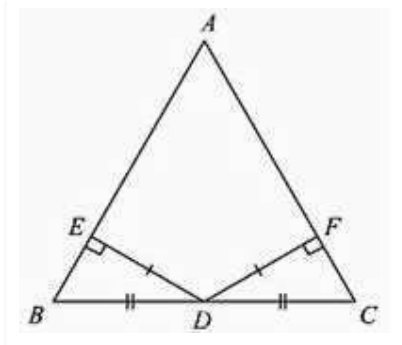


(1) ABC is a triangle and D is the mid-point of BC. The perpendiculars from D to AB and AC are equal. Prove that the triangle is isosceles.

Sol:

Given that, in two right triangles one side and acute angle of one are equal to the corresponding side and angle of the other



We have to prove that the triangles are congruent

Let us consider two right triangles such that

$$\angle B = \angle E = 90^\circ \quad \dots\dots(i)$$

$$AB = DE \dots\dots (ii)$$

$$\angle C = \angle F \dots\dots(iii)$$

Now observe the two triangles ABC and DEF

$$\angle C = \angle F \text{ [From (iii)]}$$

$$\angle B = \angle E \text{ [From (i)]}$$

$$\text{And } AB = DE \text{ [From (ii)]}$$

So, by AAS congruence criterion, we have

$$\triangle ABC \cong \triangle DEF$$

Therefore, the two triangles are congruent

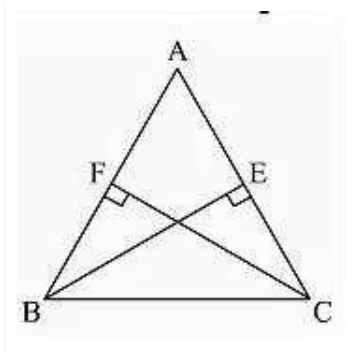
Hence proved

(2) ABC is a triangle in which BE and CF are, respectively, the perpendiculars to the sides AC and AB. If $BE = CF$, prove that $\triangle ABC$ is isosceles

Sol: Given that ABC is a triangle in which BE and CF are perpendicular to the sides AC and AS respectively such that $BE = CF$.

To prove, $\triangle ABC$ is isosceles

Now, consider $\triangle BCF$ and $\triangle CBE$,



We have

$$\angle BFC = \angle CEB = 90^\circ \text{ [Given]}$$

$$BC = CB \text{ [Common side]}$$

$$\text{And } CF = BE \text{ [Given]}$$

So, by RHS congruence criterion, we have

$$\triangle BFC \cong \triangle CEB$$

Now,

$$\angle FBC = \angle ECB \text{ [Incongruent triangles corresponding parts are equal]}$$

$$\angle ABC = \angle ACB$$

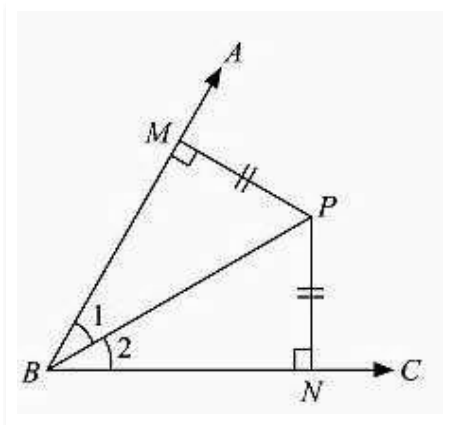
$$AC = AB \text{ [Opposite sides to equal angles are equal in a triangle]}$$

$\triangle ABC$ is isosceles

(3) If perpendiculars from any point within an angle on its arms are congruent. Prove that it lies on the bisector of that angle.

Solution:

Given that, if perpendicular from any point within, an angle on its arms is congruent, prove that it lies on the bisector of that angle.



Now,

Let us consider an angle ABC and let BP be one of the arm within the angle

Draw perpendicular PN and PM on the arms BC and BA such that they meet BC and BA in N and M respectively.

Now, in $\triangle BPM$ and $\triangle BPN$

We have $\angle BMP = \angle BNP = 90^\circ$ [given]

$BP = BP$ [Common side]

And $MP = NP$ [given]

So, by RHS congruence criterion, we have

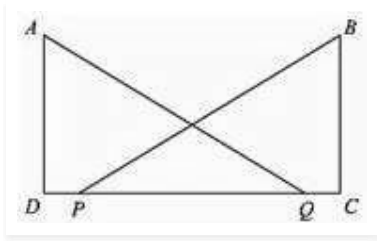
$$\triangle BPM \cong \triangle BPN$$

Now, $\angle MBP = \angle NBP$ [Corresponding parts of congruent triangles are equal]

BP is the angular bisector of $\angle ABC$.

Hence proved

(4) In fig. (10).99, $AD \perp CD$ and $CB \perp CD$. If $AQ = BP$ and $DP = CQ$, prove that $\angle DAQ = \angle CBP$.



Sol:

Given in the fig. (10).99, $AD \perp CD$ and $CB \perp CD$.

And $AQ = BP$ and $DP = CQ$,

To prove that $\angle DAQ = \angle CBP$

Given that $DP = QC$

Add PQ on both sides

$$DP + PQ = PQ + QC$$

$$DQ = PC \text{(i)}$$

Now, consider triangle DAQ and CBP ,

We have

$$\angle ADQ = \angle BCP = 90^\circ \text{ [given]} \quad AQ = BP \text{ [given]}$$

And $DQ = PC$ [From (i)]

So, by RHS congruence criterion, we have

$$\triangle DAQ \cong \triangle CBP$$

Now,

$$\angle DAQ = \angle CBP \text{ [Corresponding parts of congruent triangles are equal]}$$

Hence proved

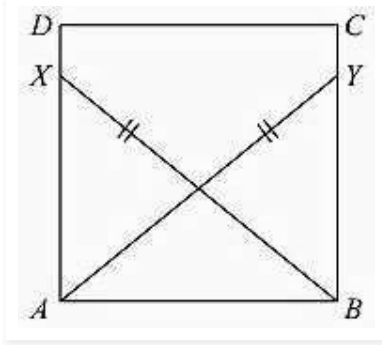
(5) $ABCD$ is a square, X and Y are points on sides AD and BC respectively such that $AY = BX$. Prove that $BY = AX$ and $\angle BAY = \angle ABX$.

Solution:

Given that $ABCD$ is a square, X and Y are points on sides AD and BC respectively such that $AY = BX$.

To prove $BY = AX$ and $\angle BAY = \angle ABX$

Join B and X, A and Y.



Since, ABCD is a square

$$\angle DAB = \angle CBA = 90^\circ$$

$$\angle XAB = \angle YAB = 90^\circ \dots\dots(i)$$

Now, consider triangle XAB and YBA

We have

$$\angle XAB = \angle YBA = 90^\circ. \text{ [From (i)] } BX = AY[\text{given}]$$

$$\text{And } AB = BA[\text{Common side}]$$

So, by RHS congruence criterion, we have $\triangle XAB \cong \triangle YBA$

Now, we know that corresponding parts of congruent triangles are equal.

$$BY = AX \text{ and } \angle BAY = \angle ABX$$

Hence proved

(6) Which of the following statements are true (T) and which are false (F):

(i) Sides opposite to equal angles of a triangle may be unequal.

(ii) Angles opposite to equal sides of a triangle are equal

(iii) The measure of each angle of an equilateral triangle is 60

(iv) If the altitude from one vertex of a triangle bisects the opposite side, then the triangle may be isosceles.

(v) The bisectors of two equal angles of a triangle are equal.

(vi) If the bisector of the vertical angle of a triangle bisects the base, then the triangle may be isosceles.

(vii) The two altitudes corresponding to two equal sides of a triangle need not be equal.

(viii) If any two sides of a right triangle are respectively equal to two sides of other right triangle, then the two triangles are congruent.

(ix) Two right-angled triangles are congruent if hypotenuse and a side of one triangle are respectively equal to the hypotenuse and a side of the other triangle.

Solution:

(i) False (F)

Reason: Sides opposite to equal angles of a triangle are equal

(ii) True (F)

Reason: Since the sides are equal, the corresponding opposite angles must be equal

(iii) True (T)

Reason: Since all the three angles of equilateral triangles are equal and sum of the three angles is 180 , each angle will be equal to $\frac{180^\circ}{3} = 60^\circ$

(iv) False (F)

Reason: Here the altitude from the vertex is also the perpendicular bisector of the opposite side.

The triangle must be isosceles and may be an equilateral triangle.

(v) True (T)

Reason: Since it an isosceles triangle, the lengths of bisectors of the two equal angles are equal

(vi) False (F)

Reason: The angular bisector of the vertex angle is also a median

=> The triangle must be an isosceles and also may be an equilateral triangle.

(vii) False (F)

Reason: Since two sides are equal, the triangle is an isosceles triangle. The two altitudes corresponding to two equal sides must be equal.

(viii) False (F)

Reason: The two right triangles may or may not be congruent

(ix) True (T)

Reason: According to RHS congruence criterion the given statement is true.

(7) Fill the blanks In the following so that each of the following statements is true.

(i) Sides opposite to equal angles of a triangle are ____

(ii) Angle opposite to equal sides of a triangle are ____

(iii) In an equilateral triangle all angles are ____

(iv) In $\triangle ABC$, if $\angle A = \angle C$, then $AB =$ ____

(v) If altitudes CE and BF of a triangle ABC are equal, then $AB =$ ____

(vi) In an isosceles triangle ABC with $AB = AC$, if BD and CE are its altitudes, then BD is ____ CE.

(vii) In right triangles ABC and DEF, if hypotenuse $AB = EF$ and side $AC = DE$, then $\triangle ABC \cong \triangle$ ____

Solution:

(i) Sides opposite to equal angles of a triangle are equal

(ii) Angles opposite to equal sides of a triangle are equal

(iii) In an equilateral triangle all angles are equal Reason: Since all sides are equal in a equilateral triangle. the angles opposite to equal sides will be equal .

(iv) In a $\triangle ABC$, if $\angle A = \angle C$, then $AB = BC$

(v) If altitudes CE and BF of a triangle ABC are equal, then $AB = AC$

(vi) In an isosceles triangle $\triangle ABC$ with $AB = AC$, if BD and CE are its altitudes, then BD is equal to CE

(vii) In right triangles ABC and DEF, if hypotenuse $AB = EF$ and side $AC = DE$, then, $\triangle ABC = \triangle EFD$.