & -2 \\ 4 & 3 \end{bmatrix}$

We have,

$|A| = -2 + 15 = 13$

Now,

$A_{11} = 2, A_{12} = 3, A_{21} = -5, A_{22} = -1$

$\therefore adjA = \begin{bmatrix} 2 & -5 \\ 3 & -1 \end{bmatrix}$

$\therefore A^{-1} = \frac{1}{|A|} adjA = \frac{1}{13} \begin{bmatrix} 2 & -5 \\ 3 & -1 \end{bmatrix}$

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Q6 Find the inverse of the matrix

$$\begin{bmatrix} -1 & 5 \\ -3 & 2 \end{bmatrix}$$

Answer. Let A= $\begin{bmatrix} -1 & 5 \\ -3 & 2 \end{bmatrix}$

We have,

$|A| = -2 + 15 = 13$

Now

$A_{11} = 2, A_{12} = 3, A_{21} = -5, A_{22} = -1$

$$\therefore adjA = \begin{bmatrix} 2 & -5 \\ 3 & -1 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|} adjA = \frac{1}{13}\begin{bmatrix} 2 & -5 \\ 3 & -1 \end{bmatrix}$$

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Q7 Find the inverse of the matrix

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 5 \end{bmatrix}$$

Answer. Let A= $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 5 \end{bmatrix}$ we have,

$|A| = 1(10 - 0) - 2(0 - 0) + 3(0 - 0) = 10$

Now,

$A_{11} = 10 - 0 = 10, A_{12} = -(0 - 0) = 0, A_{13} = 0 - 0 = 0$

$A_{21} = -(10 - 0) = -10, A_{22} = 5 - 0 = 5, A_{23} = -(0 - 0) = 0$

$A_{31} = 8 - 6 = 2, A_{12} = -(4 - 0) = -4, A_{33} = 2 - 0 = 2$

$$\therefore adjA = \begin{bmatrix} 10 & -10 & 2 \\ 0 & 5 & -4 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|} adjA = \frac{1}{10}\begin{bmatrix} 10 & -10 & 2 \\ 0 & 5 & -4 \\ 0 & 0 & 2 \end{bmatrix}$$

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Q8 Find the inverse of the matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 3 & 3 & 0 \\ 5 & 2 & -1 \end{bmatrix}$$

Answer. Let A= $\begin{bmatrix} 1 & 0 & 0 \\ 3 & 3 & 0 \\ 5 & 2 & -1 \end{bmatrix}$

We have,

$|A| = 1(-3-0) - 0 + 0 = -3$

Now,

$A_{11} = -3-0 = -3, A_{12} = -(-3-0) = 3, A_{13} = 6-15 = -9$

$A_{21} = -(0-0) = 0, A_{22} = -1-0 = -1, A_{23} = -(2-0) = -2$

$A_{31} = 0-0 = 0, A_{32} = -(0-0) = 0, A_{33} = 3-0 = 3$

$\therefore adjA = \begin{bmatrix} -3 & 0 & 0 \\ 3 & -1 & 0 \\ -9 & -2 & 3 \end{bmatrix}$

$\therefore A^{-1} = \frac{1}{|A|} \text{adj } A = -\frac{1}{3}\begin{bmatrix} -3 & 0 & 0 \\ 3 & -1 & 0 \\ -9 & -2 & 3 \end{bmatrix}$

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Q9 Find the inverse of the matrix

$\begin{bmatrix} 2 & 1 & 3 \\ 4 & -1 & 0 \\ -7 & 2 & 1 \end{bmatrix}$

Answer. Let A= $\begin{bmatrix} 2 & 1 & 3 \\ 4 & -1 & 0 \\ -7 & 2 & 1 \end{bmatrix}$

We have,

$|A| = 2(-1-0) - 1(4-0) + 3(8-7)$

$= 2(-1) - 1(4) + 3(1)$

$= -2-4+3$

$= -3$

now, $A_{11} = -1-0 = -1, A_{12} = -(4-0) = -4, A_{13} = 8-7 = 1$

$A_{21} = -(1-6) = 5, A_{22} = 2+21 = 23, A_{13} = -(4+7) = -11$

$A_{31} = 0+3 = 3, A_{12} = -(0-12) = 12, A_{33} = -2-4 = -6$

$$\therefore adjA = \begin{bmatrix} -1 & 5 & 3 \\ -4 & 23 & 12 \\ 1 & -11 & -6 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|} \text{ adj } A = -\frac{1}{3}\begin{bmatrix} -1 & 5 & 3 \\ -4 & 23 & 12 \\ 1 & -11 & -6 \end{bmatrix}$$

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Q10 Find the inverse of the matrix $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$

Answer. Let A= $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$

By expanding along C1, we have :

$|A| = 1(8-6) - 0 + 3(3-4) = 2 - 3 = -1$

Now,

$A_{11} = 8 - 6 = 2, A_{12} = -(0+9) = -9, A_3 = 0 - 6 = -6$

$A_{21} = -(-4+4) = 0, A_{22} = 4 - 6 = -2, A_{23} = -(-2+3) = -1$

$A_{31} = 3 - 4 = -1, A_{12} = -(-3-0) = 3, A_{33} = 2 - 0 = 2$

$$\therefore adjA = \begin{bmatrix} 2 & 0 & -1 \\ -9 & -2 & 3 \\ -6 & -1 & 2 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|}adjA = -\begin{bmatrix} 2 & 0 & -1 \\ -9 & -2 & 3 \\ -6 & -1 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 0 & -3 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$$

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Q11 Find the inverse of the matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & \sin\alpha & -\cos\alpha \end{bmatrix}$$

Answer. Let A= $\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & \sin\alpha & -\cos\alpha \end{bmatrix}$

We have,

$|A| = 1\left(-\cos^2\alpha - \sin^2\alpha\right) = -\left(\cos^2\alpha + \sin^2\alpha\right) = -1$

Now,

$A_{11} = -\cos^2\alpha - \sin^2\alpha = -1, A_{12} = 0, A_{13} = 0$

$A_{21} = 0, A_{22} = -\cos\alpha, A_{23} = -\sin\alpha$

$A_{31} = 0, A_{32} = -\sin\alpha, A_{33} = \cos\alpha$

$\therefore adjA = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -\cos\alpha & -\sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix}$

$\therefore A^{-1} = \frac{1}{|A|} \cdot adjA = -\begin{bmatrix} -1 & 0 & 0 \\ 0 & -\cos\alpha & -\sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & \sin\alpha & -\cos\alpha \end{bmatrix}$

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Q12 Let $A = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix} 6_{\text{and}} = \begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix}$. Verify that $(AB)^{-1} = B^{-1}A^{-1}$

Answer. Let A= $\begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}$

We have,

$|A| = 15 - 14 = 1$

Now,

$A_{11} = 5, A_{12} = -2, A_{21} = -7, A_{22} = 3$

$\therefore adjA = \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$

$\therefore A^{-1} = \frac{1}{|A|} \cdot \text{adj}A = \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$

Now, let $B = \begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix}$.

We have,

$|B| = 54 - 56 = -2$

$$\therefore \text{ adj } B = \begin{bmatrix} 9 & -8 \\ -7 & 6 \end{bmatrix} = \begin{bmatrix} -\frac{9}{2} & 4 \\ \frac{7}{2} & -3 \end{bmatrix}$$

$$B^{-1}A^{-1} = \begin{bmatrix} -\frac{9}{2} & 4 \\ \frac{7}{2} & -3 \end{bmatrix}\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$$

Now, ...(i)Then,

$$= \begin{bmatrix} -\frac{45}{2} - 8 & \frac{63}{2} + 12 \\ \frac{35}{2} + 6 & -\frac{49}{2} - 9 \end{bmatrix} = \begin{bmatrix} -\frac{61}{2} & \frac{87}{2} \\ \frac{47}{2} & -\frac{67}{2} \end{bmatrix}$$

$$AB = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} 6 & 8 \\ 2 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 18 + 49 & 24 + 63 \\ 12 + 35 & 16 + 45 \end{bmatrix}$$

$$= \begin{bmatrix} 67 & 87 \\ 47 & 61 \end{bmatrix}$$

Therefore, we have $|AB| = 67 \times 61 - 87 \times 47 = 4087 - 4089 = -2$.

Also,

$$adj(AB) = \begin{bmatrix} 61 & -87 \\ -47 & 67 \end{bmatrix}$$

$$\therefore (AB)^{-1} = \frac{1}{|AB|}\text{adj}(AB) = -\frac{1}{2}\begin{bmatrix} 61 & -87 \\ -47 & 67 \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{61}{2} & \frac{87}{2} \\ \frac{47}{2} & -\frac{67}{2} \end{bmatrix} \ldots(ii)$$

From (1) and (2), we have:

$(AB)^{-1} = B^{-1}A^{-1}$

Hence, the given result is proved.

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Q13 If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that $A^2 - 5A + 7I = O$. Hence find A^{-1}

Answer.

$A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$A^2 = A \cdot A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$

$\therefore A^2 - 5A + 7I$

$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 5 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$= \begin{bmatrix} -7 & 0 \\ 0 & -7 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Hence, $A^2 - 5A + 7I = O$

$\therefore A \cdot A - 5A = -7I$

$\Rightarrow A \cdot A\left(A^{-1}\right) - 5AA^{-1} = -7IA^{-1}$ $\left[\text{Post-multiplying by } A^{-1} \text{ as } |A| \neq 0\right]$

$\Rightarrow A\left(AA^{-1}\right) - 5I = -7A^{-1}$

$\Rightarrow AI - 5I = -7A^{-1}$

$\Rightarrow A^{-1} = -\frac{1}{7}(A - 5I)$

$\Rightarrow A^{-1} = \frac{1}{7}(5I - A)$

$= \frac{1}{7}\left(\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\right)$

$= \frac{1}{7}\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$

$\therefore A^{-1} = \frac{1}{7}\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$

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Q14 For the matrix $A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$, find the numbers a and b such that $A^2 + aA + bI = 0$.

Answer.

$A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$

$\therefore A^2 = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 9+2 & 6+2 \\ 3+1 & 2+1 \end{bmatrix} = \begin{bmatrix} 11 & 8 \\ 4 & 3 \end{bmatrix}$

Now,

$A^2 + aA + bI = O$

$\Rightarrow (AA)A^{-1} + aAA^{-1} + bIA^{-1} = O$ $\quad$ [Post-multiplying by A^{-1} as $|A| \neq 0$]

$\Rightarrow A\left(AA^{-1}\right) + aI + b\left(IA^{-1}\right) = O$

$\Rightarrow AI + aI + bA^{-1} = O$

$\Rightarrow A + aI = -bA^{-1}$

$\Rightarrow A^{-1} = -\frac{1}{b}(A + aI)$

Now,

$A^{-1} = \frac{1}{|A|}adjA = \frac{1}{1}\begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$

We have:

$$\begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix} = -\frac{1}{b}\left(\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix}\right) = -\frac{1}{b}\begin{bmatrix} 3+a & 2 \\ 1 & 1+a \end{bmatrix} = \begin{bmatrix} \frac{-3-a}{b} & -\frac{2}{b} \\ -\frac{1}{b} & \frac{-1-a}{b} \end{bmatrix}$$

Comparing the corresponding elements of the two matrices, we have:

$-\frac{1}{b} = -1 \Rightarrow b = 1$

$\frac{-3-a}{b} = 1 \Rightarrow -3 - a = 1 \Rightarrow a = -4$

Hence, -4 and 1 are the required values of a and b respectively.

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Q15 For the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$ show that $A^3 - 6A^2 + 5A + 11 = 0$.

hence ,A^{-1}

Answer.

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1+2 & 1+2-1 & 1-3+3 \\ 1+2-6 & 1+4+3 & 1-6-9 \\ 2-1+6 & 2-2-3 & 2+3+9 \end{bmatrix} = \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix}$$

$$A^3 = A^2 \cdot A = \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 4+2+2 & 4+4-1 & 4-6+3 \\ -3+8-28 & -3+16+14 & -3-24-42 \\ 7-3+28 & 7-6-14 & 7+9+42 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 7 & -69 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix}$$

$\therefore A^3 - 6A^2 + 5A + 11I$

$$= \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix} - 6\begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix} + 5\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix} + 11\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix} - \begin{bmatrix} 6 & 6 & 6 \\ 6 & 12 & -18 \\ 12 & -6 & 18 \end{bmatrix} + \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 2 & 1 \\ -3 & 13 & -14 \\ 7 & -3 & 19 \end{bmatrix} - \begin{bmatrix} 6 & 6 & 6 \\ 6 & 12 & -18 \\ 12 & -6 & 18 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -4 & -5 \\ -9 & 1 & 4 \\ -5 & 3 & 1 \end{bmatrix}$$

From equation (1), we have:

$$A^{-1} = -\frac{1}{11}\begin{bmatrix} 3 & -4 & -5 \\ -9 & 1 & 4 \\ -5 & 3 & 1 \end{bmatrix} = \frac{1}{11}\begin{bmatrix} -3 & 4 & 5 \\ 9 & -1 & -4 \\ 5 & -3 & -1 \end{bmatrix}$$

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Q16 If $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ verify that $A^3 - 6A^2 + 9A - 4I = O$ and hence find A^{-1}

Answer.

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4+1+1 & -2-2-1 & 2+1+2 \\ -2-2-1 & 1+4+1 & -1-2-2 \\ 2+1+2 & -1-2-2 & 1+1+4 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix}$$

$$A^3 = A^2A = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix}\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 12+5+5 & -6-10-5 & 6+5+10 \\ -10-6-5 & 5+12+5 & -5-6-10 \\ 10+5+6 & -5-10-6 & 5+5+12 \end{bmatrix}$$

$$= \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix}$$

now ,

$A^3 - 6A^2 + 9A - 4I$

$$= \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix} - 6\begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} + 9\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} - 4\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix} - \begin{bmatrix} 36 & -30 & 30 \\ -30 & 36 & -30 \\ 30 & -30 & 36 \end{bmatrix} + \begin{bmatrix} 18 & -9 & 9 \\ -9 & 18 & -9 \\ 9 & -9 & 18 \end{bmatrix} - \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 40 & -30 & 30 \\ -30 & 40 & -30 \\ 30 & -30 & 40 \end{bmatrix} - \begin{bmatrix} 40 & -30 & 30 \\ -30 & 40 & -30 \\ 30 & -30 & 40 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\therefore A^3 - 6A^2 + 9A - 4I = O$

Now,

$A^3 - 6A^2 + 9A - 4I = O$

$A^3 - 6A^2 + 9A - 4I = O$

$\Rightarrow (AAA)A^{-1} - 6(AA)A^{-1} + 9AA^{-1} - 4LA^{-1} = O$ [Post-multiplying by A^{-1} as $|A| \neq 0$]

$\Rightarrow AA\left(AA^{-1}\right) - 6A\left(AA^{-1}\right) + 9\left(AA^{-1}\right) = 4\left(IA^{-1}\right)$

$\Rightarrow AAI - 6AI + 9I = 4A^{-1}$

$\Rightarrow A^2 - 6A + 9I = 4A^{-1}$

$\Rightarrow A^{-1} = \frac{1}{4}\left(A^2 - 6A + 9I\right) \ldots (i)$

$A^2 - 6A + 9I$

$$= \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} - 6\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} + 9\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} - \begin{bmatrix} 12 & -6 & 6 \\ -6 & 12 & -6 \\ 6 & -6 & 12 \end{bmatrix} + \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$$

From equation (1), we have:

$$A^{-1} = \frac{1}{4}\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$$

Page : 132 , Block Name : Exercise 4.5

Q17 A be a nonsingular square matrix of order 3×3. Then $|\text{adjA}|$ is equal to

A. $|A|$ B. $|A|^2$ C. $|A|^3$ D. $|A|$

Answer. B

We know that,

$$(adjA)A = |A|I = \begin{bmatrix} |A| & 0 & 0 \\ 0 & |A| & 0 \\ 0 & 0 & |A| \end{bmatrix}$$

$$\Rightarrow |(adjA)A| = \begin{vmatrix} |A| & 0 & 0 \\ 0 & |A| & 0 \\ 0 & 0 & |A| \end{vmatrix}.$$

$$\Rightarrow |\text{adj}A||A| = |A|^3 \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = |A|^3(I)$$

$\therefore |adjA| = |A|^2$

Hence, the correct answer is B .

Page : 132 , Block Name : Exercise 4.5

Q18 If A is an invertible matrix of order 2, then det $\left(A^{-1}\right)$

A.det(A) B. $\frac{1}{\det(A)}$ C. 1 D. 0

Answer. Since A is an invertible matrix, A^{-1} exists and $A^{-1} = \frac{1}{|A|}adjA$

As matrix A is of order 2, let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Then, $|A| = ad - bc$ and $adjA = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Now,

$$A^{-1} = \frac{1}{|A|} adjA = \begin{bmatrix} \frac{d}{|A|} & \frac{-b}{|A|} \\ \frac{-c}{|A|} & \frac{a}{|A|} \end{bmatrix}$$

$$\therefore \left|A^{-1}\right| = \begin{vmatrix} \frac{d}{|A|} & \frac{-b}{|A|} \\ \frac{-c}{|A|} & \frac{a}{|A|} \end{vmatrix}$$

$$= \frac{1}{|A|^2} \begin{vmatrix} d & -b \\ -c & a \end{vmatrix} = \frac{1}{|A|^2}(ad - bc)$$

$$= \frac{1}{|A|^2} \cdot |A| = \frac{1}{|A|}$$

$$\therefore \det\left(A^{-1}\right) = \frac{1}{\det(A)}$$

Hence, the correct answer is B .

Page : 132 , Block Name : Exercise 4.5

Q1 Examine the consistency of the system of equations.

$x + 2y = 2$

$2x + 3y = 3$

Answer.

The given system of equations is:

$x + 2y = 2$

$2x + 3y = 3$

The given system of equations can be written in the form of $AX = B$, where

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

Now,

$|A| = 1(3) - 2(2) = 3 - 4 = -1 \neq 0$

$\therefore A$ is non-singular.

Therefore, A^{-1} exists.

Hence, the given system of equations is consistent.

Page : 136 , Block Name : Exercise 4.6

Q2 Examine the consistency of the system of equations.

$2x - y = 5$

$x + y = 4$

Answer.

$2x - y = 5$

$x + y = 4$

The given system of equations can be written in the form of $AX = B$, where

$$A = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$$

Now,

$|A| = 2(1) - (-1)(1) = 2 + 1 = 3 \neq 0$

$\therefore A$ is non-sinqular.

Therefore, A^{-1} exists.

Hence, the given system of equations is consistent.

Page : 136 , Block Name : Exercise 4.6

Q3 Examine the consistency of the system of equations.

$x + 3y = 5$

$2x + 6y = 8$

Answer. The given system of equations is:

$x + 3y = 5$

$2x + 6y = 8$

The given system of equations can be written in the form of AX = B, where,

$$A = \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix} \text{ and } B = \begin{bmatrix} 5 \\ 8 \end{bmatrix}$$

$|A| = 1(6) - 3(2) = 6 - 6 = 0$

$\therefore A$ is a singular matrix.

Now,

$(adjA) = \begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix}$

$(adjA)B = \begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 5 \\ 8 \end{bmatrix} = \begin{bmatrix} 30-24 \\ -10+8 \end{bmatrix} = \begin{bmatrix} 6 \\ -2 \end{bmatrix} \neq O$

Thus, the solution of the given system of equations does not exist.
Hence, the system of equations is inconsistent.

Page : 136 , Block Name : Exercise 4.6

Q4 Examine the consistency of the system of equations.
$x + y + z = 1$

$2x + 3y + 2z = 2$

$ax + ay + 2az = 4$

Answer. The given system of equations is :
$x + y + z = 1$

$2x + 3y + 2z = 2$

$ax + ay + 2az = 4$

This system of equations can be written in the form AX = B, where

$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ a & a & 2a \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$

$|A| = 1(6a - 2a) - 1(4a - 2a) + 1(2a - 3a)$

$= 4a - 2a - a = 4a - 3a = a \neq 0$

$\therefore A$ is non-singular.

Therefore, A^{-1} exists.

Hence, the given system of equations is consistent.

Page : 136 , Block Name : Exercise 4.6

Q5 Examine the consistency of the system of equations.
$3x - y - 2z = 2$

$2y - z = -1$

$3x - 5y = 3$

Answer. The given system of equations is:
$3x - y - 2z = 2$

$2y - z = -1$

$3x - 5y = 3$

This system of equations can be written in the form of AX = B, where

Now,

$|A| = 3(0 - 5) - 0 + 3(1 + 4) = -15 + 15 = 0$

$\therefore A$ is a singular matrix.

Now,

$$(adjA) = \begin{bmatrix} -5 & 6 & 3 \\ -6 & 12 & 6 \end{bmatrix}$$

$$\therefore (adjA)B = \begin{bmatrix} -5 & 10 & 5 \\ -3 & 6 & 3 \\ -6 & 12 & 6 \end{bmatrix}\begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} = \begin{bmatrix} -10 - 10 + 15 \\ -6 - 6 + 9 \\ -12 - 12 + 18 \end{bmatrix} = \begin{bmatrix} -5 \\ -3 \\ -6 \end{bmatrix} \neq O$$

Thus, the solution of the given system of equations does not exist.
Hence, the system of equations is inconsistent.

Page : 136 , Block Name : Exercise 4.6

Q6 Examine the consistency of the system of equations.

$5x - y + 4z = 5$

$2x + 3y + 5z = 2$

$5x - 2y + 6z = -1$

Answer. The given system of equations is:

$5x - y + 4z = 5$

$2x + 3y + 5z = 2$

$5x - 2y + 6z = -1$

$$A = \begin{bmatrix} 5 & -1 & 4 \\ 2 & 3 & 5 \\ 5 & -2 & 6 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 5 \\ 2 \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 5 \\ 2 \\ -1 \end{bmatrix}$$

$|A| = 5(18 + 10) + 1(12 - 25) + 4(-4 - 15)$

$= 5(28) + 1(-13) + 4(-19)$

$= 140 - 13 - 76$

$\therefore A$ is non-singular.

Therefore, A^{-1} exists.

Hence, the given system of equations is consistent.

Page : 136 , Block Name : Exercise 4.6

Q7 Solve system of linear equations, using matrix method.

$5x + 2y = 4$

$7x + 3y = 5$

Answer. The given system of equations can be written in the form of AX = B, where

$A = \begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}$ and $B = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$

Now, $|A| = 15 - 14 = 1 \neq 0$

Thus, A is non-singular. Therefore, its inverse exists.

$A^{-1} = \frac{1}{|A|}(adjA)$

$\therefore A^{-1} = \begin{bmatrix} 3 & -2 \\ -7 & 5 \end{bmatrix}$

$\therefore X = A^{-1}B = \begin{bmatrix} 3 & -2 \\ -7 & 5 \end{bmatrix}\begin{bmatrix} 4 \\ 5 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12 - 10 \\ -28 + 25 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$

Hence, x=2 and y=-3

Page : 136 , Block Name : Exercise 4.6

Q8 Solve system of linear equations, using matrix method.

$2x - y = -2$

$3x + 4y = 3$

Answer. The given system of equations can be written in the form of AX = B, where

$A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}$ and $B = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$

$|A| = 8 + 3 = 11 \neq 0$

Thus, A is non-singular. Therefore, its inverse exists.

Now,

$A^{-1} = \frac{1}{|A|}$ adj $A = \frac{1}{11}\begin{bmatrix} 4 & 1 \\ -3 & 2 \end{bmatrix}$

$\therefore X = A^{-1}B = \frac{1}{11}\begin{bmatrix} 4 & 1 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} -2 \\ 3 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{11}\begin{bmatrix} -8 + 3 \\ 6 + 6 \end{bmatrix} = \frac{1}{11}\begin{bmatrix} -5 \\ 12 \end{bmatrix} = \begin{bmatrix} -\frac{5}{11} \\ \frac{12}{11} \end{bmatrix}$

Hence, $x = \frac{-5}{11}$ and $y = \frac{12}{11}$

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Q9 Solve system of linear equations, using matrix method.

$4x - 3y = 3$

$3x - 5y = 7$

Answer. The given system of equations can be written in the form of AX = B, where

$$A = \begin{bmatrix} 4 & -3 \\ 3 & -5 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix} \text{ and } B = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$$

$|A| = -20 + 9 = -11 \neq 0$

Thus, A is non-singular. Therefore, its inverse exists.

Now ,

$$A^{-1} = \frac{1}{|A|}(adjA) = -\frac{1}{11}\begin{bmatrix} -5 & 3 \\ -3 & 4 \end{bmatrix} = \frac{1}{11}\begin{bmatrix} 5 & -3 \\ 3 & -4 \end{bmatrix}$$

$$\therefore X = A^{-1}B = \frac{1}{11}\begin{bmatrix} 5 & -3 \\ 3 & -4 \end{bmatrix}\begin{bmatrix} 3 \\ 7 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{11}\begin{bmatrix} 5 & -3 \\ 3 & -4 \end{bmatrix}\begin{bmatrix} 3 \\ 7 \end{bmatrix} = \frac{1}{11}\begin{bmatrix} 15 - 21 \\ 9 - 28 \end{bmatrix} = \frac{1}{11}\begin{bmatrix} -6 \\ 9 - 28 \end{bmatrix} = \frac{1}{11}\begin{bmatrix} -6 \\ -19 \end{bmatrix} = \begin{bmatrix} -\frac{6}{11} \\ -\frac{19}{11} \end{bmatrix}$$

Hence, $x = \frac{-6}{11}$ and $y = \frac{-19}{11}$

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Q10 Solve system of linear equations, using matrix method.

$5x + 2y = 3$

$3x + 2y = 5$

Answer. The given system of equations can be written in the form of AX = B, where

$$A = \begin{bmatrix} 5 & 2 \\ 3 & 2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix} \text{ and } B = \begin{bmatrix} 3 \\ 5 \end{bmatrix}.$$

$|A| = 10 - 6 = 4 \neq 0$

Thus, A is non-singular. Therefore, its inverse exists.

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Q11 Solve system of linear equations, using matrix method.

$2x + y + z = 1$

$x - 2y - z = \frac{3}{2}$

$3y - 5z = 9$

Answer. The given system of equations can be written in the form of AX = B, where

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & -2 & -1 \\ 0 & 3 & -5 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 \\ \frac{3}{2} \\ 9 \end{bmatrix}$$

$|A| = 2(10 + 3) - 1(-5 - 3) + 0 = 2(13) - 1(-8) = 26 + 8 = 34 \neq 0$

Now , $A_{11} = 13, A_{12} = 5, A_{13} = 3$

$A_{21} = 8, A_{22} = -10, A_{23} = -6$

$A_{31} = 1, A_{32} = 3, A_{33} = -5$

$$\therefore A^{-1} = \frac{1}{|A|}(adjA) = \frac{1}{34}\begin{bmatrix} 13 & 8 & 1 \\ 5 & -10 & 3 \\ 3 & -6 & -5 \end{bmatrix}$$

$$\therefore X = A^{-1}B = \frac{1}{34}\begin{bmatrix} 13 & 8 & 1 \\ 5 & -10 & 3 \\ 3 & -6 & -5 \end{bmatrix}\begin{bmatrix} 1 \\ \frac{3}{2} \\ 9 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{34}\begin{bmatrix} 13 + 12 + 9 \\ 5 - 15 + 27 \\ 3 - 9 - 45 \end{bmatrix}$$

$$= \frac{1}{34}\begin{bmatrix} 34 \\ 17 \\ -51 \end{bmatrix} = \begin{bmatrix} 1 \\ \frac{1}{2} \\ -\frac{3}{2} \end{bmatrix} \text{ Hence,}$$

$x = 1, y = \frac{1}{2}, \text{ and } z = -\frac{3}{2}$

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Q12 Solve system of linear equations, using matrix method.

$x - y + z = 4$

$2x + y - 3z = 0$

$x + y + z = 2$

Answer. The given system of equations can be written in the form of AX = B, where

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$$

$|A| = 1(1 + 3) + 1(2 + 3) + 1(2 - 1) = 4 + 5 + 1 = 10 \neq 0$

Thus, A is non-singular. Therefore, its inverse exists.

Now , $A_{11} = 4, A_{12} = -5, A_{13} = 1$

$A_{21} = 2, A_{22} = 0, A_{23} = -2$

$A_{31} = 2, A_{32} = 5, A_{33} = 3$

$$\therefore A^{-1} = \frac{1}{|A|}(adjA) = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$$

$$\therefore X = A^{-1}B = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ y \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 16 + 0 + 4 \\ -20 + 0 + 10 \\ 4 + 0 + 6 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

$$= \frac{1}{10}\begin{bmatrix} -10 \\ -10 \\ 10 \end{bmatrix}$$ Hence, $x = 2, y = -1,$ and $z = 1$

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Q13 Solve system of linear equations, using matrix method.

$2x + 3y + 3z = 5$

$x - 2y + z = -4$

$3x - y - 2z = 3$

Answer. The given system of equations can be written in the form AX = B, where

$$A = \begin{bmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -2 & -2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}$$

$|A| = 2(4+1) - 3(-2-3) + 3(-1+6) = 2(5) - 3(-5) + 3(5) = 10 + 15 + 15 = 40 \neq 0$

Thus, A is non-singular. Therefore, its inverse exists.

Now, $A11 = 5, A_{12} = 5, A_{13} = 5$

$A_{21} = 3, A_{22} = -13, A_{23} = 11$

$A_{31} = 9, A_{32} = 1, A_{33} = -7$

$$\therefore A^{-1} = \frac{1}{|A|}(adjA) = \frac{1}{40}\begin{bmatrix} 5 & -13 & 1 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix}$$

$$\therefore X = A^{-1}B = \frac{1}{40}\begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix}\begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{40}\begin{bmatrix} 25 + 52 + 3 \\ 25 + 52 + 3 \\ 25 - 44 - 21 \end{bmatrix}$$

$$= \frac{1}{40}\begin{bmatrix} 40 \\ 80 \\ -40 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

Hence , $x = 1, y = 2,$ and $z = -1$

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Q14 Solve system of linear equations, using matrix method.

$x - y + 2z = 7$

$3x + 4y - 5z = -5$

$2x - y + 3z = 12$

Answer. The given system of equations can be written in the form of AX = B, where

$A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}$

now, $|A| = 1(12 - 5) + 1(9 + 10) + 2(-3 - 8) = 7 + 19 - 22 = 4 \neq 0$

Thus, A is non-singular. Therefore, its inverse exists.

Now, $A_{11} = 7, A_{12} = -19, A_{13} = -11$

$A_{21} = 1, A_{22} = -1, A_{23} = -1$

$A_{31} = -3, A_{32} = 11, A_{33} = 7$

$\therefore A^{-1} = \frac{1}{|A|}(adjA) = \frac{1}{4}\begin{bmatrix} 7 & 1 & 11 \\ -11 & -1 & 7 \end{bmatrix}$

$\therefore X = A^{-1}B = \frac{1}{4}\begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}\begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \\ y \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 49 - 5 - 36 \\ -133 + 5 + 132 \\ -77 + 5 + 84 \end{bmatrix}$

$= \frac{1}{4}\begin{bmatrix} 8 \\ 4 \\ 4 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

Hence, $x = 2, y = 1,$ and $z = 3$

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Q15 If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1}. Using A^{-1} solve the system of equations

$$\begin{aligned} 2x - 3y + 5z &= 11 \\ 3x + 2y - 4z &= -5 \\ x + y - 2z &= -3 \end{aligned}$$

Answer.

$$A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$$

$\therefore |A| = 2(-4+4) + 3(-6+4) + 5(3-2) = 0 - 6 + 5 = -1 \neq 0$

Now, $A_{11} = 0, A_{12} = 2, A_{13} = 1$

$A_{21} = -1, A_{22} = -9, A_{23} = -5$

$A_{31} = 2, A_{32} = 23, A_{33} = 13$

$$\therefore A^{-1} = \frac{1}{|A|}(adjA) = -\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix} \ldots\ldots(i)$$

Now, the given system of equations can be written in the form of AX = B, where

$$A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

The solution of the system of equations is given by $X = A^{-1}B$

$X = A^{-1}B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix} \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix} \quad [\text{ Using } (i)]$$

$$= \begin{bmatrix} 0 - 5 + 6 \\ -22 - 45 + 69 \\ -11 - 25 + 39 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Hence, $x = 1, y = 2,$ and $z = 3$

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Q16 The cost of 4 kg onion, 3 kg wheat and 2 kg rice is Rs 60. The cost of 2 kg onion, 4 kg wheat and 6 kg rice is Rs 90. The cost of 6 kg onion 2 kg wheat and 3 kg rice is Rs 70. Find cost of each item per kg by matrix method.

Answer. Let the cost of onions, wheat, and rice per kg be Rs x, Rs y,and Rs z respectively.

Then, the given situation can be represented by a system of equations as:

$4x + 3y + 2z = 60$

$2x + 4y + 6z = 90$

$6x + 2y + 3z = 70$

This system of equations can be written in the form of $AX = B$, where

$$A = \begin{bmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 4 & 6 \\ 6 & 2 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix}$$

$|A| = 4(12 - 12) - 3(6 - 36) + 2(4 - 24) = 0 + 90 - 40 = 50 \neq 0$

Now, $A_{11} = 0, A_{12} = 30, A_{13} = -20$

$A_{21} = -5, A_{22} = 0, A_{23} = 10$

$A_{31} = 10, A_{32} = -20, A_{33} = 10$

$$\therefore adjA = \begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}$$

now ,

$$\therefore A^{-1} = \frac{1}{|A|} adjA = \frac{1}{50} \begin{bmatrix} 0 & -5 & 10 \\ 0 & 0 & -20 \\ 30 & 10 & -20 \\ -20 & 10 & 10 \end{bmatrix}$$

$$\Rightarrow X = \frac{1}{50} \begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix} \begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{50} \begin{bmatrix} 0 - 450 + 700 \\ 1800 + 0 - 1400 \\ -1200 + 900 + 700 \end{bmatrix}$$

$$= \frac{1}{50} \begin{bmatrix} 250 \\ 400 \\ 400 \end{bmatrix}$$

$$= \begin{bmatrix} 5 \\ 8 \\ 8 \end{bmatrix}$$

$\therefore x = 5, y = 8,$ and $z = 8$

Hence, the cost of onions is Rs 5 per kg, the cost of wheat is Rs 8 per kg, and the cost of rice is Rs 8 per kg.

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Q1 Prove that the determinant $\begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$ is independent of θ

Answer.

$$\Delta = \begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$$

$$= x\left(x^2 - 1\right) - \sin\theta(-x\sin\theta - \cos\theta) + \cos\theta(-\sin\theta + x\cos\theta)$$

$$= x^3 - x + x\sin^2\theta + \sin\theta\cos\theta - \sin\theta\cos\theta + x\cos^2\theta$$

$$= x^3 - x + x\left(\sin^2\theta + \cos^2\theta\right)$$

$$= x^3 - x + x$$

$$= x^3(\text{ Independent of } \theta)$$

Hence , Δ is independent of θ

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Q2 Without expanding the determinant, prove that

$$\begin{vmatrix} a & a^2 & bc \\ b & b^2 & ca \\ c & c^2 & ab \end{vmatrix} = \begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix}$$

Answer. L.H.S. $= \begin{vmatrix} a & a^2 & bc \\ b & b^2 & ca \\ c & c^2 & ab \end{vmatrix}$

$$= \frac{1}{abc}\begin{vmatrix} a^2 & a^3 & abc \\ b^2 & b^3 & abc \\ c^2 & c^3 & abc \end{vmatrix} \left[R_1 \to aR_1, R_2 \to bR_2, \text{ and } R_3 \to cR_3\right]$$

$$= \frac{1}{abc} \cdot abc \begin{vmatrix} a^2 & a^3 & 1 \\ b^2 & b^3 & 1 \\ c^2 & c^3 & 1 \end{vmatrix} \quad \left[\text{Taking out factor } abc \text{ from } C_3 \right]$$

$$= \begin{vmatrix} a^2 & a^3 & 1 \\ b^2 & b^3 & 1 \\ c^2 & c^3 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix} \quad [\text{Applying } C_1 \leftrightarrow C_3 \text{ and } C_2 \leftrightarrow C_3]$$

=R.H.S.

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Q3 Evaluate $\begin{vmatrix} \cos\alpha\cos\beta & \cos\alpha\sin\beta & -\sin\alpha \\ -\sin\beta & \cos\beta & 0 \\ \sin\alpha\cos\beta & \sin\alpha\sin\beta & \cos\alpha \end{vmatrix}$

Answer.

$$\Delta = \begin{vmatrix} \cos\alpha\cos\beta & \cos\alpha\sin\beta & -\sin\alpha \\ -\sin\beta & \cos\beta & 0 \\ \sin\alpha\cos\beta & \sin\alpha\sin\beta & \cos\alpha \end{vmatrix}$$

Expanding along C_3, we have:

$$\Delta = -\sin\alpha\left(-\sin\alpha\sin^2\beta - \cos^2\beta\sin\alpha\right) + \cos\alpha\left(\cos\alpha\cos^2\beta + \cos\alpha\sin^2\beta\right)$$

$$= \sin^2\alpha\left(\sin^2\beta + \cos^2\beta\right) + \cos^2\alpha\left(\cos^2\beta + \sin^2\beta\right)$$

$$= \sin^2\alpha(1) + \cos^2\alpha(1)$$

$$= 1$$

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Q4 If a, b and c are real numbers, and

$$\Delta = \begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 0$$

Show that either a + b + c = 0 or a = b = c.

Answer.

$$\Delta = \begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 0$$

Applying $R_1 \rightarrow R_1 + R_2 + R_3$,

$$\Delta = \begin{vmatrix} 2(a+b+c) & 2(a+b+c) & 2(a+b+c) \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix}$$

$$= 2(a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix}$$

Applying $C_2 \rightarrow C_2 - C_1$ and $C_3 \rightarrow C_3 - C_1$, we have:

$$\Delta = 2(a+b+c)\begin{vmatrix} 1 & 0 & 0 \\ c+a & b-c & b-a \\ a+b & c-a & c-b \end{vmatrix}$$

Expanding along R_1, we have:

$$\Delta = 2(a+b+c)(1)[(b-c)(c-b) - (b-a)(c-a)]$$

$$= 2(a+b+c)\left[-b^2 - c^2 + 2bc - bc + ba + ac - a^2 \right]$$

$$= 2(a+b+c)\left[ab + bc + ca - a^2 - b^2 - c^2\right]$$

It is given that $\Delta = 0$

$$(a+b+c)\left[ab + bc + ca - a^2 - b^2 - c^2\right] = 0$$

$\Rightarrow$ Either $a + b + c = 0$, or $ab + bc + ca - a^2 - b^2 - c^2 = 0$

Now,

$ab + bc + ca - a^2 - b^2 - c^2 = 0$

$\Rightarrow -2ab - 2bc - 2ca + 2a^2 + 2b^2 + 2c^2 = 0$

$\Rightarrow (a-b)^2 + (b-c)^2 + (c-a)^2 = 0$

$\Rightarrow (a-b)^2 = (b-c)^2 = (c-a)^2 = 0$ $\left[(a-b)^2, (b-c)^2, (c-a)^2 \text{ are non-negative } \right]$

$\Rightarrow (a-b)=(b-c)=(c-a)=0$

$\Rightarrow a=b=c$

Hence, if $\Delta=0$, then either $a+b+c=0$ or $a=b=c$.

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Q5 Solve the equations :

$$\begin{vmatrix} x+a & x & x \\ x & x+a & x \\ x & x & x+a \end{vmatrix} = 0,\ a \neq 0$$

Answer.

$$\begin{vmatrix} x+a & x & x \\ x & x+a & x \\ x & x & x+a \end{vmatrix} = 0$$

Applying $R_1 \rightarrow R_1 + R_2 + R_3$, , we get :

$$\begin{vmatrix} 3x+a & 3x+a & 3x+a \\ x & x+a & x \\ x & x & x+a \end{vmatrix} = 0$$

$$\Rightarrow (3x+a)\begin{vmatrix} 1 & 1 & 1 \\ x & x+a & x \\ x & x & x+a \end{vmatrix} = 0$$

Applying $C_2 \rightarrow C_2 - C_1$ and $C_3 \rightarrow C_3 - C_1$, , we have :

$$(3x+a)\begin{vmatrix} 1 & 0 & 0 \\ x & a & 0 \\ x & 0 & a \end{vmatrix} = 0$$

Expanding along R_1, we have:

$$(3x+a)\left[1 \times a^2\right] = 0$$

$\Rightarrow a^2(3x+a)=0$

But $a \neq 0$

Therefore, we have:

$3x+a=0$

$\Rightarrow x = -\frac{a}{3}$

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Q6 Prove that $\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2$

Answer.

$$\Delta = \begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix}$$

Taking out common factors a, b, and c from C_1, C_2, and C_3, we have:

$$\Delta = abc\begin{vmatrix} a & c & a+c \\ a+b & b & a \\ b & b+c & c \end{vmatrix}$$

Applying $R_2 \rightarrow R_2 - R_1$ and $R_3 \rightarrow R_3 - R_1$, we have:

$$\Delta = abc\begin{vmatrix} a & c & a+c \\ b & b-c & -c \\ b-a & b & -a \end{vmatrix}$$

Applying $R_2 \rightarrow R_2 + R_1$, we have :

$$\Delta = abc\begin{vmatrix} a & c & a+c \\ a+b & b & a \\ b-a & b & -a \end{vmatrix}$$

Applying $R_3 \rightarrow R_3 + R_2$

$$\Delta = abc\begin{vmatrix} a & c & a+c \\ a+b & b & a \\ 2b & 2b & 0 \end{vmatrix}$$

$$= 2ab^2c\begin{vmatrix} a & c & a+c \\ a & c & a+c \\ 1 & 1 & 0 \end{vmatrix}$$

Applying $C_2 \rightarrow C_2 - C_1$

$$\Delta = 2ab^2c\begin{vmatrix} a & c-a & a+c \\ a+b & -a & a \\ 1 & 0 & 0 \end{vmatrix}$$

$$\Delta = 2ab^2c[a(c-a) + a(a+c)]$$
$$= 2ab^2c[a(c-a) + a(a+c)]$$
$$= 2ab^2c\left[ac - a^2 + a^2 + ac\right]$$
$$= 2ab^2c(2ac)$$
$$= 4a^2b^2c^2$$

Hence, the given result is proved.

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Q7 If $A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$, find $(AB)^+$

Answer.

We know that $(AB)^{-1} = B^{-1}A^{-1}$

$$B = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

$\therefore |B| = 1 \times 3 - 2 \times (-1) - 2(2) = 3 + 2 - 4 = 5 - 4 = 1$

Now, $A_{11} = 3, A_{12} = 1, A_{13} = 2$

$A_{21} = 2, A_{22} = 1, A_{23} = 2$

$A_{31} = 6, A_{32} = 2, A_{33} = 5$

$$\therefore \text{adj}B = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$$

$$B^{-1} = \frac{1}{|B|} \cdot adjB$$

$$\therefore B^{-1} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$$

$\therefore (AB)^{-1} = B^{-1}A^{-1}$

$$= \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix} \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 9-30+30 & -3+12-12 & 3-10+12 \\ 3-15+10 & -1+6-4 & 1-5+4 \\ 6-30+25 & -2+12-10 & 2-10+10 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & -3 & 5 \\ -2 & 1 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

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Q8 Let $A = \begin{bmatrix} 1 & -2 & 1 \\ -2 & 3 & 1 \\ 1 & 1 & 5 \end{bmatrix}$ verify that

$(i)[adjA]^{-1} = \text{adj}\left(A^{-1}\right)$

$(ii)\left(A^{-1}\right)^{-1} = A$

Answer.

$$A = \begin{bmatrix} 1 & -2 & 1 \\ -2 & 3 & 1 \\ 1 & 1 & 5 \end{bmatrix}$$

$\therefore |A| = 1(15-1) + 2(-10-1) + 1(-2-3) = 14-22-5 = -13$

Now, $A_{11} = 14, A_{12} = 11, A_{13} = -5$

$A_{21} = 11, A_{22} = 4, A_{23} = -3$

$A_{31} = -5, A_{12} = -3, A_{13} = -1$

$$\therefore adjA = \begin{bmatrix} 14 & 11 & -5 \\ 11 & 4 & -3 \\ -5 & -3 & -1 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|}(adjA)$$

$$= -\frac{1}{13}\begin{bmatrix} 14 & 11 & -5 \\ 11 & 4 & -3 \\ -5 & -3 & -1 \end{bmatrix} = \frac{1}{13}\begin{bmatrix} -14 & -11 & 5 \\ -11 & -4 & 3 \\ 5 & 3 & 1 \end{bmatrix}$$

(i)

$$|adjA| = 14(-4-9) - 11(-11-15) - 5(-33+20)$$
$$= 14(-13) - 11(-26) - 5(-13)$$
$$= -182 + 286 + 65 = 169$$

We have,

$$\text{adj}(adjA) = \begin{bmatrix} -13 & 26 & -13 \\ 26 & -39 & -13 \\ -13 & -13 & -65 \end{bmatrix}$$

$$\therefore [adjA]^1 = \frac{1}{|adjA|}(adj(adjA))$$

$$= \frac{1}{169}\begin{bmatrix} -13 & 26 & -13 \\ -13 & -39 & -13 \\ -13 & -13 & -65 \end{bmatrix}$$

$$= \frac{1}{13}\begin{bmatrix} -1 & 2 & -1 \\ 2 & -3 & -1 \\ -1 & -1 & -5 \end{bmatrix}$$

$$A^{-1} = \frac{1}{13}\begin{bmatrix} -14 & -11 & 5 \\ -11 & -4 & 3 \\ 5 & 3 & 1 \end{bmatrix} = \begin{bmatrix} -\frac{14}{13} & -\frac{11}{13} & \frac{5}{13} \\ -\frac{11}{13} & -\frac{4}{13} & \frac{3}{13} \\ \frac{5}{13} & \frac{3}{13} & \frac{1}{13} \end{bmatrix}$$

$$\therefore \text{adj}\left(A^{-1}\right) = \begin{bmatrix} -\frac{4}{169} - \frac{9}{169} & -\left(-\frac{11}{169} - \frac{15}{169}\right) & -\frac{33}{169} + \frac{20}{169} \\ -\left(-\frac{11}{169} - \frac{15}{169}\right) & -\frac{14}{169} - \frac{25}{169} & \\ -\frac{33}{169} + \frac{20}{169} & -\left(-\frac{42}{169} + \frac{55}{169}\right) & \frac{56}{169} - \frac{121}{169} \end{bmatrix}$$

$$= \frac{1}{169}\begin{bmatrix} -13 & 26 & -13 \\ 26 & -39 & -13 \\ -13 & -13 & -65 \end{bmatrix} = \frac{1}{13}\begin{bmatrix} -1 & 2 & -1 \\ 2 & -3 & -1 \\ -1 & -1 & -5 \end{bmatrix}$$

Hence , $[adjA]^{-1} = adj\left(A^{-1}\right)$

(ii) We have,

$$A^{-1} = \frac{1}{13}\begin{bmatrix} -14 & -11 & 5 \\ -11 & -4 & 3 \\ 5 & 3 & 1 \end{bmatrix}$$

And, $adjA^{-1} = \frac{1}{13}\begin{bmatrix} -1 & 2 & -1 \\ 2 & -3 & -1 \\ -1 & -1 & -5 \end{bmatrix}$

$$\left|A^{-1}\right| = \left(\frac{1}{13}\right)^3[-14 \times (-13) + 11 \times (-26) + 5 \times (-13)] = \left(\frac{1}{13}\right)^3 \times (-169) = -\frac{1}{13}$$

$$\therefore \left(A^{-1}\right)^{-1} = \frac{adjA^{-1}}{\left|A^{-1}\right|} = \frac{1}{\left(-\frac{1}{13}\right)} \times \frac{1}{13}\begin{bmatrix} -1 & 2 & -1 \\ 2 & -3 & -1 \\ -1 & -1 & -5 \end{bmatrix} = \begin{bmatrix} 1 & -2 & 1 \\ -2 & 3 & 1 \\ 1 & 1 & 5 \end{bmatrix} = A$$

$$\therefore \left(A^{-1}\right)^{-1} = A$$

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Q9 Evaluate $\begin{vmatrix} x & y & x+y \\ y & x+y & x \\ x+y & x & y \end{vmatrix}$

Answer.

$$\Delta = \begin{vmatrix} x & y & x+y \\ y & x+y & x \\ x+y & x & y \end{vmatrix}$$

Applying $R_1 \rightarrow R_1 + R_2 + R_3$

$$\Delta = \begin{vmatrix} 2(x+y) & 2(x+y) & 2(x+y) \\ y & x+y & x \\ x+y & x & y \end{vmatrix}$$

$$= 2(x+y)\begin{vmatrix} 1 & x+y & x \\ y & x+y & x \\ x+y & x & y \end{vmatrix}$$

Applying $C_2 \rightarrow C_2 - C_1$ and $C_3 \rightarrow C_3 - C_1$, we have:

$$\Delta = 2(x+y)\begin{vmatrix} 1 & 0 & 0 \\ y & x & x-y \\ x+y & -y & -x \end{vmatrix}$$

*Expanding along R*1, *we have*

$$\Delta = 2(x+y)\left[-x^2 + y(x-y) \right]$$

$$= -2(x+y)\left(x^2 + y(x-y)\right]$$

$$= -2\left(x^3 + y^3\right)$$

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Q10 Evaluate $\begin{vmatrix} 1 & x & y \\ 1 & x+y & y \\ 1 & x & x+y \end{vmatrix}$

Answer.

$$\Delta = \begin{vmatrix} 1 & x & y \\ 1 & x+y & y \\ 1 & x & x+y \end{vmatrix}$$

Applying $R_2 \rightarrow R_2 - R_1$ and $R_3 \rightarrow R_3 - R_1$, we have:

$$\Delta = \begin{vmatrix} 1 & x & y \\ 0 & y & 0 \\ 0 & 0 & x \end{vmatrix}$$

*Expanding along C*1, *we have*

$\Delta = 1(xy - 0) = xy$

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Q11 Using properties of determinants, prove that:

$$\begin{vmatrix} \alpha & \alpha^2 & \beta+\gamma \\ \beta & \beta^2 & \gamma+\alpha \\ \gamma & \gamma^2 & \alpha+\beta \end{vmatrix} = (\beta-\gamma)(\gamma-\alpha)(\alpha-\beta)(\alpha+\beta+\gamma)$$

Answer.

$$\Delta = \begin{vmatrix} \alpha & \alpha^2 & \beta+\gamma \\ \beta & \beta^2 & \gamma+\alpha \\ \gamma & \gamma^2 & \alpha+\beta \end{vmatrix}$$

Applying $R_2 \rightarrow R_2 - R_1$ and $R_3 \rightarrow R_3 - R_1$, ,we have

$$= (\beta-\alpha)(\gamma-\alpha)\begin{vmatrix} \alpha & \alpha^2 & \beta+\gamma \\ 1 & \beta+\alpha & -1 \\ 1 & \gamma+\alpha & -1 \end{vmatrix}$$

Applying $R_3 \rightarrow R_3 - R_2$, we have:

$$\Delta = (\beta-\alpha)(\gamma-\alpha)\begin{vmatrix} \alpha & \alpha^2 & \beta+\gamma \\ | & \beta+\alpha & -1 \\ 0 & \gamma-\beta & 0 \end{vmatrix}$$

Expanding along R_3, we have:

$$\Delta = (\beta-\alpha)(\gamma-\alpha)[-(\gamma-\beta)(-\alpha-\beta-\gamma)]$$
$$= (\beta-\alpha)(\gamma-\alpha)(\gamma-\beta)(\alpha+\beta+\gamma)$$
$$= (\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)(\alpha+\beta+\gamma)$$

Hence, the given result is proved.

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Q12 Using properties of determinants, prove that:

$$\begin{vmatrix} x & x^2 & 1+px^3 \\ y & y^2 & 1+py^3 \\ z & z^2 & 1+pz^3 \end{vmatrix} = (1+pxyz)(x-y)(y-z)(z-x)$$

Answer.

$$\Delta = \begin{vmatrix} x & x^2 & 1+px^3 \\ y & y^2 & 1+py^3 \\ z & z^2 & 1+pz^3 \end{vmatrix}$$

Applying $R_2 \rightarrow R_2 - R_1$ and $R_3 \rightarrow R_3 - R_3$, we have:

$$\Delta = \begin{vmatrix} x & x^2 & 1+px^3 \\ y-x & y^2-x^2 & p\left(y^3-x^3\right) \\ z-x & z^2-x^2 & p\left(z^3-x^3\right) \end{vmatrix}$$

$$= (y-x)(z-x)\begin{vmatrix} x & x^2 & 1+px^3 \\ 1 & y+x & p\left(y^2+x^2+xy\right) \\ 1 & z+x & p\left(z^2+x^2+xz\right) \end{vmatrix}$$

Applying $R_3 \rightarrow R_3 - R_2$, we have:

$$\Delta = (y-x)(z-x)\begin{vmatrix} x & x^2 & 1+px^3 \\ 1 & y+x & p\left(y^2+x^2+xy\right) \\ 0 & z-y & p(z-y)(x+y+z) \end{vmatrix}$$

$$= (y-x)(z-x)(z-y)\begin{vmatrix} x & x^2 & 1+px^3 \\ 1 & y+x & p\left(y^2+x^2+xy\right) \\ 0 & 1 & p(x+y+z) \end{vmatrix}$$

Expanding along R_3, we have:

$$\Delta = (x-y)(y-z)(z-x)\left[(-1)(p)\left(xy^2+x^3+x^2y\right)+1+px^3+p(x+y+z)(xy)\right.$$

$$= (x-y)(y-z)(z-x)\left[-pxy^2-px^3-px^2y+1+px^3+px^2y+pxy^2+pxyz\right]$$

$$= (x-y)(y-z)(z-x)(1+pxyz)$$

Hence, the given result is proved.

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Q13 Using properties of determinants, prove that:

$$\begin{vmatrix} 3a & -a+b & -a+c \\ -b+a & 3b & -b+c \\ -c+a & -c+b & 3c \end{vmatrix} = 3(a+b+c)(ab+bc+ca)$$

Answer.

$$\Delta = \begin{vmatrix} 3a & -a+b-a+c & \\ -b+a & 3b & -b+c \\ -c+a & -c+b & 3c \end{vmatrix}$$

Applying $C_1 \rightarrow C_1 + C_2 + C_3$, we have:

$$\Delta = \begin{vmatrix} a+b+c & -a+b & -a+c \\ a+b+c & 3b & -b+c \\ a+b+c & -c+b & 3c \end{vmatrix}$$

$$= (a+b+c)\begin{vmatrix} 1 & -a+b & -a+c \\ 1 & 3b & -b+c \\ 1 & -c+b & 3c \end{vmatrix}$$

Applying $R_2 \rightarrow R_2 - R_1$ and $R_3 \rightarrow R_3 - R_1$, we have:

$$\Delta = (a+b+c)\begin{vmatrix} 1 & -a+b & -a+c \\ 0 & 2b+a & a-b \\ 0 & a-c & 2c+a \end{vmatrix}$$

Expanding along C_1, we have:

$$\Delta = (a+b+c)[(2b+a)(2c+a) - (a-b)(a-c)]$$
$$= (a+b+c)\left[4bc + 2ab + 2ac + a^2 - a^2 + ac + ba - bc\right]$$
$$= (a+b+c)(3ab + 3bc + 3ac)$$
$$= 3(a+b+c)(ab+bc+ca)$$

Hence, the given result is proved.

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Q14 Using properties of determinants, prove that:

$$\begin{vmatrix} 1 & 1+p & 1+p+q \\ 2 & 3+2p & 4+3p+2q \\ 3 & 6+3p & 10+6p+3q \end{vmatrix} = 1$$

Answer.

$$\Delta = \begin{vmatrix} 1 & 1+p & 1+p+q \\ 2 & 3+2p & 4+3p+2q \\ 3 & 6+3p & 10+6p+3q \end{vmatrix}$$

Applying $R_2 \rightarrow R_2 - 2R_1$ and $R_3 \rightarrow R_3 - 3R_1$, we have:

$$\Delta = \begin{vmatrix} 1 & 1+p & 1+p+q \\ 0 & 1 & 2+p \\ 0 & 3 & 7+3p \end{vmatrix}$$

Applying $R_3 \rightarrow R_3 - 3R_2$, , we have:

$$\Delta = (a+b+c)\begin{vmatrix} 1 & -a+b & -a+c \\ 0 & 2b+a & a-b \\ 0 & a-c & 2c+a \end{vmatrix}$$

Expanding along C_1, we have:

$$\Delta = (a+b+c)[(2b+a)(2c+a) - (a-b)(a-c)]$$

$$= (a+b+c)\left[4bc + 2ab + 2ac + a^2 - a^2 + ac + ba - bc\right]$$

$$= (a+b+c)(3ab + 3bc + 3ac)$$

$$= 3(a+b+c)(ab+bc+ca)$$

Hence, the given result is proved.

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Q15 Using properties of determinants, prove that:

$$\begin{vmatrix} \sin\alpha & \cos\alpha & \cos(\alpha+\delta) \\ \sin\beta & \cos\beta & \cos(\beta+\delta) \\ \sin\gamma & \cos\gamma & \cos(\gamma+\delta) \end{vmatrix} = 0$$

Answer.

$$\Delta = \begin{vmatrix} \sin\alpha & \cos\alpha & \cos(\alpha+\delta) \\ \sin\beta & \cos\beta & \cos(\beta+\delta) \\ \sin\gamma & \cos\gamma & \cos(\gamma+\delta) \end{vmatrix}$$

$$= \frac{1}{\sin\delta\cos\delta}\begin{vmatrix} \sin\alpha\sin\delta & \cos\alpha\cos\delta & \cos\alpha\cos\delta - \sin\alpha\sin\delta \\ \sin\beta\sin\delta & \cos\beta\cos\delta & \cos\beta\cos\delta - \sin\beta\sin\delta \\ \sin\gamma\sin\delta & \cos\gamma\cos\delta & \cos\gamma\cos\delta - \sin\gamma\sin\delta \end{vmatrix}$$

Applying $C_1 \rightarrow C_1 + C_3$,we have : $\Delta = \frac{1}{\sin\delta\cos\delta}\begin{vmatrix} \cos\alpha\cos\delta & \cos\alpha\cos\delta & \cos\alpha\cos\delta - \sin\alpha\sin\delta \\ \cos\beta\cos\delta & \cos\beta\cos\delta & \cos\beta\cos\delta - \sin\beta\sin\delta \\ \cos\gamma\cos\delta & \cos\gamma\cos\delta & \cos\gamma\cos\delta - \sin\gamma\sin\delta \end{vmatrix}$

Here, two columns C_1 and C_2 are identical.

$\therefore \Delta = 0$

Hence, the given result is proved.

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Q16 Solve the system of the following equations

$$\frac{2}{x}+\frac{3}{y}+\frac{10}{z}=4$$

$$\frac{4}{x}-\frac{6}{y}+\frac{5}{z}=1$$

$$\frac{6}{x}+\frac{9}{y}-\frac{20}{z}=2$$

Answer. Let $\frac{1}{x}=p, \frac{1}{y}=q, \frac{1}{z}=r$

Then the given system of equations is as follows:

$2p+3q+10r=4$

$4p-6q+5r=1$

$6p+9q-20r=2$

This system can be written in the form of $AX=B$, where

$$A=\begin{bmatrix}2 & -6 & 5\\ 4 & -6 & 5\\ 6 & 9 & -20\end{bmatrix}, X=\begin{bmatrix}p\\ r\end{bmatrix} \text{ and } B=\begin{bmatrix}4\\ 2\end{bmatrix} \text{ and } B=\begin{bmatrix}4\\ 2\end{bmatrix}$$

Now ,

$$|A| = 2(120-45)-3(-80-30)+10(36+36)$$
$$= 150+330+720$$
$$= 1200$$

Thus, A is non-singular. Therefore, its inverse exists.

$A_{11}=75, A_{12}=110, A_{13}=72$

Now , $A_{21}=150, A_{22}=-100, A_{23}=0$

$A_{31}=75, A_{32}=30, A_{33}=-24$

$$\therefore A^{-1}=\frac{1}{|A|}adjA$$

$$=\frac{1}{1200}\begin{bmatrix}75 & 150 & 75\\ 110 & -100 & 30\\ 72 & 0 & -24\end{bmatrix}$$

$X=A^{-1}B$

Now ,

$$\Rightarrow \begin{bmatrix}p\\ q\\ r\end{bmatrix}=\frac{1}{1200}\begin{bmatrix}75 & 150 & 75\\ 110 & -100 & 30\\ 72 & 0 & -24\end{bmatrix}\begin{bmatrix}4\\ 1\\ 2\end{bmatrix}$$

$$= \frac{1}{1200}\begin{bmatrix} 300 + 150 + 150 \\ 440 - 100 + 60 \\ 288 + 0 - 48 \end{bmatrix}$$

$$= \frac{1}{1200}\begin{bmatrix} 600 \\ 400 \\ 240 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{5} \end{bmatrix}$$

$\therefore p = \frac{1}{2}, q = \frac{1}{3},$ and $r = \frac{1}{5}$

Hence, $x = 2.\ y = 3.$ and $z = 5$

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Q17 Choose the correct answer.
If a, b, c, are in A.P., then the determinant

$$\begin{vmatrix} x+2 & x+3 & x+2a \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{vmatrix}$$

A. 0 B. 1 C. x D. 2x

Answer. Answer:A

$$\Delta = \begin{vmatrix} x+2 & x+3 & x+2a \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{vmatrix}$$

$$= \begin{vmatrix} x+2 & x+3 & x+2a \\ x+3 & x+4 & x+(a+c) \\ x+4 & x+5 & x+2c \end{vmatrix} \quad (2b = a + c \text{ as } a, b, \text{ and } c \text{ are in A.P. })$$

Applying $R_1 \rightarrow R_1 - R_2$ and $R_3 \rightarrow R_3 - R_2,$ we have:

$$\Delta = \begin{vmatrix} -1 & -1 & a-c \\ x+3 & x+4 & x+(a+c) \\ 1 & 1 & c-a \end{vmatrix}$$

Applying $R_1 \rightarrow R_1 + R_3$, we have:

$$\Delta = \begin{vmatrix} 0 & 0 & 0 \\ x+3 & x+4 & x+a+c \\ 1 & 1 & c-a \end{vmatrix}$$

Here, all the elements of the first row (R1) are zero.
Hence, we have $\Delta = 0$.
The correct answer is A.

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Q18 Choose the correct answer.

If x, y, z are nonzero real numbers, then the inverse of matrix $A = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$ is

A. $\begin{bmatrix} x^{-1} & 0 & 0 \\ 0 & y^{-1} & 0 \\ 0 & 0 & z^{-1} \end{bmatrix}$ B. $xyz\begin{bmatrix} x^{-1} & 0 & 0 \\ 0 & y^{-1} & 0 \\ 0 & 0 & z^{-1} \end{bmatrix}$

C. $\frac{1}{xyz}\begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$ **D.** $\frac{1}{xyz}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Answer. Answer: A

$$A = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$$

$\therefore |A| = x(yz - 0) = xyz \neq 0$

Now, $A_{11} = yz, A_{12} = 0, A_{13} = 0$

$A_{21} = 0, A_{22} = xz, A_{23} = 0$

$A_{31} = 0, A_{32} = 0, A_{33} = xy$

$$\therefore \text{adj}A = \begin{bmatrix} yz & 0 & 0 \\ 0 & xz & 0 \\ 0 & 0 & xy \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|}adjA$$

$$= \frac{1}{xyz}\begin{bmatrix} yz & 0 & 0 \\ 0 & xz & 0 \\ 0 & 0 & xy \end{bmatrix}$$

$$= \begin{bmatrix} \frac{yz}{xyz} & 0 & 0 \\ 0 & \frac{xz}{xyz} & 0 \\ 0 & 0 & \frac{xy}{xyz} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{x} & 0 & 0 \\ 0 & \frac{1}{y} & 0 \\ 0 & 0 & \frac{1}{z} \end{bmatrix} = \begin{bmatrix} 0 & y^{-1} & 0 \\ 0 & 0 & z^{-1} \end{bmatrix}$$

The correct answer is A.

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Q19 Choose the correct answer.

Let , $A = \begin{bmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{bmatrix}$ Where $0 \leq \theta \leq 2\pi$, , then

A. Det (A) = 0

B. Det (A) $\in (2, \infty)$

C. Det (A) $\in (2, 4)$

D. Det (A) $\in [2, 4]$

Answer. Answer: D

$$A = \begin{bmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{bmatrix}$$

$\therefore |A| = 1\left(1 + \sin^2\theta\right) - \sin\theta(-\sin\theta + \sin\theta) + 1\left(\sin^2\theta + 1\right)$

$= 1 + \sin^2\theta + \sin^2\theta + 1$

$= 2 + 2\sin^2\theta$

$= 2\left(1 + \sin^2\theta\right)$ Now

$0 \le \theta \le 2\pi$

$\Rightarrow 0 \le \sin^2\theta \le 1$

$\Rightarrow 1 \le 1 + \sin^2\theta \le 2$

$\Rightarrow 2 \le 2\left(1 + \sin^2\theta\right) \le 4$

$\Rightarrow -1 \le \sin\theta_{\le 1}$

$\therefore \text{Det}(A) \in [2, 4]$

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