NCERT SOLUTIONS

CLASS - 12th



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Class : 12th
Subject : Maths
Chapter : 2
Chapter Name : Inverse Trigonometric Functions

Q1 Find the principal values of the following:

$\sin^{-1}\left(-\frac{1}{2}\right)$

Answer. *Let* $\sin^{-1}\left(-\frac{1}{2}\right) = y$. Then $\sin y = -\frac{1}{2} = -\sin\left(\frac{\pi}{6}\right) = \sin\left(-\frac{\pi}{6}\right)$

We know that the range of the principal value branch of $\sin^{-1}$ is

$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and $\sin\left(-\frac{\pi}{6}\right) = -\frac{1}{2}$

Therefore, the principal value of $\sin^{-1}\left(-\frac{1}{2}\right)$ is $-\frac{\pi}{6}$.

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Q2 Find the principal values of the following:

$\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

Answer. Let $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = y$. Then, $\cos y = \frac{\sqrt{3}}{2} = \cos\left(\frac{\pi}{6}\right)$

We know that the range of the principal value branch of $\cos^{-1}$ is

$[0, \pi]$ and $\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

Therefore, the principal value of

$\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ is $\frac{\pi}{6}$

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Q3 Find the principal values of the following:

$cosec^{-1}(2)$

Answer. Let $cosec^{-1}(2) = y$. Then $cosecy = 2 = cosec\left(\frac{\pi}{6}\right)$

We know that the range of the principal value branch of cosec^{-1} is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$.

Therefore, the principal value of (2) is $\frac{\pi}{6}$.

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Q4 Find the principal values of the following:

$\tan^{-1}(-\sqrt{3})$

Answer. Let $\tan^{-1}(-\sqrt{3}) = y$. Then, $\tan y = -\sqrt{3} = -\tan\frac{\pi}{3} = \tan\left(-\frac{\pi}{3}\right)$

We know that the range of the principal value branch of $\tan^{-1}$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ and $\tan\left(-\frac{\pi}{3}\right)$ is $-\sqrt{3}$.

Therefore, the principal value of $cosec^{-1}(2)$ is $\frac{\pi}{6}$

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Q5 Find the principal values of the following:

$\cos^{-1}\left(-\frac{1}{2}\right)$

Answer. Let $\cos^{-1}\left(-\frac{1}{2}\right) = y$. Then, $\cos y = -\frac{1}{2} = -\cos\left(\frac{\pi}{3}\right) = \cos\left(\pi - \frac{\pi}{3}\right) = \cos\left(\frac{2\pi}{3}\right)$

We know that the range of the principal value branch of $\cos^{-1}$ is $[0, \pi]$ and $\cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2}$

Therefore, the principal value of $\cos^{-1}\left(-\frac{1}{2}\right)$ is $\frac{2\pi}{3}$

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Q6 Find the principal values of the following:

$\tan^{-1}(-1)$

Answer. Let $\tan^{-1}(-1) = y$. Then, $\tan y = -1 = -\tan\left(\frac{\pi}{4}\right) = \tan\left(-\frac{\pi}{4}\right)$

We know that the range of the principal value branch of $\tan^{-1}$ is

$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ and $\tan\left(-\frac{\pi}{4}\right) = -1$

Therefore, the principal value of $\tan y = -1 = -\tan\left(\frac{\pi}{4}\right) = \tan\left(-\frac{\pi}{4}\right)$

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Q7 Find the principal values of the following:

$\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$

Answer. Let

$\sec^{-1}\left(\frac{2}{\sqrt{3}}\right) = y$. Then , $\sec y = \frac{2}{\sqrt{3}} = \sec\left(\frac{\pi}{6}\right)$

We know that the range of the principal value branch of $\sec^{-1}$ is

$[0, \pi] - \left\{\frac{\pi}{2}\right\}$ and $\sec\left(\frac{\pi}{6}\right) = \frac{2}{\sqrt{3}}$

Therefore, the principal value of $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$ is $\frac{\pi}{6}$

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Q8 Find the principal values of the following:
$\cot^{-1}(\sqrt{3})$

Answer. Let

$\cot^{-1}(\sqrt{3}) = y$. Then , $\cot y = \sqrt{3} = \cot\left(\frac{\pi}{6}\right)$.

We know that the range of the principal value branch of $\cot^{-1}$ is $(0,\pi)$ and $\cot\left(\frac{\pi}{6}\right) = \sqrt{3}$

Therefore, the principal value of $\cot^{-1}(\sqrt{3})$ is $\frac{\pi}{6}$

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Q9 Find the principal values of the following:

$\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$

Answer. Let

$\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right) = y$. Then , $\cos y = -\frac{1}{\sqrt{2}} = -\cos\left(\frac{\pi}{4}\right) = \cos\left(\pi - \frac{\pi}{4}\right) = \cos\left(\frac{3\pi}{4}\right)$

We know that the range of the principal value branch of $\cot^{-1}$ is $(0,\pi)$ and $\cos\left(\frac{3\pi}{4}\right) = -\frac{1}{\sqrt{2}}$

Therefore, the principal value of $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ is $\frac{3\pi}{4}$.

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Q10 Find the principal values of the following:

$cosec^{-1}(-\sqrt{2})$

Answer. Let

$cosec^{-1}(-\sqrt{2}) = y$. Then, $\operatorname{cosec} y = -\sqrt{2} = -\csc\left(\frac{\pi}{4}\right) = \csc\left(-\frac{\pi}{4}\right)$

we know that the range of the principal value branch of $\operatorname{cosec}^{-1}$ is

$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$ and $\operatorname{cosec}\left(-\frac{\pi}{4}\right) = -\sqrt{2}$

Therefore, the principal value of $\csc^{-1}(-\sqrt{2})$ is $-\frac{\pi}{4}$

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Q11 Find the values of the following:

$\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$

Answer.

Let $\tan^{-1}(1) = x$. Then, $\tan x = 1 = \tan\frac{\pi}{4}$

$\therefore \tan^{-1}(1) = \frac{\pi}{4}$

Let $\cos^{-1}\left(-\frac{1}{2}\right) = y$. Then, $\cos y = -\frac{1}{2} = -\cos\left(\frac{\pi}{3}\right) = \cos\left(\pi - \frac{\pi}{3}\right) = \cos\left(\frac{2\pi}{3}\right)$

$\therefore \cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$

Let $\sin^{-1}\left(-\frac{1}{2}\right) = z$. Then, $\sin z = -\frac{1}{2} = -\sin\left(\frac{\pi}{6}\right) = \sin\left(-\frac{\pi}{6}\right)$

$\therefore \sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$

$\therefore \tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$

$= \frac{\pi}{4} + \frac{2\pi}{3} - \frac{\pi}{6}$

$= \frac{3\pi + 8\pi - 2\pi}{12} = \frac{9\pi}{12} = \frac{3\pi}{4}$

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Q12 Find the values of the following:
$\cos^{-1}\frac{1}{2} + 2\sin^{-1}\frac{1}{2}$

Answer.

Let $\cos^{-1}\left(\frac{1}{2}\right) = x$. Then, $\cos x = \frac{1}{2} = \cos\left(\frac{\pi}{3}\right)$

$\therefore \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$

Let $\sin^{-1}\left(\frac{1}{2}\right) = y$. Then, $\sin y = \frac{1}{2} = \sin\left(\frac{\pi}{6}\right)$

$\therefore \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$

$\therefore \cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3} + \frac{2\pi}{6} = \frac{\pi}{3} + \frac{\pi}{3} = \frac{2\pi}{3}$

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Q13 Find the value of if $\sin^{-1}x = y$, then,
(A) $0 \le y \le \pi$

(b) $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

(C) $0 < Y < \pi$

(D) $-\frac{\pi}{2} < y < \frac{\pi}{2}$

Answer. It is given that $\sin^{-1}x = y$

We know that the range of the principal value branch of $\sin^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Therefore, $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

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Q14 Find the value of $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$ is equal to

(A) π

(B) $-\frac{\pi}{3}$

(C) $\frac{\pi}{3}$

(D) $\frac{2\pi}{3}$

Answer. Let $\tan^{-1}\sqrt{3} = x$. Then, $\tan x = \sqrt{3} = \tan\frac{\pi}{3}$

We know that the range of the principal value branch of $\tan^{-1}$ is $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

$\therefore \tan^{-1}\sqrt{3} = \frac{\pi}{3}$

Let $\sec^{-1}(-2) = y$. Then, $\sec y = -2 = -\sec\left(\frac{\pi}{3}\right) = \sec\left(\pi - \frac{\pi}{3}\right) = \sec\frac{2\pi}{3}$

We know that the range of the principal value branch of $\sec^{-1}$ is $[0, \pi] - \left\{\frac{\pi}{2}\right\}$

$\therefore \sec^{-1}(-2) = \frac{2\pi}{3}$

Hence $\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2) = \frac{\pi}{3} - \frac{2\pi}{3} = -\frac{\pi}{3}$

Page : 42 , Block Name : Exercise 2.1

Q1 Prove the following:

$3\sin^{-1}x = \sin^{-1}\left(3x - 4x^3\right), x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

Answer. To prove

$3\sin^{-1}x = \sin^{-1}\left(3x - 4x^3\right), x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

Let $x = \sin\theta$. Then , $\sin^{-1}x = \theta$

We have,

R.H.S. $= \sin^{-1}\left(3x - 4x^3\right) = \sin^{-1}\left(3\sin\theta - 4\sin^3\theta\right)$

$= \sin^{-1}(\sin 3\theta)$

$= 3\theta$

$= 3\sin^{-1}x$

= L. H. S

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Q2 Prove the following:

$3\cos^{-1}x = \cos^{-1}\left(4x^3 - 3x\right), x \in \left[\frac{1}{2}, 1\right]$

Answer. To prove

$3\cos^{-1}x = \cos^{-1}\left(4x^3 - 3x\right), x \in \left[\frac{1}{2}, 1\right]$

Let $x = \cos\theta$. Then , $\cos^{-1}x = \theta$

we have.

R.H.S. $= \cos^{-1}\left(4x^3 - 3x\right)$

$= \cos^{-1}\left(4\cos^3\theta - 3\cos\theta\right)$

$= \cos^{-1}(\cos 3\theta)$

$= 3\theta$

$= 3\cos^{-1}x$

= L.H.S.

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Q3 Prove the following:

$\tan^{-1}\frac{2}{11} + \tan^{-1}\frac{7}{24} = \tan^{-1}\frac{1}{2}$

Answer. To prove

$\tan^{-1}\frac{2}{11} + \tan^{-1}\frac{1}{24} = \tan^{-1}\frac{1}{2}$

$= \tan^{-1}\frac{2}{11} + \tan^{-1}\frac{7}{24}\left[\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}\right]$

$$= \tan^{-1}\frac{\frac{2}{11}+\frac{7}{24}}{1-\frac{2}{11}\cdot\frac{7}{24}}$$

$$= \tan^{-1}\frac{\frac{48+77}{11\times 24}}{\frac{11\times 24}{11\times 24}}$$

$$= \tan^{-1}\frac{48+77}{264-14} = \tan^{-1}\frac{125}{250} = \tan^{-1}\frac{1}{2} = \text{R. H. S.}$$

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Q4 Prove the following:

$2\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{7} = \tan^{-1}\frac{31}{17}$

Answer. To prove

$2\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{7} = \tan^{-1}\frac{31}{17}$

L.H.S. $= 2\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{7}$

$$= \tan^{-1}\frac{2\cdot\frac{1}{2}}{1-\left(\frac{1}{2}\right)^2} + \tan^{-1}\frac{1}{7} \quad \left[2\tan^{-1}x = \tan^{-1}\frac{2x}{1-x^2}\right]$$

$$= \tan^{-1}\frac{1}{\left(\frac{3}{4}\right)} + \tan^{-1}\frac{1}{7}$$

$$= \tan^{-1}\frac{\frac{4}{3}+\frac{1}{7}}{1-\frac{4}{3}\cdot\frac{1}{7}} \left[\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}\right]$$

$$= \tan^{-1}\frac{\left(\frac{28+3}{21}\right)}{\left(\frac{21-4}{21}\right)}$$

$$= \tan^{-1}\frac{31}{17} = \text{R. H. S}$$

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Q5 Write the following functions in the simplest form

$\tan^{-1}\frac{\sqrt{1+x^2}-1}{x}, x \neq 0$

Answer.

$\tan^{-1}\frac{\sqrt{1+x^2}-1}{x}$

Put $x = \tan\theta \Rightarrow \theta = \tan^{-1}x$

$\therefore \tan^{-1}\frac{\sqrt{1+x^2}-1}{x} = \tan^{-1}\left(\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}\right)$

$= \tan^{-1}\left(\frac{\sec\theta-1}{\tan\theta}\right) = \tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right)$

$= \tan^{-1}\left(\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right)$

$= \tan^{-1}\left(\tan\frac{\theta}{2}\right) = \frac{\theta}{2} = \frac{1}{2}\tan^{-1}x$

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Q6 Write the following functions in the simplest form

$\tan^{-1}\frac{1}{\sqrt{x^2-1}}, \ |x| > 1$

Answer.

$\tan^{-1}\frac{1}{\sqrt{x^2-1}}, \ |x| > 1$

Put $x = \ cosec\theta \Rightarrow \theta = \ cosec^{-1}x$

$\therefore \tan^{-1}\frac{1}{\sqrt{x^2-1}} = \tan^{-1}\frac{1}{\sqrt{\cos ec^2\theta-1}}$ $\left[\ cosec^{-1}x + \sec^{-1}x = \frac{\pi}{2}\right.$

$= \tan^{-1}\left(\frac{1}{\cot\theta}\right) = \tan^{-1}(\tan\theta)$

$= \theta = \ cosec^{-1}x = \frac{\pi}{2} - \sec^{-1}x$

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Q7 Write the following functions in the simplest form

$\tan^{-1}\left(\sqrt{\frac{1-\cos x}{1+\cos x}}\right), 0 < x < \pi$

Answer.

$\tan^{-1}\left(\sqrt{\frac{1-\cos x}{1+\cos x}}\right), x < \pi$

$\tan^{-1}\left(\sqrt{\frac{1-\cos x}{1+\cos x}}\right) = \tan^{-1}\left(\sqrt{\frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}}\right)$

$= \tan^{-1}\left(\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}\right) = \tan^{-1}\left(\tan\frac{x}{2}\right)$

$= \frac{x}{2}$

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Q8 Write the following functions in the simplest form

$\tan^{-1}\left(\frac{\cos x-\sin x}{\cos x+\sin x}\right), \frac{-\pi}{4} < x < \frac{3\pi}{4}$

Answer.

$\tan^{-1}\left(\frac{\cos x-\sin x}{\cos x+\sin x}\right)$

$= \tan^{-1}\left(\frac{1-\frac{\sin x}{\cos x}}{1+\frac{\sin x}{\cos x}}\right)$

$= \tan^{-1}\left(\frac{1-\tan x}{1+\tan x}\right)$

$= \tan^{-1}(1) - \tan^{-1}(\tan x) \quad \left[\tan^{-1}\frac{x-y}{1-xy} = \tan^{-1}x - \tan^{-1}y\right]$

$= \frac{\pi}{4} - x$

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Q9 Write the following functions in the simplest form

$\tan^{-1}\frac{x}{\sqrt{a^2-x^2}},\ |x| < a$

Answer.

$\tan^{-1}\frac{x}{\sqrt{a^2-x^2}}$

Put $x = a\sin\theta \Rightarrow \frac{x}{a} = \sin\theta \Rightarrow \theta = \sin^{-1}\left(\frac{x}{a}\right)$

$\therefore \tan^{-1}\frac{x}{\sqrt{a^2-x^2}} = \tan^{-1}\left(\frac{a\sin\theta}{\sqrt{a^2-a^2\sin^2\theta}}\right)$

$= \tan^{-1}\left(\frac{a\sin\theta}{a\sqrt{1-\sin^2\theta}}\right) = \tan^{-1}\left(\frac{a\sin\theta}{a\cos\theta}\right)$

$= \tan^{-1}(\tan\theta) = \theta = \sin^{-1}\frac{x}{a}$

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Q10 Write the following functions in the simplest form

$\tan^{-1}\left(\frac{3a^2x-x^3}{a^3-3ax^2}\right), a > 0; \frac{-a}{\sqrt{3}} < x < \frac{a}{\sqrt{3}}$

Answer.

$\tan^{-1}\left(\frac{3a^2x-x^3}{a^3-3ax^2}\right)$

Put $x = a\tan\theta \Rightarrow \frac{x}{a} = \tan\theta \Rightarrow \theta = \tan^{-1}\frac{x}{a}$

$\tan^{-1}\left(\frac{3a^2x-x^3}{a^3-3ax^2}\right) = \tan^{-1}\left(\frac{3a^2\cdot a\tan\theta - a^3\tan^3\theta}{a^3-3a\cdot a^2\tan^2\theta}\right)$

$= \tan^{-1}\left(\frac{3\tan\theta-\tan^3\theta}{1-3\tan^2\theta}\right)$

$= \tan^{-1}\left(\frac{3\tan\theta-\tan^3\theta}{1-3\tan^2\theta}\right)$

$= \tan^{-1}(\tan 3\theta)$

$= 3\theta$

$= 3\tan^{-1}\frac{x}{a}$

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Q11 Find the values of each of the following:

$\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right]$

Answer. Let $\sin^{-1}\frac{1}{2} = x$. Then,$\sin x = \frac{1}{2} = \sin\left(\frac{\pi}{6}\right)$

$\therefore \sin^{-1}\frac{1}{2} = \frac{\pi}{6}$

$\therefore \tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right] = \tan^{-1}\left[2\cos\left(2 \times \frac{\pi}{6}\right)\right]$

$= \tan^{-1}\left[2\cos\frac{\pi}{3}\right] = \tan^{-1}\left[2 \times \frac{1}{2}\right]$

$= \tan^{-1}1 = \frac{\pi}{4}$

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Q12 Find the values of each of the following:

$\cot\left(\tan^{-1}a + \cot^{-1}a\right)$

Answer. $\cot\left(\tan^{-1}a + \cot^{-1}a\right)$

$= \cot\left(\frac{\pi}{2}\right) \quad \left[\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}\right]$

=0

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Q13 Find the values of each of the following:

$\tan\frac{1}{2}\left[\sin^{-1}\frac{2x}{1+x^2} + \cos^{-1}\frac{1-y^2}{1+y^2}\right], |x| < 1, y > 0$ and $xy < 1$

Answer. Let $x = \tan\theta$. Then , $\theta = \tan^{-1}x$

$\therefore \sin^{-1}\frac{2x}{1+x^2} = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right) = \sin^{-1}(\sin 2\theta) = 2\theta = 2\tan^{-1}x$

Let $y = \tan\Phi$. Then , $\Phi = \tan^{-1}y$

$\cdot \cos^{-1}\frac{1-y^2}{1+y^2} = \cos^{-1}\left(\frac{1-\tan^2\phi}{1+\tan^2\phi}\right) = \cos^{-1}(\cos 2\phi) = 2\phi = 2\tan^{-1}y$

$\therefore \tan\frac{1}{2}\left[\sin^{-1}\frac{2x}{1+x^2} + \cos^{-1}\frac{1-y^2}{1+y^2}\right]$

$= \tan\frac{1}{2}\left[2\tan^{-1}x + 2\tan^{-1}y\right]$

$= \tan\left[\tan^{-1}x + \tan^{-1}y\right]$

$= \tan\left[\tan^{-1}\left(\frac{x+y}{1-xy}\right)\right]$

$= \frac{x+y}{1-xy}$

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Q14 Find the values of each of the following:

$\sin\left(\sin^{-1}\frac{1}{5} + \cos^{-1}x\right) = 1$, then find the value of x.

Answer.

$\sin\left(\sin^{-1}\frac{1}{5} + \cos^{-1}x\right) = 1$

$\Rightarrow \sin\left(\sin^{-1}\frac{1}{5}\right)\cos\left(\cos^{-1}x\right) + \cos\left(\sin^{-1}\frac{1}{5}\right)\sin\left(\cos^{-1}x\right) = 1$

$[\sin(A + B) = \sin A\cos B + \cos A\sin B]$

$\Rightarrow \frac{1}{5} \times x + \cos\left(\sin^{-1}\frac{1}{5}\right)\sin\left(\cos^{-1}x\right) = 1$

$\Rightarrow \frac{x}{5} + \cos\left(\sin^{-1}\frac{1}{5}\right)\sin\left(\cos^{-1}x\right) = 1$ .....(1)

Now, let $\sin^{-1}\frac{1}{5} = y$

Then,$\sin y = \frac{1}{5} \Rightarrow \cos y = \sqrt{1 - \left(\frac{1}{5}\right)^2} = \frac{2\sqrt{6}}{5} \Rightarrow y = \cos^{-1}\left(\frac{2\sqrt{6}}{5}\right)$

$\therefore \sin^{-1}\frac{1}{5} = \cos^{-1}\left(\frac{2\sqrt{6}}{5}\right)$.....(2)

Let $\cos^{-1}x = z$

Then, $\cos z = x \Rightarrow \sin z = \sqrt{1-x^2} \Rightarrow z = \sin^{-1}\left(\sqrt{1-x^2}\right)$

$\therefore \cos^{-1}x = \sin^{-1}\left(\sqrt{1-x^2}\right)$.....(3)

From (1),(2), and (3) we have:

$\frac{x}{5} + \cos\left(\cos^{-1}\frac{2\sqrt{6}}{5}\right) \cdot \sin\left(\sin^{-1}\sqrt{1-x^2}\right) = 1$

$\Rightarrow \frac{x}{5} + \frac{2\sqrt{6}}{5} \cdot \sqrt{1-x^2} = 1$

$\Rightarrow x + 2\sqrt{6}\sqrt{1-x^2} = 5$

$\Rightarrow 2\sqrt{6}\sqrt{1-x^2} = 5 - x$

On squaring both sides,we get

$(4)(6)\left(1-x^2\right) = 25 + x^2 - 10x$

$\Rightarrow 24 - 24x^2 = 25 + x^2 - 10x$

$\Rightarrow 25x^2 - 10x + 1 = 0$

$\Rightarrow (5x-1)^2 = 0$

$\Rightarrow (5x-1) = 0$

$\Rightarrow x = \frac{1}{5}$

Hence, the value of x is $\frac{1}{5}$

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Q15 Find the values of each of the following:

$\tan^{-1}\frac{x-1}{x-2} + \tan^{-1}\frac{x+1}{x+2} = \frac{\pi}{4}$, then find the value of x

Answer. $\tan^{-1}\frac{x-1}{x-2} + \tan^{-1}\frac{x+1}{x+2} = \frac{\pi}{4}$

$\Rightarrow \tan^{-1}\left[\frac{\frac{x-1}{x-2}+\frac{x+1}{x+2}}{1-\left(\frac{x-1}{x-2}\right)\left(\frac{x+1}{x+2}\right)}\right] = \frac{\pi}{4} \quad \left[\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}\right]$

$$\Rightarrow \tan^{-1}\left[\frac{(x-1)(x+2)+(x+1)(x-2)}{(x+2)(x-2)-(x-1)(x+1)}\right]=\frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\left[\frac{x^2+x-2+x^2-x-2}{x^2-4-x^2+1}\right]=\frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\left[\frac{2x^2-4}{-3}\right]=\frac{\pi}{4}$$

$$\Rightarrow \tan\left[\tan^{-1}\frac{4-2x^2}{3}\right]=\tan\frac{\pi}{4}$$

$$\Rightarrow \frac{4-2x^2}{3}=1$$

$$\Rightarrow 4-2x^2=3$$

$$\Rightarrow 2x^2=4-3=1$$

$$\Rightarrow x=\pm\frac{1}{\sqrt{2}}$$

Hence the value of x is. $\pm\frac{1}{\sqrt{2}}$

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Q16 Find the values of each of the expressions $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$.

Answer. $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$

We know that $\sin^{-1}(\sin x)=x$ if $x\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$, which is the principal value branch of $\sin^{-1}x$.

Here, $\frac{2\pi}{3}\notin\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$

Now, $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$ can be written as,

$$\sin^{-1}\left(\sin\frac{2\pi}{3}\right)=\sin^{-1}\left[\sin\left(\pi-\frac{2\pi}{3}\right)\right]=\sin^{-1}\left(\sin\frac{\pi}{3}\right)\text{ where }\frac{\pi}{3}\in\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$$

$$\therefore \sin^{-1}\left(\sin\frac{2\pi}{3}\right)=\sin^{-1}\left(\sin\frac{\pi}{3}\right)=\frac{\pi}{3}$$

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Q17 Find the values of each of the expressions

$\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$

Answer. $\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$

We know that $\tan^{-1}(\tan x) = x$ if $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, which is the principal value branch of $\tan^{-1}x$.

Here, $\frac{3\pi}{4} \notin \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

Now, $\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$ can be written as:

$$\tan^{-1}\left(\tan\frac{3\pi}{4}\right) = \tan^{-1}\left[-\tan\left(\frac{-3\pi}{4}\right)\right] = \tan^{-1}\left[-\tan\left(\pi - \frac{\pi}{4}\right)\right]$$

$$= \tan^{-1}\left[-\tan\frac{\pi}{4}\right] = \tan^{-1}\left[\tan\left(-\frac{\pi}{4}\right)\right] \text{ where } -\frac{\pi}{4} \epsilon \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$$

$$\therefore \tan^{-1}\left(\tan\frac{3\pi}{4}\right) = \tan^{-1}\left[\tan\left(\frac{-\pi}{4}\right)\right] = \frac{-\pi}{4}$$

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Q18 Find the values of each of the expressions

$\tan\left(\sin^{-1}\frac{3}{5} + \cot^{-1}\frac{3}{2}\right)$

Answer. Let $\sin^{-1}\frac{3}{5} = x$. Then $x = \frac{3}{5} \Rightarrow \cos x = \sqrt{1 - \sin^2 x} = \frac{4}{5} \Rightarrow \sec x = \frac{5}{4}$

$$\therefore \tan x = \sqrt{\sec^2 x - 1} = \sqrt{\frac{25}{16} - 1} = \frac{3}{4}$$

$$\therefore x = \tan^{-1}\frac{3}{4}$$

$\therefore \sin^{-1}\frac{3}{5} = \tan^{-1}\frac{3}{4}$ … (i)

Now, $\cot^{-1}\frac{3}{2} = \tan^{-1}\frac{2}{3}$ … (ii) $\left[\tan^{-1}\frac{1}{x} = \cot^{-1}x\right]$

Hence, $\tan\left(\sin^{-1}\frac{3}{5} + \cot^{-1}\frac{3}{2}\right)$

$= \tan\left(\tan^{-1}\frac{3}{4} + \tan^{-1}\frac{2}{3}\right)$

$= \tan\left(\tan^{-1}\frac{3}{4} + \frac{2}{3}\right)$

$= \tan\left(\tan^{-1}\frac{9+8}{12-6}\right)$

$= \tan\left(\tan^{-1}\frac{17}{6}\right) = \frac{17}{6}$

$= \tan\left(\tan^{-1}\frac{17}{6}\right) = \frac{17}{6}$

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Q19 Find the value of $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$ is equal to.

(A) $\frac{7\pi}{6}$

(B) $\frac{5\pi}{6}$

(C) $\frac{5\pi}{6}$

(D) $\frac{\pi}{6}$

Answer. We know that $\cos^{-1}(\cos x) = x$ if $x \in [0, \pi]$, ,which is the principal value branch of $\cos^{-1}(x)$.

Here $\frac{7\pi}{6} \notin x \in [0, \pi]$

Now,$\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$ can be written as,

$\cos^{-1}\left(\cos\frac{7\pi}{6}\right) = \cos^{-1}\left(\cos\frac{-7\pi}{6}\right) = \cos^{-1}\left[\cos\left(2\pi - \frac{7\pi}{6}\right)\right]$[cos(2π + x) = cosx]

$= \cos^{-1}\left[\cos\frac{5\pi}{6}\right]$ where $\frac{5\pi}{6}\epsilon[0, \pi]$

$\therefore \cos^{-1}\left(\cos\frac{7\pi}{6}\right) = \cos^{-1}\left(\cos\frac{5\pi}{6}\right) = \frac{5\pi}{6}$

The correct answer is B.

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Q20 $\sin\left(\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right)$ is equal to

(A) $\frac{1}{2}$

(B) $\frac{1}{3}$

(C) $\frac{1}{4}$

(D) 1

Answer. Let $\sin^{-1}\left(\frac{-1}{2}\right) = x$. Then $x = \frac{-1}{2} = -\sin\frac{\pi}{6} = \sin\left(\frac{-\pi}{6}\right)$

We know that the range of the principal value branch of $\sin^{-1}$ is $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

$\sin^{-1}\left(\frac{-1}{2}\right) = \frac{-\pi}{6}$

$\therefore \sin\left(\frac{\pi}{3} - \sin^{-1}\left(\frac{-1}{2}\right)\right) = \sin\left(\frac{\pi}{3} + \frac{\pi}{6}\right) = \sin\left(\frac{3\pi}{6}\right) = \sin\left(\frac{\pi}{2}\right) = 1$

The correct answer is D.

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Q21 $\tan^{-1}\sqrt{3} - \cot^{-1}(-\sqrt{3})$ is equal to:

(A) π

(B) $-\frac{\pi}{2}$

(C) 0

(D) $2\sqrt{3}$

Answer. Let $\tan^{-1}\sqrt{3} = x$. Then,

$\tan x = \sqrt{3} = \tan\frac{\pi}{3}$ where $\frac{\pi}{3} \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

We know that the range of the principal value branch of

$\tan^{-1}$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\therefore \tan^{-1}\sqrt{3} = \frac{\pi}{3}$

Let $^{-1}(-\sqrt{3}) = y$

$\cot y = -\sqrt{3} = -\cot\left(\frac{\pi}{6}\right) = \cot\left(\pi - \frac{\pi}{6}\right) = \cot\frac{5\pi}{6}$ where $\frac{5\pi}{6} \in (0, \pi)$

The range of the principal value branch of $\cot^{-1}$ is $(0, \pi)$

$\therefore \cot^{-1}(-\sqrt{3}) = \frac{5\pi}{6}$

$\therefore \tan^{-1}\sqrt{3} - \cot^{-1}(-\sqrt{3}) = \frac{\pi}{3} - \frac{5\pi}{6} = \frac{2\pi - 5\pi}{6} = \frac{-3\pi}{6} = -\frac{\pi}{2}$

The correct answer is B.

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Q1 Find the value of the following:

$\cos^{-1}\left(\cos\frac{13\pi}{6}\right)$

Answer. We know that $\cos^{-1}(\cos x) = x$ if $x \in [0, \pi]$ which is the principal value branch of,$\cos^{-1}x$

Here $\frac{13\pi}{6} \notin [0, \pi]$

Now, $\cos^{-1}\left(\cos\frac{13\pi}{6}\right)$ can be written as:

$\cos^{-1}\left(\cos\frac{13\pi}{6}\right) = \cos^{-1}\left[\cos\left(2\pi + \frac{\pi}{6}\right)\right] = \cos^{-1}\left[\cos\left(\frac{\pi}{6}\right)\right]$, where $\frac{\pi}{6} \in [0, \pi]$

$\therefore \cos^{-1}\left(\cos\frac{13\pi}{6}\right) = \cos^{-1}\left[\cos\left(\frac{\pi}{6}\right)\right] = \frac{\pi}{6}$

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Q2 Find the value of the following:$\tan^{-1}\left(\tan\frac{7\pi}{6}\right)$

Answer.

We know that $\tan^{-1}(\tan x) = x$ if $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, which is the principal value branch of $\tan^{-1}x$.

Here, $\frac{7\pi}{6} \notin \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Now, $\tan^{-1}\left(\tan\frac{7\pi}{6}\right)$ can be written as,

$\tan^{-1}\left(\tan\frac{7\pi}{6}\right) = \tan^{-1}\left[\tan\left(2\pi - \frac{5\pi}{6}\right)\right] = \tan(2\pi - x) = -\tan x$

$= \tan^{-1}\left[-\tan\left(\frac{5\pi}{6}\right)\right] = \tan^{-1}\left[\tan\left(-\frac{5\pi}{6}\right)\right] = \tan^{-1}\left[\pi - \frac{5\pi}{6}\right)$

$= \tan^{-1}\left[\tan\left(\frac{\pi}{6}\right)\right]$, where $\frac{\pi}{6} \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\therefore \tan^{-1}\left(\tan\frac{7\pi}{6}\right) = \tan^{-1}\left(\tan\frac{\pi}{6}\right) = \frac{\pi}{6}$

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Q3 Prove that $2\sin^{-1}\frac{3}{5} = \tan^{-1}\frac{24}{7}$

Answer.

Let $\sin^{-1}\frac{3}{5} = x$. Then, $\sin x = \frac{3}{5}$

$\Rightarrow \cos x = \sqrt{1 - \left(\frac{3}{5}\right)^2} = \frac{4}{5}$

$\therefore \tan x = \frac{3}{4}$

$\therefore x = \tan^{-1}\frac{3}{4} \Rightarrow \sin^{-1}\frac{3}{5} = \tan^{-1}\frac{3}{4}$

L.H.S. $= 2\sin^{-1}\frac{3}{5} = 2\tan^{-1}\frac{3}{4}$

$$= \tan^{-1}\left(\frac{2\times\frac{3}{4}}{1-\left(\frac{3}{4}\right)^2}\right)$$

$$= \tan^{-1}\left(\frac{\frac{3}{2}}{\frac{16-9}{16}}\right) = \tan^{-1}\left(\frac{3}{2}\times\frac{16}{7}\right)$$

$= \tan^{-1}\frac{24}{7}$ = R. H. S

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Q4 Prove that

$\sin^{-1}\frac{8}{17} + \sin^{-1}\frac{3}{5} = \tan^{-1}\frac{77}{36}$

Answer.

Let $\sin^{-1}\frac{8}{17} = x$. Then, $\sin x = \frac{8}{17} \Rightarrow \cos x = \sqrt{1-\left(\frac{8}{17}\right)^2} = \sqrt{\frac{225}{289}} = \frac{15}{17}$

$\therefore \tan x = \frac{8}{15} \Rightarrow x = \tan^{-1}\frac{8}{15}$

$\therefore \sin^{-1}\frac{8}{17} = \tan^{-1}\frac{8}{15}$. . . . . (1)

Now , let $\sin^{-1}\frac{3}{5} = y$. Then,$\sin y = \frac{3}{5} \Rightarrow \cos y = \sqrt{1-\left(\frac{3}{5}\right)^2} = \sqrt{\frac{16}{25}} = \frac{4}{5}$.

$\therefore \tan y = \frac{3}{4} \Rightarrow y = \tan^{-1}\frac{3}{4}$

$\therefore \sin^{-1}\frac{3}{5} = \tan^{-1}\frac{3}{4}$. . . . . (2)

Now we have :

L.H.S $= \sin^{-1}\frac{8}{17} + \sin^{-1}\frac{3}{5}$

$= \tan^{-1}\frac{8}{15} + \tan^{-1}\frac{3}{4}$

Using (1) and (2)

$$= \tan^{-1}\frac{\frac{8}{15}+\frac{3}{4}}{1-\frac{8}{15}\times\frac{3}{4}}$$

$= \tan^{-1}\left(\frac{32+45}{60-24}\right)$

$= \tan^{-1}\frac{77}{36}$ = R. H. S.

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Q5 Prove that $\cos^{-1}\frac{4}{5} + \cos^{-1}\frac{12}{13} = \cos^{-1}\frac{33}{65}$

Answer.

Let $\cos^{-1}\frac{4}{5} = x$. Then, $\cos x = \frac{4}{5} \Rightarrow \sin x = \sqrt{1-\left(\frac{4}{5}\right)^2} = \frac{3}{5}$

$\therefore \tan x = \frac{3}{4} \Rightarrow x = \tan^{-1}\frac{3}{4}$

$\therefore \cos^{-1}\frac{4}{5} = \tan^{-1}\frac{3}{4}$. . . . . (1)

Now, let $\cos^{-1}\frac{12}{13} = y$. Then, $\cos y = \frac{12}{13} \Rightarrow \sin y = \frac{5}{13}$

$\therefore \tan y = \frac{5}{12} \Rightarrow y = \tan^{-1}\frac{5}{12}$

$\therefore \cos^{-1}\frac{12}{13} = \tan^{-1}\frac{5}{12}$. . . . (2)

Let $\cos^{-1}\frac{33}{65} = z$. Then, $\cos z = \frac{33}{65} \Rightarrow \sin z = \frac{56}{65}$

$\therefore \tan z = \frac{56}{33} \Rightarrow z = \tan^{-1}\frac{56}{33}$

$\therefore \cos^{-1}\frac{33}{65} = \tan^{-1}\frac{56}{33}$. . . . . (3)

Now we will prove that:

L. H.S. $= \cos^{-1}\frac{4}{5} + \cos^{-1}\frac{12}{13}$ [Using (1) and (2)]

$= \tan^{-1}\frac{3}{4} + \tan^{-1}\frac{5}{12}$

$= \tan^{-1}\frac{\frac{3}{4}+\frac{5}{12}}{1-\frac{3}{4}\cdot\frac{5}{12}}$ $\left[\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}\right]$

$= \tan^{-1}\frac{36+20}{48-15}$

$= \tan^{-1}\frac{56}{33}$

$= \tan^{-1}\frac{56}{33}$ [by (3)]

=R.H.S

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Q6 Prove that
$\cos^{-1}\frac{12}{13} + \sin^{-1}\frac{3}{5} = \sin^{-1}\frac{56}{65}$

Answer.

Let $\sin^{-1}\frac{3}{5} = x$. Then, $\sin x = \frac{3}{5} \Rightarrow \cos x = \sqrt{1 - \left(\frac{3}{5}\right)^2} = \sqrt{\frac{16}{25}} = \frac{4}{5}$

$\therefore \tan x = \frac{3}{4} \Rightarrow x = \tan^{-1}\frac{3}{4}$

$\therefore \sin^{-1}\frac{3}{5} = \tan^{-1}\frac{3}{4} \ldots . (1)$

Now, let $\cos^{-1}\frac{12}{13} = y$. Then, $\cos y = \frac{12}{13} \Rightarrow \sin y = \frac{5}{13}$

$\therefore \tan y = \frac{5}{12} \Rightarrow y = \tan^{-1}\frac{5}{12}$

$\therefore \cos^{-1}\frac{12}{13} = \tan^{-1}\frac{5}{12} \ldots . . (2)$

Let $\sin^{-1}\frac{56}{65} = z$. Then, $\sin z = \frac{56}{65} \Rightarrow \cos z = \frac{33}{65}$

$\therefore \tan z = \frac{56}{33} \Rightarrow z = \tan^{-1}\frac{56}{33}$

$\therefore \sin^{-1}\frac{56}{65} = \tan^{-1}\frac{56}{33} \ldots . (3)$

L.H.S. $= \cos^{-1}\frac{12}{13} + \sin^{-1}\frac{3}{5}$

$= \tan^{-1}\frac{5}{12} + \tan^{-1}\frac{3}{4}$ [Using (1) and (2)]

$= \tan^{-1}\frac{\frac{5}{12} + \frac{3}{4}}{1 - \frac{5}{12} \cdot \frac{3}{4}}$ $\left[\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}\right]$

$= \tan^{-1}\frac{20 + 36}{48 - 15}$

$= \tan^{-1}\frac{56}{33}$

$= \sin^{-1}\frac{56}{65}$ = R. H. S

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Q7 Prove that $\tan^{-1}\frac{63}{16} = \sin^{-1}\frac{5}{13} + \cos^{-1}\frac{3}{5}$.

Answer.

Let $\sin^{-1}\frac{5}{13} = x$. Then, $\sin x = \frac{5}{13} \Rightarrow \cos x = \frac{12}{13}$

$\therefore \tan x = \frac{5}{12} \Rightarrow x = \tan^{-1}\frac{5}{12}$

$\therefore \sin^{-1}\frac{5}{13} = \tan^{-1}\frac{5}{12} \ldots\ldots (1)$

Using (1) and (2), we have

R.H.S. $= \sin^{-1}\frac{5}{13} + \cos^{-1}\frac{3}{5}$

$= \tan^{-1}\frac{5}{12} + \tan^{-1}\frac{4}{3} \ldots . (2)$

$$= \tan^{-1}\left(\frac{\frac{5}{12}+\frac{4}{3}}{1-\frac{5}{12}\times\frac{4}{3}}\right) \quad \left[\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}\right]$$

$$= \tan^{-1}\left(\frac{15+48}{36-20}\right)$$

$$= \tan^{-1}\frac{63}{16}$$

= L. H. S

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Q8 Prove that

$\tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{8} = \frac{\pi}{4}$

Answer.

$= \tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{8}$

$$= \tan^{-1}\left(\frac{\frac{1}{5}+\frac{1}{7}}{1-\frac{1}{5}\times\frac{1}{7}}\right) + \tan^{-1}\left(\frac{\frac{1}{3}+\frac{1}{8}}{1-\frac{1}{3}\times\frac{1}{8}}\right) \quad \left[\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}\right]$$

$$= \tan^{-1}\left(\frac{7+5}{35-1}\right) + \tan^{-1}\left(\frac{8+3}{24-1}\right)$$

$$= \tan^{-1}\frac{12}{34} + \tan^{-1}\frac{11}{23}$$

$$= \tan^{-1}\frac{6}{17} + \tan^{-1}\frac{11}{23}$$

$$= \tan^{-1}\left(\frac{\frac{6}{17}+\frac{11}{23}}{1-\frac{6}{17}\times\frac{11}{23}}\right)$$

$$= \tan^{-1}\left(\frac{138+187}{391-66}\right)$$

$$= \tan^{-1}\left(\frac{325}{325}\right) = \tan^{-1}1$$

$$= \frac{\pi}{4} = \text{R. H. S}$$

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Q9 Prove that

$\tan^{-1}\sqrt{x} = \frac{1}{2}\cos^{-1}\frac{1}{1+x}, x \in [0, 1]$

Answer.

Let $x = \tan^2\theta$. Then, $\sqrt{x} = \tan\theta \Rightarrow \theta = \tan^{-1}\sqrt{x}$

$\therefore \frac{1-x}{1+x} = \frac{1-\tan^2\theta}{1+\tan^2\theta} = \cos 2\theta$

Now, we have:

R. H.S. $= \frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2}\cos^{-1}(\cos 2\theta) = \frac{1}{2}\times 2\theta = \theta = \tan^{-1}\sqrt{x} =$ L.H.S.

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Q10 Prove that

$\cot^{-1}\left(\frac{\sqrt{1+\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right) = \frac{x}{2}, x \in \left(0, \frac{\pi}{4}\right)$

Answer.

Consider $\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}$

$= \frac{(\sqrt{1+\sin x}+\sqrt{1-\sin x})^2}{(\sqrt{1+\sin x})^2-(\sqrt{1-\sin x})^2}$ (by rationalizing)

$= \frac{(1+\sin x)+(1-\sin x)+2\sqrt{(1+\sin x)(1-\sin x)}}{1+\sin x-1+\sin x}$

$= \frac{2\left(1+\sqrt{1-\sin^2 x}\right)}{2\sin x} = \frac{1+\cos x}{\sin x} = \frac{2\cos^2\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}$

$= \cot\frac{x}{2}$

$\therefore L \cdot H \cdot S = \cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right) = \cot^{-1}\left(\cot\frac{x}{2}\right) = \frac{x}{2} = R.H.S.$

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Q11 Prove that

$\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right) = \frac{\pi}{4} - \frac{1}{2}\cos^{-1}x, \ -\frac{1}{\sqrt{2}} \le x \le 1$[Hint: Put $x = \cos 2\theta$]

Answer.

Put $x = \cos 2\theta$ so that $\theta = \frac{1}{2}\cos^{-1}x$. Then, we have:

L. H.S. $= \tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)$

$= \tan^{-1}\left(\frac{\sqrt{1+\cos 2\theta}-\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}\right)$

$= \tan^{-1}\left(\frac{\sqrt{2\cos^2\theta}-\sqrt{2\sin^2\theta}}{\sqrt{2\cos^2\theta}+\sqrt{2\sin^2\theta}}\right)$

$= \tan^{-1}\left(\frac{\sqrt{2}\cos\theta-\sqrt{2}\sin\theta}{\sqrt{2}\cos\theta+\sqrt{2}\sin\theta}\right)$

$= \tan^{-1}\left(\frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}\right) = \tan^{-1}\left(\frac{1-\tan\theta}{1+\tan\theta}\right)$

$= \tan^{-1}1 - \tan^{-1}(\tan\theta)\left[\tan^{-1}\left(\frac{x-y}{1+xy}\right) = \tan^{-1}x - \tan^{-1}y\right]$

$= \frac{\pi}{4} - \theta = \frac{\pi}{4} - \frac{1}{2}\cos^{-1}x =$ R. H. S.

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Q12 Prove that

$\frac{9\pi}{8} - \frac{9}{4}\sin^{-1}\frac{1}{3} = \frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3}$

Answer.

L.H.S. $= \frac{9\pi}{8} - \frac{9}{4}\sin^{-1}\frac{1}{3}$

$= \frac{9}{4}\left(\frac{\pi}{2} - \sin^{-1}\frac{1}{3}\right)$

$= \frac{9}{4}\left(\cos^{-1}\frac{1}{3}\right) \quad \ldots(1)\left[\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}\right]$

Now, let $\cos^{-1}\frac{1}{3} = x$. Then, $\cos x = \frac{1}{3} \Rightarrow \sin x = \sqrt{1 - \left(\frac{1}{3}\right)} = \frac{2\sqrt{2}}{3}$

$\therefore x = \sin^{-1}\frac{2\sqrt{2}}{3} \Rightarrow \cos^{-1}\frac{1}{3} = \sin^{-1}\frac{2\sqrt{2}}{3}$

$\therefore$ L. H. S. $= \frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3} =$ R. H. S.

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Q13 Solve the following equations:

$2\tan^{-1}(\cos x) = \tan^{-1}(2\ cosec x)$

Answer.

$2\tan^{-1}(\cos x) = \tan^{-1}(2\ cosec x)$

$\Rightarrow \tan^{-1}\left(\frac{2\cos x}{1-\cos^2 x}\right) = \tan^{-1}(2\ cosec x) \quad \left[2\tan^{-1}x = \tan^{-1}\frac{2x}{1-x^2}\right]$

$\Rightarrow \frac{2\cos x}{1-\cos^2 x} = 2\,cosecx$

$\Rightarrow \frac{2\cos x}{\sin^2 x} = \frac{2}{\sin x}$

$\Rightarrow \cos x = \sin x$

$\Rightarrow \tan x = 1$

$x = \frac{\pi}{4}$

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Q14 Solve the following equations:$\tan^{-1}\frac{1-x}{1+x} = \frac{1}{2}\tan^{-1}x,\ (x > 0)$

Answer. $\tan^{-1}\frac{1-x}{1+x} = \frac{1}{2}\tan^{-1}x$

$\Rightarrow \tan^{-1}1 - \tan^{-1}x = \frac{1}{2}\tan^{-1}x \quad \left[\tan^{-1}x - \tan^{-1}y = \tan^{-1}\frac{x-y}{1+xy}\right]$

$\Rightarrow \frac{\pi}{4} = \frac{3}{2}\tan^{-1}x$

$\Rightarrow \tan^{-1}x = \frac{\pi}{6}$

$\Rightarrow x = \tan\frac{\pi}{6}$

$\therefore x = \frac{1}{\sqrt{3}}$

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Q15 $\sin\left(\tan^{-1}x\right),\ |x| < 1$

(A) $\frac{x}{\sqrt{1-x^2}}$

(b) $\frac{1}{\sqrt{1-x^2}}$

(C) $\frac{1}{\sqrt{1+x^2}}$

(D) $\frac{x}{\sqrt{1+x^2}}$

Answer. Let $\tan^{-1}x = y$.Then $\tan y = x \Rightarrow \sin y = \frac{x}{\sqrt{1+x^2}}$

$\therefore y = \sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right) \Rightarrow \tan^{-1}x = \sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$

$\therefore \sin\left(\tan^{-1}x\right) = \sin\left(\sin^{-1}\frac{x}{\sqrt{1+x^2}}\right) = \frac{x}{\sqrt{1+x^2}}$

The correct answer is D.

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Q16 $\sin^2(1-x) - 2\sin^{-1}x = \frac{\pi}{2}$ then x is equalto

(A) 0, $\frac{i}{2}$

(B) 1, $\frac{1}{2}$

(C) 0

(D) $\frac{1}{2}$

Answer.

$\sin^{-1}(1-x) - 2\sin^{-1}x = \frac{\pi}{2}$

$\Rightarrow -2\sin^{-1}x = \frac{\pi}{2} - \sin^{-1}(1-x)$

$\Rightarrow -2\sin^{-1}x = \cos^{-1}(1-x). \ldots . (1)$

Let $\sin^{-1}x = \theta \Rightarrow \sin\theta = x \Rightarrow \cos\theta = \sqrt{1-x^2}$

$\therefore \theta = \cos^{-1}\left(\sqrt{1-x^2}\right)$

$\therefore \quad \therefore \sin^{-1}x = \cos^{-1}\left(\sqrt{1-x^2}\right)$

Therefore, from equation (1), we have

$-2\cos^{-1}\left(\sqrt{1-x^2}\right) = \cos^{-1}(1-x)$

Put $x = \sin y$. Then, we have:

$-2\cos^{-1}\left(\sqrt{1-\sin^2 y}\right) = \cos^{-1}(1-\sin y)$

$\Rightarrow -2\cos^{-1}(\cos y) = \cos^{-1}(1-\sin y)$

$-2\cos^{-1}\left(\sqrt{1-\sin^2 y}\right) = \cos^{-1}(1-\sin y)$

$\Rightarrow -2\cos^{-1}(\cos y) = \cos^{-1}(1-\sin y)$

$\Rightarrow -2y = \cos^{-1}(1-\sin y)$

$\Rightarrow 1-\sin y = \cos(-2y) = \cos 2y$

$\Rightarrow 1-\sin y = 1-2\sin^2 y$

$\Rightarrow \sin y(2\sin y-1) = 0$

$\Rightarrow \sin y(2\sin y-1) = 0$

$\Rightarrow \sin y(2\sin y-1) = 0$

$\Rightarrow \sin y = 0 \text{ or } \frac{1}{2}$

$\therefore x = 0 \text{ or } x = \frac{1}{2}$

But, when $x = \frac{1}{2}$, , it can be observed that:

$$\text{L.H.S.} = \sin^{-1}\left(1-\frac{1}{2}\right) - 2\sin^{-1}\frac{1}{2}$$

$$= \sin^{-1}\left(\frac{1}{2}\right) - 2\sin^{-1}\frac{1}{2}$$

$$= -\sin^{-1}\frac{1}{2}$$

$$= -\frac{\pi}{6} \neq \frac{\pi}{2} \neq \text{R. H. S}$$

$\therefore x = \frac{1}{2}$ is not the solution of the given equation.

Thus, $x = 0$

Hence, the correct answer is C .

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Q17 $\tan^{-1}\left(\frac{x}{7}\right) - \sin^{-1}\frac{x-y}{x+y}$, is equal to:

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{4}$ (D) $\frac{-3\pi}{4}$

Answer. $\tan^{-1}\left(\frac{x}{y}\right) - \tan^{-1}\frac{x-y}{x+y}$

$$= \tan^{-1}\left[\frac{\frac{x}{y}-\frac{x-y}{x+y}}{1+\left(\frac{x}{y}\right)\left(\frac{x-y}{x+y}\right)}\right]\left[\tan^{-1}y-\tan^{-1}y=\tan^{-1}\frac{x-y}{1+xy}\right]$$

$$= \tan^{-1}\left[\frac{\frac{x(x+y)-y(x-y)}{y(x+y)}}{\frac{y(x+y)+x(x-y)}{y(x+y)}}\right]$$

$$= \tan^{-1}\left(\frac{x^2+xy-xy+y^2}{xy+y^2+x^2-xy}\right)$$

$$= \tan^{-1}\left(\frac{x^2+y^2}{x^2+y^2}\right) = \tan^{-1}1 = \frac{\pi}{4}$$

Here the correct answer is C.

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