(1) In fig. (10).40, it is given that RT = TS, ∠ 1 = 2 ∠ 2 and 4 = 2 ∠ (3) Prove that $\Delta RBT \cong \Delta SAT$.

Solution:

In the figure, given that

RT = TS ……(i)

∠ 1 = 2 ∠ 2 ……(ii)

And ∠ 4 = 2 ∠ 3 ……(iii)

To prove that $\Delta RBT \cong \Delta SAT$.

Let the point of intersection RB and SA be denoted by O

Since RB and SA intersect at O

∠ AOR = ∠ BOS [Vertically opposite angles]

∠ 1 = ∠ 4
2 ∠ 2 = 2 ∠ 3 [From (ii) and (iii)]
∠ 2 = ∠ 3 ……(iv)

Now we have RT =TS in Δ TRS

Δ TRS is an isosceles triangle

∠ TRS = ∠ TSR ……(v)

But we have

∠ TRS = ∠ TRB + ∠ 2 ……(vi)

∠ TSR = ∠ TSA + ∠ 3 ……(vii)

Putting (vi) and (vii) in (v) we get

∠ TRB + ∠ 2 = ∠ TSA + ∠ B

=> ∠ TRB = ∠ TSA [From (iv)]

Now consider Δ RBT and Δ SAT

RT = ST [From (i)]

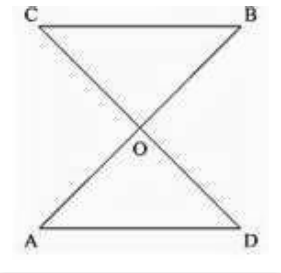
∠ TRB = ∠ TSA [From (iv)] ∠ RTB = ∠ STA [Common angle]

From ASA criterion of congruence, we have

Δ RBT = Δ SAT

(2) Two lines AB and CD intersect at O such that BC is equal and parallel to AD. Prove that the lines AB and CD bisect at O.

Solution: Given that lines AB and CD Intersect at O



Such that BC || AD and BC = AD .....(i)

We have to prove that AB and CD bisect at O.

To prove this first we have to prove that Δ AOD $\cong \Delta$ BOC

(3) BD and CE are bisectors of $\angle$ B and $\angle$ C of an isosceles Δ ABC with AB = AC. Prove that BD = CE

Solution:

Given that Δ ABC is isosceles with AB = AC and BD and CE are bisectors of $\angle$ B and $\angle$ C We have to prove BD = CE

Since AB = AC

=> Δ ABC = Δ ACB .....(i)

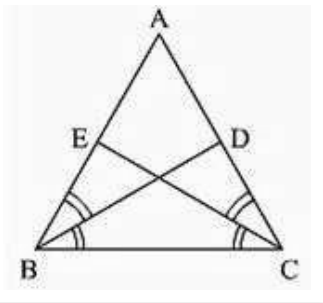
[Angles opposite to equal sides are equal]

Since BD and CE are bisectors of $\angle$ B and $\angle$ C

$\angle$ ABD = $\angle$ DBC = $\angle$ BCE = ECA = $\frac{\angle B}{2}$ = $\frac{\angle C}{2}$

Now,

Consider Δ EBC = Δ DCB



$\angle$ EBC = $\angle$ DCB [$\angle$ B = $\angle$ C] [From (i)]

BC = BC [Common side]

$\angle$ BCE = $\angle$ CBD [From (ii)]

So, by ASA congruence criterion, we have Δ EBC $\cong \Delta$ DCB

Now,

CE = BD [Corresponding parts of congruent triangles we equal]

or, BD = CE

Hence proved

Since AD || BC and transversal AB cuts at A and B respectively

$\angle$ DAO = $\angle$ OBC .....(ii) [alternate angle]

And similarly AD || BC and transversal DC cuts at D and C respectively

$\angle$ ADO = $\angle$ OBC .....(iii) [alternate angle]

Since AB end CD intersect at O.

$\angle$ AOD = $\angle$ BOC [Vertically opposite angles]

Now consider Δ AOD and Δ BOD

$\angle$ DAO = $\angle$ OBC [From (ii)]

AD = BC [From (i)]

And $\angle$ ADO = $\angle$ OCB [From (iii)]

So, by ASA congruence criterion, we have

$\Delta AOD \cong \Delta BOC$

Now,

AO= OB and DO = OC [Corresponding parts of congruent triangles are equal)

Lines AB and CD bisect at O.

Hence proved