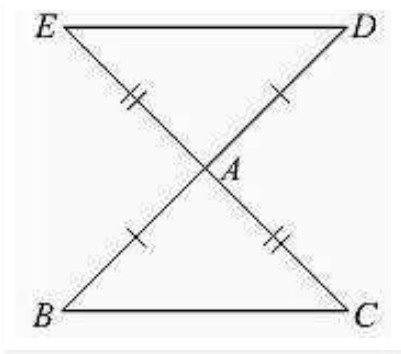


(1) In Fig. (10).22, the sides BA and CA have been produced such that: $BA = AD$ and $CA = AE$. Prove that segment $DE \parallel BC$.

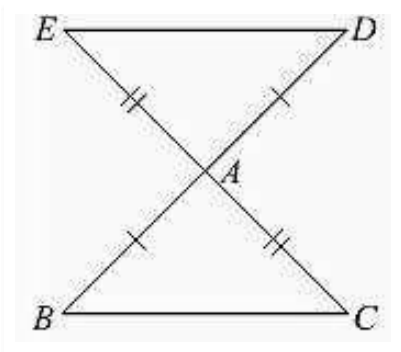


Solution:

Given that, the sides BA and CA have been produced such that $BA = AD$ and $CA = AE$ and given to prove $DE \parallel BC$
Consider triangle BAC and DAE,

We have

$BA = AD$ and $CA = AE$ [given in the data]



And also $\angle BAC = \angle DAE$ [vertically opposite angles]

So, by SAS congruence criterion, we have

$\angle BAC \cong \angle DAE$

$BC = DE$ and $\angle DEA = \angle BCA$, $\angle EDA = \angle CBA$

[Corresponding parts of congruent triangles are equal]

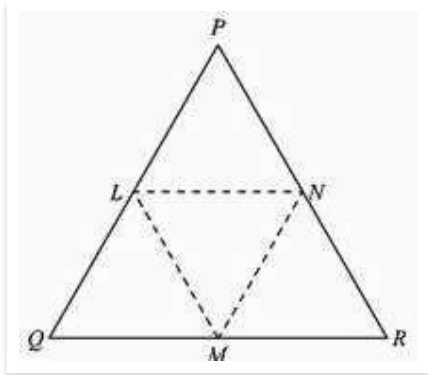
Now, DE and BC are two lines intersected by a transversal DB such that $\angle DEA = \angle BCA$ i.e., alternate angles are equal Therefore, $DE \parallel BC$.

(2) In a PQR, if $PQ = QR$ and L, M and N are the mid-points of the sides PQ, QR and RP respectively. Prove that $LN = MN$.

Solution: Given that,

In PQR, $PQ = QR$ and L, M, N are midpoints of the sides PQ, QR and RP respectively and given to prove that $LN = MN$

Here we can observe that PQR is an isosceles triangle



$$PQ = QR \text{ and } \angle QPR = \angle QRP \text{ ____ (i)}$$

And also, L and M are midpoints of PQ and QR respectively

$$PL = LQ = QM = MR = \frac{PQ}{2} = \frac{QR}{2}$$

And also, $PQ = QR$

Now, consider $\triangle LPN$ and $\triangle MRN$, $LP = MR$ [From – (2)]

$$\angle LPN = \angle MRN \text{ ____ [From – (1)]}$$

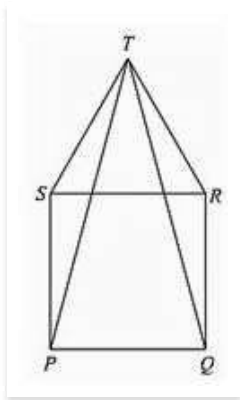
$\angle QPR$ and $\angle LPN$ and $\angle QRP$ and $\angle MRN$ are same.

$$PN = NR \text{ [N is midpoint of PR]}$$

So, by SAS congruence criterion, we have $\triangle LPN = \triangle MRN$

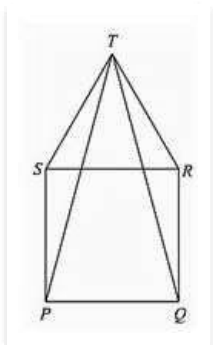
$$LN = MN \text{ [Corresponding parts of congruent triangles are equal]}$$

(3) In fig. (10).23, PQRS is a square and SRT is an equilateral triangle. Prove that (i) $PT = QT$ (ii) $\angle TQR = 15^\circ$



Solution: Given that PQRS is a square and SRT is an equilateral triangle. And given to prove that

(i) $PT = QT$ and (ii) $\angle TQR = 15^\circ$



Now, PQRS is a square

$$PQ = QR = RS = SP \text{ ---(i)}$$

$$\text{And } \angle SPQ = \angle PQR = \angle QRS = \angle RSP = 90^\circ = \text{right angle}$$

And also, SRT is an equilateral triangle.

$$SR = RT = TS \text{ ---(ii)}$$

$$\text{And } \angle TSR = \angle SRT = \angle RTS = 60^\circ$$

From (i) and (ii)

$$PQ = QR = SP = SR = RT = TS \text{ ---(iii)}$$

And also,

$$\begin{aligned} \angle TSP &= \angle TSR + \angle RSP = 60^\circ + 90^\circ + 150^\circ & \angle TRQ &= \angle TRS + \angle SRQ = 60^\circ + 90^\circ + \\ 150^\circ & \Rightarrow \angle TSR = \angle TRQ = 150^\circ & \text{--- (iv)} \end{aligned}$$

$$SP = RQ \quad [\text{From (iii)}]$$

So, by SAS congruence criterion we have

$$\triangle TSP = \triangle TRQ$$

$PT = QT$ [Corresponding parts of congruent triangles are equal] Consider $\triangle TQR$.

$$QR = TR \quad [\text{From (iii)}]$$

$\triangle TQR$ is an isosceles triangle.

$$\angle QTR = \angle TQR \text{ [angles opposite to equal sides]}$$

Now,

Sum of angles in a triangle is equal to 180°

$$\Rightarrow \angle QTR + \angle TQR + \angle TRQ = 180^\circ$$

$$\Rightarrow 2 \angle TQR + 150^\circ = 180^\circ \quad [\text{From (iv)}]$$

$$\Rightarrow 2 \angle TQR = 180^\circ - 150^\circ$$

$$\Rightarrow 2 \angle TQR = 30^\circ \quad \angle TQR = 15^\circ \dots$$

Hence proved

(4) Prove that the medians of an equilateral triangle are equal.

Solution:

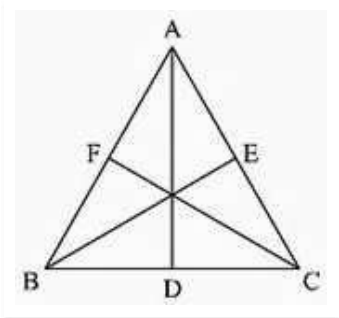
Given,

To prove the medians of an equilateral triangle are equal.

Median: The line joining the vertex and midpoint of opposite side. Now, consider an equilateral triangle ABC.

Let D, E, F are midpoints of BC, CA and AB.

Then, AD, BE and CF are medians of ABC.



Now,

$$D \text{ is midpoint of } BC \Rightarrow BD = DC = \frac{BC}{2}$$

$$\text{Similarly, } CE = EA = \frac{AC}{2}$$

$$AF = FB = \frac{AB}{2}$$

Since $\triangle ABC$ is an equilateral triangle

$$\Rightarrow AB = BC = CA \quad \text{---(i)}$$

$$\Rightarrow BD = DC = CE = EA = AF = FB = \frac{BC}{2} = \frac{AC}{2} = \frac{AB}{2} \quad \text{---(ii)}$$

$$\text{And also, } \angle ABC = \angle BCA = \angle CAB = 60^\circ \quad \text{---(iii)}$$

Now, consider $\triangle ABD$ and $\triangle BCE$ $AB = BC$ [From (i)]

$$BD = CE \quad \text{[From (ii)]}$$

Now, in $\triangle TSR$ and $\triangle TRQ$

$$TS = TR \quad \text{[From (iii)]}$$

$$\angle ABD = \angle BCE \quad \text{[From (iii)] [} \angle ABD \text{ and } \angle ABC \text{ and } \angle BCE \text{ and } \angle BCA \text{ are same]}$$

So, from SAS congruence criterion, we have

$$\triangle ABD = \triangle BCE$$

$$AD = BE \quad \text{---(iv)}$$

[Corresponding parts of congruent triangles are equal]

Now, consider $\triangle BCE$ and $\triangle CAF$, $BC = CA$ [From (i)]

$$\angle BCE = \angle CAF \quad \text{[From (ii)]}$$

[$\angle BCE$ and $\angle BCA$ and $\angle CAF$ and $\angle CAB$ are same]

$$CE = AF \quad \text{[From (ii)]}$$

So, from SAS congruence criterion, we have

$$\triangle BCE = \triangle CAF$$

$$BE = CF \text{ (v)}$$

[Corresponding parts of congruent triangles are equal]

From (iv) and (v), we have

$$AD = BE = CF$$

$$\text{Median AD} = \text{Median BE} = \text{Median CF}$$

The medians of an equilateral triangle are equal.

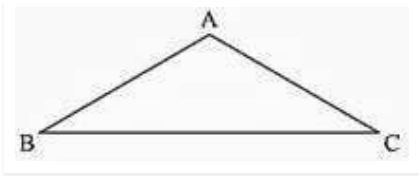
Hence proved

(5) In a $\triangle ABC$, if $\angle A = 120^\circ$ and $AB = AC$. Find $\angle B$ and $\angle C$.

Solution:

Consider a $\triangle ABC$

Given $\angle A = 120^\circ$ and $AB = AC$ and given to find $\angle B$ and $\angle C$.



We can observe that $\triangle ABC$ is an isosceles triangle since $AB = AC$

$$\angle B = \angle C \text{ (i)}$$

[Angles opposite to equal sides are equal]

We know that sum of angles in a triangle is equal to 180°

$$\Rightarrow \angle A + \angle B + \angle C = 180^\circ$$

[From (i)]

$$\Rightarrow \angle A + \angle B + \angle B = 180^\circ$$

$$\Rightarrow 120^\circ + 2\angle B = 180^\circ$$

$$\Rightarrow 2\angle B = 180^\circ - 120^\circ$$

$$\Rightarrow \angle B = \angle C = 30^\circ$$

(6) In a $\triangle ABC$, if $AB = AC$ and $\angle B = 70^\circ$. Find $\angle A$.

Solution:

Consider a $\triangle ABC$, if $AB = AC$ and $\angle B = 70^\circ$

Since, $AB = AC$ $\triangle ABC$ is an isosceles triangle

$$\angle B = \angle C \text{ [Angles opposite to equal sides are equal]}$$

$$\angle B = \angle C = 70^\circ$$

And also,

Sum of angles in a triangle = 180°

$$\angle A + \angle B + \angle C = 180^\circ$$

$$\angle A + 70^\circ + 70^\circ = 180^\circ$$

$$\angle A = 180^\circ - 140^\circ$$

$$\angle A = 40^\circ$$

(7) The vertical angle of an isosceles triangle is 100° . Find its base angles.

Solution:

Consider an isosceles $\triangle ABC$ such that $AB = AC$

Given that vertical angle A is 100°

To find the base angles

Since $\triangle ABC$ is isosceles

$$\angle B = \angle C \text{ [Angles opposite to equal sides are equal]}$$

And also,

Sum of interior angles of a triangle = 180°

$$\angle A + \angle B + \angle C = 180^\circ$$

$$(10)0^\circ + \angle B + \angle B = 180^\circ$$

$$2\angle B = 180^\circ - (10)0^\circ$$

$$\angle B = 40^\circ$$

$$\angle B = \angle C = 40^\circ$$

(8) In Fig. (10).24, $AB = AC$ and $\angle ACD = (10)5^\circ$. Find $\angle BAC$.

Solution:

Consider the given figure

We have,

$$AB = AC \text{ and } \angle ACD = (10)5^\circ$$

Since, $\angle BCD = 180^\circ = \text{Straight angle}$

$$\angle BCA + \angle ACD = 180^\circ$$

$$\angle BCA + (10)5^\circ = 180^\circ$$

$$\angle BCA = 180^\circ - (10)5^\circ$$

$$\angle BCA = 75^\circ$$

And also,

$\triangle ABC$ is an isosceles triangle [$AB = AC$]

$$\angle ABC = \angle ACB \text{ [Angles opposite to equal sides are equal]}$$

From (i), we have

$$\angle ACB = 75^\circ$$

$$\angle ABC = \angle ACB = 75^\circ$$

And also,

Sum of Interior angles of a triangle = 180°

$$\angle ABC + \angle BCA + \angle CAB = 180^\circ$$

$$75^\circ + 75^\circ + \angle CAB = 180^\circ$$

$$150^\circ + \angle BAC = 180^\circ$$

$$\angle BAC = 180^\circ - 150^\circ = 30^\circ$$

$$\angle BAC = 30^\circ$$

(9) Find the measure of each exterior angle of an equilateral triangle.

Solution:

Given to find the measure of each exterior angle of an equilateral triangle consider an equilateral triangle ABC.

We know that for an equilateral triangle

$$AB = BC = CA \text{ and } \angle ABC = \angle BCA = \angle CAB = \frac{180^\circ}{3} = 60^\circ \quad \text{---(i)}$$

Now,

Extend side BC to D, CA to E and AB to F.

Here BCD is a straight line segment

$$BCD = \text{Straight angle} = 180^\circ$$

$$\angle BCA + \angle ACD = 180^\circ \text{ [From (i)]}$$

$$60^\circ + \angle ACD = 180^\circ$$

$$\angle ACD = 120^\circ$$

Similarly, we can find $\angle FAB$ and $\angle FBC$ also as 120° because ABC is an equilateral triangle

$$\angle ACD = \angle EAB = \angle FBC = 120^\circ$$

Hence, the measure of each exterior angle of an equilateral triangle is 120°

(10) If the base of an isosceles triangle is produced on both sides, prove that the exterior angles so formed are equal to each other.

Solution:

ED is a straight line segment and B and C are points on it.

$$\angle EBC = \angle BCD = \text{straight angle} = 180^\circ$$

$$\angle EBA + \angle ABC = \angle ACB + \angle ACD$$

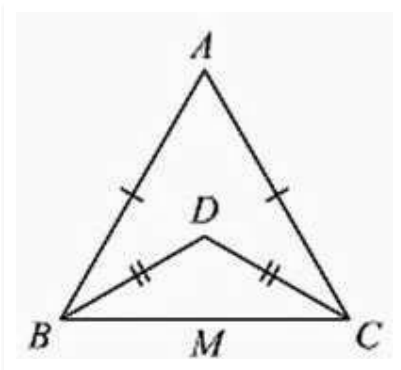
$$\angle EBA = \angle ACD + \angle ACB - \angle ABC$$

$$\angle EBA = \angle ACD \text{ [From (i) } \angle ABC = \angle ACB]$$

$$\angle ABE = \angle ACD$$

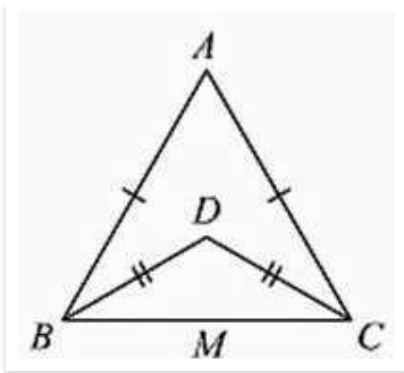
Hence proved

(11) In Fig. (10).2(5) $AB = AC$ and $DB = DC$, find the ratio $\angle ABD : \angle ACD$.



Solution:

Consider the figure



Given,

$AB = AC$, $DB = DC$ and given to find the ratio

$\angle ABD = \angle ACD$

Now, $\triangle ABC$ and $\triangle DBC$ are isosceles triangles since $AB = AC$ and $DB = DC$ respectively

$\angle ABC = \angle ACB$ and $\angle DBC = \angle DCB$ [Angles opposite to equal sides are equal]

Now consider,

$\angle ABD : \angle ACD$

$(\angle ABC - \angle DBC) : (\angle ACB - \angle DCB)$

$(\angle ABC - \angle DBC) : (\angle ABC - \angle DBC)$ [$\angle ABC = \angle ACB$ and $\angle DBC = \angle DCB$]

1:1

$ABD : ACD = 1:1$

(12) Determine the measure of each of the equal angles of a right-angled isosceles triangle.

OR

ABC is a right-angled triangle in which $\angle A = 90^\circ$ and $AB = AC$. Find $\angle B$ and $\angle C$.

Solution:

ABC is a right angled triangle

Consider on a right – angled isosceles triangle ABC such that

$\angle A = 90^\circ$ and $AB = AC$ Since,

$AB = AC \Rightarrow \angle C = \angle B$ _____(i)

[Angles opposite to equal sides are equal]

Now, Sum of angles in a triangle = 180°

$\angle A + \angle B + \angle C = 180^\circ$

$\Rightarrow 90^\circ + \angle B + \angle B = 180^\circ$

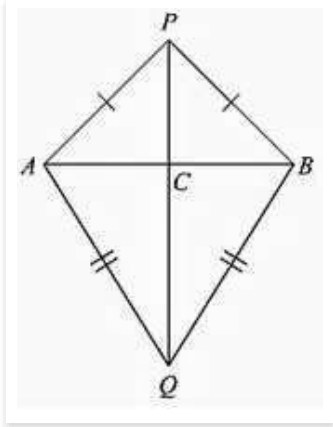
$\Rightarrow 2\angle B = 90^\circ$

$\Rightarrow \angle B = 45^\circ$

$\angle B = 45^\circ$, $\angle C = 45^\circ$

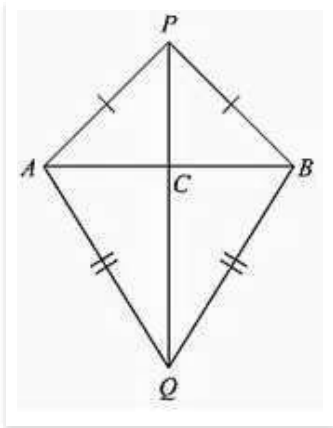
Hence, the measure of each of the equal angles of a right-angled Isosceles triangle is 45°

(13) AB is a line segment. P and Q are points on opposite sides of AB such that each of them is equidistant from the points A and B (See Fig. (10).26). Show that the line PQ is perpendicular bisector of AB.



Solution:

Consider the figure.



We have

AB is a line segment and P, Q are points on opposite sides of AB such that

$$AP = BP \quad \text{_____ (i)}$$

$$AQ = BQ \quad \text{_____ (ii)}$$

We have to prove that PQ is perpendicular bisector of AB.

Now consider $\triangle PAQ$ and $\triangle PBQ$,

We have

$$AP = BP \quad \text{[From (i)]}$$

$$AQ = BQ \quad \text{[From (ii)]}$$

$$\text{And } PQ = PQ \quad \text{[Common side]}$$

$$\triangle PAQ \cong \triangle PBQ \quad \text{_____ (iii) [From SAS congruence]}$$

Now, we can observe that $\triangle APB$ and $\triangle ABQ$ are isosceles triangles. [From (i) and (ii)]

$$\angle PAB = \angle ABQ \text{ and } \angle QAB = \angle QBA$$

Now consider $\triangle PAC$ and $\triangle PBC$

C is the point of intersection of AB and PQ

$$PA = PB \quad [\text{From (i)}]$$

$$\angle APC = \angle BPC \quad [\text{From (ii)}]$$

$$PC = PC \quad [\text{common side}]$$

So, from SAS congruency of triangle $\triangle PAC \cong \triangle PBC$

$AC = CB$ and $\angle PCA = \angle PBC$ _____(iv) [Corresponding parts of congruent triangles are equal]
And also, ACB is line segment

$$\angle ACP + \angle BCP = 180^\circ$$

$$\angle ACP = \angle PCB$$

$$\angle ACP = \angle PCB = 90^\circ$$

We have $AC = CB \Rightarrow C$ is the midpoint of AB

From (iv) and (v)

We can conclude that PC is the perpendicular bisector of AB

Since C is a point on the line PQ, we can say that PQ is the perpendicular bisector of AB.