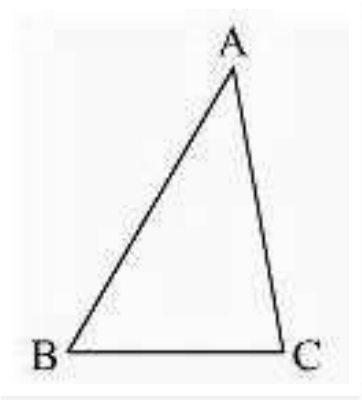
(1) In Δ ABC, if $\angle A = 40°$ and $\angle B = 60°$. Determine the longest and shortest sides of the triangle.

Solution:

Given that in Δ ABC, $\angle A = 40°$ and $\angle B = 60°$



We have to find longest and shortest side

We know that,

Sum of angles in a triangle 180°

$\angle A + \angle B + \angle C = 180°$

$40° + 60° + \angle C = 180°$

$\angle C = 180° - ((10)0°) = 80°$

$\angle C = 80°$

Now,

$\Rightarrow 40° < 60° < 80° = \angle A < \angle B < \angle C$

$\Rightarrow \angle C$ is greater angle and $\angle A$ is smaller angle.

Now, $\angle A < \angle B < \angle C$

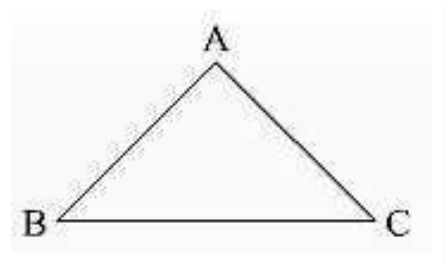
=> BC < AAC < AB [Side opposite to greater angle is larger and side opposite to smaller angle is smaller]

AB is longest and BC is smallest or shortest side.

(2) In a Δ ABC, if $\angle B = \angle C = 45°$, which is the longest side?

Solution: Given that in Δ ABC,

$\angle B = \angle C = 45°$



We have to find longest side

We know that.

Sum of angles in a triangle =180°

$\angle A + \angle B + \angle C = 180°$

$\angle A + 45° + 45° = 180°$

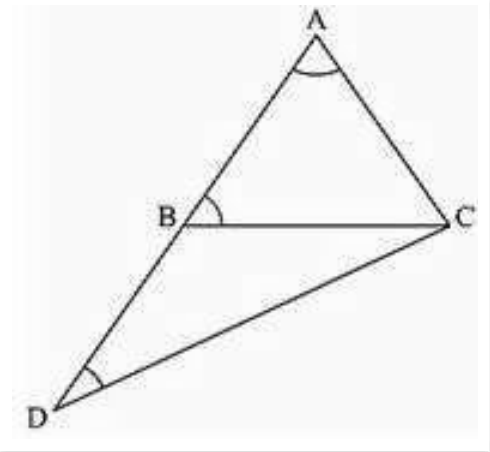
$\angle A = 180° - (45° + 45°) = 180° - 90° = 90°$

$\angle A = 90°$

(3) In Δ ABC, side AB is produced to D so that BD = BC. If $\angle B = 60°$ and $\angle A = 70°$. prove that: (i) AD > CD (ii) AD > AC

Sol: Given that, in Δ ABC, side AB is produced to D so that BD = BC.

$\angle B = 60°$, and $\angle A = 70°$



To prove,

(i) AD > CD (ii) AD > AC

First join C and D

We know that,

Sum of angles in a triangle =180°

$\angle A + \angle B + \angle C = 180°$

$70° + 60° + \angle C = 180°$

$\angle C = 180° - (130°) = 50°$

$\angle C = 50°$

$\angle ACB = 50°$……(i)

And also in Δ BDC

$\angle DBC = 180 - \angle ABC$ [ABD is a straight angle]

$180 - 60° = 120°$

and also BD = BC[given]

$\angle BCD = \angle BDC$ [Angles opposite to equal sides are equal]

Now,

$\angle DBC + \angle BCD + \angle BDC = 180°$ [Sum of angles in a triangle =180°]

=> 120° + ∠ BCD + ∠ BCD = 180°

=> 120° + 2∠ BCD = 180°

=> 2∠ BCD = 180° – 120° = 60°

=> ∠ BCD = 30°

=> ∠ BCD = ∠ BDC = 30° ….(ii)

Now, consider Δ ADC.

∠ BAC => ∠ DAC = 70° [given]

∠ BDC => ∠ ADC = 30° [From (ii)]

∠ ACD = ∠ ACB+ ∠ BCD

= 50° + 30°[From (i) and (ii)] =80°

Now, ∠ ADC < ∠ DAC < ∠ ACD

AC < DC < AD [Side opposite to greater angle is longer and smaller angle is smaller]

AD > CD and AD > AC

Hence proved

Or,

We have,

∠ ACD > ∠ DAC and ∠ ACD > ∠ ADC

AD > DC and AD> AC [Side opposite to greater angle is longer and smaller angle is smaller]

(4) Is it possible to draw a triangle with sides of length 2 cm, 3 cm and 7 cm?

Sol:

Given lengths of sides are 2cm, 3cm and 7cm.

To check whether it is possible to draw a triangle with the given lengths of sides

We know that,

A triangle can be drawn only when the sum of any two sides is greater than the third side.

So, let's check the rule.

2 + 3 $\not>$ 7 or 2 + 3 < 7

2 + 7 > 3

and 3 + 7 > 2

Here 2 + 3 $\not>$ 7

So, the triangle does not exit.

(5) O is any point in the interior of Δ ABC. Prove that

(i) AB + AC > OB + OC

(ii) AB + BC + CA > OA + QB + OC

(iii) OA + OB + OC > (1/2)(AB + BC +CA)

Solution:

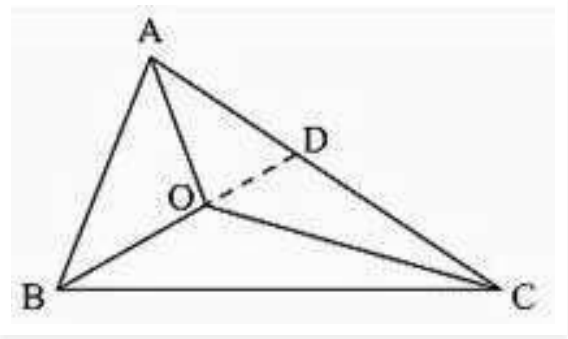
Given that O is any point in the interior of Δ ABC

To prove

(i) AB + AC > OB + OC

(ii) AB + BC + CA > OA + QB + OC

(iii) OA + OB + OC > (1/2)(AB + BC +CA)



We know that in a triangle the sum of any two sides is greater than the third side.

So, we have

In Δ ABC

AB + BC > AC

BC + AC > AB

AC + AB > BC

In Δ OBC

OB + OC > BC…….(i)

In Δ OAC

OA + OC > AC……(ii)

In Δ OAB

OA + OB > AB……(iii)

Now, extend (or) produce BO to meet AC in D.

Now, in Δ ABD, we have

AB + AD > BD

AB + AD > BO + OD …….(iv) [BD = BO + OD]

Similarly in Δ ODC, we have

OD + DC > OC……(v)

(i) Adding (iv) and (v), we get

AB + AD + OD + DC > BO + OD + OC