

Comparing Quantities

CHAPTER

7



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7.1 Recalling Ratios and Percentages

We know, ratio means comparing two quantities.

A basket has two types of fruits, say, 20 apples and 5 oranges.

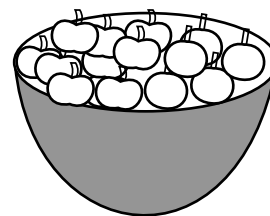
Then, the ratio of the number of oranges to the number of apples = 5 : 20.

The comparison can be done by using fractions as, $\frac{5}{20} = \frac{1}{4}$

The number of oranges is $\frac{1}{4}$ th the number of apples. In terms of ratio, this is 1 : 4, read as, “1 is to 4”

OR

Number of apples to number of oranges = $\frac{20}{5} = \frac{4}{1}$ which means, the number of apples is 4 times the number of oranges. This comparison can also be done using percentages.



There are 5 oranges out of 25 fruits.

So percentage of oranges is

$$\frac{5}{25} \times \frac{4}{4} = \frac{20}{100} = 20\%$$

[Denominator made 100].

OR

By unitary method:

Out of 25 fruits, number of oranges are 5.

So out of 100 fruits, number of oranges

$$= \frac{5}{25} \times 100 = 20.$$

Since  contains only apples and oranges,

So, percentage of apples + percentage of oranges = 100

or percentage of apples + 20 = 100

or percentage of apples = 100 – 20 = 80

Thus the basket has 20% oranges and 80% apples.

Example 1: A picnic is being planned in a school for Class VII. Girls are 60% of the total number of students and are 18 in number.

The picnic site is 55 km from the school and the transport company is charging at the rate of ₹ 12 per km. The total cost of refreshments will be ₹ 4280.

Can you tell.

1. The ratio of the number of girls to the number of boys in the class?
2. The cost per head if two teachers are also going with the class?
3. If their first stop is at a place 22 km from the school, what per cent of the total distance of 55 km is this? What per cent of the distance is left to be covered?

Solution:

1. To find the ratio of girls to boys.

Ashima and John came up with the following answers.

They needed to know the number of boys and also the total number of students.

Ashima did this

Let the total number of students

be x . 60% of x is girls.

Therefore, 60% of $x = 18$

$$\frac{60}{100} \times x = 18$$

$$\text{or, } x = \frac{18 \times 100}{60} = 30$$

Number of students = 30.

OR

John used the unitary method

There are 60 girls out of 100 students.

There is one girl out of $\frac{100}{60}$ students.

So, 18 girls are out of how many students?

$$\begin{aligned} \text{Number of students} &= \frac{100}{60} \times 18 \\ &= 30 \end{aligned}$$

So, the number of boys = $30 - 18 = 12$.

Hence, ratio of the number of girls to the number of boys is $18 : 12$ or $\frac{18}{12} = \frac{3}{2}$.
 $\frac{3}{2}$ is written as 3 : 2 and read as 3 is to 2.

2. To find the cost per person.

Transportation charge = Distance both ways $\times$ Rate

$$= ₹ (55 \times 2) \times 12$$

$$= ₹ 110 \times 12 = ₹ 1320$$

Total expenses = Refreshment charge

+ Transportation charge

$$= ₹ 4280 + ₹ 1320$$

$$= ₹ 5600$$

Total number of persons = 18 girls + 12 boys + 2 teachers

$$= 32 \text{ persons}$$

Ashima and John then used unitary method to find the cost per head.

For 32 persons, amount spent would be ₹ 5600.

$$\text{The amount spent for 1 person} = ₹ \frac{5600}{32} = ₹ 175.$$

3. The distance of the place where first stop was made = 22 km.



To find the percentage of distance:

Ashima used this method:

$$\frac{22}{55} = \frac{22}{55} \times \frac{100}{100} = 40\%$$

She is multiplying

the ratio by $\frac{100}{100} = 1$

and converting to percentage.

OR

John used the unitary method:

Out of 55 km, 22 km are travelled.

Out of 1 km, $\frac{22}{55}$ km are travelled.

Out of 100 km, $\frac{22}{55} \times 100$ km are travelled.

That is 40% of the total distance is travelled.

Both came out with the same answer that the distance from their school of the place where they stopped at was 40% of the total distance they had to travel.

Therefore, the percent distance left to be travelled = $100\% - 40\% = 60\%$.

TRY THESE

In a primary school, the parents were asked about the number of hours they spend per day

in helping their children to do homework. There were 90 parents who helped for $\frac{1}{2}$ hour

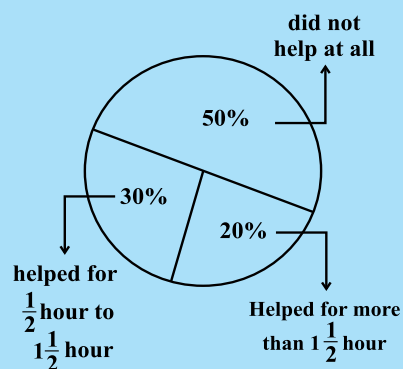
to $1\frac{1}{2}$ hours. The distribution of parents according to the time for which, they said they helped is given in the adjoining figure ; 20% helped for

more than $1\frac{1}{2}$ hours per day;

30% helped for $\frac{1}{2}$ hour to $1\frac{1}{2}$ hours; 50% did not help at all.

Using this, answer the following:

- How many parents were surveyed?
- How many said that they did not help?
- How many said that they helped for more than $1\frac{1}{2}$ hours?



EXERCISE 7.1

- Find the ratio of the following.
 - Speed of a cycle 15 km per hour to the speed of scooter 30 km per hour.
 - 5 m to 10 km
 - 50 paise to ₹ 5
- Convert the following ratios to percentages.
 - 3 : 4
 - 2 : 3
- 72% of 25 students are interested in mathematics. How many are not interested in mathematics?
- A football team won 10 matches out of the total number of matches they played. If their win percentage was 40, then how many matches did they play in all?
- If Chameli had ₹ 600 left after spending 75% of her money, how much did she have in the beginning?



6. If 60% people in a city like cricket, 30% like football and the remaining like other games, then what per cent of the people like other games? If the total number of people is 50 lakh, find the exact number who like each type of game.

7.2 Finding Discounts

Discount is a reduction given on the Marked Price (MP) of the article.

This is generally given to attract customers to buy goods or to promote sales of the goods. You can find the discount by subtracting its sale price from its marked price.

So, Discount = Marked price – Sale price



Example 2: An item marked at ₹ 840 is sold for ₹ 714. What is the discount and discount %?

Solution:

$$\begin{aligned}\text{Discount} &= \text{Marked Price} - \text{Sale Price} \\ &= ₹ 840 - ₹ 714 \\ &= ₹ 126\end{aligned}$$

Since discount is on marked price, we will have to use marked price as the base.

On marked price of ₹ 840, the discount is ₹ 126.

On MP of ₹ 100, how much will the discount be?

$$\text{Discount} = \frac{126}{840} \times 100\% = 15\%$$

You can also find discount when discount % is given.

Example 3: The list price of a frock is ₹ 220.

A discount of 20% is announced on sales. What is the amount of discount on it and its sale price.

Solution: Marked price is same as the list price.

20% discount means that on ₹ 100 (MP), the discount is ₹ 20.

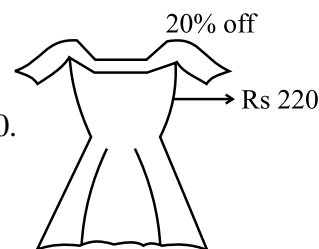
By unitary method, on ₹ 1 the discount will be ₹ $\frac{20}{100}$.

$$\text{On ₹ 220, discount} = ₹ \frac{20}{100} \times 220 = ₹ 44$$

The sale price = (₹ 220 – ₹ 44) or ₹ 176

Rehana found the sale price like this —

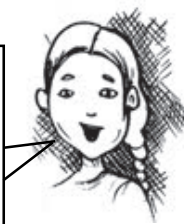
A discount of 20% means for a MP of ₹ 100, discount is ₹ 20. Hence the sale price is ₹ 80. Using unitary method, when MP is ₹ 100, sale price is ₹ 80;



When MP is ₹ 1, sale price is ₹ $\frac{80}{100}$.

Hence when MP is ₹ 220, sale price = ₹ $\frac{80}{100} \times 220 = ₹ 176$.

Even though the discount was not found, I could find the sale price directly.



TRY THESE

- A shop gives 20% discount. What would the sale price of each of these be?
 - A dress marked at ₹ 120
 - A pair of shoes marked at ₹ 750
 - A bag marked at ₹ 250
- A table marked at ₹ 15,000 is available for ₹ 14,400. Find the discount given and the discount per cent.
- An almirah is sold at ₹ 5,225 after allowing a discount of 5%. Find its marked price.

7.2.1 Estimation in percentages

Your bill in a shop is ₹ 577.80 and the shopkeeper gives a discount of 15%. How would you estimate the amount to be paid?

(i) Round off the bill to the nearest tens of ₹ 577.80, i.e., to ₹ 580.

(ii) Find 10% of this, i.e., ₹ $\frac{10}{100} \times 580 = ₹ 58$.

(iii) Take half of this, i.e., $\frac{1}{2} \times 58 = ₹ 29$.

(iv) Add the amounts in (ii) and (iii) to get ₹ 87.

You could therefore reduce your bill amount by ₹ 87 or by about ₹ 85, which will be ₹ 495 approximately.

- Try estimating 20% of the same bill amount.
- Try finding 15% of ₹ 375.

7.3 Sales Tax/Value Added Tax/Goods and Services Tax

The teacher showed the class a bill in which the following heads were written.

Bill No.			Date	
Menu				
S.No.	Item	Quantity	Rate	Amount
		Bill amount + ST (5%)		
	Total			



Sales tax (ST) is charged by the government on the sale of an item. It is collected by the shopkeeper from the customer and given to the government. This is, therefore, always on the selling price of an item and is added to the value of the bill. There is another type of tax which is included in the prices known as **Value Added Tax (VAT)**.

From July 1, 2017, Government of India introduced GST which stands for Goods and Services Tax which is levied on supply of goods or services or both.

Example 4: (Finding Sales Tax) The cost of a pair of roller skates at a shop was ₹ 450. The sales tax charged was 5%. Find the bill amount.

Solution: On ₹ 100, the tax paid was ₹ 5.

$$\begin{aligned}\text{On ₹ 450, the tax paid would be} &= ₹ \frac{5}{100} \times 450 \\ &= ₹ 22.50\end{aligned}$$

$$\text{Bill amount} = \text{Cost of item} + \text{Sales tax} = ₹ 450 + ₹ 22.50 = ₹ 472.50.$$



Example 5: (Value Added Tax (VAT)) Waheeda bought an air cooler for ₹ 3300 including a tax of 10%. Find the price of the air cooler before VAT was added.

Solution: The price includes the VAT, i.e., the value added tax. Thus, a 10% VAT means if the price without VAT is ₹ 100 then price including VAT is ₹ 110.

Now, when price including VAT is ₹ 110, original price is ₹ 100.

$$\text{Hence when price including tax is ₹ 3300, the original price} = ₹ \frac{100}{110} \times 3300 = ₹ 3000.$$

Example 6: Salim bought an article for ₹ 784 which included GST of 12%. What is the price of the article before GST was added?

Solution: Let original price of the article be ₹ 100. GST = 12%.

$$\text{Price after GST is included} = ₹ (100 + 12) = ₹ 112$$

$$\text{When the selling price is ₹ 112 then original price} = ₹ 100.$$

$$\text{When the selling price is ₹ 784, then original price} = ₹ \frac{100}{112} \times 784 = ₹ 700$$



THINK, DISCUSS AND WRITE

1. Two times a number is a 100% increase in the number. If we take half the number what would be the decrease in per cent?
2. By what per cent is ₹ 2,000 less than ₹ 2,400? Is it the same as the per cent by which ₹ 2,400 is more than ₹ 2,000?



EXERCISE 7.2

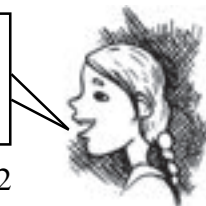
1. During a sale, a shop offered a discount of 10% on the marked prices of all the items. What would a customer have to pay for a pair of jeans marked at ₹ 1450 and two shirts marked at ₹ 850 each?
2. The price of a TV is ₹ 13,000. The sales tax charged on it is at the rate of 12%. Find the amount that Vinod will have to pay if he buys it.
3. Arun bought a pair of skates at a sale where the discount given was 20%. If the amount he pays is ₹ 1,600, find the marked price.
4. I purchased a hair-dryer for ₹ 5,400 including 8% VAT. Find the price before VAT was added.
5. An article was purchased for ₹ 1239 including GST of 18%. Find the price of the article before GST was added?



7.4 Compound Interest

You might have come across statements like “one year interest for FD (fixed deposit) in the bank @ 9% per annum” or ‘Savings account with interest @ 5% per annum’.

Interest is the extra money paid by institutions like banks or post offices on money deposited (kept) with them. Interest is also paid by people when they borrow money. We already know how to calculate **Simple Interest**.



Example 7: A sum of ₹ 10,000 is borrowed at a rate of interest 15% per annum for 2 years. Find the simple interest on this sum and the amount to be paid at the end of 2 years.

Solution: On ₹ 100, interest charged for 1 year is ₹ 15.

$$\text{So, on ₹ 10,000, interest charged} = \frac{15}{100} \times 10000 = ₹ 1500$$

$$\text{Interest for 2 years} = ₹ 1500 \times 2 = ₹ 3000$$

$$\begin{aligned} \text{Amount to be paid at the end of 2 years} &= \text{Principal} + \text{Interest} \\ &= ₹ 10000 + ₹ 3000 = ₹ 13000 \end{aligned}$$

TRY THESE

Find interest and amount to be paid on ₹ 15000 at 5% per annum after 2 years.

My father has kept some money in the post office for 3 years. Every year the money increases as more than the previous year.

We have some money in the bank. Every year some interest is added to it, which is shown in the passbook. This interest is not the same, each year it increases.





Normally, the interest paid or charged is never simple. The interest is calculated on the amount of the previous year. This is known as interest compounded or **Compound Interest (C.I.)**.

Let us take an example and find the interest year by year. Each year our sum or principal changes.

Calculating Compound Interest

A sum of ₹ 20,000 is borrowed by Heena for 2 years at an interest of 8% compounded annually. Find the Compound Interest (C.I.) and the amount she has to pay at the end of 2 years.

Aslam asked the teacher whether this means that they should find the interest year by year. The teacher said 'yes', and asked him to use the following steps :

1. Find the Simple Interest (S.I.) for one year.

Let the principal for the first year be P_1 . Here, $P_1 = ₹ 20,000$

$$SI_1 = \text{SI at } 8\% \text{ p.a. for 1st year} = ₹ \frac{20000 \times 8}{100} = ₹ 1600$$

2. Then find the amount which will be paid or received. This becomes principal for the next year.

$$\begin{aligned} \text{Amount at the end of 1st year} &= P_1 + SI_1 = ₹ 20000 + ₹ 1600 \\ &= ₹ 21600 = P_2 \text{ (Principal for 2nd year)} \end{aligned}$$

3. Again find the interest on this sum for another year.

$$\begin{aligned} SI_2 &= \text{SI at } 8\% \text{ p.a. for 2nd year} = ₹ \frac{21600 \times 8}{100} \\ &= ₹ 1728 \end{aligned}$$

4. Find the amount which has to be paid or received at the end of second year.

$$\begin{aligned} \text{Amount at the end of 2nd year} &= P_2 + SI_2 \\ &= ₹ 21600 + ₹ 1728 \\ &= ₹ 23328 \end{aligned}$$

$$\begin{aligned} \text{Total interest given} &= ₹ 1600 + ₹ 1728 \\ &= ₹ 3328 \end{aligned}$$

Reeta asked whether the amount would be different for simple interest. The teacher told her to find the interest for two years and see for herself.

$$\text{SI for 2 years} = ₹ \frac{20000 \times 8 \times 2}{100} = ₹ 3200$$

Reeta said that when compound interest was used Heena would pay ₹ 128 more.

Let us look at the difference between simple interest and compound interest. We start with ₹ 100. Try completing the chart.

		Under Simple Interest	Under Compound Interest
First year	Principal	₹ 100.00	₹ 100.00
	Interest at 10%	₹ 10.00	₹ 10.00
	Year-end amount	₹ 110.00	₹ 110.00
Second year	Principal	₹ 100.00	₹ 110.00
	Interest at 10%	₹ 10.00	₹ 11.00
	Year-end amount	₹(110 + 10) = ₹ 120	₹ 121.00
Third year	Principal	₹ 100.00	₹ 121.00
	Interest at 10%	₹ 10.00	₹ 12.10
	Year-end amount	₹(120 + 10) = ₹ 130	₹ 133.10

Which means you pay interest on the interest accumulated till then!

Note that in 3 years,

Interest earned by Simple Interest = ₹ (130 – 100) = ₹ 30, whereas,

Interest earned by Compound Interest = ₹ (133.10 – 100) = ₹ 33.10

Note also that the Principal remains the same under Simple Interest, while it changes year after year under compound interest.

7.5 Deducing a Formula for Compound Interest

Zubeda asked her teacher, ‘Is there an easier way to find compound interest?’ The teacher said ‘There is a shorter way of finding compound interest. Let us try to find it.’

Suppose P_1 is the sum on which interest is compounded annually at a rate of $R\%$ per annum.

Let $P_1 = ₹ 5000$ and $R = 5$. Then by the steps mentioned above

$$1. \quad SI_1 = ₹ \frac{5000 \times 5 \times 1}{100} \quad \text{or} \quad SI_1 = ₹ \frac{P_1 \times R \times 1}{100}$$

$$\text{so, } A_1 = ₹ 5000 + \frac{5000 \times 5 \times 1}{100} \quad \text{or} \quad A_1 = P_1 + SI_1 = P_1 + \frac{P_1 R}{100}$$

$$= ₹ 5000 \left(1 + \frac{5}{100} \right) = P_2 \quad = P_1 \left(1 + \frac{R}{100} \right) = P_2$$

$$\begin{aligned}
 2. \quad SI_2 &= ₹ 5000 \left(1 + \frac{5}{100}\right) \times \frac{5 \times 1}{100} & \text{or} & \quad SI_2 = \frac{P_2 \times R \times 1}{100} \\
 &= ₹ \frac{5000 \times 5}{100} \left(1 + \frac{5}{100}\right) & & \quad = P_1 \left(1 + \frac{R}{100}\right) \times \frac{R}{100} \\
 & & & \quad = \frac{P_1 R}{100} \left(1 + \frac{R}{100}\right)
 \end{aligned}$$

$$\begin{aligned}
 A_2 &= ₹ 5000 \left(1 + \frac{5}{100}\right) + ₹ \frac{5000 \times 5}{100} \left(1 + \frac{5}{100}\right) & A_2 &= P_2 + SI_2 \\
 &= ₹ 5000 \left(1 + \frac{5}{100}\right) \left(1 + \frac{5}{100}\right) & & \quad = P_1 \left(1 + \frac{R}{100}\right) + P_1 \frac{R}{100} \left(1 + \frac{R}{100}\right) \\
 &= ₹ 5000 \left(1 + \frac{5}{100}\right)^2 = P_3 & & \quad = P_1 \left(1 + \frac{R}{100}\right) \left(1 + \frac{R}{100}\right) \\
 & & & \quad = P_1 \left(1 + \frac{R}{100}\right)^2 = P_3
 \end{aligned}$$

Proceeding in this way the amount at the end of n years will be

$$A_n = P_1 \left(1 + \frac{R}{100}\right)^n$$

Or, we can say $A = P \left(1 + \frac{R}{100}\right)^n$

So, Zubeda said, but using this we get only the formula for the amount to be paid at the end of n years, and not the formula for compound interest.

Aruna at once said that we know $CI = A - P$, so we can easily find the compound interest too.

Example 8: Find CI on ₹ 12600 for 2 years at 10% per annum compounded annually.

Solution: We have, $A = P \left(1 + \frac{R}{100}\right)^n$, where Principal (P) = ₹ 12600, Rate (R) = 10,
Number of years (n) = 2

$$= ₹ 12600 \left(1 + \frac{10}{100}\right)^2 = ₹ 12600 \left(\frac{11}{10}\right)^2$$

$$= ₹ 12600 \times \frac{11}{10} \times \frac{11}{10} = ₹ 15246$$

$$CI = A - P = ₹ 15246 - ₹ 12600 = ₹ 2646$$

TRY THESE

1. Find CI on a sum of ₹ 8000 for 2 years at 5% per annum compounded annually.

7.6 Applications of Compound Interest Formula

There are some situations where we could use the formula for calculation of amount in CI. Here are a few.

- (i) Increase (or decrease) in population.
- (ii) The growth of a bacteria if the rate of growth is known.
- (iii) The value of an item, if its price increases or decreases in the intermediate years.

Example 9: The population of a city was 20,000 in the year 1997. It increased at the rate of 5% p.a. Find the population at the end of the year 2000.

Solution: There is 5% increase in population every year, so every new year has new population. Thus, we can say it is increasing in compounded form.

Population in the beginning of 1998 = 20000 (we treat this as the principal for the 1st year)



$$\text{Increase at 5\%} = \frac{5}{100} \times 20000 = 1000$$

$$\text{Population in 1999} = 20000 + 1000 = 21000$$

Treat as the Principal for the 2nd year.

$$\text{Increase at 5\%} = \frac{5}{100} \times 21000 = 1050$$

$$\text{Population in 2000} = 21000 + 1050$$

$$= 22050$$

Treat as the Principal for the 3rd year.

$$\text{Increase at 5\%} = \frac{5}{100} \times 22050$$

$$= 1102.5$$

$$\text{At the end of 2000 the population} = 22050 + 1102.5 = 23152.5$$

$$\text{or, Population at the end of 2000} = 20000 \left(1 + \frac{5}{100}\right)^3$$

$$= 20000 \times \frac{21}{20} \times \frac{21}{20} \times \frac{21}{20}$$

$$= 23152.5$$

So, the estimated population = 23153.

Aruna asked what is to be done if there is a decrease. The teacher then considered the following example.

Example 10: A TV was bought at a price of ₹ 21,000. After one year the value of the TV was depreciated by 5% (Depreciation means reduction of value due to use and age of the item). Find the value of the TV after one year.

Solution:

$$\begin{aligned}\text{Principal} &= ₹ 21,000 \\ \text{Reduction} &= 5\% \text{ of ₹ 21000 per year} \\ &= ₹ \frac{21000 \times 5 \times 1}{100} = ₹ 1050\end{aligned}$$

value at the end of 1 year = ₹ 21000 – ₹ 1050 = ₹ 19,950



Alternately, We may directly get this as follows:

$$\begin{aligned}\text{value at the end of 1 year} &= ₹ 21000 \left(1 - \frac{5}{100}\right) \\ &= ₹ 21000 \times \frac{19}{20} = ₹ 19,950\end{aligned}$$

TRY THESE

1. A machinery worth ₹ 10,500 depreciated by 5%. Find its value after one year.
2. Find the population of a city after 2 years, which is at present 12 lakh, if the rate of increase is 4%.



EXERCISE 7.3

1. The population of a place increased to 54,000 in 2003 at a rate of 5% per annum
 - (i) find the population in 2001.
 - (ii) what would be its population in 2005?
2. In a Laboratory, the count of bacteria in a certain experiment was increasing at the rate of 2.5% per hour. Find the bacteria at the end of 2 hours if the count was initially 5,06,000.
3. A scooter was bought at ₹ 42,000. Its value depreciated at the rate of 8% per annum. Find its value after one year.



WHAT HAVE WE DISCUSSED?

1. **Discount** is a reduction given on marked price.
 $\text{Discount} = \text{Marked Price} - \text{Sale Price}.$
2. Discount can be calculated when discount percentage is given.
 $\text{Discount} = \text{Discount \% of Marked Price}$
3. Additional expenses made after buying an article are included in the cost price and are known as **overhead expenses**.
 $\text{CP} = \text{Buying price} + \text{Overhead expenses}$
4. Sales tax is charged on the sale of an item by the government and is added to the Bill Amount.
 $\text{Sales tax} = \text{Tax\% of Bill Amount}$
5. GST stands for Goods and Services Tax and is levied on supply of goods or services or both.
6. Compound interest is the interest calculated on the previous year's amount ($A = P + I$)



NOTES