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School of Technology and Computer Science

Instructions for the written test

There are two streams in the School of Technology and Computer Science:

1. Computer Science.
2. Systems Science.

Topics covered in the two streams, as well as some sample questions, are given below.

The question paper will have three parts. Part A is common to both the streams. It will test the general mathematical aptitude of the candidate. There is no prescribed sylabus for Part A. Part B will be oriented towards the topics listed under 'Computer Science' below; and Part C will be oriented towards topics listed under 'Systems Science' below. Only one of Parts B, C, should be attempted. The duration of the written test will be **three hours**. The test will be of **multiple choice type**, with negative marking for incorrect answers. The use of calculators will not be allowed during the test.

Computer Science

1. Discrete Mathematics: Sets and Relations, Combinatorics (Counting) and Elementary Probability Theory, Graph Theory, Propositional and Predicate Logic.
2. Formal Languages, Automata Theory and Computability.
3. Data Structures and Algorithms: Arrays, Lists and Trees, Sorting and Searching, Graph algorithms, Complexity of problems and NP-completeness.
4. Fundamentals of Programming Languages and Compilers: Control structures, Parameter passing mechanisms, Recursion, Parsing and type checking, Memory management.
5. Operating Systems and Concurrency
6. Switching Theory and Digital Circuits
7. Theory of Databases

Sample Questions [Computer Science]

1. A function $f : \{0,1\}^n \to \{0,1\}$ is called *symmetric* if for every $x_1, x_2, \ldots, x_n \in \{0,1\}$ and every permutation σ of $\{1,2,\ldots,n\}$, we have

$$f(x_1, x_2, \ldots, x_n) = f(x_{\sigma(1)}, x_{\sigma(2)}, \ldots, x_{\sigma(n)}).$$

   The number of such symmetric functions is:

   (a) 2^{n+1} (b) 2^n (c) $2^{2^n}/n!$ (d) 2^{2^n} (e) n!

2. Let r, s and t be regular expressions. Which of the following is wrong?

   (a) $(r+s)^* = (r^*s^*)^*$ (b) $r(s+t) = rs + rt$
   (c) $(r+s)^* = (s+r)^*$ (d) $(rs+r)^*r = r(sr+r)^*$ (e) All are correct.

3. Consider the following program

```
x:=0; y:=1; z:=1;
while y <= N do
begin
x:=x+1; y:=y+z+2; z:=z+2;
end
```

   Which of the following holds on termination of the program?

   (a) $(x+1)^2 = N$ (b) $x = \sqrt{N}$
   (c) $x^2 = N$ (d) $x^2 \leq N < (x+1)^2$ (e) $x^2 < N \leq (x+1)^2$.

4. The maximum height of a rooted binary tree (all nodes have either two children or none) with N nodes is

   (a) N (b) $\log N$ (c) $(N-1)/2$ (d) $(N^2)/2$ (e) $N(N-1)/2$.

5. If a graph G has n vertices and m edges then the depth first traversal of G can be carried out in time

   (a) $O(n+m)$ (b) $O(nm)$ but not $O(n+m)$
   (c) $O(n^2)$ but not $O(n+m)$ (d) $O(n)$ (e) $O(m)$

Systems Science

1. Engineering Mathematics: Complex Analysis, Linear Algebra, Elementary Numerical Analysis, Basic Optimization Theory and Algorithms, Introduction to Probability Theory and Statistics.

2. Electrical and Computer Sciences: Introduction to Signals and Linear Systems Analysis, Control Systems, Digital Signal Processing, Basic Circuit Theory, Introduction to Digital Communications, Digital Computer Fundamentals, Introduction to Computer Programming.

Sample Questions [Systems Science]

1. The probability density of a random variable is

$$f(x) = ax^2 \exp^{-kx} \ (k > 0, 0 \leq x \leq \infty)$$

Then, the coefficient a equals

(a) $k^3/2$ (b) k^3 (c) k^2 (d) k (e) $2k/\pi$.

2. Discrete sequences $x(n)$ is non-zero for $0 \leq n \leq N_x$ and $y(n)$ for $0 \leq n \leq N_y$. The sequence $z(n)$ is obtained by convolving $x(n)$ and $y(n)$. $z(n)$ assumes nonzero values for $N_1 \leq n \leq N_2$, where N_1 and N_2 can be expressed in terms of N_x and N_y as,

(a) $N_1 = 0; N_2 = MAX(N_x, N_y)$
(b) $N_1 = N_x; N_2 = N_y$
(c) $N_1 = MIN(N_x, N_y); N_2 = N_x + N_y$
(d) $N_1 = 0; N_2 = N_x + N_y$
(e) $N_1 = MIN(N_x, N_y); N_2 = MAX(N_x, N_y)$

3. This is a portion of FORTRAN-77 program for assigning values to a $N \times N$ matrix **A**:

```
DO I=1,N
    DO J=I,N
        A(I,J) = ABS(I−J)+1
    ENDDO
ENDDO
```

What is the matrix **A** called ?

(a) Anti-symmetric (b) Sparse (c) Upper triangular (d) Toeplitz (e) Irregular.

4. $log_b(log_b x)$ equals

(a) $(\ln \ln x - \ln \ln b)/\ln b$
(b) $(\ln x - \ln b)/\ln b$
(c) $(\ln \ln x - \ln \ln b)$
(d) $(\ln x - \ln b)/[(\ln x)(\ln b)]$
(e) None of the Above.

5. The Laplace Transform $G(s)$ of the transfer function of a linear time invariant system is given by

$$G(s) = \frac{1}{(s+a)^2 + b^2}$$

For the system to be stable it is necessary that

(a) $a < 0$ (b) $a \geq 0$ (c) $a = b$ (d) $b = 0$ (e) $a = -b$.