Shivaji University , Kolhapur

Seat No.	

Paper Code(ENT202013)

M.Sc Entrance Exam , MAY-2020

58716-MATHS

Day And Date : Friday 20/11/2020 **Total Marks :** 100

Time : 13:00 To 14:30

Marks

1 A matrix whose each element is zero is called a : 1

A] Symmetric matrix B] Scalar Matrix

C] Null Matrix D] Identity Marix

2 The geometric series $\sum_{n=0}^{\infty} x^n$ converges only if 1

(a) $x \geq 1$

(b) $0 < x < 1$

(c) $x \leq -1$

(d) for all $x \in \mathbb{R}$

3 Consider the following data: 1

x	-3	-1	1
f	1	-2	3

Then f(0)----------

a) -0.5 b) 0.5 c) 0.7 d) -0.7

4 $\int_1^{\infty} e^{-\alpha x} dx$ converges if 1

(a) $\alpha > 0$

(b) $\alpha < 0$

(c) for any $\alpha \in \mathbb{R}$

(d) for any $\alpha \in \mathbb{R} - \{0\}$

5 Open ball $B\left(\frac{1}{4}, \frac{1}{2}\right)$ *in* $[0,1]$ with respect to absolute value of metric is …….. 1

a) $\left(-\frac{1}{2}, \frac{3}{4}\right)$ b) $\left(-\frac{1}{2}, -\frac{3}{4}\right]$ c) $\left[0, \frac{3}{4}\right)$ d) $\left(0, -\frac{3}{4}\right]$

6 If $u = r^2 cos2\theta$ is harmonic function then $v =$ ____. 1

A) $rcos2\theta$ B) $rsin2\theta$ C) $r^2 sin2\theta$ D) $r^3 sin2\theta$

7 The singular point of the function $f(z) = \frac{z+5}{z^2-5z+6}$ are _____. 1

A) $z = -1, 2$ B) $z = 1, 2$ C) $z = -2, -3$ D) $z = 2, 3$

8 The function $f(z) = |z|^2$ is _____. 1

A) continuous everywhere but nowhere differentiable except at the origin

B) continuous everywhere and differentiable for every z

C) discontinuous everywhere and differentiable for every z

D) neither continuous nor differentiable

9 If $f(z) = u + iv$ is analytic function in a finite region and $u = x^3 - 3xy^2$ then $v =$ _____. 1

A) $3xy - y^3$ B) $3x^2y^2 + c$ C) $3x^2y - y^3 + c$ D) $3x^2y^2 - y^3 + c$

10 A measure of how much the conic section deviates from being circular is termed as 1

A] Radius of curvature B] Pedal equation

C] Eccentricity D] Polar forms.

11 1

Function $f(x) = \begin{cases} x & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -x & \text{if } x < 0 \end{cases}$ is _____.

A) continuous and differentiable at $x = 0$
B) not continuous but differentiable at $x = 0$
C) continuous but not differentiable at $x = 0$
D) neither continuous nor differentiable at $x = 0$

12 At $z = 0$ the function $f(z) = \overline{z}$, $z \in \mathbb{C}$ is 1

(a) not continuous
(b) analytic
(c) not differentiable
(d) differentiable

13 "Every square matrix satisfy its characteristic equations" This statement is related to ---- 1

A] Bolzano–Weierstrass theorem B] Hamiltonion Method

C] Cayley Hamilton Theorem D] Reduction Method.

14 1

Polar form of complex number $-5+i5$ is

(a) $5\sqrt{2}\ e^{\frac{3\pi i}{4}}$

(b) $5\sqrt{2}\ e^{\frac{\pi i}{4}}$

(c) $5\sqrt{2}\ e^{\frac{\pi i}{2}}$

(d) $5\sqrt{2}\ e^{2\pi i}$

15 1

Zeros of $f(z)=\left(\frac{z+1}{z^2+1}\right)^2$

(a) $-1,-1$

(b) ± 1

(c) $\pm i$

(d) $\pm 1, \pm i$

16 1

The regular functions are called _____

A) Isomorphic

B) Homomorphic

C)Holomorphic

D)conformal

17 Let $f(x)$ is a continuous function on a closed interval $[a, b]$. Consider the statements 1

i) $f(x)$ is bounded in $[a, b]$.

ii) $f(x)$ is uniformly continuous in $[a, b]$.

Then

A) i) is false and ii) is true

B) i) is true and ii) is false

C) both i) and ii) are false

D) both i) and ii) are true

18 Which of the following is invalid property for congruence modulo? 1

A) $a \equiv b(mod\ n)$ if $n|(a-b)$

B) $a \equiv b(mod\ n)$ implies $b \equiv a(mod\ n)$

C) $a \equiv b(mod\ n)$ and $b \equiv c(mod\ n)$ implies $a \equiv c(mod\ n)$

D) $a \equiv b(mod\ n)$ implies $ax \equiv by(mod\ n)$

19 If R is a relation such that aRb if and only if $a|b$ means that a is a divisor of b, then the relation R is ---------- 1

A) reflexive, transitive and not symmetric B) transitive and symmetric

C) reflexive, transitive and symmetric D) reflexive and symmetric

20 For any integer x, $gcd(a, b) =$ ---------- 1

A) $gcd(a, b + ax)$ B) $gcd(b, a + bx)$

C) $x \cdot gcd(b, ax)$ D) $x \cdot gcd(a, bx)$

21 In the differential equation $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$, P, Q, R are 1

A) Direction ratios of the normal to the curve

B) Direction cosines of the normal to the curve

C) Direction ratios of the tangent to the curve

D) Direction cosines of the tangent to the curve

22 Total differential equation can be solved by 1

A) Changing dependent variable B) Changing independent variable

C) Method of multipliers D) Taking one variable as constant

23 The alternating series $\sum_{n=0}^{\infty}(-1)^{n+1}a_n$ always converges only if 1

(a) $\lim_{n\to\infty} a_n = 0$ and $\{a_n\}_{n=1}^{\infty}$ is non-increasing

(b) $\lim_{n\to\infty} a_n = 0$ and $\{a_n\}_{n=1}^{\infty}$ is non-decreasing

(c) $\lim_{n\to\infty} a_n = \infty$ and $\{a_n\}_{n=1}^{\infty}$ is non-increasing

(d) $\lim_{n\to\infty} a_n = -\infty$ and $\{a_n\}_{n=1}^{\infty}$ is non-decreasing

24 1

Polar form of conic $(x-3)^2+(y+4)^2=25$ is

A] $r = 6\cos\theta + 8\sin\theta$ B] $r = 3\cos\theta + 4\sin\theta$

C] $r = 6\cos\theta - 8\sin\theta$ D] $1 = 1 + e\cos\theta$

25 The equivalence class of element $a \in A$ w.r.t. equivalence relation R is---------- 1

A) $[a] = \{x \in A/(x,a) \notin R\}$ B) $[a] = \{x \in A/(x,a) \notin A\}$

C) $[a] = \{x \in A/(x,a) \in R\}$ D) $[a] = \{x \in A/(x,a) \in A\}$

26 Let $\{s_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ are bounded sequences of real numbers. Consider the statements 1

(I) $\limsup_{n\to\infty}(s_n + t_n) \le \limsup_{n\to\infty} s_n + \limsup_{n\to\infty} t_n$

(I) $\liminf_{n\to\infty}(s_n + t_n) \ge \liminf_{n\to\infty} s_n + \liminf_{n\to\infty} t_n$.

Then,

(a) only (I) is true

(b) only (II) is true

(c) both (I) and (II) are true

(d) both (I) and (II) are false

27 A formula for the sequence $1, 3, 6, 10, 15\cdots$ is 1

(a) $s_n = s_{n-1} - n,\ n \in \mathbb{N}$

(b) $s_n = s_{n-1} + n,\ n \in \mathbb{N}$

(c) $s_n = s_{n-1} + n^2,\ n \in \mathbb{N}$

(d) $s_n = s_{n-1} - n^2,\ n \in \mathbb{N}$

28 Consider the characteristic function $\chi : [0,1] \to \mathbb{R}$. Then $\int_0^1 \chi =$ 1

(a) -1

(b) 1

(c) 0

(d) ∞

29 39 A monoid is a group if and only if ····· hold in it . 1
a. distributive laws b. cancellation laws c. associative laws d.closure law

30 40 If G is a group of prime order then it will have ····· subgroups . 1
a. three b. $|G|$ c. two d. four

31 43 Homomorphic image of an abelian group is ····· group. 1
a. abelian b. cyclic c. finite d. non abelian

32 44 Smallest non commutative ring is of order ····· . 1
a. four b. two c. three d. five

33 A homomorphism $f : G \to G'$ is one-one if and only if ····· . 1
a. Ker$f = \{G'\}$ b. Ker$f = \{G\}$ c. Ker$f = \{0\}$ d. Ker$f = \{e\}$

34 Every group G is isomorphic to a ····· . 1
a. abelian group b. cyclic group c. permutation group d.dyhedral group

35 If $x = r\cos\theta$ and $y = r\sin\theta$ then $\frac{\partial(x,y)}{\partial(r,\theta)} \cdot \frac{\partial(r,\theta)}{\partial(x,y)} =$ _____. 1

A) 1
B) r
C) $\frac{1}{r}$
D) 0

36 A non zero element a in Z_n is unit if and only if a and n are ····· . 1
a. prime b. relatively prime c. multiple of others d. a divides n

37 Any ring of prime order is ····· . 1
a. commutative b. non commutative c. division ring d. field

38 If a ∈ M, M is a metric space and {a} is both open and closed in M, then M = _____. 1

a) ℓ^2 b) ℓ^∞ c) R_d d) R

39 If a subset A of R_d contains only a finite number of points, then A is _____. 1

a) unbounded b) infinite
c) totally bounded d) need not be closed

40 If < M, ρ > is a metric space, a ∈ M and r > 0 then we define B[a : r] to be the set of all points x ∈ M whose distance to _____. 1

a) a is less than or equal to r b) a is less than r
c) r is less than or equal to a d) a is greater than r

41 The metric space $\langle M, \rho \rangle$ is compact if it is _____. 1

a) both complete and bounded b) both complete and totally bounded
c) both complete and connected d) not complete and bounded

42 If A is not a bounded subset of a metric space(M, ρ) then diamA is …….. 1

a) infinity c) less than one
b) zero d) greater than zero

43 The function f from set A into a metric space M is said to be bounded if …… 1

a) the co-domain M of f is unbounded c) set A is only bounded
b) set A is only bounded above d) its range f(A) is bounded subset of M

44 If $T : V \to W$ is a linear transformation , then the set of all vectors in W that are images under T of at least one vector in V is called _____ . 1

A) a kernel of T B) a range of T C) nullity of T D) none of these.

45 A non empty subset W of a vector space V(F) is a subspace of V iff _____ . 1

A) $\alpha x - \beta y \in W$, for $\alpha , \beta \in F$ and $x, y \in V$

B) $\alpha x + \beta y \notin W$, for $\alpha , \beta \in F$ and $x, y \in W$

C) $\alpha x - \beta y \notin W$, for $\alpha , \beta \in F$ and $x, y \in W$

D) $\alpha x + \beta y \in W$, for $\alpha , \beta \in F$ and $x, y \in W$

46 $f(x, y) = x^2 + y^2 + 6x + 12$ has _____ 1

A) minimum value 3 at $(-3,0)$

B) maximum value 3 at $(-3,0)$

C) minimum value -3 at $(0, -3)$

D) maximum value -3 at $(0, -3)$

47 The norm of the vector $u = (1, -2, 5)$ is _____ . 1

A) $\sqrt{30}$ B) $\sqrt{22}$ C) $2\sqrt{5}$ D) $6\sqrt{5}$

48 If $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$, then characteristic polynomial of A is _____ . 1

A) x^2 B) $x^2 + 1$ C) $x^2 - 1$ D) $(x - 1)^2$

49 A linear transformation $T : V \to W$ is non singular if _____. 1

A) T is not one- one B) T is not one- one C) ker T = {0} D) none of these

50 The sum of two subspaces of a vector space is _____ . 1

A) not a subspace B) a subspace C) field D) ring

51 Dimension of a vector space is 1

A) Number of elements in the basis

B) Number of elements in a vector space

C) Number of elements in the set that span the vector space

D) always finite

52 A linear transformation $T : V \longrightarrow V$ is one-one if and only if ····· . 1

a. Ker$T = \{V\}$ b. Ker$T = \{U\}$ c. T is onto d. Ker$T = \{V \cap U\}$

53 Gram-Schmidt process is used to convert given basis to ····· . 1

a. orthonormal basis b. linear span c. orthogonal basis d. linearly independent set

54 The value of integral $\int_C \frac{e^{2z}dz}{(z-3)(z-4)}$ where C is circle $|z| = 2$ is _____. 1

A) 0 B) 2πi C) πi D) $-2\pi i$

55 If the function f(z) is analytic in the entire complex plane then f(z) is called_____. 1

A) integral function B) meromorphic function C) analytic continuation D) convex function

56 1

If $\bar{f}$ is a constant vector, then

A) $div\ \bar{f} = 0$ and $curl\ \bar{f} \neq \bar{0}$

B) $div\ \bar{f} \neq 0$ and $curl\ \bar{f} \neq \bar{0}$

C) $div\ \bar{f} = 0$ and $curl\ \bar{f} = \bar{0}$

D) $div\ \bar{f} \neq 0$ and $curl\ \bar{f} = \bar{0}$

57 If the vector $\bar{f} = (3x + y)\hat{\imath} + (3y - z)\hat{\jmath} + (xy + 3az)\hat{k}$ is solenoidal, then constant a =_____. 1

A) 1
B) -1
C) 2
D) -2

58 Geometrical interpretation of $Pdx + Qdy + Rdz = 0$ is -------- 1

A) P, Q, R are direction ratios of the normal to the curve

B) P, Q, R are direction cosines of the normal to the curve

C) P, Q, R are direction ratios of the tangent to the curve

D) P, Q, R are direction cosines of the tangent to the curve

59 Ordinary simultaneous differential equation can be solved by ------- 1

A) Changing dependent variable B) Changing independent variable

C)Taking one variable as constant D) Method of multipliers

60 The number of elements in the adjacency matrix of a graph having 7 vertices is ---------- 1

A) 7 B) 14 C) 36 D) 49

61 $P \rightarrow (Q \rightarrow R)$ is equivalent to 1

A) $(P \wedge Q) \rightarrow R$ B) $(P \wedge Q) \rightarrow \sim R$

C) $(P \vee Q) \rightarrow R$ D) $(P \vee Q) \rightarrow \sim R$

62 Let $A, B \subseteq \mathbb{R}$. Then $\chi_{AUB}=$ 1

(a) $\chi_A - \chi_B$
(b) $\chi_A + \chi_B$
(c) $\max\{\chi_A, \chi_B\}$
(d) $\min\{\chi_A, \chi_B\}$

63 Which of the following sequence is Monotone? 1

(a) $\{\sin n\}_{n=1}^{\infty}$
(b) $\{\cos n\}_{n=1}^{\infty}$
(c) $\{\tan n\}_{n=1}^{\infty}$
(d) $\{\frac{1}{1+n^2}\}_{n=1}^{\infty}$

64 Let $s_n = (-1)^n \left(1 + \frac{1}{n}\right)$. Then $\limsup_{n\to\infty} s_n =$ 1

(a) ∞
(b) -1
(c) 1
(d) 0

65 The l.u.b. of the set $\{\pi + 1, \pi + \frac{1}{2}, \pi + \frac{1}{3}, \pi + \frac{1}{4}, \cdots\}$ is 1

(a) $\pi + 1$
(b) π
(c) $\pi + \frac{1}{2}$
(d) ∞

66 The normalizer is a ····· of G . 1
a. subgroup b. supergroup c. group d. none of these

67 The generator of cyclic group is ····· . 1
a. unique b. not necessarily unique c. exact two d. $|G|$

68 Let $\phi(x, y, z)$ be a scaler function. Then $\nabla\phi = \frac{\partial\phi}{\partial x}\hat{\imath} + \frac{\partial\phi}{\partial y}\hat{\jmath} + \frac{\partial\phi}{\partial z}\hat{k}$ is called as _____ 1

A) curl of ϕ
B) gradient of ϕ
C) divergence of ϕ
D) irrotational vector

69 In a metric space intersection of an infinite number of open sets_____. 1

a) is always open b) is both open and closed

c) is always closed d) need not be open

70 The interval $\left[0, \frac{1}{2}\right)$ in R^1 is _____. 1

a) is always open b) is both open and closed

c) is always closed d) not an open subset of R^1

71 In a metric space union of an infinite number of open sets 1

a) is always closed c) is always open

b) need not be closed d) neither open nor closed

72 If $T: R^4 \to R^3$ is defined by 1

$T(x, y, z, w) = (x - y, y - z, z - w) for\ (x, y, z, w) \in R^4$ then the dimension of Kernel T is .....

a) 0 b) 1 c) 2 d) 3

73 Number of solutions to the equation $(1 - i)^x = 2^x$ is : 1

A] 1

B] 2

C] 3

D] no solution

74 If , arg(z) < 0, then arg(-z) – arg(z) = 1

A] π

B] $\frac{\pi}{4}$

C] $-\frac{\pi}{4}$

D] $\frac{\pi}{2}$

75 If f(x) satisfies all conditions of Rolle's mean value theorem in [a, b] then there exists a point c ∈ (a, b) such that…… 1

A] f(a) = f(b) B] f'(c) > 0

C] f'(c) < 0 D] f'(c) = 0

76 $\lim_{x\to 0} \frac{e^x-1}{x}$ = ……. 1

A] 0 B] 1 C] - 1 D] log x

77 The infinite series 1- x + x^2- x^3+…….is of …… 1

A] e^x B] $\frac{1}{1-x}$ C] $\frac{1}{1+x}$ D] $\frac{1}{x-1}$

78 Right hand limit of $\lim_{x\to 0} \frac{e^{\frac{1}{x}}}{e^{\frac{1}{x}}+1}$ is …… 1

A] 0 B] 1 C] ∞ D] -∞

79 _____ is the solution of $\frac{d^2y}{dx^2} + P\frac{dy}{dx} = 0$ where P is function of x. 1

A) $y = \int e^{-\int P\,dx} dx$

B) $y = \int e^{\int P\,dx} dx$

C) $y = e^{-\int P\,dx}$

D) $y = e^{\int P\,dx}$

80 If p, q, r are functions of u, v, w and u, v, w are functions of x, y, z and if $J_1 = \frac{\partial(p,q,r)}{\partial(x,y,z)}$, $J_2 = \frac{\partial(p,q,r)}{\partial(u,v,w)}$ and $J_3 = \frac{\partial(u,v,w)}{\partial(x,y,z)}$, then 1

A) $J_3 = J_1 \cdot J_2$
B) $J_2 = J_1 \cdot J_3$
C) $J_1 = J_2 \cdot J_3$
D) $J_1 = J_2 = J_3$

81 If f(-0.4)=0.6 and f(-0.2)= 0.8 then the approximate value of $f'(-0.2)$ is----- 1

a) 0.6 b) 0.7 c) 1.0 d) -0.75

82 The particular integral of $(D^2 - 5DD' + 4D'^2)z = \sin(4x + y)$ is 1

(A) $\frac{1}{3}\cos(4x + y)$
(B) $-\frac{1}{3}x\cos(4x + y)$
(C) $-\frac{5x}{3}\sin(4x + y)$
(D) $-\frac{5x}{2}\sin(4x + y)$

83 The partial differetntial equation obtained by eliminating arbitrary constants a and b from the equation $2z = (ax + y)^2 + b$ is 1

(A) $yp - xq = p$
(B) $px + qy = q^2$
(C) $p + pq = zq$
(D) $p - pq = z$

84 The complementary function of $(D^3 - 4D^2D' + 4DD'^2)z = 0$ is 1

(A) $z = \phi_1(y + x) + \phi_2(y + 2x) + x\,\phi_3(y + 2x)$
(B) $z = \phi_1(y) + \phi_2(y + 2x) + \phi_3(y - 2x)$
(C) $z = \phi_1(y) + \phi_2(y + 2x) + x\,\phi_3(y + 2x)$
(D) $z = \phi_1(y + x) + \phi_2(y + 2x) + x\,\phi_3(y - 2x)$

85 If I_n is length of nth interval in bisection method then $\lim_{n\to\infty} I_n =$ _____ . 1

A) a+b
B) 2
C) 1
D) 0

86 In Jacobi's method to solve system of non-homogeneous linear equation, the condition that, coefficient matrix is diagonally row dominant is _____. 1

A) Necessary but not sufficient
B) Sufficient but not necessary
C) Necessary and sufficient
D) Neither necessary nor sufficient

87 If f(0)=1, f(0.5)=0.6 and if trapezium rule gives $\int_0^1 f(x)dx = 0.675$ then f(1)=-------- 1

a) 0 b) 0.4 c) 0.7 d) 0.5

88 The divided difference $f[x, y]$ =---------- 1

a) $f'(z)$, for some z between x and y b) $\frac{f[x]-f[y]}{x-y}$

c) both a and b d) neither a not b

89 The partial differential equation obtained by eliminating arbitrary function ϕ from the equation $z = y^2 + 2\,\phi\left(\frac{1}{x} + \log y\right)$ is 1

(A) $px^2 + qy = 2y^2$

(B) $xp + yq = 0$

(C) $px^2 + qy = 1$

(D) $p - q = 2y^2$

90 1

Solution of $\frac{d^2y}{dx^2} - y = 0$ is _____.

A) $y = (c_1x + c_2)e^x$

B) $y = (c_1x + c_2)e^{-x}$

C) $y = c_1e^x + c_2e^{-x}$

D) $y = c_1 \cos x + c_2 \sin x$

91 If $\lambda^2 + a\lambda + b = 0$ is characteristic equation of matrix $A = \begin{bmatrix} 2 & \sqrt{2} \\ \sqrt{2} & 1 \end{bmatrix}$ then values of a and b are respectively _____. 1

A) 2, 0

B) 3, 1

C) -3, 1

D) -3, 0

92 Gauss-Elimination to solve system of linear equations fails if pivot element is _____. 1

A) Numerically largest in row

B) Numerically smallest in row

C) 1

D) 0

93 _____ method is also called back substitution method. 1

A) Gauss Elimination

B) Gauss-Jordon

C) Jacobi's

D) Gauss-Seidal

94 Eigen values of matrix $A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ are _____. 1

A) (1, 2, 3)
B) (5, 4, -3)
C) (4, 3, -1)
D) (2, 2, 2)

95 Power method is useful to find _____ eigen value. 1

A) Largest
B) Smallest
C) Numerically second largest
D) Numerically largest value

96 Consider the following data: 1

x	-2	0	1	2	4
f	0	5	3	7	1

If $l_4(x)$ is the Lagrangian polynomial for this data then $l_4(1) = ------$

a) 5 b)1 c) -1 d) 0

97 The general integral of $(x^2 + 2y^2)p - xyq = xz$ is 1

(A) $F(x^2 - y - z, y^2x^2 + y^4) = 0$
(B) $F(xy - z^2, yz) = 0$
(C) $F(x^4y - z^2, xyz) = 0$
(D) $F(yz,\ y^2x^2 + y^4) = 0$

98 The complete solution of $(D^2 - 6DD' + 9D'^2)z = 6x + 2y$ is 1

(A) $z = \phi_1(y + 3x) + x\,\phi_2(y + 3x) + x^2(y + 3x)$
(B) $z = \phi_1(y + 3x) + \phi_2(y - 3x) + x^2(y + 3x)$
(C) $z = \phi_1(y + 3x) + x\,\phi_2(y + 3x) + (2y + 6x)$
(D) $z = \phi_1(y - 3x) + x\,\phi_2(y + 3x) + (y + 3x)$

99 Consider the data 1

x	-1	1	3
f	5	9.2	-1

Then the approximate value of $f''(1) = ----------$

a) 9.2 b) 18.4 c) -3.6 d) -4.15

100 Consider the initial value problem $y' = -y^2, y(0) = y_0 = -0.5$. If the Euler method gives $y_1 = y(h) = -0.55$ then the step size $h = -------$ 1

a) 1/2 b) 3/4 c) 3/2 d) 1/5