

In each of the following, use factor theorem to find whether polynomial $g(x)$ is a factor of polynomial $f(x)$, or not :
(1 - 7)

Q1. $f(x) = x^3 - 6x^2 + 11x - 6$, $g(x) = x - 3$

Sol :

Here , $f(x) = x^3 - 6x^2 + 11x - 6$

$$g(x) = x - 3$$

To prove that $g(x)$ is the factor of $f(x)$,

we should show $\Rightarrow f(3) = 0$

here , $x - 3 = 0$

$$\Rightarrow x = 3$$

Substitute the value of x in $f(x)$

$$f(3) = 3^3 - 6 * (3)^2 + 11(3) - 6$$

$$= 27 - (6*9) + 33 - 6$$

$$= 27 - 54 + 33 - 6$$

$$= 60 - 60$$

$$= 0$$

Since, the result is 0 $g(x)$ is the factor of $f(x)$

Q2. $f(x) = 3x^4 + 17x^3 + 9x^2 - 7x - 10$, $g(x) = x + 5$

Sol :

Here , $f(x) = 3x^4 + 17x^3 + 9x^2 - 7x - 10$

$$g(x) = x + 5$$

To prove that $g(x)$ is the factor of $f(x)$,

we should show $\Rightarrow f(-5) = 0$

here , $x + 5 = 0$

$$\Rightarrow x = -5$$

Substitute the value of x in $f(x)$

$$f(-5) = 3(-5)^4 + 17(-5)^3 + 9(-5)^2 - 7(-5) - 10$$

$$= (3 * 625) + (12 * (-125)) + (9*25) + 35 - 10$$

$$= 1875 - 2125 + 225 + 35 - 10$$

$$= 2135 - 2135$$

$$= 0$$

Since, the result is 0 $g(x)$ is the factor of $f(x)$

Q3. $f(x) = x^5 + 3x^4 - x^3 - 3x^2 + 5x + 15$, $g(x) = x + 3$

Sol :

$$\text{Here , } f(x) = x^5 + 3x^4 - x^3 - 3x^2 + 5x + 15$$

$$g(x) = x + 3$$

To prove that $g(x)$ is the factor of $f(x)$,

we should show $\Rightarrow f(-3) = 0$

$$\text{here , } x + 3 = 0$$

$$\Rightarrow x = -3$$

Substitute the value of x in $f(x)$

$$f(-3) = (-3)^5 + 3(-3)^4 - (-3)^3 - 3(-3)^2 + 5(-3) + 15$$

$$= -243 + 243 + 27 - 27 - 15 + 15$$

$$= 0$$

Since, the result is 0 $g(x)$ is the factor of $f(x)$

$$\text{Q4. } f(x) = x^3 - 6x^2 - 19x + 84, g(x) = x - 7$$

Sol :

$$\text{Here , } f(x) = x^3 - 6x^2 - 19x + 84$$

$$g(x) = x - 7$$

To prove that $g(x)$ is the factor of $f(x)$,

we should show $\Rightarrow f(7) = 0$

$$\text{here , } x - 7 = 0$$

$$\Rightarrow x = 7$$

Substitute the value of x in $f(x)$

$$f(7) = 7^3 - 6(7)^2 - 19(7) + 84$$

$$= 343 - (6 \cdot 49) - (19 \cdot 7) + 84$$

$$= 343 - 294 - 133 + 84$$

$$= 427 - 427$$

$$= 0$$

Since, the result is 0 $g(x)$ is the factor of $f(x)$

$$\text{Q5. } f(x) = 3x^3 + x^2 - 20x + 12, g(x) = 3x - 2$$

Sol :

$$\text{Here , } f(x) = 3x^3 + x^2 - 20x + 12$$

$$g(x) = 3x - 2$$

To prove that $g(x)$ is the factor of $f(x)$,

we should show $\Rightarrow f(\frac{2}{3}) = 0$

here , $3x - 2 = 0$

$$\Rightarrow 3x = 2$$

$$\Rightarrow x = \frac{2}{3}$$

Substitute the value of x in f(x)

$$f(\frac{2}{3}) = 3(\frac{2}{3})^3 + (\frac{2}{3})^2 - 20(\frac{2}{3}) + 12$$

$$= 3(\frac{8}{27}) + \frac{4}{9} - \frac{40}{3} + 12$$

$$= \frac{8}{9} + \frac{4}{9} - \frac{40}{3} + 12$$

$$= \frac{12}{9} - \frac{40}{3} + 12$$

Taking L.C.M

$$= \frac{12-120+108}{9}$$

$$= \frac{120-120}{9}$$

$$= 0$$

Since, the result is 0 g(x) is the factor of f(x)

$$\text{Q6. } f(x) = 2x^3 - 9x^2 + x + 13, g(x) = 3 - 2x$$

Sol :

$$\text{Here , } f(x) = 2x^3 - 9x^2 + x + 13$$

$$g(x) = 3 - 2x$$

To prove that g(x) is the factor of f(x) ,

To prove that g(x) is the factor of f(x) ,

we should show $\Rightarrow f(\frac{3}{2}) = 0$

here , $3 - 2x = 0$

$$\Rightarrow -2x = -3$$

$$\Rightarrow 2x = 3$$

$$\Rightarrow x = \frac{3}{2}$$

Substitute the value of x in f(x)

$$f(\frac{3}{2}) = 2(\frac{3}{2})^3 - 9(\frac{3}{2})^2 + (\frac{3}{2}) + 13$$

$$= 2(\frac{27}{8}) - 9(\frac{9}{4}) + \frac{3}{2} + 12$$

$$= (\frac{27}{4}) - (\frac{81}{4}) + \frac{3}{2} + 12$$

Taking L.C.M

$$= \frac{21-81+6+48}{4}$$

$$= \frac{81-81}{4}$$

$$= 0$$

Since, the result is 0 $g(x)$ is the factor of $f(x)$

$$Q7. f(x) = x^3 - 6x^2 + 11x - 6, g(x) = x^2 - 3x + 2$$

Sol :

$$\text{Here, } f(x) = x^3 - 6x^2 + 11x - 6$$

$$g(x) = x^2 - 3x + 2$$

First we need to find the factors of $x^2 - 3x + 2$

$$\Rightarrow x^2 - 2x - x + 2$$

$$\Rightarrow x(x - 2) - 1(x - 2)$$

$$\Rightarrow (x - 1) \text{ and } (x - 2) \text{ are the factors}$$

To prove that $g(x)$ is the factor of $f(x)$,

The results of $f(1)$ and $f(2)$ should be zero

$$\text{Let, } x - 1 = 0$$

$$x = 1$$

substitute the value of x in $f(x)$

$$f(1) = 1^3 - 6(1)^2 + 11(1) - 6$$

$$= 1 - 6 + 11 - 6$$

$$= 12 - 12$$

$$= 0$$

$$\text{Let, } x - 2 = 0$$

$$x = 2$$

substitute the value of x in $f(x)$

$$f(2) = 2^3 - 6(2)^2 + 11(2) - 6$$

$$= 8 - (6 * 4) + 22 - 6$$

$$= 8 - 24 + 22 - 6$$

$$= 30 - 30$$

$$= 0$$

Since, the results are 0 $g(x)$ is the factor of $f(x)$

$$Q8. \text{ Show that } (x - 2), (x + 3) \text{ and } (x - 4) \text{ are the factors of } x^3 - 3x^2 - 10x + 24$$

Sol :

$$\text{Here , } f(x) = x^3 - 3x^2 - 10x + 24$$

The factors given are $(x - 2)$, $(x + 3)$ and $(x - 4)$

To prove that $g(x)$ is the factor of $f(x)$,

The results of $f(2)$, $f(-3)$ and $f(4)$ should be zero

$$\text{Let , } x - 2 = 0$$

$$\Rightarrow x = 2$$

Substitute the value of x in $f(x)$

$$f(2) = 2^3 - 3(2)^2 - 10(2) + 24$$

$$= 8 - (3 * 4) - 20 + 24$$

$$= 8 - 12 - 20 + 24$$

$$= 32 - 32$$

$$= 0$$

$$\text{Let , } x + 3 = 0$$

$$\Rightarrow x = -3$$

Substitute the value of x in $f(x)$

$$f(-3) = (-3)^3 - 3(-3)^2 - 10(-3) + 24$$

$$= -27 - 3(9) + 30 + 24$$

$$= -27 - 27 + 30 + 24$$

$$= 54 - 54$$

$$= 0$$

$$\text{Let , } x - 4 = 0$$

$$\Rightarrow x = 4$$

Substitute the value of x in $f(x)$

$$f(4) = (4)^3 - 3(4)^2 - 10(4) + 24$$

$$= 64 - (3*16) - 40 + 24$$

$$= 64 - 48 - 40 + 24$$

$$= 84 - 84$$

$$= 0$$

Since, the results are 0 $g(x)$ is the factor of $f(x)$

Q9. Show that $(x + 4)$, $(x - 3)$ and $(x - 7)$ are the factors of $x^3 - 6x^2 - 19x + 84$

Sol :

$$\text{Here , } f(x) = x^3 - 6x^2 - 19x + 84$$

The factors given are $(x + 4)$, $(x - 3)$ and $(x - 7)$

To prove that $g(x)$ is the factor of $f(x)$,

The results of $f(-4)$, $f(3)$ and $f(7)$ should be zero

Let, $x + 4 = 0$

$$\Rightarrow x = -4$$

Substitute the value of x in $f(x)$

$$f(-4) = (-4)^3 - 6(-4)^2 - 19(-4) + 84$$

$$= -64 - (6 * 16) - (19 * (-4)) + 84$$

$$= -64 - 96 + 76 + 84$$

$$= 160 - 160$$

$$= 0$$

Let, $x - 3 = 0$

$$\Rightarrow x = 3$$

Substitute the value of x in $f(x)$

$$f(3) = (3)^3 - 6(3)^2 - 19(3) + 84$$

$$= 27 - (6 * 9) - (19 * 3) + 84$$

$$= 27 - 54 - 57 + 84$$

$$= 111 - 111$$

$$= 0$$

Let, $x - 7 = 0$

$$\Rightarrow x = 7$$

Substitute the value of x in $f(x)$

$$f(7) = (7)^3 - 6(7)^2 - 19(7) + 84$$

$$= 343 - (6 * 49) - (19 * 7) + 84$$

$$= 343 - 294 - 133 + 84$$

$$= 427 - 427$$

$$= 0$$

Since, the results are 0 $g(x)$ is the factor of $f(x)$

Q10. For what value of a is $(x - 5)$ a factor of $x^3 - 3x^2 + ax - 10$

Sol :

$$\text{Here, } f(x) = x^3 - 3x^2 + ax - 10$$

By factor theorem

If $(x - 5)$ is the factor of $f(x)$ then , $f(5) = 0$

$$\Rightarrow x - 5 = 0$$

$$\Rightarrow x = 5$$

Substitute the value of x in f(x)

$$f(5) = 5^3 - 3(5)^2 + a(5) - 10$$

$$= 125 - (3 * 25) + 5a - 10$$

$$= 125 - 75 + 5a - 10$$

$$= 5a + 40$$

Equate f(5) to zero

$$f(5) = 0$$

$$\Rightarrow 5a + 40 = 0$$

$$\Rightarrow 5a = -40$$

$$\Rightarrow a = \frac{-40}{5}$$

$$= -8$$

When a = -8, (x - 5) will be factor of f(x)

Q11. Find the value of a such that (x - 4) is a factor of $5x^3 - 7x^2 - ax - 28$

Sol :

$$\text{Here, } f(x) = 5x^3 - 7x^2 - ax - 28$$

By factor theorem

If (x - 4) is the factor of f(x) then, f(4) = 0

$$\Rightarrow x - 4 = 0$$

$$\Rightarrow x = 4$$

Substitute the value of x in f(x)

$$f(4) = 5(4)^3 - 7(4)^2 - a(4) - 28$$

$$= 5(64) - 7(16) - 4a - 28$$

$$= 320 - 112 - 4a - 28$$

$$= 180 - 4a$$

Equate f(4) to zero, to find a

$$f(4) = 0$$

$$\Rightarrow 180 - 4a = 0$$

$$\Rightarrow -4a = -180$$

$$\Rightarrow 4a = 180$$

$$\Rightarrow a = \frac{180}{4}$$

$$\Rightarrow a = 45$$

When $a = 45$, $(x - 4)$ will be factor of $f(x)$

Q12. Find the value of a , if $(x + 2)$ is a factor of $4x^4 + 2x^3 - 3x^2 + 8x + 5a$

Sol :

$$\text{Here, } f(x) = 4x^4 + 2x^3 - 3x^2 + 8x + 5a$$

By factor theorem

If $(x + 2)$ is the factor of $f(x)$ then, $f(-2) = 0$

$$\Rightarrow x + 2 = 0$$

$$\Rightarrow x = -2$$

Substitute the value of x in $f(x)$

$$f(-2) = 4(-2)^4 + 2(-2)^3 - 3(-2)^2 + 8(-2) + 5a$$

$$= 4(16) + 2(-8) - 3(4) - 16 + 5a$$

$$= 64 - 16 - 12 - 16 + 5a$$

$$= 5a + 20$$

equate $f(-2)$ to zero

$$f(-2) = 0$$

$$\Rightarrow 5a + 20 = 0$$

$$\Rightarrow 5a = -20$$

$$\Rightarrow a = \frac{-20}{5}$$

$$\Rightarrow a = -4$$

When $a = -4$, $(x + 2)$ will be factor of $f(x)$

Q13. Find the value of k if $x - 3$ is a factor of $k^2x^3 - kx^2 + 3kx - k$

Sol :

$$\text{Let } f(x) = k^2x^3 - kx^2 + 3kx - k$$

From factor theorem if $x - 3$ is the factor of $f(x)$ then $f(3) = 0$

$$\Rightarrow x - 3 = 0$$

$$\Rightarrow x = 3$$

Substitute the value of x in $f(x)$

$$f(3) = k^2(3)^3 - k(3)^2 + 3k(3) - k$$

$$= 27k^2 - 9k + 9k - k$$

$$= 27k^2 - k$$

$$= k(27k - 1)$$

Equate $f(3)$ to zero, to find k

$$\Rightarrow f(3) = 0$$

$$\Rightarrow k(27k - 1) = 0$$

$$\Rightarrow k = 0 \text{ and } 27k - 1 = 0$$

$$\Rightarrow k = 0 \text{ and } 27k = 1$$

$$\Rightarrow k = 0 \text{ and } k = \frac{1}{27}$$

When $k = 0$ and $\frac{1}{27}$, $(x - 3)$ will be the factor of $f(x)$

Q14. Find the values of a and b , if $x^2 - 4$ is a factor of $ax^4 + 2x^3 - 3x^2 + bx - 4$

Sol :

$$\text{Given, } f(x) = ax^4 + 2x^3 - 3x^2 + bx - 4$$

$$g(x) = x^2 - 4$$

first we need to find the factors of $g(x)$

$$\Rightarrow x^2 - 4$$

$$\Rightarrow x^2 = 4$$

$$\Rightarrow x = \sqrt{4}$$

$$\Rightarrow x = \pm 2$$

$(x - 2)$ and $(x + 2)$ are the factors

By factor theorem if $(x - 2)$ and $(x + 2)$ are the factors of $f(x)$ the result of $f(2)$ and $f(-2)$ should be zero

$$\text{Let, } x - 2 = 0$$

$$\Rightarrow x = 2$$

Substitute the value of x in $f(x)$

$$f(2) = a(2)^4 + 2(2)^3 - 3(2)^2 + b(2) - 4$$

$$= 16a + 2(8) - 3(4) + 2b - 4$$

$$= 16a + 2b + 16 - 12 - 4$$

$$= 16a + 2b$$

Equate $f(2)$ to zero

$$\Rightarrow 16a + 2b = 0$$

$$\Rightarrow 2(8a + b) = 0$$

$$\Rightarrow 8a + b = 0 \text{ ---- 1}$$

$$\text{Let, } x + 2 = 0$$

$$\Rightarrow x = -2$$

Substitute the value of x in $f(x)$

$$f(-2) = a(-2)^4 + 2(-2)^3 - 3(-2)^2 + b(-2) - 4$$

$$= 16a + 2(-8) - 3(4) - 2b - 4$$

$$= 16a - 2b - 16 - 12 - 4$$

$$= 16a - 2b - 32$$

$$= 16a - 2b - 32$$

Equate $f(2)$ to zero

$$\Rightarrow 16a - 2b - 32 = 0$$

$$\Rightarrow 2(8a - b) = 32$$

$$\Rightarrow 8a - b = 16 \text{ ----- } 2$$

Solve equation 1 and 2

$$8a + b = 0$$

$$8a - b = 16$$

$$16a = 16$$

$$a = 1$$

substitute a value in eq 1

$$8(1) + b = 0$$

$$\Rightarrow b = -8$$

The values are $a = 1$ and $b = -8$

Q15. Find α , β if $(x + 1)$ and $(x + 2)$ are the factors of $x^3 + 3x^2 - 2\alpha x + \beta$

Sol:

Given, $f(x) = x^3 + 3x^2 - 2\alpha x + \beta$ and the factors are $(x + 1)$ and $(x + 2)$

From factor theorem, if they are the factors of $f(x)$ then results of $f(-2)$ and $f(-1)$ should be zero

$$\text{Let, } x + 1 = 0$$

$$\Rightarrow x = -1$$

Substitute value of x in $f(x)$

$$f(-1) = (-1)^3 + 3(-1)^2 - 2\alpha(-1) + \beta$$

$$= -1 + 3 + 2\alpha + \beta$$

$$= 2\alpha + \beta + 2 \text{ ----- } 1$$

$$\text{Let, } x + 2 = 0$$

$$\Rightarrow x = -2$$

Substitute value of x in $f(x)$

$$f(-2) = (-2)^3 + 3(-2)^2 - 2\alpha(-2) + \beta$$

$$= -8 + 12 + 4\alpha + \beta$$

$$= 4\alpha + \beta + 4 \text{ ----- } 2$$

Solving 1 and 2 i.e (1 - 2)

$$\Rightarrow 2\alpha + \beta + 2 - (4\alpha + \beta + 4) = 0$$

$$\Rightarrow -2\alpha - 2 = 0$$

$$\Rightarrow 2\alpha = -2$$

$$\Rightarrow \alpha = -1$$

Substitute $\alpha = -1$ in equation 1

$$\Rightarrow 2(-1) + \beta = -2$$

$$\Rightarrow \beta = -2 + 2$$

$$\Rightarrow \beta = 0$$

The values are $\alpha = -1$ and $\beta = 0$

Q16. Find the values of p and q so that $x^4 + px^3 + 2x^2 - 3x + q$ is divisible by $(x^2 - 1)$

Sol :

$$\text{Here, } f(x) = x^4 + px^3 + 2x^2 - 3x + q$$

$$g(x) = x^2 - 1$$

first, we need to find the factors of $x^2 - 1$

$$\Rightarrow x^2 - 1 = 0$$

$$\Rightarrow x^2 = 1$$

$$\Rightarrow x = \pm 1$$

$$\Rightarrow (x + 1) \text{ and } (x - 1)$$

From factor theorem, if $x = 1, -1$ are the factors of $f(x)$ then $f(1) = 0$ and $f(-1) = 0$

Let us take, $x + 1$

$$\Rightarrow x + 1 = 0$$

$$\Rightarrow x = -1$$

Substitute the value of x in $f(x)$

$$f(-1) = (-1)^4 + p(-1)^3 + 2(-1)^2 - 3(-1) + q$$

$$= 1 - p + 2 + 3 + q$$

$$= -p + q + 6 \text{ ---- } 1$$

Let us take, $x - 1$

$$\Rightarrow x - 1 = 0$$

$$\Rightarrow x = 1$$

Substitute the value of x in $f(x)$

$$f(1) = (1)^4 + p(1)^3 + 2(1)^2 - 3(1) + q$$

$$= 1 + p + 2 - 3 + q$$

$$= p + q \text{ ---- } 2$$

Solve equations 1 and 2

$$-p + q = -6$$

$$p + q = 0$$

$$2q = -6$$

$$q = -3$$

substitute q value in equation 2

$$p + q = 0$$

$$p - 3 = 0$$

$$p = 3$$

the values of are $p = 3$ and $q = -3$

Q17. Find the values of a and b so that $(x + 1)$ and $(x - 1)$ are the factors of $x^4 + ax^3 - 3x^2 + 2x + b$

Sol :

$$\text{Here, } f(x) = x^4 + ax^3 - 3x^2 + 2x + b$$

The factors are $(x + 1)$ and $(x - 1)$

From factor theorem , if $x = 1, -1$ are the factors of $f(x)$ then $f(1) = 0$ and $f(-1) = 0$

Let , us take $x + 1$

$$\Rightarrow x + 1 = 0$$

$$\Rightarrow x = -1$$

Substitute value of x in $f(x)$

$$f(-1) = (-1)^4 + a(-1)^3 - 3(-1)^2 + 2(-1) + b$$

$$= 1 - a - 3 - 2 + b$$

$$= -a + b - 4 \text{ --- 1}$$

Let , us take $x - 1$

$$\Rightarrow x - 1 = 0$$

$$\Rightarrow x = 1$$

Substitute value of x in $f(x)$

$$f(1) = (1)^4 + a(1)^3 - 3(1)^2 + 2(1) + b$$

$$= 1 + a - 3 + 2 + b$$

$$= a + b \text{ --- 2}$$

Solve equations 1 and 2

$$-a + b = 4$$

$$a + b = 0$$

$$2b = 4$$

$$b = 2$$

substitute value of b in eq 2

$$a + 2 = 0$$

$$a = -2$$

the values are $a = -2$ and $b = 2$

Q18. If $x^3 + ax^2 - bx + 10$ is divisible by $x^3 - 3x + 2$, find the values of a and b

Sol :

$$\text{Here, } f(x) = x^3 + ax^2 - bx + 10$$

$$g(x) = x^3 - 3x + 2$$

first, we need to find the factors of $g(x)$

$$g(x) = x^3 - 3x + 2$$

$$= x^3 - 2x - x + 2$$

$$= x(x - 2) - 1(x - 2)$$

$$= (x - 1) \text{ and } (x - 2) \text{ are the factors}$$

From factor theorem, if $x = 1, 2$ are the factors of $f(x)$ then $f(1) = 0$ and $f(2) = 0$

Let, us take $x - 1$

$$\Rightarrow x - 1 = 0$$

$$\Rightarrow x = 1$$

Substitute the value of x in $f(x)$

$$f(1) = 1^3 + a(1)^2 - b(1) + 10$$

$$= 1 + a - b + 10$$

$$= a - b + 11 \text{ --- 1}$$

Let, us take $x - 2$

$$\Rightarrow x - 2 = 0$$

$$\Rightarrow x = 2$$

Substitute the value of x in $f(x)$

$$f(2) = 2^3 + a(2)^2 - b(2) + 10$$

$$= 8 + 4a - 2b + 10$$

$$= 4a - 2b + 18$$

Equate $f(2)$ to zero

$$\Rightarrow 4a - 2b + 18 = 0$$

$$\Rightarrow 2(2a - b + 9) = 0$$

$$\Rightarrow 2a - b + 9 \text{ ---- 2}$$

Solve 1 and 2

$$a - b = -11$$

$$2a - b = -9$$

$$(-) (+) (+)$$

$$-a = -2$$

$$a = 2$$

substitute a value in eq 1

$$\Rightarrow 2 - b = -11$$

$$\Rightarrow -b = -11 - 2$$

$$\Rightarrow -b = -13$$

$$\Rightarrow b = 13$$

The values are $a = 2$ and $b = 13$

Q19. If both $(x + 1)$ and $(x - 1)$ are the factors of $ax^3 + x^2 - 2x + b$, Find the values of a and b

Sol:

$$\text{Here, } f(x) = ax^3 + x^2 - 2x + b$$

$(x + 1)$ and $(x - 1)$ are the factors

From factor theorem, if $x = 1, -1$ are the factors of $f(x)$ then $f(1) = 0$ and $f(-1) = 0$

$$\text{Let, } x - 1 = 0$$

$$\Rightarrow x = 1$$

Substitute x value in $f(x)$

$$f(1) = a(1)^3 + (1)^2 - 2(1) + b$$

$$= a + 1 - 2 + b$$

$$= a + b - 1 \text{ ---- } 1$$

$$\text{Let, } x + 1 = 0$$

$$\Rightarrow x = -1$$

Substitute x value in $f(x)$

$$f(-1) = a(-1)^3 + (-1)^2 - 2(-1) + b$$

$$= -a + 1 + 2 + b$$

$$= -a + b + 3 \text{ ---- } 2$$

Solve equations 1 and 2

$$a + b = 1$$

$$-a + b = -3$$

$$2b = -2$$

$$\Rightarrow b = -1$$

substitute b value in eq 1

$$\Rightarrow a - 1 = 1$$

$$\Rightarrow a = 1 + 1$$

$$\Rightarrow a = 2$$

The values are $a = 2$ and $b = -1$

Q20. What must be added to $x^3 - 3x^2 - 12x + 19$ so that the result is exactly divisible by $x^2 + x - 6$

Sol :

$$\text{Here, } p(x) = x^3 - 3x^2 - 12x + 19$$

$$g(x) = x^2 + x - 6$$

by division algorithm, when $p(x)$ is divided by $g(x)$, the remainder will be a linear expression in x

let, $r(x) = ax + b$ is added to $p(x)$

$$\Rightarrow f(x) = p(x) + r(x)$$

$$= x^3 - 3x^2 - 12x + 19 + ax + b$$

$$f(x) = x^3 - 3x^2 + x(a - 12) + 19 + b$$

$$\text{We know that, } g(x) = x^2 + x - 6$$

First, find the factors for $g(x)$

$$g(x) = x^2 + 3x - 2x - 6$$

$$= x(x + 3) - 2(x + 3)$$

$$= (x + 3)(x - 2) \text{ are the factors}$$

From, factor theorem when $(x + 3)$ and $(x - 2)$ are the factors of $f(x)$ the $f(-3) = 0$ and $f(2) = 0$

$$\text{Let, } x + 3 = 0$$

$$\Rightarrow x = -3$$

Substitute the value of x in $f(x)$

$$f(-3) = (-3)^3 - 3(-3)^2 + (-3)(a - 12) + 19 + b$$

$$= -27 - 27 - 3a + 24 + 19 + b$$

$$= -3a + b + 1 \text{ ---- 1}$$

$$\text{Let, } x - 2 = 0$$

$$\Rightarrow x = 2$$

Substitute the value of x in $f(x)$

$$f(2) = (2)^3 - 3(2)^2 + (2)(a - 12) + 19 + b$$

$$= 8 - 12 + 2a - 24 + b$$

$$= 2a + b - 9 \text{ ---- 2}$$

Solve equations 1 and 2

$$-3a + b = -1$$

$$2a + b = 9$$

$$(-) \quad (-) \quad (-)$$

$$-5a = -10$$

$$a = 2$$

substitute the value of a in eq 1

$$\Rightarrow -3(2) + b = -1$$

$$\Rightarrow -6 + b = -1$$

$$\Rightarrow b = -1 + 6$$

$$\Rightarrow b = 5$$

$$\therefore r(x) = ax + b$$

$$= 2x + 5$$

$\therefore x^3 - 3x^2 - 12x + 19$ is divided by $x^2 + x - 6$ when it is added by $2x + 5$

Q21. What must be added to $x^3 - 6x^2 - 15x + 80$ so that the result is exactly divisible by $x^2 + x - 12$

Sol :

$$\text{Let, } p(x) = x^3 - 6x^2 - 15x + 80$$

$$q(x) = x^2 + x - 12$$

by division algorithm, when $p(x)$ is divided by $q(x)$ the remainder is a linear expression in x .

so, let $r(x) = ax + b$ is subtracted from $p(x)$, so that $p(x) - q(x)$ is divisible by $q(x)$

$$\text{let } f(x) = p(x) - q(x)$$

$$q(x) = x^2 + x - 12$$

$$= x^2 + 4x - 3x - 12$$

$$= x(x + 4) - 3(x + 4)$$

$$= (x+4), (x - 3)$$

clearly, $(x - 3)$ and $(x + 4)$ are factors of $q(x)$

so, $f(x)$ will be divisible by $q(x)$ if $(x - 3)$ and $(x + 4)$ are factors of $q(x)$

from , factor theorem

$$f(-4) = 0 \text{ and } f(3) = 0$$

$$\Rightarrow f(3) = 3^3 - 6(3)^2 - 3(a + 15) + 80 - b = 0$$

$$= 27 - 54 - 3a - 45 + 80 - b$$

$$= -3a - b + 8 = 0$$

Similarly,

$$f(-4) = 0$$

$$\Rightarrow f(-4) \Rightarrow (-4)^3 - 6(-4)^2 - (-4)(a + 15) + 80 - b = 0$$

$$\Rightarrow -64 - 96 - 4a + 60 + 80 - b = 0$$

$$\Rightarrow 4a - b - 20 = 0 \text{ ---- 2}$$

Subtract eq 1 and 2

$$\Rightarrow 4a - b - 20 - 8 + 3a + b = 0$$

$$\Rightarrow 7a - 28 = 0$$

$$\Rightarrow a = \frac{28}{7}$$

$$\Rightarrow a = 4$$

Put $a = 4$ in eq 1

$$\Rightarrow -3(4) - b = -8$$

$$\Rightarrow -b - 12 = -8$$

$$\Rightarrow -b = -8 + 12$$

$$\Rightarrow b = -4$$

Substitute a and b values in $r(x)$

$$\Rightarrow r(x) = ax + b$$

$$= 4x - 4$$

Hence, $p(x)$ is divisible by $q(x)$, if $r(x) = 4x - 4$ is subtracted from it

Q22. What must be added to $3x^3 + x^2 - 22x + 9$ so that the result is exactly divisible by $3x^2 + 7x - 6$

Sol :

$$\text{Let, } p(x) = 3x^3 + x^2 - 22x + 9 \text{ and } q(x) = 3x^2 + 7x - 6$$

By division theorem, when $p(x)$ is divided by $q(x)$, the remainder is a linear equation in x .

Let, $r(x) = ax + b$ is added to $p(x)$, so that $p(x) + r(x)$ is divisible by $q(x)$

$$f(x) = p(x) + r(x)$$

$$\Rightarrow f(x) = 3x^3 + x^2 - 22x + 9(ax + b)$$

$$\Rightarrow = 3x^3 + x^2 + x(a - 22) + b + 9$$

We know that,

$$q(x) = 3x^2 + 7x - 6$$

$$= 3x^2 + 9x - 2x - 6$$

$$= 3x(x+3) - 2(x+3)$$

$$= (3x-2)(x+3)$$

So, $f(x)$ is divided by $q(x)$ if $(3x-2)$ and $(x+3)$ are the factors of $f(x)$

From, factor theorem

$$f\left(\frac{2}{3}\right) = 0 \text{ and } f(-3) = 0$$

$$\text{let, } 3x - 2 = 0$$

$$3x = 2$$

$$x = \frac{2}{3}$$

$$\Rightarrow f\left(\frac{2}{3}\right) = 3\left(\frac{2}{3}\right)^3 + \left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)(a - 22) + b + 9$$

$$= 3\left(\frac{8}{27}\right) + \frac{4}{9} + \frac{2}{3}a - \frac{44}{3} + b + 9$$

$$= \frac{12}{9} + \frac{2}{3}a - \frac{44}{3} + b + 9$$

$$= \frac{12+6a-132+9b+81}{9}$$

Equate to zero

$$\Rightarrow \frac{12+6a-132+9b+81}{9} = 0$$

$$\Rightarrow 6a + 9b - 39 = 0$$

$$\Rightarrow 3(2a + 3b - 13) = 0$$

$$\Rightarrow 2a + 3b - 13 = 0 \text{ ---- 1}$$

Similarly,

$$\text{Let, } x + 3 = 0$$

$$\Rightarrow x = -3$$

$$\Rightarrow f(-3) = 3(-3)^3 + (-3)^2 + (-3)(a - 22) + b + 9$$

$$= -81 + 9 - 3a + 66 + b + 9$$

$$= -3a + b + 3$$

Equate to zero

$$-3a + b + 3 = 0$$

Multiply by 3

$$-9a + 3b + 9 = 0 \text{ ---- 2}$$

Subtract eq 1 from 2

$$\Rightarrow -9a + 3b + 9 - 2a - 3b + 13 = 0$$

$$\Rightarrow -11a + 22 = 0$$

$$\Rightarrow -11a = -22$$

$$\Rightarrow a = \frac{22}{11}$$

$$\Rightarrow a = 2$$

Substitute a value in eq 1

$$\Rightarrow -3(2) + b = -3$$

$$\Rightarrow -6 + b = -3$$

$$\Rightarrow b = -3 + 6$$

$$\Rightarrow b = 3$$

Put the values in $r(x)$

$$r(x) = ax + b$$

$$= 2x + 3$$

Hence, $p(x)$ is divisible by $q(x)$, if $r(x) = 2x + 3$ is added to it

Q23. If $x - 2$ is a factor of each of the following two polynomials, find the value of a in each case :

$$1. x^3 - 2ax^2 + ax - 1$$

$$2. x^5 - 3x^4 - ax^3 + 3ax^2 + 2ax + 4$$

Sol :

$$(1) \text{ let } f(x) = x^3 - 2ax^2 + ax - 1$$

from factor theorem

if $(x - 2)$ is the factor of $f(x)$ the $f(2) = 0$

$$\text{let, } x - 2 = 0$$

$$\Rightarrow x = 2$$

Substitute x value in $f(x)$

$$f(2) = 2^3 - 2a(2)^2 + a(2) - 1$$

$$= 8 - 8a + 2a - 1$$

$$= -6a + 7$$

Equate $f(2)$ to zero

$$\Rightarrow -6a + 7 = 0$$

$$\Rightarrow -6a = -7$$

$$\Rightarrow a = \frac{7}{6}$$

When, $(x - 2)$ is the factor of $f(x)$ then $a = \frac{7}{6}$

$$(2) \text{ Let, } f(x) = x^5 - 3x^4 - ax^3 + 3ax^2 + 2ax + 4$$

from factor theorem

if $(x - 2)$ is the factor of $f(x)$ the $f(2) = 0$

$$\text{let, } x - 2 = 0$$

$$\Rightarrow x = 2$$

Substitute x value in $f(x)$

$$f(2) = 2^5 - 3(2)^4 - a(2)^3 + 3a(2)^2 + 2a(2) + 4$$

$$= 32 - 48 - 8a + 12 + 4a + 4$$

$$= 8a - 12$$

Equate $f(2)$ to zero

$$\Rightarrow 8a - 12 = 0$$

$$\Rightarrow 8a = 12$$

$$\Rightarrow a = \frac{12}{8}$$

$$= \frac{3}{2}$$

So, when $(x - 2)$ is a factor of $f(x)$ then $a = \frac{3}{2}$

Q24. In each of the following two polynomials, find the value of a , if $(x - a)$ is a factor :

$$1. x^6 - ax^5 + x^4 - ax^3 + 3x - a + 2$$

$$2. x^5 - a^2x^3 + 2x + a + 1$$

Sol :

$$(1) x^6 - ax^5 + x^4 - ax^3 + 3x - a + 2$$

$$\text{let, } f(x) = x^6 - ax^5 + x^4 - ax^3 + 3x - a + 2$$

$$\text{here, } x - a = 0$$

$$\Rightarrow x = a$$

Substitute the value of x in $f(x)$

$$f(a) = a^6 - a(a)^5 + (a)^4 - a(a)^3 + 3(a) - a + 2$$

$$= a^6 - a^6 + (a)^4 - a^4 + 3(a) - a + 2$$

$$= 2a + 2$$

Equate to zero

$$\Rightarrow 2a + 2 = 0$$

$$\Rightarrow 2(a + 1) = 0$$

$$\Rightarrow a = -1$$

So, when $(x - a)$ is a factor of $f(x)$ then $a = -1$

$$(2) x^5 - a^2x^3 + 2x + a + 1$$

$$\text{let, } f(x) = x^5 - a^2x^3 + 2x + a + 1$$

$$\text{here, } x - a = 0$$

$$\Rightarrow x = a$$

Substitute the value of x in $f(x)$

$$f(a) = a^5 - a^2a^3 + 2(a) + a + 1$$

$$= a^5 - a^5 + 2a + a + 1$$

$$= 3a + 1$$

Equate to zero

$$\Rightarrow 3a + 1 = 0$$

$$\Rightarrow 3a = -1$$

$$\Rightarrow a = \frac{-1}{3}$$

So, when $(x - a)$ is a factor of $f(x)$ then $a = \frac{-1}{3}$

Q25. In each of the following two polynomials, find the value of a , if $(x + a)$ is a factor :

1. $x^3 + ax^2 - 2x + a + 4$

2. $x^4 - a^2x^2 + 3x - a$

Sol :

(1) $x^3 + ax^2 - 2x + a + 4$

let, $f(x) = x^3 + ax^2 - 2x + a + 4$

here, $x + a = 0$

$$\Rightarrow x = -a$$

Substitute the value of x in $f(x)$

$$f(-a) = (-a)^3 + a(-a)^2 - 2(-a) + a + 4$$

$$= (-a)^3 + a^3 - 2(-a) + a + 4$$

$$= 3a + 4$$

Equate to zero

$$\Rightarrow 3a + 4 = 0$$

$$\Rightarrow 3a = -4$$

$$\Rightarrow a = \frac{-4}{3}$$

So, when $(x + a)$ is a factor of $f(x)$ then $a = \frac{-4}{3}$

(2) $x^4 - a^2x^2 + 3x - a$

let, $f(x) = x^4 - a^2x^2 + 3x - a$

here, $x + a = 0$

$$\Rightarrow x = -a$$

Substitute the value of x in $f(x)$

$$f(-a) = (-a)^4 - a^2(-a)^2 + 3(-a) - a$$

$$= a^4 - a^4 - 3(a) - a$$

$$= -4a$$

Equate to zero

$$\Rightarrow -4a = 0$$

$$\Rightarrow a = 0$$

So, when $(x + a)$ is a factor of $f(x)$ then $a = 0$