

23/12/2020

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ch-10

Trigonometric identities

Q.1 (i)

$$\sin A = \frac{2}{3}$$

$$\sin A = \frac{P}{H} = \frac{2}{3}$$

$$P = 2k$$

$$H = 3k$$

$$B = \sqrt{H^2 - P^2}$$

$$= \sqrt{3k^2 - 2k^2}$$

$$= \sqrt{9k^2 - 4k^2}$$

$$= \sqrt{5k^2} = \sqrt{5}k$$

(Pythagoras theorem)

$$\cos A = \frac{B}{H} = \frac{\sqrt{5}k}{3k} = \frac{\sqrt{5}}{3}$$

$$\sec A = \frac{1}{\cos A} = \frac{3}{\sqrt{5}}$$

$$\tan A = \frac{P}{B} = \frac{2k}{\sqrt{5}k} = \frac{2}{\sqrt{5}}$$

$$\operatorname{cosec} A = \frac{1}{\sin A} = \frac{3}{2}$$

$$\cot A = \frac{B}{P} = \frac{\sqrt{5}k}{2k} = \frac{\sqrt{5}}{2}$$

Q.9

$$\tan \theta = \frac{P}{B} = \frac{a}{b}$$

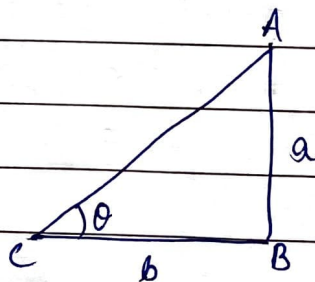
$$P = a$$

$$B = b$$

$$H = \sqrt{P^2 + B^2}$$

$$H = \sqrt{a^2 + b^2}$$

Pythagoras theorem



$$\sin \theta = \frac{P}{H} = \frac{a}{\sqrt{a^2 + b^2}}$$

$$\cos \theta = \frac{B}{H} = \frac{b}{\sqrt{a^2 + b^2}}$$

$$\frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta}$$

$$\Rightarrow \frac{b}{\sqrt{a^2+b^2}} + \frac{a}{\sqrt{a^2+b^2}} = \frac{b+a}{\sqrt{a^2+b^2}}$$

$$\frac{b}{\sqrt{a^2+b^2}} - \frac{a}{\sqrt{a^2+b^2}} = \frac{b-a}{\sqrt{a^2+b^2}}$$

$$= \frac{b+a}{\sqrt{a^2+b^2}} \times \frac{\sqrt{a^2+b^2}}{b-a} = \boxed{\frac{b+a}{b-a}}$$

Q.11 $3 \cot \theta = 2$

$$\cot \theta = \frac{2}{3}$$

$$\frac{4 \sin \theta - 3 \cos \theta}{2 \sin \theta + 6 \cos \theta}$$

Dividing by $\sin \theta$

$$\frac{4 \frac{\sin \theta}{\sin \theta} - 3 \frac{\cos \theta}{\sin \theta}}{2 \frac{\sin \theta}{\sin \theta} + 6 \frac{\cos \theta}{\sin \theta}} = \frac{4 - 3 \cot \theta}{2 + 6 \cot \theta}$$

$$= \frac{4 - 3 \left(\frac{2}{3} \right)}{2 + 6 \left(\frac{2}{3} \right)} = \frac{4 - 2}{2 + 4} = \frac{2}{6} = \boxed{\frac{1}{3}}$$

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Ch-10 Trigonometric Identities

Q.13

$$\sec \theta = \frac{13}{5} = \frac{H}{B}$$

$$\cos \theta = \frac{1}{\sec \theta} = \frac{5}{13}$$

$$H = 13k$$

$$B = 5k$$

$$P = \sqrt{H^2 - B^2} \text{ (Pythagoras theorem)}$$

$$P = \sqrt{13^2 k^2 - 5^2 k^2}$$

$$P = \sqrt{169k^2 - 25k^2}$$

$$P = \sqrt{144k^2}$$

$$P = 12k$$

$$\sin \theta = \frac{P}{H} = \frac{12k}{13k} = \frac{12}{13}$$

$$\frac{2 \sin \theta - 3 \cos \theta}{4 \sin \theta - 9 \cos \theta} = \frac{2 \left(\frac{12}{13} \right) - 3 \left(\frac{5}{13} \right)}{4 \left(\frac{12}{13} \right) - 9 \left(\frac{5}{13} \right)}$$

$$= \frac{24 - 15}{13} = \frac{9}{13} \times \frac{13}{13} = \frac{9}{13}$$

$$\frac{48 - 45}{13}$$

$$= \boxed{3} \\ = \text{RHS}$$

Hence, proved.

Q.15

$$\cot \theta = \frac{1}{\sqrt{3}} = \frac{B}{P}$$

$$B = 1k$$

$$P = \sqrt{3}k$$

$$H = \sqrt{P^2 + B^2}$$

$$H = \sqrt{(\sqrt{3}R)^2 + (4R)^2}$$

$$H = \sqrt{3R^2 + 16R^2}$$

$$H = \sqrt{19R^2}$$

$$H = 2R$$

$$\sin \theta = \frac{P}{H} = \frac{\sqrt{3}R}{2R} = \frac{\sqrt{3}}{2}$$

$$\frac{1 - \cos^2 \theta}{2 - \sin^2 \theta} = \frac{\sin^2 \theta}{2 - \sin^2 \theta} \quad [\sin^2 \theta + \cos^2 \theta = 1]$$

$$= \frac{\left(\frac{\sqrt{3}}{2}\right)^2}{2 - \left(\frac{\sqrt{3}}{2}\right)^2} = \frac{3}{8 - 3} = \frac{3}{5} = \text{RHS}$$

Hence, proved.

$$\text{Q.17} \quad \sec \theta = \frac{5}{4} = \frac{H}{B}$$

$$H = 5R$$

$$B = 4R$$

$$P = \sqrt{H^2 - B^2} \quad (\text{Pythagoras theorem})$$

$$P = \sqrt{(5R)^2 - (4R)^2}$$

$$P = \sqrt{25R^2 - 16R^2}$$

$$P = \sqrt{9R^2}$$

$$P = 3R$$

$$\sin \theta = \frac{P}{H} = \frac{3R}{5R} = \frac{3}{5}$$

$$\cos \theta = \frac{1}{\sec \theta} = \frac{4}{5}$$

$$\tan \theta = \frac{P}{B} = \frac{3R}{4R} = \frac{3}{4}$$

$$\cot \theta = \frac{1}{\tan \theta} = \frac{4}{3}$$

$$\frac{\sin \theta - 2 \cos \theta}{\tan \theta - \cot \theta} = \frac{3}{5} - 2 \left(\frac{4}{5} \right)$$

$$\frac{3}{4} - \frac{4}{3}$$

$$= \frac{3 - 8}{5} = \frac{-5}{5} \times \frac{12}{7} = \boxed{\frac{-12}{7}}$$

$$\frac{9 - 16}{12}$$

Q. 19 $\cos \theta = \frac{3}{5} = \frac{B}{H}$

$$B = 3k$$

$$H = 5k$$

$$P = \sqrt{H^2 - B^2}$$

$$P = \sqrt{(5k)^2 - (3k)^2}$$

$$P = \sqrt{25k^2 - 9k^2}$$

$$P = \sqrt{16k^2}$$

$$P = 4k$$

} Pythagoras theorem

$$\sin \theta = \frac{P}{H} = \frac{4k}{5k} = \frac{4}{5}$$

$$\tan \theta = \frac{P}{B} = \frac{4k}{3k} = \frac{4}{3}$$

$$\cot \theta = \frac{1}{\tan \theta} = \frac{3}{4}$$

$$\frac{\sin \theta - 1}{\tan \theta} = \frac{\sin \theta - \cot \theta}{2 \tan \theta}$$

$$2 \tan \theta =$$

$$= \frac{4}{5} - \frac{3}{4} = \frac{16 - 15}{20} = \frac{1}{20} \times \frac{3}{8} = \boxed{\frac{3}{160}}$$

$$2 \left(\frac{4}{3} \right) \quad \frac{8}{3}$$

$$Q.20 \quad \sin \theta = \frac{3}{5} = \frac{P}{H}$$

$$P = 3k$$

$$H = 5k$$

$$B = \sqrt{H^2 - P^2}$$

$$B = \sqrt{(5k)^2 - (3k)^2}$$

$$B = \sqrt{25k^2 - 9k^2}$$

$$B = \sqrt{16k^2}$$

$$B = 4k$$

} Pythagoras theorem

$$\cos \theta = \frac{B}{H} = \frac{4k}{5k} = \frac{4}{5}$$

$$\cot \theta = \frac{B}{P} = \frac{4k}{3k} = \frac{4}{3}$$

$$\frac{\cos \theta - 1}{2 \cot \theta} = \frac{4}{5} \cdot \frac{\cos \theta - \cot \theta}{2 \cot \theta}$$

$$= \frac{4}{5} - \frac{4}{3} = \frac{12 - 20}{15} = \frac{-8}{15} \times \frac{3}{3} = \frac{-1}{5}$$

$$Q.22 \quad \sin \theta = \frac{a}{b} = \frac{P}{H}$$

$$P = a$$

$$H = b$$

$$B = \sqrt{H^2 - P^2}$$

$$= \sqrt{b^2 - a^2}$$

} Pythagoras theorem

$$\sec \theta = \frac{H}{B} = \frac{b}{\sqrt{b^2 - a^2}}$$

$$\tan \theta = \frac{P}{B} = \frac{a}{\sqrt{b^2 + a^2}}$$

$$\begin{aligned} \sec \theta + \tan \theta &= \frac{b}{\sqrt{b^2 + a^2}} + \frac{a}{\sqrt{b^2 + a^2}} \\ &= \frac{b+a}{\sqrt{b^2 + a^2}} = \frac{(\sqrt{b+a})^2}{\sqrt{(b+a)(b-a)}} = \frac{(\sqrt{b+a})(\sqrt{b+a})}{(\sqrt{b+a})(\sqrt{b-a})} \\ &= \boxed{\frac{\sqrt{b+a}}{\sqrt{b-a}}} \end{aligned}$$

Q.24

$$\tan = \frac{20}{21} = \frac{P}{B}$$

$$P = 20k$$

$$B = 21k$$

$$H = \sqrt{P^2 + B^2}$$

$$H = \sqrt{(20k)^2 + (21k)^2}$$

$$H = \sqrt{400k^2 + 441k^2}$$

$$H = \sqrt{841k^2}$$

$$H = 29k$$

} Pythagoras theorem

$$\sin \theta = \frac{P}{H} = \frac{20k}{29k} = \frac{20}{29}$$

$$\cos \theta = \frac{B}{H} = \frac{21k}{29k} = \frac{21}{29}$$

$$\begin{aligned} \frac{1 - \sin \theta + \cos \theta}{1 + \sin \theta + \cos \theta} &= \frac{1 - \frac{20}{29} + \frac{21}{29}}{1 + \frac{20}{29} + \frac{21}{29}} = \frac{29 - 20 + 21}{29 + 20 + 21} \\ &= \frac{29 - 20 + 21}{29} = \frac{29 + 20 + 21}{29} \end{aligned}$$

$$= \frac{30}{29} \times \frac{29}{70} = \frac{3}{7}$$

= RHS

Hence, proved



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ch - 10
Exercise 10.1

Q.25 $\operatorname{cosec} A = \frac{2}{1} = \frac{H}{P}$

$H = 2k$

$P = 1k$

$B = \sqrt{(2k)^2 - (1k)^2}$

$B = \sqrt{4k^2 - 1k^2}$

$B = \sqrt{3k^2}$

$B = \sqrt{3}k$

} Pythagoras theorem

$\sin A = \frac{1}{2} = \frac{1}{2}$

$\cos A = \frac{B}{H} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$

$\cot A = \frac{B}{P} = \frac{\sqrt{3}k}{1k} = \sqrt{3}$

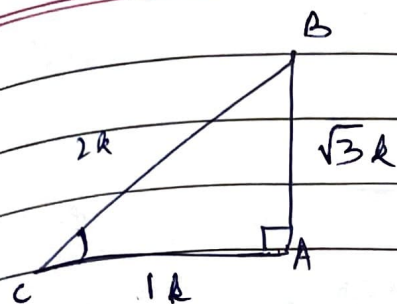
$$\frac{1}{\tan A} + \frac{\sin A}{1 + \cos A} = \cot A + \frac{\sin A}{1 + \cos A}$$

$$= \sqrt{3} + \frac{1}{2} = \sqrt{3} + \frac{1}{2}$$

$$\frac{1 + \sqrt{3}}{2} \quad \frac{2 + \sqrt{3}}{2}$$

$= \sqrt{3} + \frac{1}{2 + \sqrt{3}} = \frac{2\sqrt{3} + 3 + 1}{2 + \sqrt{3}}$

$= \frac{2\sqrt{3} + 4}{2 + \sqrt{3}} = \frac{2(\sqrt{3} + 2)}{2 + \sqrt{3}} = \boxed{2}$



$$\tan C = \frac{\sqrt{3}}{1} = \frac{P}{B} = \frac{AB}{AC}$$

$$AB = \sqrt{3}R$$

$$AC = 1R$$

$$BC = \sqrt{AB^2 + AC^2}$$

$$BC = \sqrt{(\sqrt{3}R)^2 + (1R)^2}$$

$$BC = \sqrt{3R^2 + 1R^2}$$

$$BC = \sqrt{4R^2}$$

$$BC = 2R$$

$$\sin B = \frac{P}{H} = \frac{AC}{BC} = \frac{1R}{2R} = \frac{1}{2}$$

$$\cos B = \frac{B}{H} = \frac{AB}{BC} = \frac{\sqrt{3}R}{2R} = \frac{\sqrt{3}}{2}$$

$$\sin C = \frac{P}{H} = \frac{AB}{BC} = \frac{\sqrt{3}R}{2R} = \frac{\sqrt{3}}{2}$$

$$\cos C = \frac{B}{H} = \frac{AC}{BC} = \frac{1R}{2R} = \frac{1}{2}$$

$$(\sin B) \cos C + (\sin C) \cos B$$

$$= \left(\frac{1}{2} \times \frac{1}{2} \right) + \left(\frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} \right) = \frac{1}{4} + \frac{3}{4} = \frac{4}{4} = 1$$

Q.28 NCERT Ques

Q.30

$$\cos \theta = \frac{5}{13} = \frac{B}{H}$$

$$H = 13R$$

$$B = 5R$$

$$P = \sqrt{(H)^2 - (B)^2}$$

$$= \sqrt{(13R)^2 - (5R)^2}$$

$$= \sqrt{169R^2 - 25R^2}$$

$$= \sqrt{144R^2}$$

$$= 12R$$

$$\sin \theta = \frac{P}{H} = \frac{12R}{13R} = \frac{12}{13}$$

$$\tan \theta = \frac{P}{B} = \frac{12R}{5R} = \frac{12}{5}$$

$$\frac{\sin^2 \theta - \cos^2 \theta}{2 \sin \theta \cos \theta} \times \frac{1}{\tan^2 \theta}$$

$$= \left(\frac{12}{13} \right)^2 - \left(\frac{5}{13} \right)^2$$

$$\frac{2 \times 12}{13} \times \frac{5}{13}$$

$$= \frac{144}{169} - \frac{25}{169} \times \frac{25}{144} = \frac{144 - 25}{169} \times \frac{25}{144}$$

$$\frac{120}{169}$$

$$\frac{120}{169}$$

$$= \frac{119 \times 255}{24 \times 120 \times 144}$$

$$= \frac{595}{3456}$$

$$24 \times 120 \times 144$$

$$3456$$

Q.31 $\sec A = \frac{5}{4} = \frac{H}{B}$

$$H = 5k$$

$$B = 4k$$

$$\begin{aligned} P &= \sqrt{(H)^2 - (B)^2} \\ &= \sqrt{(5k)^2 - (4k)^2} \\ &= \sqrt{25k^2 - 16k^2} \\ &= \sqrt{9k^2} \\ &= 3k \end{aligned}$$

$$\sin A = \frac{P}{H} = \frac{3k}{5k} = \frac{3}{5}$$

$$\cos A = \frac{1}{\sec A} = \frac{4}{5}$$

$$\tan A = \frac{P}{B} = \frac{3k}{4k} = \frac{3}{4}$$

$$\frac{3 \sin A - 4 \sin^3 A}{4 \cos^3 A - 3 \cos A} = \frac{3 \tan A - \tan^3 A}{1 - 3 \tan^2 A}$$

$$\begin{aligned} \text{LHS} &= \frac{3 \sin A - 4 \sin^3 A}{4 \cos^3 A - 3 \cos A} = \frac{3 \left(\frac{3}{5} \right) - 4 \left(\frac{3}{5} \right)^3}{4 \left(\frac{4}{5} \right)^3 - 3 \left(\frac{4}{5} \right)} \end{aligned}$$

$$\begin{aligned} &= \frac{9}{5} - \frac{108}{125} = \frac{225 - 108}{125} = \frac{-117}{125} \quad \text{--- (1)} \\ &\quad \frac{256}{125} - \frac{12}{5} = \frac{256 - 300}{125} \end{aligned}$$

$$\begin{aligned} \text{RHS} &= \frac{3 \tan A - \tan^3 A}{1 - 3 \tan^2 A} = \frac{3 \left(\frac{3}{4} \right) - \left(\frac{3}{4} \right)^3}{1 - 3 \left(\frac{3}{4} \right)^2} \end{aligned}$$

$$= \frac{9 - 27}{4 \cdot 64} = \frac{144 - 24}{64}$$

$$\frac{1 - 27}{16} \quad \frac{16 - 27}{16}$$

$$= \frac{117}{64} = \frac{117}{644} \times \frac{161}{-11} = \frac{-117}{44} \quad \text{--- (2)}$$

$$\frac{-11}{16}$$

From (1) & (2)

$$\underline{\text{LHS} = \text{RHS}}$$

Hence, proved

Q.32 $\sin \theta = \frac{3}{4} = \frac{P}{H}$

$$P = 3k$$

$$H = 4k$$

$$B = \sqrt{(H)^2 - (P)^2}$$

$$= \sqrt{(4k)^2 - (3k)^2}$$

$$= \sqrt{16k^2 - 9k^2}$$

$$= \sqrt{7k^2}$$

$$= \sqrt{7}k$$

$$\operatorname{cosec} \theta = \frac{1}{\sin \theta} = \frac{4}{3}$$

$$\sec \theta = \frac{H}{B} = \frac{4k}{\sqrt{7}k} = \frac{4}{\sqrt{7}}$$

$$\cot \theta = \frac{B}{P} = \frac{\sqrt{7}k}{3k} = \frac{\sqrt{7}}{3}$$

$$\sqrt{\frac{\operatorname{cosec}^2 \theta - \cot^2 \theta}{\sec^2 \theta - 1}} = \frac{\sqrt{7}}{3}$$

$$\begin{aligned}
 \text{LHS} &= \frac{\csc^2 \theta - \cot^2 \theta}{\sec^2 \theta - 1} \\
 &= \frac{\left(\frac{4}{3}\right)^2 - \left(\frac{\sqrt{7}}{3}\right)^2}{\left(\frac{4}{\sqrt{7}}\right)^2 - 1} \\
 &= \frac{\frac{16}{9} - \frac{7}{9}}{\frac{16}{7} - 1} = \frac{\frac{9}{9}}{\frac{9}{7}} = \sqrt{\frac{7}{9}} \\
 &= \frac{\sqrt{7}}{3} = \text{RHS}
 \end{aligned}$$

Hence, proved

Q.33 $\sec A = \frac{17}{8} = \frac{H}{B}$

$$H = 17k$$

$$B = 8k$$

$$\begin{aligned}
 P &= \sqrt{H^2 - B^2} \\
 &= \sqrt{(17k)^2 - (8k)^2} \\
 &= \sqrt{289k^2 - 64k^2} \\
 &= \sqrt{225k^2} = 15k
 \end{aligned}$$

$$\sin A = \frac{P}{H} = \frac{15k}{17k} = \frac{15}{17}$$

$$\cos A = \frac{1}{\sec A} = \frac{8}{17}$$

$$\tan A = \frac{P}{B} = \frac{15k}{8k} = \frac{15}{8}$$

$$\frac{3 - 4 \sin^2 A}{4 \cos^2 A - 3} = \frac{3 - \tan^2 A}{1 - 3 \tan^2 A}$$

$$\underline{\text{LHS}} = \frac{3 - 4 \sin^2 A}{4 \cos^2 A - 3} = \frac{3 - 4 \left(\frac{15}{17} \right)^2}{4 \left(\frac{8}{17} \right)^2 - 3}$$

$$= \frac{3 - 4 \left(\frac{225}{289} \right)}{4 \left(\frac{64}{289} \right) - 3} = \frac{867 - 900}{289 - 1156}$$

$$= \frac{-33}{-611} = \frac{33}{611} \quad \text{--- ①}$$

$$\underline{\text{RHS}} = \frac{3 - \tan^2 A}{1 - 3 \tan^2 A} = \frac{3 - \left(\frac{15}{8} \right)^2}{1 - 3 \left(\frac{15}{8} \right)^2}$$

$$= \frac{3 - \frac{225}{64}}{1 - 3 \left(\frac{225}{64} \right)} = \frac{192 - 225}{64 - 675} = \frac{-33}{-611} = \frac{33}{611} \quad \text{--- ②}$$

From ① & ②,

$$\underline{\text{LHS}} = \underline{\text{RHS}}$$

Hence, proved

Q.34 $\cot \theta = \frac{3}{4} = \frac{B}{P}$

$$B = 3k$$

$$P = 4k$$

$$H = \sqrt{P^2 + B^2}$$

$$= \sqrt{(4k)^2 + (3k)^2} = \sqrt{16k^2 + 9k^2} = \sqrt{25k^2} = 5k$$

$$\sqrt{\frac{\sec \theta - \operatorname{cosec} \theta}{\sec \theta + \operatorname{cosec} \theta}} = \frac{1}{\sqrt{7}}$$

$$\sec \theta = \frac{H}{B} = \frac{5}{3} = \frac{5}{3}$$

$$\operatorname{cosec} \theta = \frac{H}{P} = \frac{5}{4} = \frac{5}{4}$$

$$\text{LHS} = \sqrt{\frac{\sec \theta - \operatorname{cosec} \theta}{\sec \theta + \operatorname{cosec} \theta}}$$

$$= \sqrt{\frac{5/3 - 5/4}{5/3 + 5/4}}$$

$$= \sqrt{\frac{\frac{20-15}{12}}{\frac{20+15}{12}}}$$

$$= \sqrt{\frac{5}{35}} = \sqrt{\frac{1}{7}}$$

$$= \text{RHS}$$

Hence proved

Q.35 $3 \cos \theta - 4 \sin \theta = 2 \cos \theta + \sin \theta$

$$3 \cos \theta - 2 \cos \theta = \sin \theta + 4 \sin \theta$$

$$\cos \theta = 5 \sin \theta$$

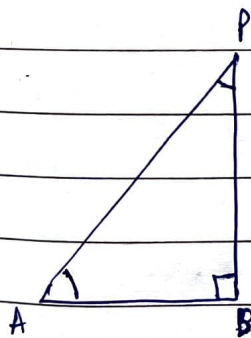
$$\frac{\cos \theta}{\sin \theta} = 5$$

$$\frac{\sin \theta}{\cos \theta} = \frac{1}{5}$$

$$\frac{\sin \theta}{\cos \theta} = \frac{1}{5}$$

$$\frac{\sin \theta}{\cos \theta} = \frac{1}{5}$$

$$\therefore \tan \theta = \frac{1}{5}$$



$$\tan A = \tan P \quad (\text{given})$$

$$\tan A = \frac{BP}{AB}, \quad \tan P = \frac{AB}{BP}$$

$$\therefore \frac{BP}{AB} = \frac{AB}{BP}$$

$\Rightarrow AB^2 = BP^2$

$\Rightarrow AB = BP$

$\therefore \underline{LA = LP}$ (LS opp. to equal sides are equal)

Hence, proved