SECTION 1 (Maximum Marks: 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated **according to the following marking scheme**:

Full Marks : $+3$ If **ONLY** the correct option is chosen;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : -1 In all other cases.

Q.1 Let $\vec{a}$, $\vec{b}$ be two vectors, and let P, Q and R be the points with position vectors $\vec{a}$, $\vec{b}$ and $\vec{a}+\vec{b}$, respectively, with respect to the origin O. If $|\vec{a}+\vec{b}| = \sqrt{21}$, $|\vec{a}-\vec{b}| = 3$, and $\vec{a}$ and $(\vec{a}-\vec{b})$ are perpendicular to each other, then the area of the triangle OPR is

(A)	$\sqrt{3}$	(B)	$\frac{\sqrt{3}}{2}$	(C)	$\frac{3\sqrt{3}}{2}$	(D)	$\frac{3}{2}$

Answer Q1: C

Q.2 Let T be the tangent to the parabola $y^2 = 16x$ at the point $(64, 32)$. Let L be the tangent to the same parabola at another point (x_1, y_1) on the parabola. If L and T are perpendicular to each other, then the distance between the point (x_1, y_1) and the focus of the parabola, is

(A)	$\frac{15}{4}$	(B)	4	(C)	$\frac{17}{4}$	(D)	5

Answer Q2: C

Q.3 Let $y : (-\infty, \infty) \to (0, \infty)$ be the solution of the differential equation

$$\frac{dy}{dx} = \frac{e^{5x}y^3 + y^3}{e^x + e^x y^4},$$

satisfying $y(0) = \frac{1}{\sqrt{2}}$. Then the value of $y(\log_e 2)$ is

(A)	$\sqrt{\frac{5+\sqrt{35}}{2}}$	(B)	$\sqrt{\frac{7+\sqrt{53}}{2}}$
(C)	$\frac{7+\sqrt{53}}{2}$	(D)	$\frac{5+\sqrt{35}}{2}$

Answer Q3: B

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Q.4 The value of the definite integral

$$\int_0^2 \frac{1}{3^x + 3} dx$$

is

(A)	$\frac{1}{2}$	(B)	$\frac{1}{3}$	(C)	$\frac{\log_e 3}{3}$	(D)	$\frac{\log_e 3}{2}$

Answer Q4: B

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SECTION 2 (Maximum Marks: 20)

- This section contains **FIVE (05)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated **according to the following marking scheme**:

  Full Marks : $+4$ **ONLY** if (all) the correct option(s) is(are) chosen;
  Partial Marks : $+3$ If all the four options are correct but **ONLY** three options are chosen;
  Partial Marks : $+2$ If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
  Partial Marks : $+1$ If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
  Negative Marks : -1 In all other cases.
- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
  choosing ONLY (A), (B) and (D) will get $+4$ marks;
  choosing ONLY (A) and (B) will get $+2$ marks;
  choosing ONLY (A) and (D) will get $+2$ marks;
  choosing ONLY (B) and (D) will get $+2$ marks;
  choosing ONLY (A) will get +1 mark;
  choosing ONLY (B) will get +1 mark;
  choosing ONLY (D) will get +1 mark;
  choosing no option (i.e. the question is unanswered) will get 0 marks; and
  choosing any other combination of options will get -1 marks.

Q.5 Let $\mathbb{R}$ denote the set of all real numbers. Consider the polynomial function $f:\mathbb{R}\to\mathbb{R}$ defined by

$$f(x)=\frac{d^{10}}{dx^{10}}((x^2-1)^{10}), \qquad \text{for all } x\in\mathbb{R}.$$

Here $\frac{d^{10}}{dx^{10}}((x^2-1)^{10})$ is the 10th order derivative of the function $(x^2-1)^{10}$.

Then which of the following statements is (are) TRUE ?

(A)	The coefficient of x^8 in the polynomial $f(x)$ is $(-10)\left(\frac{18!}{8!}\right)$
(B)	The value of $f(1)+f(-1)$ is equal to $10!\,2^{11}$
(C)	The degree of the polynomial $f(x)$ is 10
(D)	The constant term of the polynomial $f(x)$ is $-\left(\frac{10!}{5!}\right)$

Answer Q5: ABC

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Q.6

Let $a,\ b,\ c$ be positive integers in arithmetic progression such that the equation

$$ax^2 + bx + c = 0$$

has only integer solutions.

Then which of the following statements is (are) TRUE ?

(A)	$c - b$ is an integer multiple of a
(B)	Both the roots of the equation $ax^2 + bx + c = 0$ are odd integers
(C)	If $c = 15$, then $ab = 8$
(D)	If $b = 8$, then $x = 3$ is a root of the equation $ax^2 + bx + c = 0$

Answer Q6: ABC

Q.7

Let L be the straight line joining the points $P(1, 2, -1)$ and $Q(2, 3, 1)$. Let S be the foot of the perpendicular drawn from the point $R(4, -1, 5)$ to the line L. Another line passing through R intersects L at a point T such that the point S divides the line segment PT internally in the ratio $|PS| : |ST| = 1 : 2$, where $|PS|$ and $|ST|$ are the lengths of the line segments PS and ST, respectively.

Then which of the following statements is (are) TRUE ?

(A)	The orthocentre of the triangle PRT is $\left(\frac{23}{5}, -4, \frac{31}{5}\right)$
(B)	The orthocentre of the triangle PRT is $(4, 3, 5)$
(C)	The area of the triangle PRT is $6\sqrt{5}$
(D)	The area of the triangle PRT is $18\sqrt{5}$

Answer Q7: AD

Q.8

Let $y = f(x)$ be the real valued function defined on the interval $(0, \infty)$, satisfying $y(1) = 0$ and the differential equation

$$x\frac{dy}{dx} = y - x^3 .$$

Then which of the following statements is (are) TRUE ?

(A)	The function f has a local minimum at $x = \frac{1}{\sqrt{3}}$
(B)	The function f has a local maximum at $x = \frac{1}{\sqrt{3}}$
(C)	The function f is increasing in the interval $(1,\ 2)$
(D)	If $g(x) = 4x^3 - 5x^2 + \frac{3}{2}x$ for $x > 0$, then the number of elements in the set $\{x \in (0, \infty) :\ f(x) = g(x)\}$ is 2

Answer Q8: BD

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Q.9

Let $\mathbb{R}$ denote the set of all real numbers and let $i = \sqrt{-1}$. Consider the matrices

$$S = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \text{ and } T = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}.$$

Let a, b, c, d be real numbers such that

$$ST = \begin{bmatrix} a & b \\ c & d \end{bmatrix}.$$

Let

$$H = \{ x + iy : \ x, y \in \mathbb{R} \text{ and } y > 0 \}.$$

Then which of the following statements is (are) TRUE ?

(A)	$\frac{b+ia}{d+ic} = i$
(B)	If $\omega = \frac{-1+i\sqrt{3}}{2}$, then $\frac{a\omega+b}{c\omega+d} = \omega$
(C)	If m is an integer greater than 2 such that $(ST)^2 = (ST)^m$, then m is an integer multiple of 8
(D)	If $z \in H$, then $\frac{az+b}{cz+d} \in H$

Answer Q9: BD

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SECTION 3 (Maximum Marks: 20)

- This section contains **FIVE (05)** questions.
- The answer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct numerical value corresponding to the answer in the designated place using the mouse and the on-screen virtual numeric keypad.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated **according to the following marking scheme**:

  Full Marks : $+4$ If **ONLY** the correct numerical value is entered in the designated place;
  Zero Marks : 0 In all other cases.

Q.10 Let $\mathbb{N}$ denote the set of all positive integers. Consider the sets

$$A = \{1, 2, 3, 4, 5\} \text{ and } B = \{1, 2, 3, 4, 5, 6, 7\}.$$

Let S be the set of all functions $f: A \to B$ such that $f(2) \neq 2$ and $f(4) \neq 4$. Consider the set

$$T = \{f \in S : \text{there exists a function } g : B \to \mathbb{N} \text{ such that } g(f(x)) = 2^x \text{ for all } x \in A\}.$$

Then the number of elements in the set T is _____.

Answer Q10: [1860]

Q.11 A bookshelf contains 6 distinct books of Mathematics and 5 distinct books of Physics. From these 11 books, 6 books are chosen at random. Let X be the absolute value of the difference between the number of Mathematics books chosen and the number of Physics books chosen. If α is the mean of the random variable X, then the value of 77α is _____.

Answer Q11: [99 to 101]

Q.12 Consider a data consisting of 10 observations $x_1, x_2, \ldots, x_{10}$, whose mean is 5 and variance is 7. If the mean and the variance of the first 8 observations $x_1, x_2, \ldots, x_8$ are 4 and 3.5, respectively, and $x_9 < x_{10}$, then the value of $3x_9 + 2x_{10}$ is _____.

Answer Q12: [44]

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Q.13

Consider the ellipse E given by $\frac{x^2}{18}+\frac{y^2}{12}=1$. Let H be the hyperbola whose eccentricity is the reciprocal of the eccentricity of E and whose foci are the same as that of E. Let P and Q be the points of intersection of H and the parabola $\sqrt{5}\,y=x^2$ in the first quadrant. Let d be the distance between P and Q.

If a and b are the integers such that $d^2=a+b\sqrt{5}$, then the value of $a-b$ is _____.

Answer Q13: [18]

Q.14

For a real number α, let $[\alpha]$ denote the greatest integer less than or equal to α. For a finite set S, let $|S|$ denote the number of elements in the set S.

Consider the functions $f:(-3,3)\to(-\infty,\ \infty)$ and $g:(-3,3)\to(-\infty,\ \infty)$ defined by

$$f(x)=[x^3]\log_e\left(1+\sin^2\left(\pi(x-[x])\right)\right)$$

and

$$g(x)=x^3\sin^2(\pi\log_e(1+x-[x])).$$

Let

$$A=\{x\in(-3,3):f \text{ is discontinuous at } x\}$$

and

$$B=\{x\in(-3,3):g \text{ is discontinuous at } x\}.$$

Then the value of $|A|+2|B|-|A\cap B|$ is _____.

Answer Q14: [56]

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SECTION 4 (Maximum Marks: 8)

- This section contains **TWO (02)** question stems.
- There are **TWO (02)** questions corresponding to each question stem.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value corresponding to the answer in the designated place using the mouse and the on-screen virtual numeric keypad.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated **according to the following marking scheme**:
  Full Marks : +2 If **ONLY** the correct numerical value is entered at the designated place;
  Zero Marks : 0 In all other cases.

Question Stem for Question Nos. 15 and 16

Consider the curve C_1 given by

$$y = e^{-x} \quad \text{for } x \in [0, 10\pi],$$

and the curve C_2 given by

$$y = e^{-x}(\sin x + \cos x) \quad \text{for } x \in [0, 10\pi].$$

Let n be the total number of points of intersection of the curves C_1 and C_2.

Suppose that $\alpha_1, \alpha_2, \ldots, \alpha_n \in [0, 10\pi]$ are the x - coordinates of the points of intersection of the curves C_1 and C_2 such that

$$\alpha_1 < \alpha_2 < \cdots < \alpha_n.$$

Q.15 The value of n is _____.

Answer Q15: [11]

Q.16 Let β be the area of the region enclosed between the curves C_1, C_2, and the lines $x = \alpha_1$ and $x = \alpha_4$. Then the value of

$$-\frac{1}{\pi}\log_e\left(\beta - 2\,e^{-\frac{\pi}{2}}\right)$$

is _____.

Answer Q16: [2.4 to 2.6]

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Question Stem for Question Nos. 17 and 18

Consider the ellipses given by

$$x^2 + 4y^2 = 1 \quad \text{and} \quad 4x^2 + y^2 = 1 \ .$$

Q.17 Let P be the point in the first quadrant where the given ellipses intersect. If θ is the acute angle between the tangents to the given ellipses at the point P, then the value of $4\tan\theta$ is _____.

Answer Q17: [7.4 to 7.6]

Q.18 If α is the area of the common region that lies inside both the given ellipses, then the value of $\cot\alpha$ is _____.

Answer Q18: [0.74 to 0.76]

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SECTION 1 (Maximum Marks: 12)

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- Answer to each question will be evaluated **according to the following marking scheme**:

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Negative Marks : -1 In all other cases.

Q.1 A metal wire of cross-sectional area 0.5 mm^2 and length 100 m is connected across a battery of e.m.f. 2 V and internal resistance 1 Ω. The density, atomic mass and electrical conductivity of the metal are 6.35×10^3 kg m^{-3}, 63.5 gm/mole and 2×10^8 mho m^{-1}, respectively. Assuming one conduction electron per atom of the metal, the drift velocity (in mm s^{-1}) of the electrons in the wire is:
[Take Avogadro's number as 6×10^{23} and charge of the electron as 1.6×10^{-19} C.]

(A)	0.052	(B)	0.104	(C)	0.208	(D)	0.156

Answer Q1: C

Q.2 A nuclear reactor starts producing a radioactive nuclide X from $t = 0$, at a constant rate of α per second. Each decay of X produces energy E_0, which is utilized to heat a liquid of mass m and specific heat s. Assuming no heat loss from the liquid and taking λ as the decay constant of X, the rate of increase in the temperature of the liquid is:

(A)	$\frac{\alpha E_0}{m\,s}(1 - e^{-\lambda t})$	(B)	$\frac{\alpha E_0}{m\,s}(e^{\lambda t} - 1)$
(C)	$\frac{\lambda E_0}{m\,s}(1 - e^{-\lambda t})$	(D)	$\frac{E_0}{m\,s}(\alpha - \lambda e^{-\lambda t})$

Answer Q2: A

Q.3 A beam of polychromatic light passes through a thin prism of prism angle 6°. The refractive index of the material of the prism varies with wavelength (λ) as $n(\lambda) = \alpha\lambda + \frac{\beta}{\lambda^2}$, where $\alpha = 3\ \mu\text{m}^{-1}$ and $\beta = 0.096\ \mu\text{m}^2$. If λ_{min} is the wavelength at which the angle of minimum deviation D_m is smallest, then the correct value of D_m at λ_{min} is

(A)	6.4°	(B)	4.8°	(C)	3.2°	(D)	2.4°

Answer Q3: B

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Q.4 A particle of mass m, and angular momentum ℓ is moving in a circular orbit of radius r_0 under the influence of an attractive force $\vec{F}(r) = -\frac{k}{r^2}\hat{r}$. Keeping its angular momentum unchanged, the particle is displaced radially by a small distance $\delta r \ll r_0$, due to which its radial distance varies periodically. The corresponding time period is:

(A)	$\frac{2\pi\ell^3}{mk^2}$	(B)	$2\pi\sqrt{\frac{m}{k}}$	(C)	$\frac{2\pi\ell^3}{3mk^2}$	(D)	$\frac{2\pi\ell^3}{5mk^2}$

Answer Q4: A

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SECTION 2 (Maximum Marks: 20)

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- For each question, choose the option(s) corresponding to (all) the correct answer(s).
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  Partial Marks : $+3$ If all the four options are correct but **ONLY** three options are chosen;
  Partial Marks : $+2$ If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
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  choosing ONLY (A), (B) and (D) will get $+4$ marks;
  choosing ONLY (A) and (B) will get $+2$ marks;
  choosing ONLY (A) and (D) will get $+2$ marks;
  choosing ONLY (B) and (D) will get $+2$ marks;
  choosing ONLY (A) will get +1 mark;
  choosing ONLY (B) will get +1 mark;
  choosing ONLY (D) will get +1 mark;
  choosing no option (i.e. the question is unanswered) will get 0 marks; and
  choosing any other combination of options will get -1 marks.

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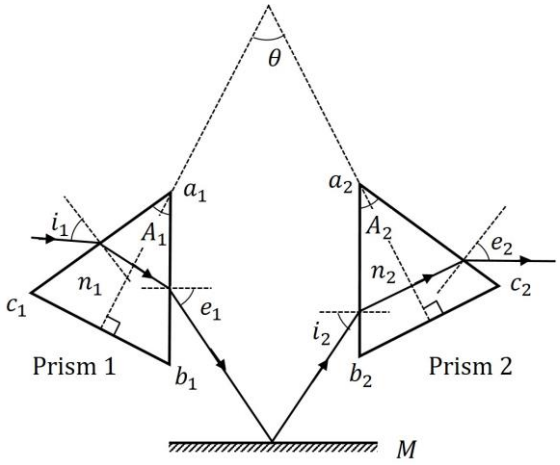
Q.5

Consider two isosceles prisms 1 and 2 with prism angles A_1 and A_2 and refractive indices n_1 and n_2, respectively, as shown in the figure. The faces a_1b_1 and a_2b_2 are parallel to each other and perpendicular to the mirror M. If a ray of light is incident on the face a_1c_1 and emerges from the face a_2c_2, then the correct statement(s) is/are:



(A)	If both the prisms are at minimum deviation condition, then $\frac{n_2}{n_1} = \sin\left(\frac{A_1}{2}\right) / \sin\left(\frac{A_2}{2}\right)$.
(B)	If prism 2 is at minimum deviation condition, then $\sin i_1 = n_2 \sin\left(\frac{A_2}{2}\right)$ is always true.
(C)	If both the prisms 1 and 2 are thin and are at minimum deviation condition with angles of deviation δ_{m1} and δ_{m2}, respectively, then $\theta = \frac{\delta_{m1}}{2(n_1-1)} + \frac{\delta_{m2}}{2(n_2-1)}$.
(D)	If prism 1 is at minimum deviation condition, then $\sin i_2 = n_1 \sin\left(\frac{A_1}{2}\right)$ is always true.

Answer Q5: ACD

Q.6

In a vacuum chamber, a particle of charge 1 μC and mass 1 mg is projected with a velocity $(\hat{\imath} + 2\hat{\jmath})$ ms^{-1} from the XZ plane at time $t = 0$ in an electric field of $1\hat{\imath}$ Vm^{-1}. At $t = 0.2$ s, the electric field is switched off and a magnetic field of $6\hat{\jmath}$ T is switched on. The acceleration due to gravity is $-10\hat{\jmath}$ ms^{-2}. Correct option(s) is/are:

(A)	The vertical distance of the particle from the XZ plane at $t = 0.3$ s is 15 cm.
(B)	The vertical distance of the particle from the XZ plane at $t = 0.4$ s is 10 cm.
(C)	The radius of the trajectory of the particle for $t > 0.2$ s is 20 cm.
(D)	The particle will be in the XZ plane at $t = 0.35$ s.

Answer Q6: AC

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Q.7 Two charges $Q_1 = q$ and $Q_2 = mq$ are placed at the points $P_1(a, b)$ and $P_2\ (ma, mb)$, respectively, in the XY plane, where $a, b \neq 0$ and $m \neq 0, 1$. If V_1 is the potential at a point in the XY plane due to charge Q_1 and V_2 is the potential at that point due to charge Q_2. Correct statement(s) for the points at which $|V_1| = |V_2|$ is/are:

(A)	For $m = -1$, locus of these points is $ax + by = 0$.
(B)	For $m = 2$, the locus of these points is a circle of radius $\frac{2}{3}\sqrt{a^2 + b^2}$ centered at $\left(\frac{2}{3}a, \frac{2}{3}b\right)$
(C)	For $m = -2$, the locus of these points is a circle of radius $2\sqrt{a^2 + b^2}$ centered at $(2a, 2b)$
(D)	For $m = -3$, locus of these points is $3bx + 3ay = 0$.

Answer Q7: ABC

Q.8 Consider an electric dipole comprising two charges $+q$ and $-q$ each with mass m, separated by a fixed distance d and initially at rest with its dipole moment pointing along $\hat{\imath}$. A uniform electric field $E\hat{\jmath}$ is turned on at time $t = 0$ and it is turned off at $t = t_f$, when the dipole moment makes an angle θ_f with $\hat{\imath}$. Neglecting any sources of energy loss, correct option(s) is/are:

(A)	The center of mass of the dipole is deflected towards $\hat{\jmath}$ in the presence of the field.
(B)	If the magnitude of the final angular velocity $\omega_f = \sqrt{\frac{2qE}{md}}$, then $\theta_f = \frac{\pi}{6}$.
(C)	If $\theta_f = \pi/3$, then the change in kinetic energy of the dipole is given by $2\sqrt{3}\, qEd$.
(D)	For $\theta_f = \pi/4$, the dipole rotates around its center of mass with a constant angular velocity after $t > t_f$.

Answer Q8: BD or D

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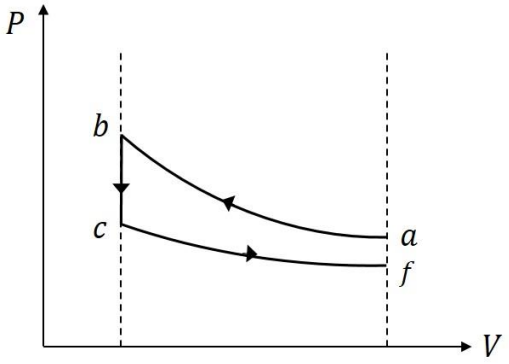
Q.9

Ten moles of an ideal monoatomic gas, initially in state $\boldsymbol{a}$ at atmospheric pressure and temperature $T_a = 27°C$, is enclosed in a metal cylinder of volume V_0 fitted with a frictionless piston. The gas is suddenly compressed to state $\boldsymbol{b}$ with volume $V_0/3$. Now, keeping the piston stationary, the cylinder is submerged in a water bath of temperature 11°C until the gas reaches the temperature of the water bath, which is denoted as state $\boldsymbol{c}$. Finally, while still in the water bath, the piston is brought slowly to its initial position, which is denoted as state $\boldsymbol{f}$. If R is universal gas constant, then the correct option(s) is/are:

[Given: $9^{1/3} = 2.08$]

(A)	The schematic P-V diagram of the processes described above is: 
(B)	The change in internal energy in going from state $\boldsymbol{a}$ to $\boldsymbol{b}$ is 4860R.
(C)	The net change in the internal energy in the whole process is -240R.
(D)	The pressure and temperature of the state $\boldsymbol{b}$ are 2.08 times the atmospheric pressure and 624 K, respectively.

Answer Q9: ABC

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SECTION 3 (Maximum Marks: 20)

- This section contains **FIVE (05)** questions.
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- For each question, enter the correct numerical value corresponding to the answer in the designated place using the mouse and the on-screen virtual numeric keypad.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated **according to the following marking scheme**:

  Full Marks : $+4$ If **ONLY** the correct numerical value is entered in the designated place;
  Zero Marks : 0 In all other cases.

Q.10 Two thin wires, Wire-1 of diameter 0.650 mm and Wire-2 of unknown diameter d are given. To obtain the value of d, the diameters of the two wires are measured with a screw gauge. The screw gauge has a pitch of 0.5 mm and there are 100 divisions on the circular scale (CS). The smallest division on the linear scale (LS) is 0.5 mm. The table shows the readings of LS and CS for the measurements. The value of d (in μm) is:

	Readings	
	LS (mm)	CS
Wire-1	0.5	42
Wire-2	1.5	95

Answer Q10: [1915]

Q.11 In a single slit diffraction experiment, a slit of width (0.016 ± 0.002) mm is used to measure the wavelength of a monochromatic light source. In the diffraction pattern, the angular distance between the central maximum and first minimum is measured to be $(2° \pm 40')$. The value of the fractional error in the measurement of wavelength is:
[Given: $\sin(2°) = 0.035$]

Answer Q11: [0.43 to 0.50]

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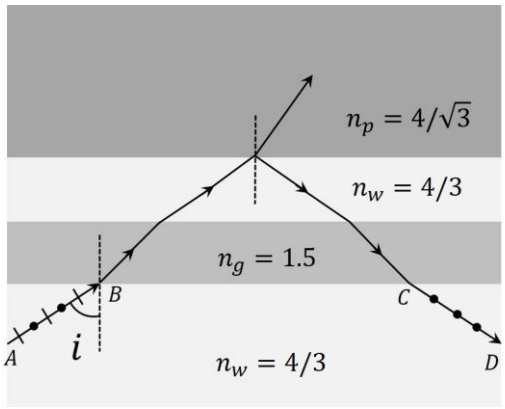
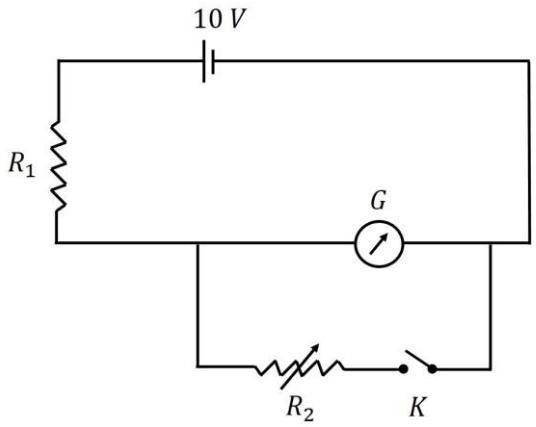
Q.12 As shown in the figure, a ray AB of unpolarized light enters from water of refractive index $n_w = 4/3$ into a medium of refractive index $n_p = 4/\sqrt{3}$ after passing through a glass plate of refractive index $n_g = 1.5$ and a layer of water. At a particular incident angle i the reflected ray CD is polarized in the direction as shown in the figure. The value of i (in degrees) is:



Answer Q12: [60]

Q.13 As shown in the figure, the resistance of a galvanometer G can be found by the half-deflection method. Here the resistance R_2 is adjusted such that when the key K is closed the deflection in the galvanometer becomes half of the value as compared to when K is open. Half-deflection is obtained at $R_2 = 4\ \Omega$ and thus the galvanometer resistance is found to be 6 Ω. In this half-deflection condition the current (in mA) through the resistor R_1 is:



Answer Q13: [690 to 710]

Q.14 In a new system of units, the units of mass, length, time and current are 5 kg, 5 m, 5 s and 5 A, respectively. If μ_0 and ϵ_0 are the permeability and permittivity of free space, respectively, then in this new system of units, the magnitude of one SI unit of $\sqrt{\mu_0/\epsilon_0}$, is:

Answer Q14: [25]

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SECTION 4 (Maximum Marks: 8)

- This section contains **TWO (02)** question stems.
- There are **TWO (02)** questions corresponding to each question stem.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value corresponding to the answer in the designated place using the mouse and the on-screen virtual numeric keypad.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated **according to the following marking scheme**:
  Full Marks : +2 If **ONLY** the correct numerical value is entered at the designated place;
  Zero Marks : 0 In all other cases.

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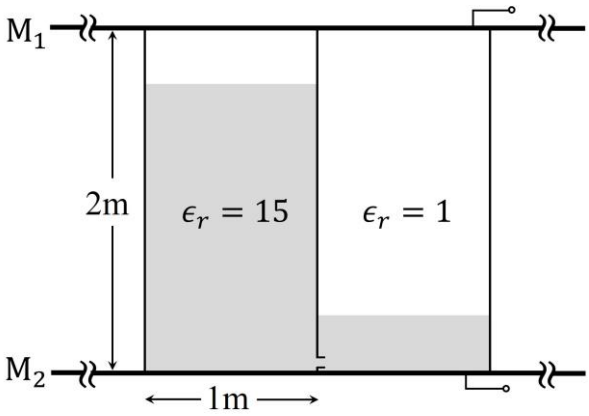
Question Stem for Question Nos. 15 and 16

A container of height 2 m, length 2 m and breadth 1 m is made of insulating vertical walls and two large area horizontal metal plates (M_1 and M_2) which extend far beyond the vertical walls in all directions. The container is partitioned into two equal chambers with a thin insulating vertical wall. The partition wall contains a small hole of cross-sectional area $\sqrt{10}$ cm^2 near its bottom edge. Initially the hole is closed and the left chamber of the container is completely filled with a liquid of dielectric constant $\epsilon_r = 15$ and the right chamber is empty ($\epsilon_r = 1$). At time $t = 0$, the hole is opened and the liquid flows from the left chamber to the right chamber. In both the chambers, the space above the liquid has $\epsilon_r = 1$ and is maintained at atmospheric pressure. The schematic of the container at a time $t > 0$ is shown in the figure.
[Given: acceleration due to gravity is 10 ms^{-2}.]



Q.15 The height (in m) of the liquid in left chamber at $t = 500$ s is:

Answer Q15: [1.25]

Q.16 The difference in the capacitance (in F) between the metal plates at $t = 0$ and that at $t = 500$ s is $(8 - n)\epsilon_0$, where ϵ_0 is the permittivity of free space. The value of n is:

Answer Q16: [1.90 to 2.10]

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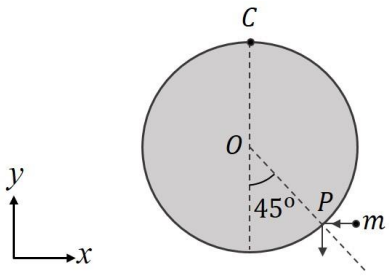
Question Stem for Question Nos. 17 and 18

A uniform circular disk of radius 0.2 m and mass 1 kg is pivoted at its top point C such that it can rotate freely around C in the XY plane, as shown in the figure. Initially, when the disk is at rest, a particle of mass 20 g, travelling along negative x direction in the XY plane with speed 100 ms^{-1}, hits the circumference of the disk at a point P. After collision the particle moves along negative y direction at a speed of 90 ms^{-1}.
[Given: the acceleration due to gravity (g) $= -10\,\hat{j}\ \text{ms}^{-2}$]



Q.17 After the collision the disk starts to rotate around point C in the XY plane. The maximum change in the height (in m) of its center O is:

Answer Q17: [0.14 to 0.16]

Q.18 Amount of energy loss (in J) in the collision is:

Answer Q18: [17 to 18]

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SECTION 1 (Maximum Marks: 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated **according to the following marking scheme**:

Full Marks : +3 If **ONLY** the correct option is chosen;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : −1 In all other cases.

Q.1 At 300 K, the molar conductivities of the aqueous solutions of three salts at two different concentrations are given below:

Salt	Concentration (M)	Molar conductivity (S cm^2 mol^{-1})
$NaNO_3$	0.01	111
	0.04	101
NaCl	0.01	117
	0.04	107
$AgNO_3$	0.01	125
	0.04	116

The conductivity of a saturated aqueous solution of AgCl is 1.40×10^{-6} S cm^{-1} at 300 K. If the solubility of AgCl in water at 300 K is $\boldsymbol{X}$ mol L^{-1}, then $\log_{10}(\boldsymbol{X}^{-1})$ is

(Assume that AgCl dissolved in water ionizes completely and that the molar conductivity of saturated AgCl solution is equal to its limiting molar conductivity.)

(A)	3
(B)	4
(C)	5
(D)	6

Answer Q1: C

Q.2 The correct order of ONO bond angle in the given species is

(A)	$NO_2^+ < NO_2 < NO_3^- < NO_2^-$
(B)	$NO_2^- < NO_3^- < NO_2 < NO_2^+$
(C)	$NO_3^- < NO_2^- < NO_2 < NO_2^+$
(D)	$NO_2^- < NO_3^- < NO_2^+ < NO_2$

Answer Q2: B

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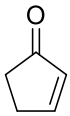
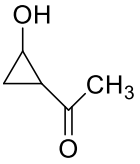
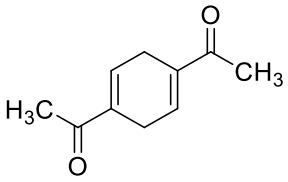
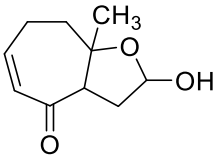
Q.3 Natural rubber on complete ozonolysis (O_3/Zn-H_2O) gives compound **X** as the major product. **X** gives positive iodoform and Tollen's tests. **X** on heating with aqueous NaOH gives **Y** as the major product. **Y** is

(A)		(B)	
(C)		(D)	

Answer Q3: A

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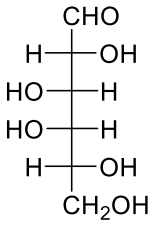
Q.4

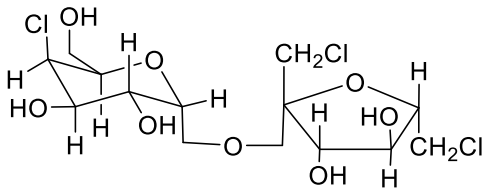
A known artificial sweetener **X** is composed of 4-chloro-4-deoxy-α-D-galactose and 1,6-dichloro-1,6-dideoxy-β-D-fructose joined by a glycosidic linkage. Structure of D-galactose is given below:



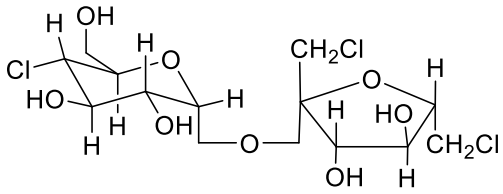
= D-galactose

The correct structure of **X** is

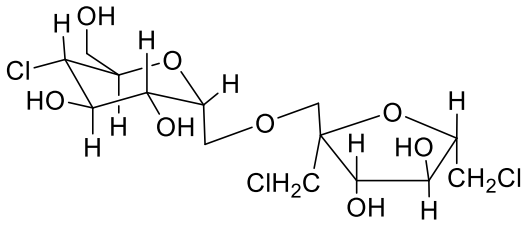
(A)



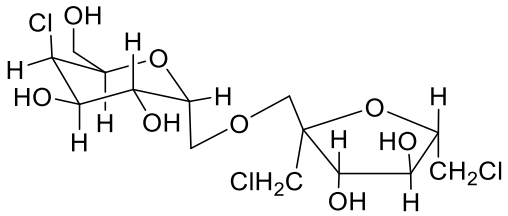
(B)



(C)



(D)



Answer Q4: A

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SECTION 2 (Maximum Marks: 20)

- This section contains **FIVE (05)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated **according to the following marking scheme**:

  Full Marks : $+4$ **ONLY** if (all) the correct option(s) is(are) chosen;
  Partial Marks : $+3$ If all the four options are correct but **ONLY** three options are chosen;
  Partial Marks : $+2$ If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
  Partial Marks : $+1$ If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
  Negative Marks : -1 In all other cases.

- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
  choosing ONLY (A), (B) and (D) will get $+4$ marks;
  choosing ONLY (A) and (B) will get $+2$ marks;
  choosing ONLY (A) and (D) will get $+2$ marks;
  choosing ONLY (B) and (D) will get $+2$ marks;
  choosing ONLY (A) will get +1 mark;
  choosing ONLY (B) will get +1 mark;
  choosing ONLY (D) will get +1 mark;
  choosing no option (i.e. the question is unanswered) will get 0 marks; and
  choosing any other combination of options will get -1 marks.

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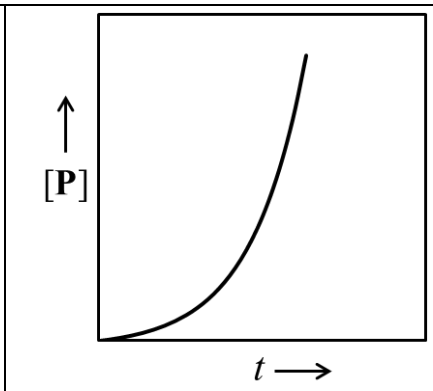
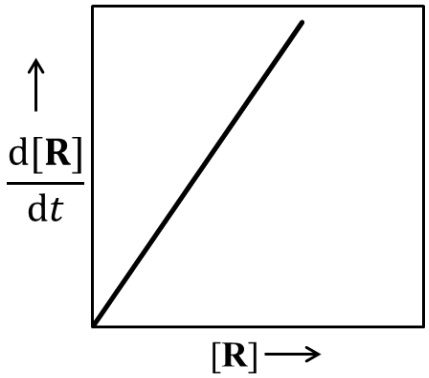
Q.5 For a first-order reaction **R** ⟶ **P** at a given temperature, k is the rate constant. For this reaction, at the given temperature, the concentrations of **R** and **P** at a time t are [**R**] and [**P**], respectively. The correct graphical representation(s) for this reaction is(are)

(A)



(B)



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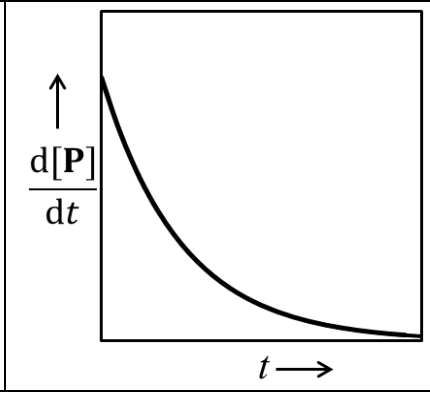
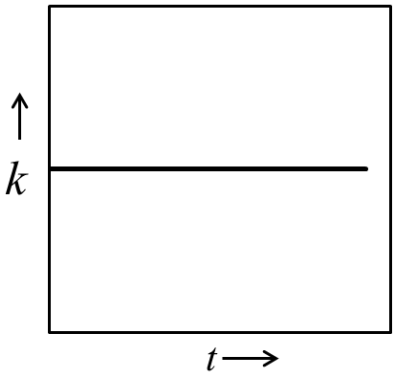


(D)



Answer Q5: CD

Q.6 Correct statement(s) about the compounds **P**, **Q** and **R** is(are)

$$\underset{(1:5\ \text{ratio})}{Xe(g) + F_2(g)} \xrightarrow{873\ K,\ 7\ bar} \mathbf{P}$$

$$\mathbf{P} + O_2F_2 \xrightarrow{143\ K} \mathbf{Q} + O_2$$

$$\mathbf{Q} + H_2O \xrightarrow{\text{complete hydrolysis}} \mathbf{R} + HF$$

(A)	**P** has two lone pairs of electrons on the central atom.
(B)	**Q** has a perfect octahedral geometry.
(C)	**Q** can act as a fluorinating agent.
(D)	The molecular structure of **R** is trigonal pyramidal.

Answer Q6: ACD

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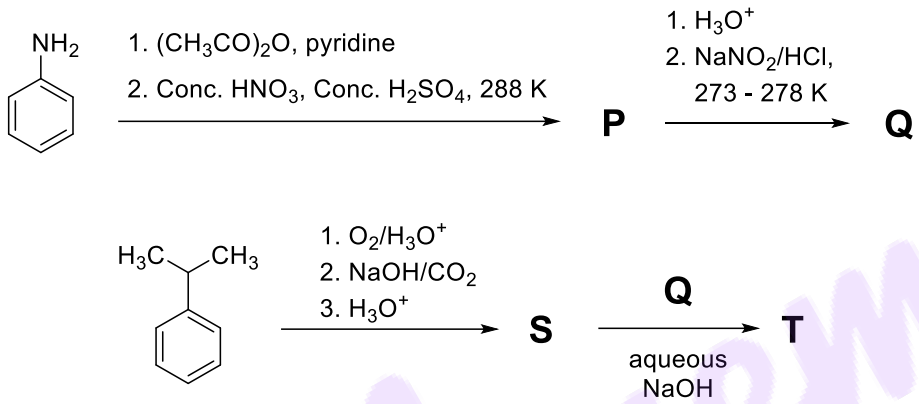
Q.7 The correct statement(s) regarding the periodic properties of elements is(are)

(A)	Second ionization enthalpy of carbon atom is less than that of boron atom.
(B)	Increasing order of ionic radii: $Al^{3+} < Mg^{2+} < Na^{+}$
(C)	Under identical conditions, in solid state, the density of potassium metal is more than density of sodium metal.
(D)	The H–H bond is weaker than F–F bond.

Answer Q7: AB

Q.8 In the following reaction sequence, **P**, **Q**, **S** and **T** are the major products.



The correct statement(s) about **P**, **Q**, **S** and **T** is(are)

(A)	**Q** on treatment with ethanol generates an aromatic aldehyde.
(B)	**S** gives positive phthalein dye test.
(C)	**P** is a dinitro compound.
(D)	**T** is a coloured compound.

Answer Q8: BD

Q.9 The correct statement(s) regarding sugars is(are)

Given: Specific rotations of L-(−)-glucose and L-(+)-fructose are −52.5° and +92.5°, respectively.

(A)	On treatment with HNO_3, gluconic acid is oxidized to saccharic acid, whereas glucose is not oxidized to saccharic acid.
(B)	Fructose gives a positive Fehling's test because it isomerises to glucose and another aldohexose in the presence of Fehling's reagent.
(C)	Invert sugar is an equimolar mixture of D-glucose and D-fructose formed after hydrolysis of the corresponding disaccharide.
(D)	Specific rotation of invert sugar is −40°.

Answer Q9: BC

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SECTION 3 (Maximum Marks: 20)

- This section contains **FIVE (05)** questions.
- The answer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct numerical value corresponding to the answer in the designated place using the mouse and the on-screen virtual numeric keypad.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated **according to the following marking scheme**:

  Full Marks : +4 If **ONLY** the correct numerical value is entered in the designated place;
  Zero Marks : 0 In all other cases.

Q.10 $\mathbf{X}^{a+}$ and $\mathbf{Y}^{b+}$ are hydrogen-like species. The wavelength of light absorbed during the transition between the states with principal quantum numbers $n = 1$ and $n = 2$ of $\mathbf{X}^{a+}$ is λ. The wavelength of light absorbed during the transition between the states with principal quantum numbers $n = 2$ and $n = 4$ of $\mathbf{Y}^{b+}$ is 9λ. The lowest possible value of $(\boldsymbol{a}+\boldsymbol{b})$ is ____.

Answer Q10: [3]

Q.11 At a given temperature, 0.45 g of acetic acid in 50 mL of water is shaken with 1.0 g of charcoal and the pH of the resulting solution is 3.0. Assume, the adsorption of acetic acid from the aqueous solution by charcoal follows Freundlich isotherm,

$$\frac{x}{m} = kC^{1/n}$$

If the plot of $\log_{10}(x/m)$ against $\log_{10}C$ gives a straight line with slope 1, the value of k in L mol^{-1} is ____.

Given: The molar mass of acetic acid is 60 g mol^{-1}.
The acid dissociation constant of acetic acid is 1.0×10^{-5} at the given temperature.
x is the mass (in grams) of acetic acid adsorbed.
m is the mass (in grams) of charcoal.
C is the equilibrium concentration of acetic acid in the solution after the adsorption is complete.
k and n are constants for acetic acid–charcoal system at the given temperature.

Answer Q11: [1.45 to 1.52]

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Q.12 In a solvent **S**, a compound **B** is partially dissociated into **C** and **D** as given below:

$$\mathbf{B} \rightleftharpoons 2\mathbf{C} + 2\mathbf{D}$$

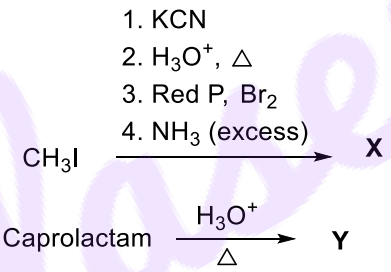
B, **C** and **D** are non-volatile in nature. The molar mass of **B** is 10 times the molar mass of **S**. The standard boiling point and the standard enthalpy of vaporization of **S** are 400 K and $10R$ J mol^{-1}, respectively (R is the gas constant in J K^{-1} mol^{-1}). A solution of **B** in **S** with an initial concentration of **B** as 0.25% (mass/mass) has a boiling point of 408 K at 1 bar pressure. In this solution, the mole percent of **B** that has been dissociated is ____.

Answer Q12: [31.60 to 33.33]

Q.13 Consider that the coordinating atoms of the ligands in *cis*-$[Co(NH_3)_4Cl_2]Cl$ and *mer*-$[Co(NH_3)_3Cl_3]$ octahedral complexes are at the vertices of an octahedron. The sum of total number of the triangular faces in both the complexes having one N atom and two Cl atoms at their corners is____.

Answer Q13: [6]

Q.14 In the following reaction sequence, major products **X** and **Y** are acyclic monomers.



500 mol of **X** completely reacts with 500 mol of **Y** to give 1 mol of a single biodegradable acyclic copolymer **Z** as the only product. The amount of **Z** formed in grams is ____.

Given:
Atomic mass (in amu): H : 1, C : 12, N : 14, O : 16, Br : 80

Answer Q14: [85018 to 85138.02]

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SECTION 4 (Maximum Marks: 8)

- This section contains **TWO (02)** question stems.
- This section contains **TWO (02)** questions corresponding to each question stem.
- The answer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct numerical value corresponding to the answer in the designated place using the mouse and the on-screen virtual numeric keypad.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated **according to the following marking scheme**:

Full Marks : +2 If **ONLY** the correct numerical value is entered in the designated place;
Zero Marks : 0 In all other cases.

Question Stem for Question Nos. 15 and 16

Two volatile liquids **A** and **B** form an ideal solution. Consider a 5 molal solution of **B** in **A** inside a closed container having a total vapour pressure of 100 mm Hg at 300 K. The vapour pressure of pure **A** at 300K is 105 mm Hg. Assume that **A** and **B** behave as ideal gases in the vapour phase.

Given:
The gas constant R = 0.08 L atm K^{-1} mol^{-1}
Molar mass of **A** is 50 g mol^{-1}
Molar mass of **B** is 57 g mol^{-1}
Density of liquid **B** at 300 K is 0.5 g/mL
1 atm = 760 mm Hg

Q.15 At 300 K, the ratio of the molar volume of pure **B** in vapour phase to its molar volume in liquid phase is ____.

Answer Q15: [1991 to 2053]

Q.16 The mole fraction of **B** in vapour phase which is in equilibrium with this solution is ____.

Answer Q16: [0.16]

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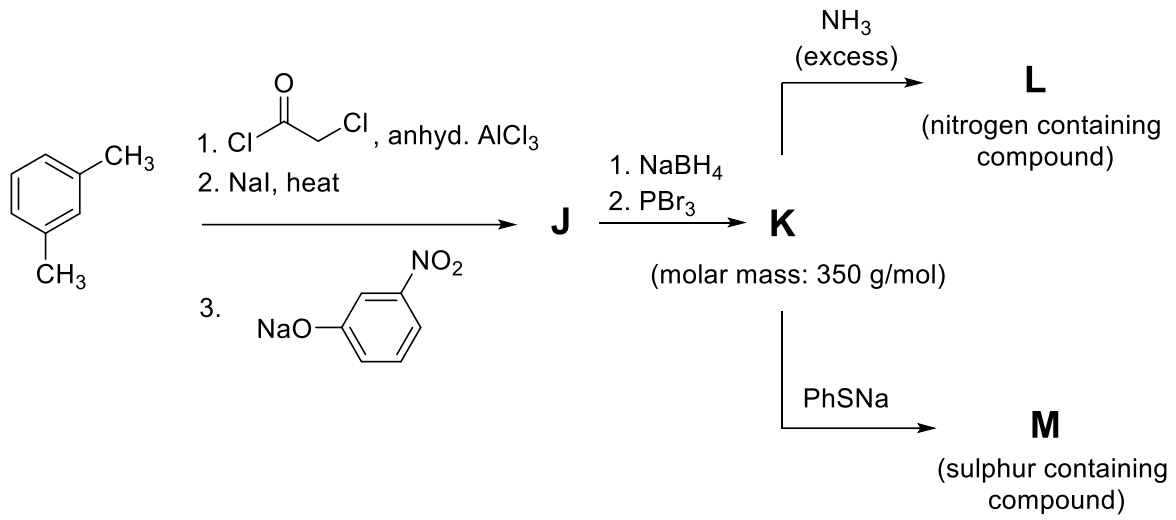
Question Stem for Question Nos. 17 and 18

Consider the following reaction sequence in which **J**, **K**, **L** and **M** are the major products.



Given:
Atomic mass (in amu): H : 1, C : 12, N : 14, O : 16, S : 32, Br : 80, Ba : 137

Q.17 The volume of 1 M aqueous H_2SO_4 required to completely neutralize the ammonia evolved from 5.72 g of **L** in Kjeldahl's method of nitrogen estimation is ____ mL.

Answer Q17: [10]

Q.18 In sulphur estimation by Carius method, the amount of $BaSO_4$ formed from 3.79 g of **M** is ____ g.

Answer Q18: [2.33]

END OF THE QUESTION PAPER

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