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Exercise 10.3

Q.1

(i) $\frac{\sin 20^\circ}{\cos 70^\circ}$ ~~$= \sin 20^\circ$~~

$= \frac{\sin 20^\circ}{\sin(90^\circ - 70^\circ)} = \frac{\sin 20^\circ}{\sin 20^\circ} = \boxed{1}$

(ii) $\frac{\cos 19^\circ}{\sin 71^\circ}$

$= \frac{\cos 19^\circ}{\cos(90^\circ - 71^\circ)} = \frac{\cos 19^\circ}{\cos 19^\circ} = \boxed{1}$

(iii) $\frac{\sin 21^\circ}{\cos 69^\circ}$

$= \frac{\sin 21^\circ}{\sin(90^\circ - 69^\circ)} = \frac{\sin 21^\circ}{\sin 21^\circ} = \boxed{1}$

(iv) $\frac{\tan 10^\circ}{\cot 80^\circ}$

$= \frac{\tan 10^\circ}{\tan(90^\circ - 80^\circ)} = \frac{\tan 10^\circ}{\tan 10^\circ} = \boxed{1}$

(v) $\frac{\sec 11^\circ}{\operatorname{cosec} 79^\circ}$

$= \frac{\sec 11^\circ}{\sec(90^\circ - 79^\circ)} = \frac{\sec 11^\circ}{\sec 11^\circ} = \boxed{1}$

Q.2

(i) $\left(\frac{\sin 49^\circ}{\cos 41^\circ}\right)^2 + \left(\frac{\cos 41^\circ}{\sin 49^\circ}\right)^2$

$= \left(\frac{\sin 49^\circ}{\sin(90^\circ - 41^\circ)}\right)^2 + \left(\frac{\cos 41^\circ}{\cos(90^\circ - 49^\circ)}\right)^2$

$= \left(\frac{\sin 49^\circ}{\sin 49^\circ}\right)^2 + \left(\frac{\cos 41^\circ}{\cos 41^\circ}\right)^2$

$= (1)^2 + (1)^2 = 1 + 1 = \boxed{2}$

(ii) $\cos 48^\circ - \sin 42^\circ$

$= \cos 48^\circ - \cos(90^\circ - 42^\circ)$

$= \cos 48^\circ - \cos 48^\circ$

$= \boxed{0}$

(iii) $\frac{\cot 40^\circ}{\tan 50^\circ} - \frac{1}{2}\left(\frac{\cos 35^\circ}{\sin 55^\circ}\right)$

$= \frac{\cot 40^\circ}{\cot(90^\circ - 50^\circ)} - \frac{1}{2}\left[\frac{\cos 35^\circ}{\cos(90^\circ - 55^\circ)}\right]$

$= \frac{\cot 40^\circ}{\cot 40^\circ} - \frac{1}{2}\left(\frac{\cos 35^\circ}{\cos 35^\circ}\right)$

$= 1 - \frac{1}{2} = \boxed{\frac{1}{2}}$

(iv) $\left(\frac{\sin 27^\circ}{\cos 63^\circ}\right)^2 - \left(\frac{\cos 63^\circ}{\sin 27^\circ}\right)^2$

$= \left[\frac{\sin 27^\circ}{\sin(90^\circ - 63^\circ)}\right]^2 - \left[\frac{\cos 63^\circ}{\cos(90^\circ - 27^\circ)}\right]^2$

$= \left(\frac{\sin 27^\circ}{\sin 27^\circ}\right)^2 - \left(\frac{\cos 63^\circ}{\cos 63^\circ}\right)^2$

$= (1)^2 - (1)^2 = 1 - 1 = \boxed{0}$

(v) $\frac{\tan 35^\circ}{\cot 55^\circ} + \frac{\cot 78^\circ}{\tan 12^\circ} - 1$

$= \frac{\tan 35^\circ}{\tan(90^\circ - 55^\circ)} + \frac{\cot 78^\circ}{\cot(90^\circ - 12^\circ)} - 1$

$= \frac{\tan 35^\circ}{\tan 35^\circ} + \frac{\cot 78^\circ}{\cot 78^\circ} - 1$

$= 1 + 1 - 1 = 2 - 1 = \boxed{1}$

(vi) $\frac{\sec 70^\circ}{\operatorname{cosec} 20^\circ} + \frac{\sin 59^\circ}{\cos 31^\circ}$

$= \frac{\sec 70^\circ}{\sec(90^\circ - 20^\circ)} + \frac{\sin 59^\circ}{\sin(90^\circ - 31^\circ)}$

$= \frac{\sec 70^\circ}{\sec 70^\circ} + \frac{\sin 59^\circ}{\sin 59^\circ} = 1 + 1 = \boxed{2}$

(vii) $\operatorname{cosec} 31^\circ - \sec 59^\circ$

$= \operatorname{cosec} 31^\circ - \operatorname{cosec}(90^\circ - 59^\circ)$

$= \operatorname{cosec} 31^\circ - \operatorname{cosec} 31^\circ = \boxed{0}$

(viii) $(\sin 72^\circ + \cos 18^\circ)(\sin 72^\circ - \cos 18^\circ)$

$= (\sin 72^\circ)^2 - (\cos 18^\circ)^2$

$= (\sin 72^\circ)^2 - [\sin(90^\circ - 18^\circ)]^2$

$= \sin^2 72^\circ - \sin^2 72^\circ$

$= \boxed{0}$

(ix) $\sin 35^\circ \sin 55^\circ - \cos 35^\circ \cos 55^\circ$

$= \sin 35^\circ \sin(90^\circ - 55^\circ) - \cos 35^\circ \cos(90^\circ - 55^\circ)$

$= \sin 35^\circ \sin 35^\circ$

(ix) $\sin 35^\circ \sin 55^\circ - \cos 35^\circ \cos 55^\circ$

$= \sin 35^\circ \cos(90^\circ - 55^\circ) - \cos 35^\circ \sin(90^\circ - 55^\circ)$

$= \sin 35^\circ \cos 35^\circ - \cos 35^\circ \sin 35^\circ$

$= \boxed{0}$

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Q.3

(i) $\sin 59° + \cos 56°$

$= \cos(90° - 59°) + \sin(90° - 56°)$

$= \cancel{\cos(31°)}$ $\boxed{\cos 31° + \sin 34°}$

(ii) $\tan 65° + \cot 49°$

$= \cot(90° - 65°) + \tan(90° - 49°)$

$= \boxed{\cot 25° + \tan 41°}$

(iii) $\sec 76° + \operatorname{cosec} 52°$

$= \operatorname{cosec}(90° - 76°) + \sec(90° - 52)$

$= \boxed{\operatorname{cosec} 14° + \sec 38°}$

(iv) $\cos 78° + \sec 78°$

$= \sin(90° - 78°) + \operatorname{cosec}(90° - 78°)$

$= \boxed{\sin 12° + \operatorname{cosec} 12°}$

(v) $\operatorname{cosec} 54° + \sin 72°$

$= \sec(90° - 54) + \cos(90° - 72°)$

$= \boxed{\sec 36° + \cos 18°}$

(vi) $\cot 85° + \cos 75°$

$= \tan(90° - 85°) + \sin(90° - 75°)$

$= \boxed{\tan 5° + \sin 15°}$

(vii) $\sin 67° + \cos 75°$

$= \cos(90° - 67°) + \sin(90° - 75°)$

$= \boxed{\cos 23° + \sin 15°}$

Q.4 $\cos 75° + \cot 75°$

$= \sin(90° - 75°) + \tan(90° - 75°)$

$= \boxed{\sin 15° + \tan 15°}$

Q.5 $\sin 3A = \cos(A - 26°)$

$\sin 3A = \sin(90° - A + 26°)$

$3A = 116° - A$

$3A + A = 116°$

$A = \frac{116°}{4}\ 29$

$\boxed{A = 29°}$

Q.6 A, B, C are interior angles of ΔABC,

$\therefore \angle A + \angle B + \angle C = 180°$ (Angle sum prop of Δ)

(i) $\tan\left(\frac{C+A}{2}\right) = \cot\frac{B}{2}$

$\angle A + \angle B + \angle C = 180°$

$\frac{A}{2} + \frac{B}{2} + \frac{C}{2} = 90°$ —— (1)

$\tan\left(\frac{C}{2} + \frac{A}{2}\right) = \tan\left(90° - \frac{B}{2}\right)$

$\frac{C}{2} + \frac{A}{2} = 90° - \frac{B}{2}$

$\frac{A}{2} + \frac{B}{2} + \frac{C}{2} = 90°$ —— (2)

(1) & (2) are equal.

Hence proved

(ii) $\sin\left(\frac{B+C}{2}\right) = \cos\left(\frac{A}{2}\right)$

$\sin\left(\frac{B}{2}+\frac{C}{2}\right) = \sin\left(90^\circ - \frac{A}{2}\right)$

$\frac{B}{2}+\frac{C}{2} = 90^\circ - \frac{A}{2}$

$\frac{A}{2}+\frac{B}{2}+\frac{C}{2} = 90^\circ$ —— ①

$\angle A + \angle B + \angle C = 180^\circ$ (Angle sum prop. of Δ)

$\frac{A}{2}+\frac{B}{2}+\frac{C}{2} = 90^\circ$ —— ②

① & ② are equal.

Hence, proved

(iii) $\angle A = 90^\circ$ (given)

$\angle A + \angle B + \angle C = 180^\circ$ (Angle sum property of Δ).

$\angle B + \angle C = 180^\circ - \angle A$

$\frac{B+C}{2} = \frac{180^\circ}{2} - \frac{A}{2}$

$\tan\left(\frac{B+C}{2}\right) = \tan\left(90^\circ - \frac{A}{2}\right)$

$\tan\left(\frac{B+C}{2}\right) = \cot\left(\frac{A}{2}\right) = \cot\left(\frac{90^\circ}{2}\right) = \cot 45^\circ$

$\therefore \tan\left(\frac{B+C}{2}\right) = \boxed{1}$ $(\cot 45^\circ = 1)$

Q.7 (i) $\tan 20^\circ \tan 35^\circ \tan 45^\circ \tan 55^\circ \tan 70^\circ = 1$

LHS = $\tan 20° \tan 35° \tan 45° \tan 55° \tan 70°$

$= \cot(90°-20°) \cot(90°-35°) \tan 45° \tan 55° \tan 70°$

$= \cot 70° \cot 55° \tan 45° \tan 55° \tan 70°$

$= \tan 45°$

$= \boxed{1} = RHS$

Hence proved

(ii) $\sin 48° \sec 42° + \cos 48° \operatorname{cosec} 42° = 2$

LHS = $\sin 48° \sec 42° + \cos 48° \operatorname{cosec} 42°$

$= \sin 48° \operatorname{cosec}(90°-42°) + \cos 48° \sec(90°-42°)$

$= \sin 48° \operatorname{cosec} 48° + \cos 48° \sec 48°$

$= 1 + 1 = \boxed{2} = RHS$

Hence proved

(iii) $\frac{\sin 70°}{\cos 20°} + \frac{\operatorname{cosec} 20°}{\sec 70°} - 2\cos 70° \operatorname{cosec} 20° = 0$

LHS = $\frac{\sin 70°}{\cos 20°} + \frac{\operatorname{cosec} 20°}{\sec 70°} - 2\cos 70° \operatorname{cosec} 20°$

$= \frac{\sin 70°}{\sin(90°-20°)} + \frac{\operatorname{cosec} 20°}{\sec(90°-20°)} - 2\sin(90°-20°) \operatorname{cosec} 20°$

$= \frac{\sin 70°}{\sin 70°} + \frac{\operatorname{cosec} 20°}{\operatorname{cosec} 20°} - 2\sin 70° \operatorname{cosec} 20°$

$= 1 + 1 - 2 = 2 - 2 = \boxed{0} = RHS$

Hence, proved

(iv) $\frac{\cos 80°}{\sin 10°} + \cos 59° \operatorname{cosec} 31° = 2$

LHS = $\frac{\cos 80°}{\sin 10°} + \cos 59° \operatorname{cosec} 31°$

$= \frac{\cos 80^\circ}{\cos(90^\circ - 10^\circ)} + \cos 59^\circ \sec(90^\circ - 31^\circ)$

$= \frac{\cancel{\cos 80^\circ}}{\cancel{\cos 80^\circ}} + \cancel{\cos 59^\circ \sec 59^\circ}$

$= 1 + 1 = \boxed{2} = \text{RHS}$

Hence, proved

Q.8(i) $\sin\theta \sin(90^\circ - \theta) - \cos\theta \cos(90^\circ - \theta) = 0$

LHS $= \sin\theta \sin(90^\circ - \theta) - \cos\theta \cos(90^\circ - \theta)$

$= (\sin\theta \cos\theta) - (\cos\theta \sin\theta)$

$= \boxed{0} = \text{RHS}$

Hence, proved

(ii) $\frac{\cos(90^\circ - \theta) \sec(90^\circ - \theta) \tan\theta}{\operatorname{cosec}(90^\circ - \theta) \sin(90^\circ - \theta) \cot(90^\circ - \theta)} + \frac{\tan(90^\circ - \theta)}{\cot\theta} = 2$

LHS $= \frac{\cos(90^\circ - \theta) \sec(90^\circ - \theta) \tan\theta}{\operatorname{cosec}(90^\circ - \theta) \sin(90^\circ - \theta) \cot(90^\circ - \theta)} + \frac{\tan(90^\circ - \theta)}{\cot\theta}$

$= \frac{\cancel{\sin\theta} \operatorname{cosec}\theta \tan\theta}{\cancel{\sec\theta} \cancel{\cos\theta} \tan\theta} + \frac{\cot\theta}{\cot\theta} = \frac{\cancel{\tan\theta}}{\cancel{\tan\theta}} + \frac{\cancel{\cot\theta}}{\cancel{\cot\theta}}$

$= 1 + 1 = \boxed{2} = \text{RHS}$

Hence, proved

(iii) $\frac{\tan(90^\circ - A) \cot A}{\operatorname{cosec}^2 A} - \cos^2 A = 0$

LHS $= \frac{\tan(90^\circ - A) \cot A}{\operatorname{cosec}^2 A} - \cos^2 A$

$= \frac{\cot A \cot A}{\operatorname{cosec}^2 A} - \cos^2 A$

$$= \frac{\cos^2 A}{\sin^2 A} - \cos^2 A$$

$$\frac{1}{\sin^2 A}$$

$$= \cos^2 A - \cos^2 A$$

$$= \boxed{0} = \text{RHS}$$

Hence, proved

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Q.8
(iv) $\frac{\cos(90°-A)\cdot\sin(90°-A)}{\tan(90°-A)} = \sin^2 A$

LHS $= \frac{\cos(90°-A)\sin(90°-A)}{\tan(90°-A)}$

$= \frac{\sin A \cos A}{\cot A} = \frac{\sin A \cos A}{\frac{\cos A}{\sin A}}$

$= \frac{\sin A \cancel{\cos A} \sin A}{\cancel{\cos A}} = \boxed{\sin^2 A} = RHS$

Hence, proved

(v) $\sin(50°+\theta) - \cos(40°-\theta) + \tan 1° \tan 10° \tan 20° \tan 70° \tan 80° \tan 89° = 1$

LHS $= \sin(50°+\theta) - \cos(40°-\theta) + \tan 1° \tan 10° \tan 20° \tan 70° \tan 80° \tan 89°$

$= \cos(90°-50°-\theta) - \cos(40°-\theta) + \tan 1° \tan 10° \tan 20° \cot(90°-70°) \cot(90°-80°) \cot(90°-89°)$

$= \cancel{\cos(40°-\theta)} - \cancel{\cos(40°-\theta)} + \tan 1° \tan 10° \tan 20° \cot 20° \cot 10° \cot 1°$

$= 1 \times 1 \times 1 = \boxed{1} = RHS$

Hence, proved

Q.9
(vi) $\frac{2}{3}(\cos^4 30° - \sin^4 45°) - 3(\sin^2 60° - \sec^2 45°) + \frac{1}{4}\cot^2 30°$

$$= \frac{2}{3}\left[\left(\frac{\sqrt{3}}{2}\right)^4 - \left(\frac{1}{\sqrt{2}}\right)^4\right] - 3\left[\left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{\sqrt{2}}\right)^4\right] + \frac{1}{4}(\sqrt{3})^2$$

$$= \frac{2}{3}\left(\frac{9}{16} - \frac{1}{4}\right) - 3\left(\frac{3}{4} - \frac{1}{2}\right) + \frac{1}{4}(3)$$

$$= \frac{2}{3}\left(\frac{9-4}{16}\right) - 3\left(\frac{3-2}{4}\right) + \frac{3}{4}$$

$$= \frac{2}{3}\left(\frac{5}{16_8}\right) - 3\left(\frac{1}{4}\right) + \frac{3}{4}$$

$$= \frac{5}{24} + \frac{15}{4} + \frac{3}{4} = \frac{5+90+18}{24} = \boxed{\frac{113}{24}}$$

(ii) $4(\sin^4 30° + \cos^4 60°) - \frac{2}{3}(\sin^2 60° - \cos^2 45°) + \frac{1}{2}\tan^2 60°$

$$= 4\left[\left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^4\right] - \frac{2}{3}\left[\left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{\sqrt{2}}\right)^2\right] + \frac{1}{2}(\sqrt{3})^2$$

$$= 4\left(\frac{1}{16} + \frac{1}{16}\right) - \frac{2}{3}\left(\frac{3}{4} - \frac{1}{2}\right) + \frac{1}{2}(3)$$

$$= 4\left(\frac{2}{16_8}\right) - \frac{2}{3}\left(\frac{3-2}{4_2}\right) + \frac{3}{2}$$

$$= \frac{1}{2} - \frac{1}{6} + \frac{3}{2} = \frac{3-1+9}{6} = \boxed{\frac{11}{6}}$$

(iii) $\frac{\sin 50°}{\cos 40°} + \frac{\operatorname{cosec} 40°}{\sec 50°} - 4\cos 50° \operatorname{cosec} 40°$

$$= \frac{\sin 50^\circ}{\sin(90^\circ - 40^\circ)} + \frac{\operatorname{cosec} 40^\circ}{\operatorname{cosec}(90^\circ - 50^\circ)} - 4\cos 50^\circ \sec(90^\circ - 40^\circ)$$

$$= \frac{\sin 50^\circ}{\sin 50^\circ} + \frac{\operatorname{cosec} 40^\circ}{\operatorname{cosec} 40^\circ} - 4\cos 50^\circ \sec 50^\circ$$

$$= 1 + 1 - 4(1) = 2 - 4 = \boxed{-2}$$

(iv) $\tan 35^\circ \tan 40^\circ \tan 45^\circ \tan 50^\circ \tan 55^\circ$

$$= \tan 35^\circ \tan 40^\circ \tan 45^\circ \cot(90^\circ - 50^\circ) \cot(90^\circ - 55^\circ)$$

$$= \tan 35^\circ \tan 40^\circ \tan 45^\circ \cot 40^\circ \cot 35^\circ$$

$$= \tan 45^\circ = \boxed{1}$$

(v) $\operatorname{cosec}(65^\circ + \theta) - \sec(25^\circ - \theta) - \tan(55^\circ - \theta) + \cot(35^\circ + \theta)$

$$= \operatorname{cosec}(65^\circ + \theta) - \operatorname{cosec}(90^\circ - 25^\circ + \theta) - \tan(55^\circ - \theta) + \tan(90^\circ - 35^\circ - \theta)$$

$$= \operatorname{cosec}(65^\circ + \theta) - \operatorname{cosec}(65^\circ + \theta) - \tan(55^\circ - \theta) + \tan(55^\circ - \theta)$$

$$= \boxed{0}$$

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Q.9 (vi) $\tan 7^\circ \tan 23^\circ \tan 60^\circ \tan 67^\circ \tan 83^\circ$

$= \tan 7^\circ \tan 23^\circ \tan 60^\circ \cot(90^\circ - 67^\circ) \cot(90^\circ - 83^\circ)$

$= \tan 7^\circ \tan 23^\circ \tan 60^\circ \cot 23^\circ \cot 7^\circ$

$= \tan 60^\circ = \boxed{\sqrt{3}}$

(vii) $\dfrac{2\sin 68^\circ}{\cos 22^\circ} - \dfrac{2\cot 15^\circ}{5\tan 75^\circ} - \dfrac{3\tan 45^\circ \tan 20^\circ \tan 40^\circ \tan 50^\circ \tan}{5}$

$= \dfrac{2\sin 68^\circ}{\sin(90^\circ - 22^\circ)} - \dfrac{2\cot 15^\circ}{5\cot(90^\circ - 75^\circ)} - \dfrac{3 \times 1 \times \tan 20^\circ \tan 40^\circ \cot(90^\circ -}{\cot(90^\circ - 70^\circ)}$

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$= \dfrac{2\sin 68^\circ}{\sin 68^\circ} - \dfrac{2\cot 15^\circ}{5\cot 15^\circ} - \dfrac{3\tan 20^\circ \tan 40^\circ \cot 40^\circ \cot 20^\circ}{5}$

$= 2 - \dfrac{2}{5} - \dfrac{3}{5} = 2 - \dfrac{5}{5} = 2 - 1 = \boxed{1}$

(viii) $\dfrac{3\cos 55^\circ}{7\sin 35^\circ} - \dfrac{4(\cos 70^\circ \operatorname{cosec} 20^\circ)}{7(\tan 5^\circ \tan 25^\circ \tan 45^\circ \tan 65^\circ \tan 85^\circ)}$

$= \dfrac{3\cos 55^\circ}{7\cos(90^\circ - 35^\circ)} - \dfrac{4[\cos 70^\circ - \sec(90^\circ - 20^\circ)]}{7[\tan 5^\circ \tan 25^\circ \tan 45^\circ \cot(90^\circ - 65^\circ) \cot(90^\circ - 85^\circ)}$

$= \dfrac{3\cos 55^\circ}{7\cos 55^\circ} - \dfrac{4(\cos 70^\circ - \sec 70^\circ)}{7(\tan 5^\circ \tan 25^\circ \tan 45^\circ \cot 25^\circ \cot 5^\circ)}$

$= \dfrac{3}{7} - \dfrac{4(1)}{7(1)} = \dfrac{3-4}{7} = \boxed{\dfrac{-1}{7}}$

(ix) $\dfrac{\sin 18^\circ}{\cos 72^\circ} + \sqrt{3}(\tan 10^\circ \tan 30^\circ \tan 40^\circ \tan 50^\circ \tan 80^\circ)$

$= \dfrac{\sin 18^\circ}{\sin(90^\circ - 72^\circ)} + \sqrt{3}\left[\tan 10^\circ \times \dfrac{1}{\sqrt{3}} \times \tan 40^\circ \times \cot(90^\circ - 50^\circ) \times \cot(90^\circ - 80^\circ\right.$

$= \frac{\sin 18^\circ}{\sin 18^\circ} 1 + \sqrt{3}\left[\frac{1}{\sqrt{3}} \times \tan 10^\circ \tan 40^\circ \cot 10^\circ \cot 40^\circ\right]$

$= 1 + 1 = \boxed{2}$

(x) $\frac{\cos 58^\circ + \sin 22^\circ}{\sin 32^\circ \cos 68^\circ} - \frac{\cos 38^\circ \operatorname{cosec} 52^\circ}{\tan 18^\circ \tan 35^\circ \tan 60^\circ \tan 72^\circ \tan 55^\circ}$

$= \frac{\cos 58^\circ}{\cos(90^\circ - 32^\circ)} + \frac{\sin 22^\circ}{\sin(90^\circ - 68^\circ)} - \frac{\cos 38^\circ \sec(90^\circ - 52^\circ)}{\tan 18^\circ \tan 35^\circ \tan 60^\circ \cot(90^\circ - 72^\circ) \cot(90^\circ - 55^\circ)}$

$= \frac{\cos 58^\circ}{\cos 58^\circ} + \frac{\sin 22^\circ}{\sin 22^\circ} - \frac{\cos 38^\circ \sec 38^\circ}{\tan 18^\circ \tan 35^\circ \tan 60^\circ \cot 18^\circ \cot 35^\circ}$

$= 1 + 1 - \frac{1}{\sqrt{3}} = 2 - \frac{1}{\sqrt{3}} = \frac{2\sqrt{3} - 1}{\sqrt{3}}$

$= \frac{(2\sqrt{3} - 1)\sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \boxed{\frac{6 - \sqrt{3}}{3}}$

Q.10 $\sin\theta = \cos(\theta - 45^\circ)$

$\cos(90^\circ - \theta) = \cos(\theta - 45^\circ)$

$90^\circ - \theta = \theta - 45^\circ$

$\theta + \theta = 90^\circ + 45^\circ$

$2\theta = 135^\circ$

$\theta = \frac{135^\circ}{2}$

$\boxed{\theta = 67.5^\circ}$

Q.11 A, B, C are interior $\angle$s of $\triangle ABC$

$\therefore \angle A + \angle B + \angle C = 180^\circ$ (Angle sum prop of $\triangle$)

$\Rightarrow \frac{A}{2} + \frac{B}{2} + \frac{C}{2} = 90^\circ$ — (1)

(i) $\tan\frac{B+C}{2} = \cot\frac{A}{2}$

LHS $= \tan \frac{B+C}{2}$

$= \tan\left(\frac{B}{2} + \frac{C}{2}\right)$

$= \tan\left(90° - \frac{A}{2}\right)$ (From ①)

$= \boxed{\cot\left(\frac{A}{2}\right)}$ = RHS

Hence, proved

(ii) $\cos\left(\frac{B+C}{2}\right) = \sin\left(\frac{A}{2}\right)$

LHS $= \cos \frac{B+C}{2}$

$= \cos\left(\frac{B}{2} + \frac{C}{2}\right)$

$= \cos\left(90° - \frac{A}{2}\right)$ (From ①)

$= \boxed{\sin\left(\frac{A}{2}\right)}$ = RHS

Hence, proved

Q.12 $\sin(2\theta + 45°) = \cos(30° - \theta)$

$\sin(2\theta + 45°) = \sin(90° - 30° + \theta)$

$\sin(2\theta + 45°) = \sin(60° + \theta)$

$2\theta + 45° = 60° + \theta$

$2\theta - \theta = 60° - 45°$

$\boxed{\theta = 15°}$

Q.13 $\sec\theta = \operatorname{cosec} 60°$

$\sec\theta = \sec(90° - 60°)$

$\sec\theta = \sec 30°$

$\theta = 30°$ ——— ①

$2\cos^2\theta - 1$

$= 2\cos^2 30° - 1$ (from (1))

$= 2\left(\frac{\sqrt{3}}{2}\right)^2 - 1$

$= 2\left(\frac{3}{4_2}\right) - 1 = \frac{3}{2} - 1$

$= \frac{3-2}{2} = \boxed{\frac{1}{2}}$

Q.14 $\cos 2\theta = \sin 4\theta$

$\cos 2\theta = \cos(90° - 4\theta)$

$2\theta = 90° - 4\theta$

$2\theta + 4\theta = 90°$

$6\theta = 90°$

$\theta = \frac{90°}{6}\ 15$

$\boxed{\theta = 15°}$

Q.15 $\sin 3\theta = \cos(\theta - 6°)$

$\sin 3\theta = \sin(90° - \theta + 6°)$

$\sin 3\theta = \sin(96° - \theta)$

$3\theta = 96° - \theta$

$3\theta + \theta = 96°$

$4\theta = 96°$

$\theta = \frac{96°}{4}\ 24°$

$\boxed{\theta = 24°}$

Q.16 $\sec 4A = \operatorname{cosec}(A - 20°)$

$\sec 4A = \sec(90° - A + 20°)$

$\sec 4A = \sec(110° - A)$

$$4A = 110° - A$$

$$4A + A = 110°$$

$$A = \frac{110° 22}{5}$$

$$\boxed{A = 22°}$$

Q.17 $\sec 2A = \text{cosec}\,(A - 42°)$

$$\sec 2A = \sec(90° - A + 42°)$$

$$\sec 2A = \sec(132° - A)$$

$$2A = 132° - A$$

$$2A + A = 132°$$

$$3A = 132°$$

$$A = \frac{132° 44°}{3}$$

$$\boxed{A = 44°}$$

Q.18 $\tan 2A = \cot(A - 18°)$

$$\tan 2A = \tan(90° - A + 18°)$$

$$\tan 2A = \tan(108° - A)$$

$$2A = 108° - A$$

$$2A + A = 108°$$

$$3A = 108°$$

$$A = \frac{108° 36°}{3}$$

$$\boxed{A = 36°}$$