

Q1) In a $\triangle ABC$, if $\angle A = 55^\circ$, $\angle B = 40^\circ$, Find $\angle C$.

Solution:

Given Data:

$$\angle A = 55^\circ, \angle B = 40^\circ, \text{ then } \angle C = ?$$

We know that

In a $\triangle ABC$ sum of all angles of a triangle is 180°

$$\text{i.e., } \angle A + \angle B + \angle C = 180^\circ$$

$$\Rightarrow 55^\circ + 40^\circ + \angle C = 180^\circ$$

$$\Rightarrow 95^\circ + \angle C = 180^\circ$$

$$\Rightarrow \angle C = 180^\circ - 95^\circ$$

$$\Rightarrow \angle C = 85^\circ$$

Q2) If the angles of a triangle are in the ratio 1:2:3, determine three angles.

Solution:

Given that,

Angles of a triangle are in the ratio 1:2:3

Let the angles be $x, 2x, 3x$

$\therefore$ We know that,

Sum of all angles of triangles is 180°

$$x + 2x + 3x = 180^\circ$$

$$\Rightarrow 6x = 180^\circ$$

$$\Rightarrow x = \frac{180^\circ}{6}$$

$$\Rightarrow x = 30^\circ$$

$$\text{Since } x = 30^\circ$$

$$2x = 2(30^\circ) = 60^\circ$$

$$3x = 3(30^\circ) = 90^\circ$$

Therefore, angles are $30^\circ, 60^\circ, 90^\circ$

Q3) The angles of a triangle are $(x - 40^\circ)$, $(x - 20^\circ)$ and $(\frac{1}{2}x - 10^\circ)$. Find the value of x .

Solution:

Given that,

The angles of a triangle are

$$(x - 40^\circ), (x - 20^\circ) \text{ and } (\frac{1}{2}x - 10^\circ)$$

We know that,

Sum of all angles of triangle is 180^0

$$\therefore (x - 40^0) + (x - 20^0) + (\frac{1}{2}x - 10^0) = 180^0$$

$$2x + \frac{1}{2}x - 70^0 = 180^0$$

$$\frac{5}{2}x = 180^0 + 70^0$$

$$5x = 2(250)^0$$

$$x = \frac{500^0}{5}$$

$$\therefore x = 100^0$$

Q4) The angles of a triangle are arranged in ascending order of magnitude. If the difference between two consecutive angles is 10^0 , find the three angles.

Solution:

Given that,

The difference between two consecutive angles is 10^0

Let $x, x+10^0, x+20^0$ be the consecutive angles that differ by 10^0

We know that,

Sum of all angles in a triangle is 180^0

$$x+x+10^0+x+20^0 = 180^0$$

$$3x+30^0 = 180^0$$

$$\Rightarrow 3x = 180^0 - 30^0$$

$$\Rightarrow 3x = 150^0$$

$$\Rightarrow x = 50^0$$

Therefore, the required angles are

$$x = 50^0$$

$$x+10^0 = 50^0 + 10^0 = 60^0$$

$$x+20^0 = 50^0 + 20^0 = 70^0$$

As the difference between two consecutive angles is 10^0 , the three angles are $50^0, 60^0, 70^0$.

Q5) Two angles of a triangle are equal and the third angle is greater than each of those angles by 30^0 . Determine all the angles of the triangle.

Solution:

Given that,

Two angles of a triangle are equal and the third angle is greater than each of those angles by 30^0 .

Let $x, x, x+30^0$ be the angles of a triangle

We know that,

Sum of all angles in a triangle is 180^0

$$x + x + x + 30^0 = 180^0$$

$$3x + 30^0 = 180^0$$

$$3x = 180^0 - 30^0$$

$$3x = 150^0$$

$$x = 50^0$$

Therefore, the three angles are $50^0, 50^0, 80^0$.

Q6) If one angle of a triangle is equal to the sum of the other two, show that the triangle is a right angle triangle.

Solution:

If one angle of a triangle is equal to the sum of the other two angles

$$\Rightarrow \angle B = \angle A + \angle C$$

In $\triangle ABC$,

Sum of all angles of a triangle is 180^0

$$\Rightarrow \angle A + \angle B + \angle C = 180^0$$

$$\Rightarrow \angle B + \angle B = 180^0 [\angle B = \angle A + \angle C]$$

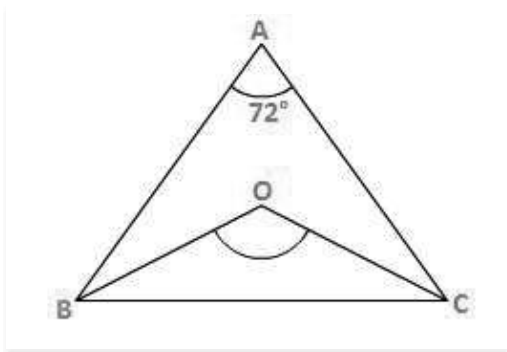
$$\Rightarrow 2\angle B = 180^0$$

$$\Rightarrow \angle B = \frac{180^0}{2}$$

$$\Rightarrow \angle B = 90^0$$

Therefore, ABC is a right angled triangle.

Q7) ABC is a triangle in which $\angle A = 72^0$, the internal bisectors of angles B and C meet in O. Find the magnitude of $\angle BOC$.



Solution:

Given,

ABC is a triangle where $\angle A = 72^0$ and the internal bisector of angles B and C meeting O.

In $\triangle ABC$,

$$\angle A + \angle B + \angle C = 180^\circ$$

$$\Rightarrow 72^\circ + \angle B + \angle C = 180^\circ$$

$$\Rightarrow \angle B + \angle C = 180^\circ - 72^\circ$$

Dividing both sides by '2'

$$\Rightarrow \frac{\angle B}{2} + \frac{\angle C}{2} = \frac{108^\circ}{2}$$

$$\Rightarrow \angle OBC + \angle OCB = 54^\circ$$

$$\text{Now, In } \triangle BOC \Rightarrow \angle OBC + \angle OCB + \angle BOC = 180^\circ$$

$$\Rightarrow 54^\circ + \angle BOC = 180^\circ$$

$$\Rightarrow \angle BOC = 180^\circ - 54^\circ = 126^\circ$$

$$\therefore \angle BOC = 126^\circ$$

Q8) The bisectors of base angles of a triangle cannot enclose a right angle in any case.

Solution:

In $\triangle XYZ$,

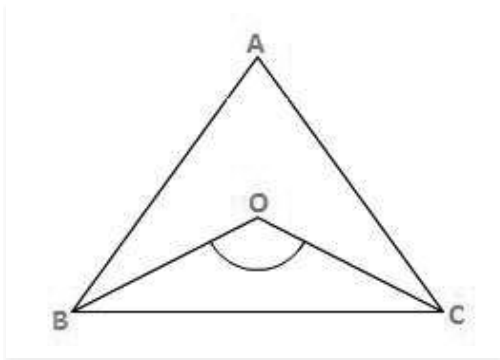
Sum of all angles of a triangle is 180°

$$\text{i.e., } \angle X + \angle Y + \angle Z = 180^\circ$$

Dividing both sides by '2'

$$\Rightarrow \frac{1}{2}\angle X + \frac{1}{2}\angle Y + \frac{1}{2}\angle Z = 90^\circ$$

$$\Rightarrow \frac{1}{2}\angle X + \angle OYZ + \angle OYZ = 90^\circ \quad [\because OY, OZ, \angle Y \text{ and } \angle Z]$$



$$\Rightarrow \angle OYZ + \angle OZY = 90^\circ - \frac{1}{2}\angle X$$

Now in $\triangle YOZ$

$$\therefore \angle YOZ + \angle OYZ + \angle OZY = 180^\circ$$

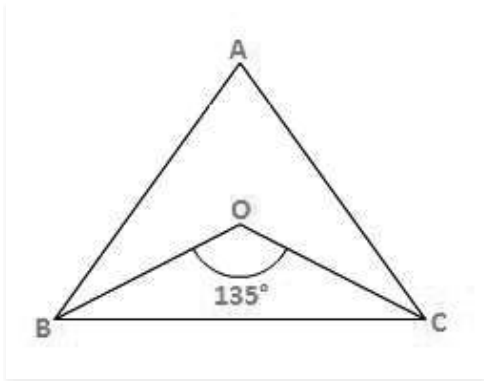
$$\Rightarrow \angle YOZ + 90^\circ - \frac{1}{2}\angle X = 180^\circ$$

$$\Rightarrow \angle YOZ = 90^\circ - \frac{1}{2}\angle X$$

Therefore, the bisectors of a base angle cannot enclose right angle.

Q9) If the bisectors of the base angles of a triangle enclose an angle of 135° , prove that the triangle is a right angle.

Solution:



Given the bisectors of the base angles of a triangle enclose an angle of 135°

i.e., $\angle BOC = 135^\circ$

But, We know that

$$\angle BOC = 90^\circ + \frac{1}{2}\angle A$$

$$\Rightarrow 135^\circ = 90^\circ + \frac{1}{2}\angle A$$

$$\Rightarrow \frac{1}{2}\angle A = 135^\circ - 90^\circ$$

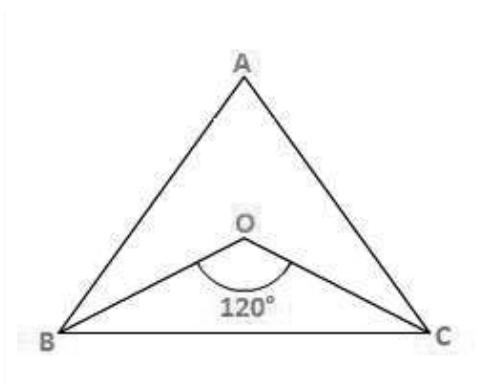
$$\Rightarrow \angle A = 45^\circ(2)$$

$$\Rightarrow \angle A = 90^\circ$$

Therefore, $\triangle ABC$ is a right angle triangle that is right angled at A.

Q10) In a $\triangle ABC$, $\angle ABC = \angle ACB$ and the bisectors of $\angle ABC$ and $\angle ACB$ intersect at O such that $\angle BOC = 120^\circ$. Show that $\angle A = \angle B = \angle C = 60^\circ$.

Solution:



Given,

In $\triangle ABC$,

$$\angle ABC = \angle ACB$$

Dividing both sides by '2'

$$\frac{1}{2}\angle ABC = \frac{1}{2}\angle ACB$$

$$\Rightarrow \angle OBC = \angle OCB \quad [\because OB, OC \text{ bisects } \angle B \text{ and } \angle C]$$

Now,

$$\angle BOC = 90^\circ + \frac{1}{2}\angle A$$

$$\Rightarrow 120^\circ - 90^\circ = \frac{1}{2}\angle A$$

$$\Rightarrow 30^\circ * (2) = \angle A$$

$$\Rightarrow \angle A = 60^\circ$$

Now in $\triangle ABC$

$$\angle A + \angle ABC + \angle ACB = 180^\circ \quad (\text{Sum of all angles of a triangle})$$

$$\Rightarrow 60^\circ + 2\angle ABC = 180^\circ \quad [\because \angle ABC = \angle ACB]$$

$$\Rightarrow 2\angle ABC = 180^\circ - 60^\circ$$

$$\Rightarrow \angle ABC = \frac{120^\circ}{2} = 60^\circ$$

$$\Rightarrow \angle ABC = \angle ACB$$

$$\therefore \angle ACB = 60^\circ$$

Hence Proved.

Q11) Can a triangle have:

(i) Two right angles?

(ii) Two obtuse angles?

(iii) Two acute angles?

(iv) All angles more than 60° ?

(v) All angles less than 60° ?

(vi) All angles equal to 60° ?

Justify your answer in each case.

Sol:

(i) No,

Two right angles would up to 180° . So the third angle becomes zero. This is not possible, so a triangle cannot have two right angles. [Since sum of angles in a triangle is 180°]

(ii) No,

A triangle can't have 2 obtuse angles. Obtuse angle means more than 90° So that the sum of the two sides will exceed 180° which is not possible. As the sum of all three angles of a triangle is 180° .

(iii) Yes

A triangle can have 2 acute angles. Acute angle means less the 90° angle.

(iv) No

Having angles more than 60° make that sum more than 180° . This is not possible. [Since the sum of all the internal angles of a triangle is 180°]

(v) No

Having all angles less than 60° will make that sum less than 180° which is not possible. [Therefore, the sum of all the internal angles of a triangle is 180°]

(vi) Yes

A triangle can have three angles equal to 60° . Then the sum of three angles equal to the 180° . Such triangles are called as equilateral triangle. [Since, the sum of all the internal angles of a triangle is 180°]

Q12) If each angle of a triangle is less than the sum of the other two, show that the triangle is acute angled.

Solution

Given each angle of a triangle less than the sum of the other two

$$\therefore \angle X + \angle Y + \angle Z$$

$$\Rightarrow \angle X + \angle X < \angle X + \angle Y + \angle Z$$

$$\Rightarrow 2\angle X < 180^\circ \quad [\text{Sum of all the angles of a triangle}]$$

$$\Rightarrow \angle X < 90^\circ$$

Similarly $\angle Y < 90^\circ$ and $\angle Z < 90^\circ$

Hence, the triangles are acute angled.